18th Mediterranean Conference on Control & Automation
Congress Palace Hotel, Marrakech, Morocco
June 23-25, 2010

Model-Based Control using a Lifting Approach.

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***Abstract*—In this paper, discrete-time Model-Based Networked Control Systems (MB-NCS) are studied. A lifting process is applied to a general MB-NCS configuration in which the controller is connected to the actuator and plant by means of a digital communication network resulting in a multirate system; the controller is updated every nT time units while the plant is updated every mT time units. Here, necessary and sufficient conditions for asymptotic stability are derived in terms of the parameters n and m. The lifting process is also applied to the MB-NCS configuration when only sensor data is sent over the network. In both cases a Linear Time-Invariant (LTI) system is obtained after applying lifting techniques. Necessary and sufficient conditions are given for the asymptotic stability of the system with instantaneous feedback in terms of h, the periodic update constant, and in terms of h and τ for the intermittent feedback case, where τ is the time interval in which the loop remains closed.**

I. Introduction

IN recent years, control networks have been replacing traditional point-to-point wired systems. In networked control systems, the different elements, plants, controllers, sensors, and actuators are connected through a digital communication network with limited bandwidth. The new challenges that this implementation has brought are well documented [1]-[4]. Perhaps the most relevant is the limitation on bandwidth; many researchers have studied different problems related to bandwidth restrictions such the state estimation problem under limited network capacity [5] or the minimum bit rate required to stabilize a Network Control System (NCS) [6], [7]. Other authors have focused on reducing network communication maintaining the system stable or keeping some level of performance. Georgiev and Tilbury [8] use the packet structure more efficiently, that is, reduction on communication is obtained by sending packets of information using all data bits available; for the sequence of sensor data received, the controller needs to find a control sequence instead of a single control value. Otanez *et al*. [9] use deadbands at each node to record the last value sent to the network and compares that value to the current one, making a decision on sending the current information or not. Walsh, *et al.* [10] introduced a network control protocol Try-Once-Discard (TOD) to allocate network resources to the different nodes in a Networked Control System.

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The support of the National Science Foundation under Grant No. CCF-0819865 is gratefully acknowledged.

A type of NCS called Model Based Networked Control Systems (MB-NCS) aims to reduce communication over the network by incorporating an explicit model of the system to be controlled. The state of this model is used for control when no feedback is available (open loop). When the loop is closed, the state of the model is updated with new information, namely, the state of the real system. The MB-NCS framework is able to reduce network communication; consequently, the network is available for other uses, reducing time delays and bandwidth limitations.

Work in MB-NCS by Montestruque and Antsaklis [11], [12] provided necessary and sufficient conditions for stability for the case when the update intervals are constant; the output feedback and network delay case were also studied. In an extension, the same authors [13] also presented results when the update intervals are time-varying and follow different probabilistic distributions. In a related work Estrada, *et al.* [14] introduced MB-NCS to intermittent feedback control resulting in improved performance and longer permissible update intervals. Recently, the intermittent control concept has been successfully applied to control systems [15]-[18].

In all the above work on MB-NCS it is assumed that the network exists only between the sensor and the controller node while the controller is connected directly to the actuator and plant, that is the input generated by the controller is available to the plant at all times without delays or losses.

The work presented in this paper has strong connections with multirate systems [22]-[26]. Such systems arise mainly due to the limitation in sensing some variables fast enough, while the control variables can be adjusted faster. The main difference in this paper with respect to the control strategies used in multirate systems is the implementation of an explicit model to generate estimates of the state between sampling times. Note that, [24] offers a similar implementation, aiming to compare certain characteristics against a fast single rate control system. By contrast, we aim to find the specific rates that result in a stable multirate system.

In this paper lifting techniques are used to derive necessary and sufficient conditions for stability when there is also a communication network between the controller and the plant, a more general and flexible implementation that uses the network in both sides of the control loop. In addition, lifting techniques are also used to derive necessary and sufficient conditions for stability for traditional MB-NCS configurations, thus verifying existing results.

II. PROBLEM STATEMENT

MB-NCS make use of an explicit model of the plant which is added to the controller node to compute the control input based on the state of the model rather than on the plant state. Fig. 1 shows a basic MB-NCS configuration, where the network exists only on the sensor-controller side while the controller is connected directly to the actuator and the plant.

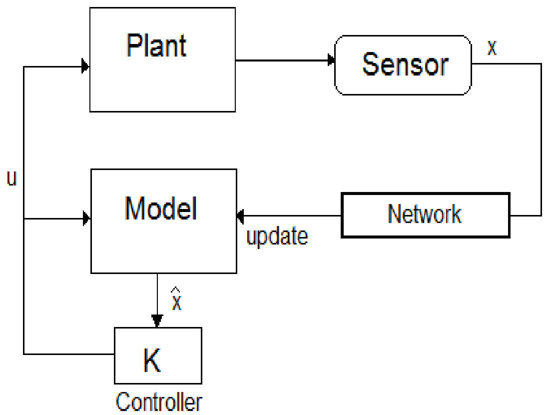


Fig. 1. Representation of a Model-Based Networked Control System.

The dynamics of the plant and the model are given respectively by:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ \hat{x}(k+1) &= \hat{A}\hat{x}(k) + \hat{B}u(k) \end{aligned} \tag{1}$$

where x is the state of the plant, $\hat{x}$ is the state of the model, A, B are the state space parameters of the physical system and $\hat{A}, \hat{B}$ represent the model of the system.

The input u for the case of instantaneous feedback can be expressed as:

$$u(k) = \begin{cases} Kx(k) & k = ih \\ K\hat{x}(k) & k = ih + j \end{cases} \tag{2}$$

for $i=0,1,2,\ldots$; h is the number of samples of the real plant that the sensor must wait in order to broadcast a measurement update, therefore h is an integer; and $0<j<h$ (j is also an integer).

In this paper we will analyze system (1) with feedback (2) using a lifting approach. The lifting process has the purpose of extending the input and output spaces properly in order to obtain a Linear Time Invariant (LTI) system description for sampled-data, multi-rate, or linear time-varying periodic systems. Since the lifted system is a LTI system, the available tools and results for LTI systems are applicable to the lifted system as well.

Suppose there exist two periods h and hs in a discrete-time set up and they are related by $hs=h/r$, where r is some positive integer. For a discrete-time signal $v(k)$ referred to the sub-period h/r, that is, $v(0)$ occurs at time $t=0$, $v(1)$ at $t=h/r$, $v(2)$ at $t=2h/r$ and so on, the lifted signal $\underline{v}$ is defined as follows:

If $v = \{v(0), v(1), v(2), \ldots\}$ then

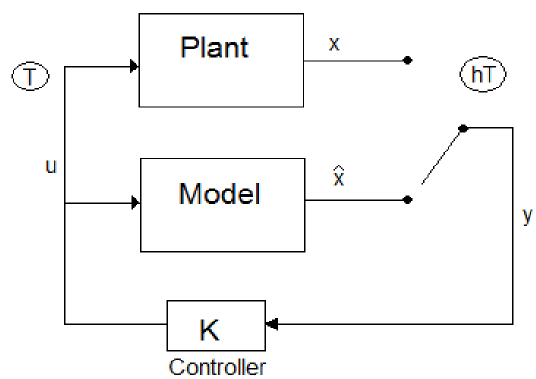
$$\underline{v} = \left\{ \begin{bmatrix} v(0) \\ v(1) \\ : \\ v(r-1) \end{bmatrix}, \begin{bmatrix} v(r) \\ v(r+1) \\ : \\ v(2r-1) \end{bmatrix}, \ldots \right\}$$

The dimension of the lifted signal $\underline{v}(k)$ is r times the dimension of the original signal $v(k)$ and is regarded to the base period h, i.e. $\underline{v}(k)$ occurs at time $t=kh$.

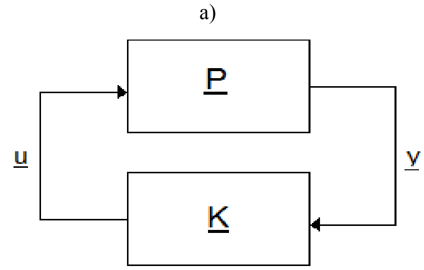
For a detailed treatment on lifting signals and systems the reader is referred to [19]. See also [21], [27], and [28].

III. INSTANTANEOUS AND INTERMITTENT FEEDBACK

The MB-NCS of Fig. 1 can be seen as the linear time-varying system shown in part a) of Fig. 2, by considering an output y that is equal to $\hat{x}$ when the loop is open and equal to x when we have an update (closed loop). The system after applying lifting is represented in part b) of the same figure, and is regarded as a LTI system with higher dimension input and output.



a)



b)

Fig. 2. Equivalent systems to a MB-NCS. a) Linear time-varying system. b) Lifted system.

Note that the original period of the system is denoted by T and the period of the network by hT. Then we have that for this case $r=h$ since $hT/h=T$. The input $\underline{u}$ for the lifted system $\underline{P}$ and its output $\underline{y}$ are given by the equations,

$$\underline{u}(kh)=\begin{bmatrix} u(kh) \\ u(kh+1) \\ : \\ u(kh+h-1) \end{bmatrix}=\begin{bmatrix} Kx(kh) \\ K\hat{x}(kh+1) \\ : \\ K\hat{x}(kh+h-1) \end{bmatrix} \tag{3}$$

$$\underline{y}(kh)=\begin{bmatrix} I \\ \hat{A} \\ \hat{A}^2 \\ : \\ \hat{A}^{h-1} \end{bmatrix}x(kh)+\begin{bmatrix} 0 & 0 & ... & 0 \\ \hat{B} & 0 & ... & 0 \\ \hat{A}\hat{B} & \hat{B} & ... & 0 \\ : & & & : \\ \hat{A}^{h-2}\hat{B} & \hat{A}^{h-3}\hat{B} & ... & 0 \end{bmatrix}\underline{u}(kh) \tag{4}$$

The dimension of the state is preserved, and the state equation expressed in terms of the lifted input is given by:

$$x((k+1)h)=A^h x(kh)+\begin{bmatrix} A^{h-1}B & A^{h-2}B & ... & B \end{bmatrix}\underline{u}(kh) \tag{5}$$

Theorem 1. The lifted system is asymptotically stable if only if the eigenvalues of

$$A^h+\sum_{j=0}^{h-1}A^{h-1-j}BK(\hat{A}+\hat{B}K)^j \tag{6}$$

lie strictly inside the unit circle.

Proof: To prove this theorem we note that (5) is the same as the state equation that characterizes the autonomous linear time invariant system:

$$x((k+1)h)=(A^h+\sum_{j=0}^{h-1}A^{h-1-j}BK(\hat{A}+\hat{B}K)^j)x(kh) \tag{7}$$

Equation (7) can be obtained by directly substituting (3) in (5), and then substituting the value of each individual output by its equivalent in terms of the state $x(kh)$, i.e.

$$\begin{aligned} \hat{x}(kh+1)&=\hat{A}x(kh)+\hat{B}u(kh)=(\hat{A}+\hat{B}K)x(kh) \\ \hat{x}(kh+2)&=(\hat{A}+\hat{B}K)^2x(kh) \\ &: \end{aligned} \tag{8}$$

The resulting equation can be simply expressed as (7). Since the lifted system is a LTI system we simply apply basic results for LTI systems, i.e. a system given by $x(k+1)=Ax(k)$ is asymptotically stable if only if the eigenvalues of A lie inside the unit circle.■

For the intermittent feedback case [14] the input u of the MB-NCS of Fig. 1 is defined as:

$$u(k)=\begin{cases} Kx(k) & h_l \le k \le h_l+\tau_l \\ K\hat{x}(k) & h_l+\tau_l < k < h_{l+1} \end{cases} \tag{9}$$

In this work we consider intermittent feedback with constant updates and constant closed loop times, that means, $h_{l+1}-h_l=h$, which represents how often we close the loop between the sensor and the controller, and $\tau_l=\tau<h$, which represents the constant number of clock ticks that the loop remains closed, h and τ are positive integer numbers.

Theorem 2. The lifted system with intermittent feedback (9) is asymptotically stable if only if the eigenvalues of:

$$(A^{h-\tau}+\sum_{j=0}^{h-\tau-1}A^{h-\tau-1-j}BK(\hat{A}+\hat{B}K)^j)(A+BK)^\tau \tag{10}$$

lie strictly inside the unit circle.
The proof for this theorem is similar to the one for theorem 1 and it can be found in [20].

IV. Double Network Path MB-NCS

We call double network path MB-NCS a NCS in which not only the path from the sensor to the controller is implemented using a digital network but also the path from the controller to the actuator as well. This configuration offers more flexibility to the designer since there is no need to place the model/controller and actuator/plant in the same node, or directly connect them using a dedicated wire. Given the circumstances of the problem it is preferable many times to use the network to implement this connection. The configuration is shown in Fig. 3; here the two switches are closed at different constant rates, giving rise to the constant update intervals n and m. As a starting point we wish to find the bounding values of m and n that preserve stability of the MB-NCS using instantaneous feedback. Note that in this case, between input updates, the input to the plant is held constant in the actuator and it is equal to the last received value.

Assume that $n \ge m$ (low measurement rate, which is typical in many implementations of physical systems) and $p=n/m$ is assumed to be an integer.

Theorem 3: The lifted system corresponding to Fig. 3 is asymptotically stable if only if the eigenvalues of:

$$A^n+\sum_{i=0}^{p-1}H_i\Gamma^i \tag{11}$$

lie strictly inside the unit circle, where

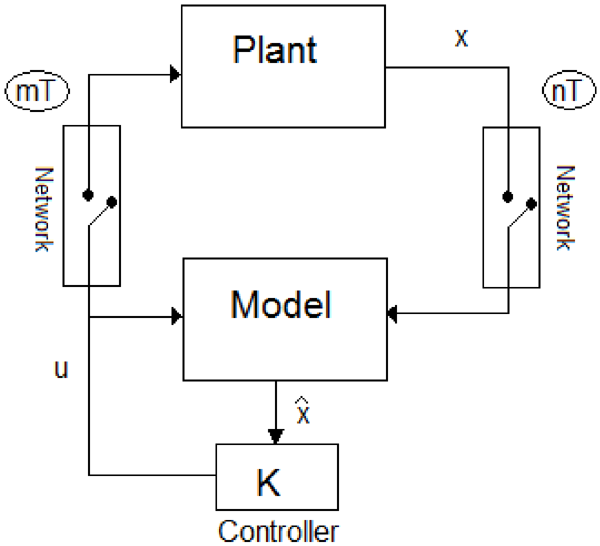


Fig. 3. Model-Based Networked Control System with communication network between sensor-controller and controller-actuator (plant).

$$H_i = \sum_{j=1}^{m} A^{n-im-j} BK \tag{12}$$

$$\Gamma = (\hat{A} + \hat{B}K)^m \tag{13}$$

Proof: Since $p=n/m$ is an integer the period of the networked system in Fig. 3 is n. Taking a similar approach as in the last two cases we obtain a LTI system. In order to find the state equations of the lifted system let us describe the response of the system as a function of the input updates that take place every m clock ticks. From equation (5) we obtain:

$$\begin{aligned} x(kn+n) = A^n x(kn) + [A^{n-1} \quad A^{n-2} \quad \ldots \quad A^{n-m}]Bu(kn) + \\ [A^{n-m-1} \quad A^{n-m-2} \quad \ldots \quad A^{n-2m}]Bu(kn+m) + \\ [A^{n-2m-1} \quad A^{n-2m-2} \quad \ldots \quad A^{n-3m}]Bu(kn+2m) + \ldots \end{aligned} \tag{14}$$

The input u is a function of the state of the plant at times kn and a function of the state of the model otherwise. The state of the model between sensor updates can be expressed in terms of the state of the plant at times kn as follows:

$$\begin{aligned} &u(kn) = Kx(kn) \\ &u(kn+m) = K\hat{x}(kn+m) = K(\hat{A} + \hat{B}K)^m x(kn) \\ &u(kn+2m) = K\hat{x}(kn+2m) = K(\hat{A} + \hat{B}K)^{2m} x(kn) \\ &\vdots \end{aligned} \tag{15}$$

Note that the model has access to the input that it generates at all times as it can be deduced from Fig. 3. The network connection is between the controller and the plant, and the model is part of the controller. Equation (14) becomes:

$$\begin{aligned} x((k+1)n) &= (A^n + H_0\Gamma^0 + H_1\Gamma^1 + H_2\Gamma^2 + \ldots + H_{p-1}\Gamma^{p-1})x(kn) \\ &= (A^n + \sum_{i=0}^{p-1} H_i\Gamma^i)x(kn) \end{aligned} \tag{16}$$

where H_i and Γ are given by (12) and (13). Equation (16) represents a discrete-time LTI, therefore asymptotic stability is achieved when the eigenvalues of (11) lie inside the unit circle. ■

V. Examples

In this section we provide some simulation examples that complement the results obtained in the past two sections. An illustrative way to proceed is to plot the eigenvalues of the corresponding test equation and obtain the time response of the system for different values of the parameters involved in the lifted system, those parameters could be h, τ, n, or m, depending on the type of feedback and the implemented configuration.

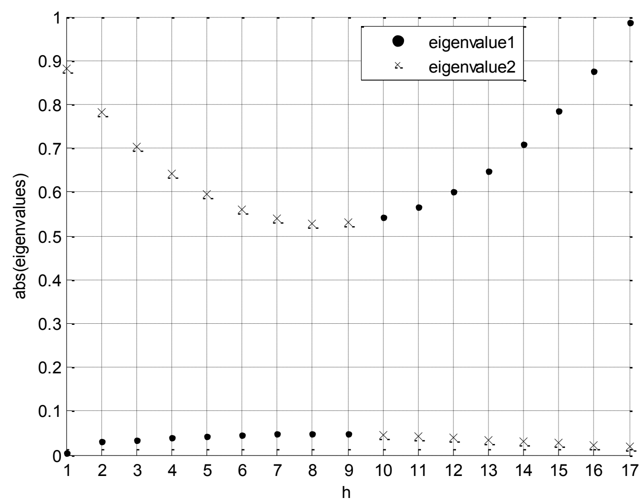


Fig. 4. Absolute value of the eigenvalues of equation (6), showing the values of h for which the system remains stable.

Example 1. Consider the following plant and model implemented using a traditional MB-NCS configuration with network only from sensor to controller and using instantaneous feedback with $T=0.1$ seconds. The model is a random perturbation of the real parameters, representing some existing uncertainty in most typical implementations of real systems.

$$A = \begin{pmatrix} 1.15 & 0 \\ 0 & 0.9 \end{pmatrix} \qquad B = \begin{pmatrix} 1.01 \\ 1.1 \end{pmatrix}$$

$$\hat{A} = \begin{pmatrix} 1.1497 & 0.0196 \\ 0.0086 & 0.9272 \end{pmatrix} \qquad \hat{B} = \begin{pmatrix} 1.0109 \\ 1.1018 \end{pmatrix}$$

$$K = \begin{pmatrix} -1.2225 & 0.0633 \end{pmatrix}$$

Fig. 4 shows the absolute value of the eigenvalues of (6) in theorem 1. Note thet h takes values only in the integers. We can implement our controller and model over a network, receive measurements every h clock ticks and remain stable as long as that h produces eigenvalues of equation (6) with absolute value less than one. Fig. 5 shows the response of the system for particular values of h. and for initial conditions equal to $x_0 = [0.5 \quad 0.2]^T$ in both cases.

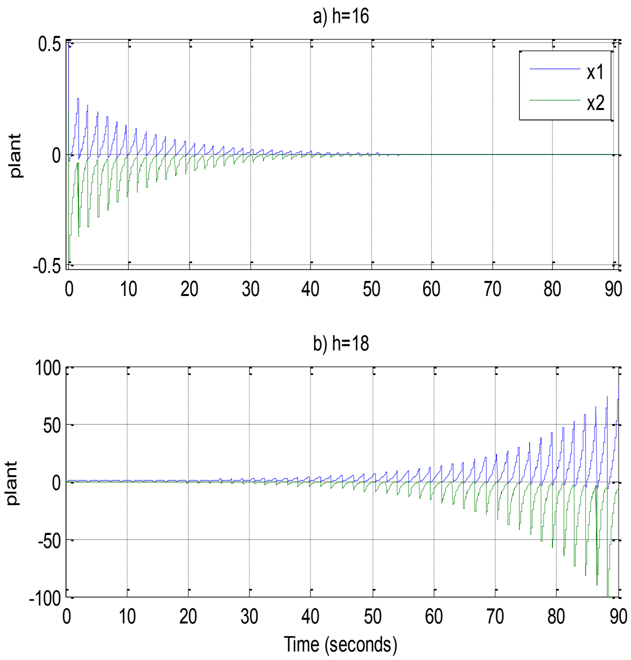


Fig. 5. Response of the plant for different values of h: a) $h=16$ system is still stable, b) $h=18$ system is unstable.

Example 2. Consider now the following plant and model implemented as in Fig. 3 where the network is also used to connect the controller to the plant. $T=1$second.

$$A = \begin{pmatrix} 0.9 & 0.1 \\ 0 & 1.07 \end{pmatrix} \quad B = \begin{pmatrix} 0.01 \\ 0.01 \end{pmatrix}$$

$$\hat{A} = \begin{pmatrix} 0.9117 & 0.1054 \\ 0.0360 & 1.0672 \end{pmatrix} \quad \hat{B} = \begin{pmatrix} 0.0109 \\ 0.0117 \end{pmatrix}$$

$$K = \begin{pmatrix} -2.3294 & -17.6266 \end{pmatrix}$$

In this case we have two variables, n and m, and, intuitively, we expect a decrease in the necessary value of n as m increases. An appropriate way to proceed here then is as follows: first find the largest value of n for $m=1$, that is, find the value of h in theorem 1; then select a value for n less than the value of h that we just found and find the divisors of n; with this information we can plot the eigenvalues of equation (11) as a function of m. For the example in hand, the highest value of h, from theorem 1, is 29, so for illustrative purposes we can fix $n=24$ (the choice of n is taken considering the existence of a large number of divisors to get $p=n/m$ an integer). For this case the eigenvalues of equation (11) are shown in Fig. 6, note that the horizontal axis contains those values of m that result in p an integer. The response of the plant is shown in Fig. 7. The first plot represents the response of the system by choosing parameters n,m according to Fig. 6 for which all eigenvalues of (11) have magnitude less than one (inside the unit circle). In the second plot the selected parameters result in an unstable system since not all eigenvalues of (11) lie inside the unit circle.

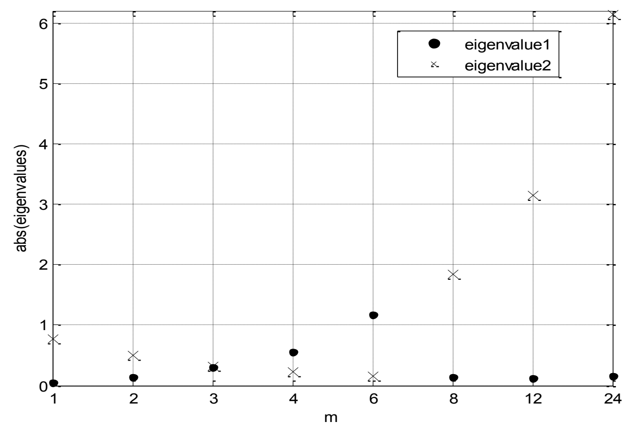


Fig. 6. Absolute value of the eigenvalues of equation (11) with $n=24$.

Fig. 8 shows all admissible pairs (n,m) in the range $n<30$, i.e. those pairs that result in p having an integer value, it also shows which of those pairs provide stability for the system in example 2.

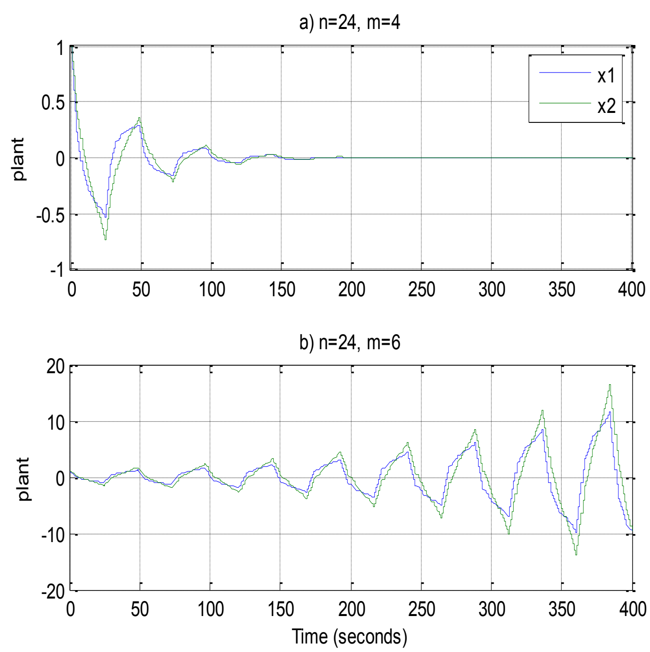


Fig. 7. Response of the plant for n=24 and different values of m for which: a) stability is still preserved and b) system becomes unstable.

VI. Conclusions and Future Work

The Model-Based control of networked systems has been revisited in this paper making use of lifting procedures. These techniques were applied to different configurations of Networked Control Systems that use a model of the plant to generate an estimate of the state between updates. The typical MB-NCS configurations (those in which the network is implemented only between the sensor and the controller) were analyzed and necessary and sufficient conditions for asymptotic stability were obtained. In addition, the more general configuration where the network is also present between the controller and the plant was also studied and analogous results were derived. These results represent the main contributions of the paper.

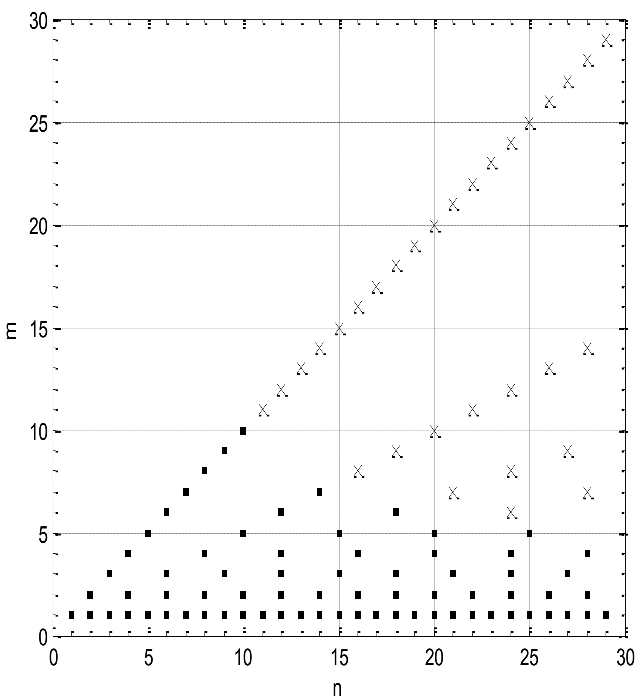


Fig.8. Sets of admissible values of *n* and *m* for example 2. (■) represent the pairs (*n,m*) that result in a stable system. (x) represent the pairs (*n,m*) that result in an unstable system, the rest are inadmissible rates.

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