The Control Handbook

Second Edition

CONTROL SYSTEM ADVANCED METHODS

Edited by

William S. Levine

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Control System Advanced Methods

The Electrical Engineering Handbook Series

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Control System Fundamentals

Control System Applications

Control System Advanced Methods

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Preface to the Second Edition

As you may know, the first edition of *The Control Handbook* was very well received. Many copies were sold and a gratifying number of people took the time to tell me that they found it useful. To the publisher, these are all reasons to do a second edition. To the editor of the first edition, these same facts are a modest disincentive. The risk that a second edition will not be as good as the first one is real and worrisome. I have tried very hard to insure that the second edition is at least as good as the first one was. I hope you agree that I have succeeded.

I have made two major changes in the second edition. The first is that all the *Applications* chapters are new. It is simply a fact of life in engineering that once a problem is solved, people are no longer as interested in it as they were when it was unsolved. I have tried to find especially inspiring and exciting applications for this second edition.

Secondly, it has become clear to me that organizing the *Applications* book by academic discipline is no longer sensible. Most control applications are interdisciplinary. For example, an automotive control system that involves sensors to convert mechanical signals into electrical ones, actuators that convert electrical signals into mechanical ones, several computers and a communication network to link sensors and actuators to the computers does not belong solely to any specific academic area. You will notice that the applications are now organized broadly by application areas, such as automotive and aerospace.

One aspect of this new organization has created a minor and, I think, amusing problem. Several wonderful applications did not fit into my new taxonomy. I originally grouped them under the title Miscellaneous. Several authors objected to the slightly pejorative nature of the term "miscellaneous." I agreed with them and, after some thinking, consulting with literate friends and with some of the library resources, I have renamed that section "Special Applications." Regardless of the name, they are all interesting and important and I hope you will read those articles as well as the ones that did fit my organizational scheme.

There has also been considerable progress in the areas covered in the *Advanced Methods* book. This is reflected in the roughly two dozen articles in this second edition that are completely new. Some of these are in two new sections, "Analysis and Design of Hybrid Systems" and "Networks and Networked Controls."

There have even been a few changes in the *Fundamentals*. Primarily, there is greater emphasis on sampling and discretization. This is because most control systems are now implemented digitally.

I have enjoyed editing this second edition and learned a great deal while I was doing it. I hope that you will enjoy reading it and learn a great deal from doing so.

William S. Levine

Acknowledgments

The people who were most crucial to the second edition were the authors of the articles. It took a great deal of work to write each of these articles and I doubt that I will ever be able to repay the authors for their efforts. I do thank them very much.

The members of the advisory/editorial board for the second edition were a very great help in choosing topics and finding authors. I thank them all. Two of them were especially helpful. Davor Hrovat took responsibility for the automotive applications and Richard Braatz was crucial in selecting the applications to industrial process control.

It is a great pleasure to be able to provide some recognition and to thank the people who helped bring this second edition of *The Control Handbook* into being. Nora Konopka, publisher of engineering and environmental sciences for Taylor & Francis/CRC Press, began encouraging me to create a second edition quite some time ago. Although it was not easy, she finally convinced me. Jessica Vakili and Kari Budyk, the project coordinators, were an enormous help in keeping track of potential authors as well as those who had committed to write an article. Syed Mohamad Shajahan, senior project executive at Techset, very capably handled all phases of production, while Richard Tressider, project editor for Taylor & Francis/CRC Press, provided direction, oversight, and quality control. Without all of them and their assistants, the second edition would probably never have appeared and, if it had, it would have been far inferior to what it is.

Most importantly, I thank my wife Shirley Johannesen Levine for everything she has done for me over the many years we have been married. It would not be possible to enumerate all the ways in which she has contributed to each and everything I have done, not just editing this second edition.

William S. Levine

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Editor

William S. Levine received B.S., M.S., and Ph.D. degrees from the Massachusetts Institute of Technology. He then joined the faculty of the University of Maryland, College Park where he is currently a research professor in the Department of Electrical and Computer Engineering. Throughout his career he has specialized in the design and analysis of control systems and related problems in estimation, filtering, and system modeling. Motivated by the desire to understand a collection of interesting controller designs, he has done a great deal of research on mammalian control of movement in collaboration with several neurophysiologists.

He is co-author of *Using MATLAB to Analyze and Design Control Systems*, March 1992. Second Edition, March 1995. He is the coeditor of *The Handbook of Networked and Embedded Control Systems,* published by Birkhauser in 2005. He is the editor of a series on control engineering for Birkhauser. He has been president of the IEEE Control Systems Society and the American Control Council. He is presently the chairman of the SIAM special interest group in control theory and its applications.

He is a fellow of the IEEE, a distinguished member of the IEEE Control Systems Society, and a recipient of the IEEE Third Millennium Medal. He and his collaborators received the Schroers Award for outstanding rotorcraft research in 1998. He and another group of collaborators received the award for outstanding paper in the *IEEE Transactions on Automatic Control*, entitled "Discrete-Time Point Processes in Urban Traffic Queue Estimation."

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I

Analysis Methods for MIMO Linear Systems

1

Numerical and Computational Issues in Linear Control and System Theory*

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1.1 Introduction

This chapter provides a survey of various aspects of the numerical solution of selected problems in linear systems, control, and estimation theory. Space limitations preclude an exhaustive survey and extensive list of references. The interested reader is referred to [1,4,10,14] for sources of additional detailed information.

* This material is based on a paper written by the same authors and published in Patel, R.V., Laub, A.J., and Van Dooren, P.M., Eds., *Numerical Linear Algebra Techniques for Systems and Control*, Selected Reprint Series, IEEE Press, New York, pp. 1–29 1994, copyright 1994 IEEE.

Many of the problems considered in this chapter arise in the study of the "standard" linear model

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1.1}$$

$$y(t) = Cx(t) + Du(t). \tag{1.2}$$

Here, $x(t)$ is an n-vector of states, $u(t)$ is an m-vector of controls or inputs, and $y(t)$ is a p-vector of outputs. The standard discrete-time analog of Equations 1.1 and 1.2 takes the form

$$x_{k+1} = Ax_k + Bu_k, \tag{1.3}$$

$$y_k = Cx_k + Du_k. \tag{1.4}$$

Of course, considerably more elaborate models are also studied, including time-varying, stochastic, and nonlinear versions of the above, but these are not discussed in this chapter. In fact, the above linear models are usually derived from linearizations of nonlinear models regarding selected nominal points.

The matrices considered here are, for the most part, assumed to have real coefficients and to be small (of order a few hundred or less) and dense, with no particular exploitable structure. Calculations for most problems in classical single-input, single-output control fall into this category. Large sparse matrices or matrices with special exploitable structures may significantly involve different concerns and methodologies than those discussed here.

The systems, control, and estimation literature is replete with *ad hoc* algorithms to solve the computational problems that arise in the various methodologies. Many of these algorithms work quite well on some problems (e.g., "small-order" matrices) but encounter numerical difficulties, often severe, when "pushed" (e.g., on larger order matrices). The reason for this is that little or no attention has been paid to the way algorithms perform in "finite arithmetic," that is, on a finite word length digital computer.

A simple example by Moler and Van Loan [14, p. 649]* illustrates a typical pitfall. Suppose it is desired to compute the matrix e^A in single precision arithmetic on a computer which gives six decimal places of precision in the fractional part of floating-point numbers. Consider the case

$$A = \begin{bmatrix} -49 & 24 \\ -64 & 31 \end{bmatrix}$$

and suppose the computation is attempted with the Taylor series formula

$$e^A = \sum_{k=0}^{+\infty} \frac{1}{k!} A^k. \tag{1.5}$$

This is easily coded and it is determined that the first 60 terms in the series suffice for the computation, in the sense that the terms for $k \geq 60$ of the order 10^{-7} no longer add anything significant to the sum. The resulting answer is

$$\begin{bmatrix} -22.2588 & -1.43277 \\ -61.4993 & -3.47428 \end{bmatrix}.$$

Surprisingly, the true answer is (correctly rounded)

$$\begin{bmatrix} -0.735759 & 0.551819 \\ -1.47152 & 1.10364 \end{bmatrix}.$$

What happened here was that the intermediate terms in the series became very large before the factorial began to dominate. The 17th and 18th terms, for example, are of the order of 10^7 but of opposite signs so

* The page number indicates the location of the appropriate reprint in [14].

that the less significant parts of these numbers, while significant for the final answer, are "lost" because of the finiteness of the arithmetic.

For this particular example, various fixes and remedies are available. However, in more realistic examples, one seldom has the luxury of having the "true answer" available so that it is not always easy to simply inspect or test a computed solution and determine that it is erroneous. Mathematical analysis (truncation of the series, in the example above) alone is simply not sufficient when a problem is analyzed or solved in finite arithmetic (truncation of the arithmetic). Clearly, a great deal of care must be taken.

The finiteness inherent in representing real or complex numbers as floating-point numbers on a digital computer manifests itself in two important ways: floating-point numbers have only finite precision and finite range. The degree of attention paid to these two considerations distinguishes many reliable algorithms from more unreliable counterparts.

The development in systems, control, and estimation theory of stable, efficient, and reliable algorithms that respect the constraints of finite arithmetic began in the 1970s and still continues. Much of the research in numerical analysis has been directly applicable, but there are many computational issues in control (e.g., the presence of hard or structural zeros) where numerical analysis does not provide a ready answer or guide. A symbiotic relationship has developed, especially between numerical linear algebra and linear system and control theory, which is sure to provide a continuing source of challenging research areas.

The abundance of numerically fragile algorithms is partly explained by the following observation:

> If an algorithm is amenable to "easy" manual calculation, it is probably a poor method if implemented in the finite floating-point arithmetic of a digital computer.

For example, when confronted with finding the eigenvalues of a 2×2 matrix, most people would find the characteristic polynomial and solve the resulting quadratic equation. But when extrapolated as a general method for computing eigenvalues and implemented on a digital computer, this is a very poor procedure for reasons such as roundoff and overflow/underflow. The preferred method now would generally be the double Francis QR algorithm (see [17] for details) but few would attempt that manually, even for very small-order problems.

Many algorithms, now considered fairly reliable in the context of finite arithmetic, are not amenable to manual calculations (e.g., various classes of orthogonal similarities). This is a kind of converse to the observation quoted above. Especially in linear system and control theory, we have been too easily tempted by the ready availability of closed-form solutions and numerically naive methods to implement those solutions. For example, in solving the initial value problem

$$\dot{x}(t) = Ax(t); \quad x(0) = x_0, \tag{1.6}$$

it is not at all clear that one should explicitly compute the intermediate quantity e^{tA}. Rather, it is the vector $e^{tA}x_0$ that is desired, a quantity that may be computed by treating Equation 1.6 as a system of (possibly stiff) differential equations and using an implicit method for numerically integrating the differential equation. But such techniques are definitely not attractive for manual computation.

The awareness of such numerical issues in the mathematics and engineering community has increased significantly in the last few decades. In fact, some of the background material well known to numerical analysts has already filtered down to undergraduate and graduate curricula in these disciplines. This awareness and education has affected system and control theory, especially linear system theory. A number of numerical analysts were attracted by the wealth of interesting numerical linear algebra problems in linear system theory. At the same time, several researchers in linear system theory turned to various methods and concepts from numerical linear algebra and attempted to modify them in developing reliable algorithms and software for specific problems in linear system theory. This cross-fertilization has been greatly enhanced by the widespread use of software packages and by developments over the last couple of decades in numerical linear algebra. This process has already begun to have a significant impact on the future directions and development of system and control theory, and on applications, as evident

from the growth of computer-aided control system design (CACSD) as an intrinsic tool. Algorithms implemented as mathematical software are a critical "inner" component of a CACSD system.

In the remainder of this chapter, we survey some results and trends in this interdisciplinary research area. We emphasize numerical aspects of the problems/algorithms, which is why we also spend time discussing appropriate numerical tools and techniques. We discuss a number of control and filtering problems that are of widespread interest in control.

Before proceeding further, we list here some notations to be used:

$\mathbb{F}^{n\times m}$	the set of all $n \times m$ matrices with coefficients in the field $\mathbb{F}$ ($\mathbb{F}$ is generally $\mathbb{R}$ or $\mathbb{C}$)
A^T	the transpose of $A \in \mathbb{R}^{n\times m}$
A^H	the complex-conjugate transpose of $A \in \mathbb{C}^{n\times m}$
A^+	the Moore–Penrose pseudoinverse of A
$\|A\|$	the spectral norm of A (i.e., the matrix norm subordinate to the Euclidean vector norm: $\|A\| = \max_{\|x\|_2=1} \|Ax\|_2$)
diag $(a_1, \ldots, a_n)$	the diagonal matrix $\begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix}$
$\Lambda(A)$	the set of eigenvalues $\lambda_1, \ldots, \lambda_n$ (not necessarily distinct) of $A \in \mathbb{F}^{n\times n}$
$\lambda_i(A)$	the ith eigenvalue of A
$\Sigma(A)$	the set of singular values $\sigma_1, \ldots, \sigma_m$ (not necessarily distinct) of $A \in \mathbb{F}^{n\times m}$
$\sigma_i(A)$	the ith singular value of A

Finally, let us define a particular number to which we make frequent reference in the following. The *machine epsilon* or *relative machine precision* is defined, roughly speaking, as the smallest positive number ϵ that, when added to 1 on our computing machine, gives a number greater than 1. In other words, any machine representable number δ less than ϵ gets " rounded off" when (floating-point) added to 1 to give exactly 1 again as the rounded sum. The number ϵ, of course, varies depending on the kind of computer being used and the precision of the computations (single precision, double precision, etc.). But the fact that such a positive number ϵ exists is entirely a consequence of finite word length.

1.2 Numerical Background

In this section, we give a very brief discussion of two concepts fundamentally important in numerical analysis: *numerical stability* and *conditioning*. Although this material is standard in textbooks such as [8], it is presented here for completeness and because the two concepts are frequently confused in the systems, control, and estimation literature.

Suppose we have a mathematically defined problem represented by f which acts on data d belonging to some set of data $\mathcal{D}$, to produce a solution $f(d)$ in a solution set $\mathcal{S}$. These notions are kept deliberately vague for expository purposes. Given $d \in \mathcal{D}$, we desire to compute $f(d)$. Suppose d^* is some approximation to d. If $f(d^*)$ is "near" $f(d)$, the problem is said to be well conditioned. If $f(d^*)$ may potentially differ greatly from $f(d)$ even when d^* is near d, the problem is said to be ill-conditioned. The concept of "near" can be made precise by introducing norms in the appropriate spaces. We can then define the condition of the problem f with respect to these norms as

$$\kappa[f(d)] = \lim_{\delta\to 0} \sup_{d_2(d,d^*)=\delta} \left[\frac{d_1(f(d), f(d^*))}{\delta}\right], \tag{1.7}$$

where $d_i(\cdot,\cdot)$ are distance functions in the appropriate spaces. When $\kappa[f(d)]$ is infinite, the problem of determining $f(d)$ from d is *ill-posed* (as opposed to *well-posed*). When $\kappa[f(d)]$ is finite and *relatively large* (or *relatively small*), the problem is said to be *ill-conditioned* (or *well-conditioned*).

A simple example of an ill-conditioned problem is the following. Consider the $n \times n$ matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & \cdot & \cdot & \cdot & 0 \\ \cdot & & & & \cdot & \cdot & 1 \\ 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix},$$

with n eigenvalues at 0. Now, consider a small perturbation of the data (the n^2 elements of A) consisting of adding the number 2^{-n} to the first element in the last (nth) row of A. This perturbed matrix then has n distinct eigenvalues $\lambda_1, \ldots, \lambda_n$ with $\lambda_k = 1/2 \exp(2k\pi j/n)$, where $j := \sqrt{-1}$. Thus, we see that this small perturbation in the data has been magnified by a factor on the order of 2^n resulting in a rather large perturbation in solving the problem of computing the eigenvalues of A. Further details and related examples can be found in [9,17].

Thus far, we have not mentioned how the problem f above (computing the eigenvalues of A in the example) was to be solved. Conditioning is a function solely of the problem itself. To solve a problem numerically, we must implement some numerical procedures or algorithms which we denote by f^*. Thus, given $d, f^*(d)$ is the result of applying the algorithm to d (for simplicity, we assume d is "representable"; a more general definition can be given when some approximation d^{**} to d must be used). The algorithm f^* is said to be numerically (backward) stable if, for all $d \in \mathcal{D}$, there exists $d^* \in \mathcal{D}$ near d so that $f^*(d)$ is near $f(d^*)$, ($f(d^*)$ = the exact solution of a nearby problem). If the problem is well-conditioned, then $f(d^*)$ is near $f(d)$ so that $f^*(d)$ is near $f(d)$ if f^* is numerically stable. In other words, f^* does not introduce any more sensitivity to perturbation than is inherent in the problem. Example 1.1 further illuminates this definition of stability which, on a first reading, can seem somewhat confusing.

Of course, one cannot expect a stable algorithm to solve an ill-conditioned problem any more accurately than the data warrant, but an unstable algorithm can produce poor solutions even to well-conditioned problems. Example 1.2, illustrates this phenomenon. There are thus two separate factors to consider in determining the accuracy of a computed solution $f^*(d)$. First, if the algorithm is stable, $f^*(d)$ is near $f(d^*)$, for some d^*, and second, if the problem is well conditioned, then, as above, $f(d^*)$ is near $f(d)$. Thus, $f^*(d)$ is near $f(d)$ and we have an "accurate" solution.

Rounding errors can cause unstable algorithms to give disastrous results. However, it would be virtually impossible to account for every rounding error made at every arithmetic operation in a complex series of calculations. This would constitute a *forward* error analysis. The concept of *backward* error analysis based on the definition of numerical stability given above provides a more practical alternative. To illustrate this, let us consider the singular value decomposition (SVD) of an arbitrary $m \times n$ matrix A with coefficients in $\mathbb{R}$ or $\mathbb{C}$ [8] (see also Section 1.3.3),

$$A = U\Sigma V^H. \tag{1.8}$$

Here U and V are $m \times m$ and $n \times n$ unitary matrices, respectively, and Σ is an $m \times n$ matrix of the form

$$\Sigma = \left[\begin{array}{c|c} \Sigma_r & 0 \\ \hline 0 & 0 \end{array}\right]; \quad \Sigma_r = \operatorname{diag}\{\sigma_1, \ldots, \sigma_r\} \tag{1.9}$$

with the *singular values* σ_i positive and satisfying $\sigma_1 \geq \sigma_2 \cdots \geq \sigma_r > 0$. The computation of this decomposition is, of course, subject to rounding errors. Denoting computed quantities by an overbar, for some

error matrix E_A,

$$\overline{A} = A + E_A = \overline{U\Sigma V}^H. \tag{1.10}$$

The computed decomposition thus corresponds exactly to a *perturbed* matrix $\overline{A}$. When using the SVD algorithm available in the literature [8], this perturbation can be bounded by

$$\| E_A \| \leq \pi\epsilon \| A \|, \tag{1.11}$$

where ϵ is the machine precision and π is some quantity depending on the dimensions m and n, but reasonably close to 1 (see also [14, p. 74]). Thus, the *backward error* E_A induced by this algorithm has roughly the same norm as the *input error* E_i resulting, for example, when reading the data A into the computer. Then, according to the definition of numerical stability given above, when a bound such as that in Equation 1.11 exists for the error induced by a numerical algorithm, the algorithm is said to be *backward stable* [17]. Note that backward stability does not guarantee any bounds on the errors in the result $\overline{U}$, $\overline{\Sigma}$, and $\overline{V}$. In fact, this depends on how perturbations in the data (namely, $E_A = \overline{A} - A$) affect the resulting decomposition (namely, $E_U = \overline{U} - U, E_\Sigma = \overline{\Sigma} - \Sigma$, and $E_V = \overline{V} - V$). This is commonly measured by the condition $\kappa[f(A)]$.

Backward stability is a property of an algorithm, and the condition is associated with a problem and the specific data for that problem. The errors in the result depend on the stability of the algorithm used and the condition of the problem solved. A *good* algorithm should, therefore, be backward stable because the size of the errors in the result is then mainly due to the condition of the problem, not to the algorithm. An unstable algorithm, on the other hand, may yield a large error even when the problem is well conditioned.

Bounds of the type Equation 1.11 are obtained by an error analysis of the algorithm used, and the condition of the problem is obtained by a sensitivity analysis; for example, see [9,17].

We close this section with two simple examples to illustrate some of the concepts introduced.

Example 1.1:

Let x and y be two floating-point computer numbers and let $fl(x * y)$ denote the result of multiplying them in floating-point computer arithmetic. In general, the product $x * y$ requires more precision to be represented exactly than was used to represent x or y. But for most computers

$$fl(x * y) = x * y(1 + \delta), \tag{1.12}$$

where $|\delta| < \epsilon$ (= relative machine precision). In other words, $fl(x * y)$ is $x * y$ correct to within a unit in the last place. Another way to write Equation 1.12 is as follows:

$$fl(x * y) = x(1 + \delta)^{1/2} * y(1 + \delta)^{1/2}, \tag{1.13}$$

where $|\delta| < \epsilon$. This can be interpreted as follows: the computed result $fl(x * y)$ is the exact product of the two slightly perturbed numbers $x(1 + \delta)^{1/2}$ and $y(1 + \delta)^{1/2}$. The slightly perturbed data (not unique) may not even be representable as floating-point numbers. The representation of Equation 1.13 is simply a way of accounting for the roundoff incurred in the algorithm by an initial (small) perturbation in the data.

Example 1.2:

Gaussian elimination with no pivoting for solving the linear system of equations

$$Ax = b \tag{1.14}$$

is known to be numerically unstable; see for example [8] and Section 1.3. The following data illustrate this phenomenon. Let

$$A = \begin{bmatrix} 0.0001 & 1.000 \\ 1.000 & -1.000 \end{bmatrix}, \quad b = \begin{bmatrix} 1.000 \\ 0.000 \end{bmatrix}.$$

All computations are carried out in four-significant-figure decimal arithmetic. The "true answer" $x = A^{-1}b$ is

$$\begin{bmatrix} 0.9999 \\ 0.9999 \end{bmatrix}.$$

Using row 1 as the "pivot row" (i.e., subtracting 10,000 $\times$ row 1 from row 2) we arrive at the equivalent triangular system

$$\begin{bmatrix} 0.0001 & 1.000 \\ 0 & -1.000 \times 10^4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.000 \\ -1.000 \times 10^4 \end{bmatrix}.$$

The coefficient multiplying x_2 in the second equation should be $-10,001$, but because of roundoff, becomes $-10,000$. Thus, we compute $x_2 = 1.000$ (a good approximation), but back substitution in the equation

$$0.0001x_1 = 1.000 - fl(1.000 * 1.000)$$

yields $x_1 = 0.000$. This extremely bad approximation to x_1 is the result of numerical instability. The problem, it can be shown, is quite well conditioned.

1.3 Fundamental Problems in Numerical Linear Algebra

In this section, we give a brief overview of some of the fundamental problems in numerical linear algebra that serve as building blocks or "tools" for the solution of problems in systems, control, and estimation.

1.3.1 Linear Algebraic Equations and Linear Least-Squares Problems

Probably the most fundamental problem in numerical computing is the calculation of a vector x which satisfies the linear system

$$Ax = b, \tag{1.15}$$

where $A \in \mathbb{R}^{n\times n}$(or $\mathbb{C}^{n\times n}$) and has rank n. A great deal is now known about solving Equation 1.15 in finite arithmetic both for the general case and for a large number of special situations, for example, see [8,9].

The most commonly used algorithm for solving Equation 1.15 with general A and small n (say $n \leq 1000$) is Gaussian elimination with some sort of pivoting strategy, usually "partial pivoting." This amounts to factoring some permutation of the rows of A into the product of a unit lower triangular matrix L and an upper triangular matrix U. The algorithm is effectively stable, that is, it can be proved that the computed solution is near the exact solution of the system

$$(A + E)x = b \tag{1.16}$$

with $|e_{ij}| \leq \phi(n)\,\gamma\,\beta\,\epsilon$, where $\phi(n)$ is a modest function of n depending on details of the arithmetic used, γ is a "growth factor" (which is a function of the pivoting strategy and is usually—but not always—small), β behaves essentially like $\|A\|$, and ϵ is the machine precision. In other words, except for moderately pathological situations, E is "small"—on the order of $\epsilon\,\|A\|$.

The following question then arises. If, because of rounding errors, we are effectively solving Equation 1.16 rather than Equation 1.15, what is the relationship between $(A+E)^{-1}b$ and $A^{-1}b$? To answer this question, we need some elementary perturbation theory and this is where the notion of condition number arises. A condition number for Equation 1.15 is given by

$$\kappa(A) := \|A\|\,\|A^{-1}\|. \tag{1.17}$$

Simple perturbation results can show that perturbation in A and/or b can be magnified by as much as $\kappa(A)$ in the computed solution. Estimating $\kappa(A)$ (since, of course, A^{-1} is unknown) is thus a crucial aspect of assessing solutions of Equation 1.15 and the particular estimating procedure used is usually the principal difference between competing linear equation software packages. One of the more sophisticated and reliable condition estimators presently available is implemented in LINPACK [5] and its successor LAPACK [2]. LINPACK and LAPACK also feature many codes for solving Equation 1.14 in case A has certain special structures (e.g., banded, symmetric, or positive definite).

Another important class of linear algebra problems, and one for which codes are available in LINPACK and LAPACK, is the linear least-squares problem

$$\min \|Ax-b\|_2, \tag{1.18}$$

where $A \in \mathbb{R}^{m\times n}$ and has rank k, with (in the simplest case) $k = n \le m$, for example, see [8]. The solution of Equation 1.18 can be written formally as $x = A^{+}b$. The method of choice is generally based on the QR factorization of A (for simplicity, let $\text{rank}(A) = n$)

$$A = QR, \tag{1.19}$$

where $R \in \mathbb{R}^{n\times n}$ is upper triangular and $Q \in \mathbb{R}^{m\times n}$ has orthonormal columns, that is, $Q^TQ = I$. With special care and analysis, the case $k < n$ can also be handled similarly. The factorization is effected through a sequence of Householder transformations H_i applied to A. Each H_i is symmetric and orthogonal and of the form $I - 2uu^T/u^Tu$, where $u \in \mathbb{R}^m$ is specially chosen so that zeros are introduced at appropriate places in A when it is premultiplied by H_i. After n such transformations,

$$H_nH_{n-1}\ldots H_1A = \begin{bmatrix} R \\ 0 \end{bmatrix},$$

from which the factorization Equation 1.19 follows. Defining c and d by

$$\begin{bmatrix} c \\ d \end{bmatrix} := H_nH_{n-1}\ldots H_1b,$$

where $c \in \mathbb{R}^n$, it is easily shown that the least-squares solution x of Equation 1.18 is given by the solution of the linear system of equations

$$Rx = c\,. \tag{1.20}$$

The above algorithm is numerically stable and, again, a well-developed perturbation theory exists from which condition numbers can be obtained, this time in terms of

$$\kappa(A) := \|A\|\,\|A^{+}\|.$$

Least-squares perturbation theory is fairly straightforward when $\text{rank}(A) = n$, but is considerably more complicated when A is rank deficient. The reason for this is that, although the inverse is a continuous

function of the data (i.e., the inverse is a continuous function in a neighborhood of a nonsingular matrix), the pseudoinverse is discontinuous. For example, consider

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = A^+$$

and perturbations

$$E_1 = \begin{bmatrix} 0 & 0 \\ \delta & 0 \end{bmatrix} \quad \text{and} \quad E_2 = \begin{bmatrix} 0 & 0 \\ 0 & \delta \end{bmatrix}$$

with δ being small. Then

$$(A + E_1)^+ = \begin{bmatrix} \dfrac{1}{1+\delta^2} & \dfrac{\delta}{1+\delta^2} \\ 0 & 0 \end{bmatrix},$$

which is close to A^+ but

$$(A + E_2)^+ = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{1}{\delta} \end{bmatrix},$$

which gets arbitrarily far from A^+ as δ is decreased toward 0.

In lieu of Householder transformations, Givens transformations (elementary rotations or reflections) may also be used to solve the linear least-squares problem [8]. Givens transformations have received considerable attention for solving linear least-squares problems and systems of linear equations in a parallel computing environment. The capability of introducing zero elements selectively and the need for only local interprocessor communication make the technique ideal for "parallelization."

1.3.2 Eigenvalue and Generalized Eigenvalue Problems

In the algebraic eigenvalue/eigenvector problem for $A \in \mathbb{R}^{n\times n}$, one seeks nonzero solutions $x \in \mathbb{C}^n$ and $\lambda \in \mathbb{C}$, which satisfy

$$Ax = \lambda x. \tag{1.21}$$

The classic reference on the numerical aspects of this problem is Wilkinson [17]. A briefer textbook introduction is given in [8].

Quality mathematical software for eigenvalues and eigenvectors is available; the EISPACK [7,15] collection of subroutines represents a pivotal point in the history of mathematical software. The successor to EISPACK (and LINPACK) is LAPACK [2], in which the algorithms and software have been restructured to provide high efficiency on vector processors, high-performance workstations, and shared memory multiprocessors.

The most common algorithm now used to solve Equation 1.21 for general A is the QR algorithm of Francis [17]. A shifting procedure enhances convergence and the usual implementation is called the double-Francis-QR algorithm. Before the QR process is applied, A is initially reduced to upper Hessenberg form $A_H (a_{ij} = 0 \text{ if } i - j \geq 2)$. This is accomplished by a finite sequence of similarities of the Householder form discussed above. The QR process then yields a sequence of matrices orthogonally similar to A and converging (in some sense) to a so-called quasi-upper triangular matrix S also called the real Schur form (RSF) of A. The matrix S is block upper triangular with 1×1 diagonal blocks corresponding to real eigenvalues of A and 2×2 diagonal blocks corresponding to complex-conjugate pairs of eigenvalues. The quasi-upper triangular form permits all arithmetic to be real rather than complex as would be necessary for convergence to an upper triangular matrix. The orthogonal transformations from both the Hessenberg reduction and the QR process may be accumulated in a single orthogonal transformation U so that

$$U^T A U = R \tag{1.22}$$

compactly represents the entire algorithm. An analogous process can be applied in the case of symmetric A, and considerable simplifications and specializations result.

Closely related to the QR algorithm is the QZ algorithm for the generalized eigenvalue problem

$$Ax = \lambda Mx, \tag{1.23}$$

where $A, M \in \mathbb{R}^{n\times n}$. Again, a Hessenberg-like reduction, followed by an iterative process, is implemented with orthogonal transformations to reduce Equation 1.23 to the form

$$QAZy = \lambda QMZy, \tag{1.24}$$

where QAZ is quasi-upper triangular and QMZ is upper triangular. For a review and references to results on stability, conditioning, and software related to Equation 1.23 and the QZ algorithm, see [8]. The generalized eigenvalue problem is both theoretically and numerically more difficult to handle than the ordinary eigenvalue problem, but it finds numerous applications in control and system theory [14, p. 109].

1.3.3 The Singular Value Decomposition and Some Applications

One of the basic and most important tools of modern numerical analysis, especially numerical linear algebra, is the SVD. Here we make a few comments about its properties and computation as well as its significance in various numerical problems.

Singular values and the SVD have a long history, especially in statistics and numerical linear algebra. These ideas have found applications in the control and signal processing literature, although their use there has been overstated somewhat in certain applications. For a survey of the SVD, its history, numerical details, and some applications in systems and control theory, see [14, p. 74].

The fundamental result was stated in Section 1.2 (for the complex case). The result for the real case is similar and is stated below.

Theorem 1.1:

Let $A \in \mathbb{R}^{m\times n}$ with rank$(A) = r$. Then there exist orthogonal matrices $U \in \mathbb{R}^{m\times m}$ and $V \in \mathbb{R}^{n\times n}$ so that

$$A = U\Sigma V^T, \tag{1.25}$$

where

$$\Sigma = \begin{bmatrix} \Sigma_r & 0 \\ 0 & 0 \end{bmatrix}$$

and $\Sigma_r = diag\{\sigma_1, \ldots, \sigma_r\}$ with $\sigma_1 \geq \cdots \geq \sigma_r > 0$.

The proof of Theorem 1.1 is straightforward and can be found, for example, in [8]. Geometrically, the theorem says that bases can be found (separately) in the domain and codomain spaces of a linear map with respect to which the matrix representation of the linear map is diagonal. The numbers $\sigma_1, \ldots, \sigma_r$, together with $\sigma_{r+1} = 0, \ldots, \sigma_n = 0$, are called the singular values of A, and they are the positive square roots of the eigenvalues of A^TA. The columns $\{u_k, k = 1, \ldots, m\}$ of U are called the left singular vectors of A (the orthonormal eigenvectors of AA^T), while the columns $\{v_k, k = 1, \ldots, n\}$ of V are called the right singular vectors of A (the orthonormal eigenvectors of A^TA). The matrix A can then be written (as a dyadic expansion) also in terms of the singular vectors as follows:

$$A = \sum_{k=1}^{r} \sigma_k u_k v_k^T.$$

The matrix A^T has m singular values, the positive square roots of the eigenvalues of AA^T. The r [$=$ rank(A)] nonzero singular values of A and A^T are, of course, the same. The choice of A^TA rather than

AA^T in the definition of singular values is arbitrary. Only the nonzero singular values are usually of any real interest and their number, given the SVD, is the rank of the matrix. Naturally, the question of how to distinguish nonzero from zero singular values in the presence of rounding error is a nontrivial task.

It is not generally advisable to compute the singular values of A by first finding the eigenvalues of A^TA, tempting as that is. Consider the following example, where μ is a real number with $|\mu| < \sqrt{\epsilon}$ (so that $fl(1+\mu^2) = 1$, where $fl(\cdot)$ denotes floating-point computation). Let

$$A = \begin{bmatrix} 1 & 1 \\ \mu & 0 \\ 0 & \mu \end{bmatrix}.$$

Then

$$fl(A^TA) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

So we compute $\hat{\sigma}_1 = \sqrt{2},\ \hat{\sigma}_2 = 0$ leading to the (erroneous) conclusion that the rank of A is 1. Of course, if we could compute in infinite precision, we would find

$$A^TA = \begin{bmatrix} 1+\mu^2 & 1 \\ 1 & 1+\mu^2 \end{bmatrix}$$

with $\sigma_1 = \sqrt{2+\mu^2}, \sigma_2 = |\mu|$ and thus $\text{rank}(A) = 2$. The point is that by working with A^TA we have unnecessarily introduced μ^2 into the computations. The above example illustrates a potential pitfall in attempting to form and solve the normal equations in a linear least-squares problem and is at the heart of what makes square root filtering so attractive numerically. Very simplistically, square root filtering involves working directly on an "A-matrix," for example, updating it, as opposed to updating an "A^TA-matrix."

Square root filtering is usually implemented with the QR factorization (or some closely related algorithm) as described previously rather than SVD. Moreover, critical information may be lost irrecoverably by simply forming A^TA.

Returning now to the SVD, two features of this matrix factorization make it attractive in finite arithmetic: first, it can be computed in a numerically stable way, and second, singular values are well conditioned. Specifically, there is an efficient and numerically stable algorithm by Golub and Reinsch [8] which works directly on A to give the SVD. This algorithm has two phases. In the first phase, it computes orthogonal matrices U_1 and V_1 so that $B = {U_1}^T A V_1$ is in bidiagonal form, that is, only the elements on its diagonal and first superdiagonal are nonzero. In the second phase, the algorithm uses an iterative procedure to compute orthogonal matrices U_2 and V_2 so that ${U_2}^T B V_2$ is diagonal and nonnegative. The SVD defined in Equation 1.25 is then $\Sigma = U^TBV$, where $U = U_1U_2$ and $V = V_1V_2$. The computed U and V are orthogonal approximately to the working precision, and the computed singular values are the exact σ_i's for $A+E$, where $\|E\|/\|A\|$ is a modest multiple of ϵ. Fairly sophisticated implementations of this algorithm can be found in [5,7]. The well-conditioned nature of the singular values follows from the fact that if A is perturbed to $A+E$, then it can be proved that

$$\|\sigma_i(A+E) - \sigma_i(A)\| \leq \|E\|.$$

Thus, the singular values are computed with small absolute error although the relative error of sufficiently small singular values is not guaranteed to be small.

It is now acknowledged that the singular value decomposition is the most generally reliable method of determining rank numerically (see [14, p. 589] for a more elaborate discussion). However, it is considerably more expensive to compute than, for example, the QR factorization which, with column pivoting [5], can usually give equivalent information with less computation. Thus, while the SVD is a useful theoretical tool, its use for actual computations should be weighed carefully against other approaches.

The problem of numerical determination of rank is now well understood. The essential idea is to try to determine a "gap" between "zero" and the "smallest nonzero singular value" of a matrix A. Since the computed values are exact for a matrix near A, it makes sense to consider the ranks of all matrices in some δ-ball (with respect to the spectral norm $\|\cdot\|$, say) around A. The choice of δ may also be based on measurement errors incurred in estimating the coefficients of A, or the coefficients may be uncertain because of rounding errors incurred in a previous computation. However, even with SVD, numerical determination of rank in finite arithmetic is a difficult problem.

That other methods of rank determination are potentially unreliable is demonstrated by the following example. Consider the Ostrowski matrix $A \in \mathbb{R}^{n\times n}$ whose diagonal elements are all -1, whose upper triangle elements are all $+1$, and whose lower triangle elements are all 0. This matrix is clearly of rank n, that is, is invertible. It has a good "solid" upper triangular shape. All of its eigenvalues (-1) are well away from zero. Its determinant $(-1)^n$ is definitely not close to zero. But this matrix is, in fact, very nearly singular and becomes more nearly so as n increases. Note, for example, that

$$\begin{bmatrix} -1 & +1 & \cdots & & \cdots & +1 \\ 0 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & \ddots & & \\ & & & \ddots & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & +1 \\ 0 & \cdots & & \cdots & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2^{-1} \\ \vdots \\ 2^{-n+1} \end{bmatrix} = \begin{bmatrix} -2^{-n+1} \\ -2^{-n+1} \\ \vdots \\ -2^{-n+1} \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} (n \to +\infty).$$

Moreover, adding 2^{-n+1} to every element in the first column of A gives an exactly singular matrix. Arriving at such a matrix by, say, Gaussian elimination would give no hint as to the near singularity. However, it is easy to check that $\sigma_n(A)$ behaves as 2^{-n+1}. A corollary for control theory is that eigenvalues do not necessarily give a reliable measure of "stability margin." It is useful to note that in this example of an invertible matrix, the crucial quantity, $\sigma_n(A)$, which measures nearness to singularity, is simply $1/\|A^{-1}\|$, and the result is familiar from standard operator theory. There is nothing intrinsic about singular values in this example and, in fact, $\|A^{-1}\|$ might be more cheaply computed or estimated in other matrix norms.

Because rank determination, in the presence of rounding error, is a nontrivial problem, the same difficulties naturally arise in any problem equivalent to, or involving, rank determination, such as determining the independence of vectors, finding the dimensions of certain subspaces, etc. Such problems arise as basic calculations throughout systems, control, and estimation theory. Selected applications are discussed in more detail in [14, p. 74] and in [1,4,10].

Finally, let us close this section with a brief example illustrating a totally inappropriate use of SVD. The rank condition

$$\text{rank } [B, AB, \ldots, A^{n-1}B] = n \tag{1.26}$$

for the controllability of Equation 1.1 is too well known. Suppose

$$A = \begin{bmatrix} 1 & \mu \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ \mu \end{bmatrix}$$

with $|\mu| < \sqrt{\epsilon}$. Then

$$fl[B, AB] = \begin{bmatrix} 1 & 1 \\ \mu & \mu \end{bmatrix},$$

and now even applying SVD, the erroneous conclusion of uncontrollability is reached. Again the problem is in just forming AB; not even SVD can come to the rescue after that numerical *faux pas*.

References

1 1.6 Concluding Remarks

1. Varga, A. et al. The amazing power of numerical awareness in Control, IEEE Control Systems Magazine pp. 14–76, 24, February 2004.

2. Anderson, E., Bai, Z., Bischof, C., Blackford, S., Demmel, J., Dongarra, J., DuCroz, J., et al. LAPACK Users' Guide (3rd edition), SIAM, Philadelphia, PA, 1999.

3. Bunse-Gerstner, A., Byers, R., andMehrmann, V., Numericalmethods for algebraic Riccati equations, in Proceedings of Workshop on the Riccati Equation in Control, Systems, and Signals (Como, Italy) Bittanti, S., Ed., Pitagora, Bologna, Italy, pp. 107–116, 1989.

4. Datta, B.N., Linear Algebra and Applications (2nd edition), SIAM, Philadelphia, PA, 2010.

5. Dongarra, J.J., Bunch, J.R., Moler, C.B., and Stewart, G.W., LINPACK Users' Guide, SIAM, Philadelphia, PA, 1979.

6. Dorato, P. and Yedavalli, R.K., Eds., Recent Advances in Robust Control, Selected Reprint Series, IEEE Press, New York, 1990.

7. Garbow, B.S., Boyle, J.M., Dongarra, J.J., and Moler, C.B., Matrix Eigensystem Routines—EISPACK Guide Extension, in Lecture Notes in Computer Science, Springer (vol. 51) New York, 1977.

8. Golub, G.H. and Van Loan, C.F., Matrix Computations (3rd edition), Johns Hopkins University Press, Baltimore, MD, 1996.

9. Higham, N.J., Accuracy and Stability of Numerical Algorithms (2nd edition), SIAM, Philadelphia, PA, 2002.

10. Higham, N.J., Functions of Matrices. Theory and Computation, SIAM, Philadelphia, PA, 2008.

11. Laub, A.J., Survey of computational methods in control theory, in Electric Power Problems: The Mathematical Challenge, Erisman, A.M., Neves, K.W., and Dwarakanath, M.H., Eds., SIAM, Philadelphia, PA, pp. 231–260, 1980.

12. Laub, A.J., Invariant subspace methods for the numerical solution of Riccati equations, in The Riccati

Equation, Bittanti, S., Laub, A.J., and Willems, J.C., Eds., Springer, Berlin, pp. 163–196, 1991.

13. Pandey, P. and Laub, A.J., Numerical issues in robust control design techniques, inControl andDynamic Systems—Advances in Theory and Applications: Digital and Numeric Techniques and Their Applications in Control Systems, vol. 55, Leondes, C.T., Ed., Academic, San Diego, CA, pp. 25–50, 1993. “73648_C001” #25

14. Patel, R.V., Laub, A.J., and Van Dooren, P.M., Eds., Numerical Linear Algebra Techniques for Systems and Control, Selected Reprint Series, IEEE Press, New York, 1994.

15. Smith, B.T, Boyle, J.M., Dongarra, J.J., Garbow, B.S., Ikebe, Y., Klema, V.C., and Moler, C.B., Matrix Eigensystem Routines—EISPACK Guide, in Lecture Notes in Computer Science, vol. 6, Springer, New York, 1976.

16. Van Dooren, P., Numerical aspects of system and control algorithms, Journal A, 30, pp. 25–32, 1989.

17. Wilkinson, J.H., The Algebraic Eigenvalue Problem, Oxford University Press, Oxford, England, 1965. “73648_C001” #26

2 2.4 Multivariable Transmission Zeros

1. Kailath, T., Linear Systems, Prentice Hall, Englewood Cliffs, NJ, 1980.

2. MacFarlane, A.G.J. and Karcanias, N., Poles and Zeros of Linear Multivariable Systems: A Survey of the Algebraic, Geometric, and Complex Variable Theory, Int. J. Control, 24, 33-74, 1976.

3. Rosenbrock, H.H., State-Space and Multivariable Theory, John Wiley & Sons, New York, 1973.

4. Sain, M.K. and Schrader, C.B., The Role of Zeros in the Performance of Multiinput, Multioutput Feedback Systems, IEEE Trans. Education, 33(3), 1990.

5. Schrader, C.B. and Sain, M.K., Research on System Zeros: A Survey, Int. J. Control, 50(4), 1989.

3 3.5 Defining Terms

1. Chen, C. T., Linear System Theory and Design, Holt, Rinehart, and Winston, New York, 1984.

2. Kamen, E.W., The poles and zeros of a linear time-varying system, Linear Algebra Appl., 98, 263–289, 1988. “73648_C003” #32

3. Kamen, E.W. andHafez, K.M., Algebraic theory of linear time-varying systems, SIAM J. Control Optim., 17, 500–510, 1979.

4. Kloet, P. van der and Neerhoff, F.L., The Cauchy–Floquet factorization by successive Riccati transformations, Proc. IEEE Int. Sym. Circuits Systems, Phoenix, AZ, 257–260, 2002.

5. Magnus, W., On the exponential solution of differential equations for a linear operator, Commun. Pure Appl. Math., VII, 649–673, 1954.

6. Marinescu, B. and Bourlès, H., An intrinsic algebraic setting for poles and zeros of linear time-varying systems, Systems Control Lett., 58, 248–253, 2009.

7. O’Brien, R.T. and Iglesias, P. A., On the poles and zeros of linear, time-varying systems, IEEE Trans. Circuits Systems-I: Fundam. Theory Appl., 48, 565–577, 2001.

8. Rugh, W.J., Linear System Theory, 2nd edn, Prentice-Hall, Englewood Cliffs, NJ, 1996.

9. Silverman, L.M. and Meadows, H.E., Controllability and observability in time-variable linear systems, SIAM J. Control Optim., 5, 64–73, 1967.

10. Silverman, L.M., Transformation of time-variable systems to canonical (Phase-variable) form, IEEE Trans. Automat. Control, AC-11, 300, 1966.

11. Valasek, M. and Olgac, N., An efficient pole placement technique for linear time-variant SISO systems, IEE Proc. Control Theory Appl. D, 451–458, 1995.

12. Valasek, M. and Olgac, N., Pole placement for linear time-varying non-lexicographically fixed MIMO systems, Automatica, 35, 101–108, 1999.

13. Zhu, J.J., Well-defined series and parallel d-spectra

for linear time-varying systems, Proc. Am. Control Conf., Baltimore, MD, 734–738,1994.

Further Reading

There are a large number of research papers and textbooks that contain results on the theory of linear

time-varying systems. Only a small portion of the existing work is mentioned here, with the emphasis on

fundamentals of linear time-varying systems include the following:

14. Zadeh, L.A. and Desoer, C.A., Linear System Theory, McGraw-Hill, New York, 1963.

15. Brockett, R.W., Finite Dimensional Linear Systems, Wiley & Sons, New York, 1970.

16. D'Angelo, H., Linear Time-Varying Systems, Allyn and Bacon, Boston, MA, 1970.

17. Kailath, T., Linear Systems, Prentice-Hall, Englewood Cliffs, NJ, 1980.

18. Sontag, E.D., Mathematical Control Theory, Springer-Verlag, New York, 1990.

19. Antsaklis, P.J. and Michel, A.N., Linear Systems, Birkhauser, Boston, MA, 2006.

For textbooks on H-infinity and H2 control of time-varying systems, see

20. Peters,M.A. and Iglesias, P.A.,MinimumEntropyControl for Time-Varying Systems, Birkhauser, Boston, MA, 1997.

21. Ichikawa, A. and Katayama, H., Linear Time-Varying and Sampled-Data Systems, Lecture Notes in Control and Information Science, 265, Springer-Verlag, London, 2001.

An approach to linear time-varying systems given in terms of matrix algebra and analytic function theory

is developed in the following textbook:

22. Dewilde, P. and Veen, A. van der, Time-Varying Systems and Computations, Kluwer, Boston, MA, 1998.

For textbooks on the adaptive control of time-varying systems and observers for time-varying systems,

see

23. Tsakalis, K.S. and Ioannou, P.A., Linear Time-Varying Plants: Control and Adaptation, Prentice-Hall, Englewood Cliffs, NJ, 1993.

24. O'Reilly, J., Observers for Linear Systems, Academic Press, New York, 1983.

Many textbooks exist on the stability of time-varying differential equations and systems. Examples are

25. Bellman, R., Stability of Differential Equations, McGraw-Hill, New York, 1953.

26. Hahn, W., Stability of Motion, Springer-Verlag, New York, 1967.

27. Harris, C.J. and Miles, J.F., Stability of Linear Systems, Academic Press, New York, 1980.

28. Lukes, D.L., Differential Equations: Classical to Controlled, Academic Press, New York, 1982.

29. Miller, R.K. and Michel, A.N., Ordinary Differential Equations, Academic Press, New York, 1982. "73648_C003" #33

30. Michel, A.N., Liu, D., and Hou, L., Stability of Dynamical Systems: Continuous, Discontinuous, and Discrete Systems, Birkhauser Verlag, Boston, MA, 2008.

For an in-depth treatment of the stability of second-order linear time-varying differential equations, see

31. Duc, L.H., Ilchmann, A., Siegmund, S., and Taraba, P., On stability of linear time-varying second-order differential equations, Quart. Appl. Math. 64, 137–151, 2006.

There are a number of papers on the study of time-varying systems given in terms of rings of differential

are as follows:

32. Kamen, E.W., Khargonekar, P.P., and Poolla, K.R., A transfer function approach to linear time-varying discrete-time systems, SIAM J. Control Optim., 23, 550–565,

1985.

33. Poolla, K. R. and Khargonekar, P.P., Stabilizability and stable proper factorizations for linear timevarying systems, SIAM J. Control Optim., 25, 723-736, 1987.

34. Fliess, M., Some basic structural properties of generalized linear systems, Systems Control Lett., 15, 391-396, 1990. “73648_C003” #34

1. A.C. Antoulas, Approximation of Large-Scale Dynamical Systems, SIAM, Philadelphia, 2005.

2. A. Astolfi, Model reduction by moment matching, Proc. 7th IFAC NOLCOS, Pretoria, South Africa, 95–102, August 2007.

3. J. Batt, Nonlinear compact mappings and their adjoints, Math. Ann., 189, 5–25, 1970.

4. C. L. Beck, J. Doyle, and K. Glover, Model reduction of multi-dimensional and uncertain systems, IEEE Trans. Automat. Control, AC-41, 10, 1466–1477, 1996.

5. P. Benner, V. Mehrmann, and D.C. Sorensen, Eds., Dimension Reduction of i Large-Scale Systems, Springer-Verlag, Berlin, 2005.

6. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol. 1, Interscience Publishers, New York, 1953.

7. U. B. Desai and D. Pal, A transformation approach to stochastic model reduction, IEEE Trans. Automat. Control, 29, 1097–1100, 1984.

8. K.V. Fernando and H. Nicholson, Singular pertubational model reduction of balanced systems, IEEE Trans. Automat. Control, AC-27, 466–468, 1982.

9. K. Fujimoto, What are singular values of nonlinear operators?, Proc. 43rd IEEE Conf. on Decision and Control, The Bahamas, 1623–1628, 2004.

10. K. Fujimoto, J. M. A. Scherpen, andW. S. Gray, Hamiltonian realizations of nonlinear adjoint operators, Automatica, 38, 10, 1769–1775, 2002.

11. K. Fujimoto and J.M.A. Scherpen, Nonlinear balanced realization based on singular value analysis of Hankel operators, Proc. 42nd IEEE Conf. on Decision and Control, Maui, HI, 6072–6077, 2003.

12. K. Fujimoto and J.M.A. Scherpen, Nonlinear input-normal realizations based on the differential eigenstructure of Hankel operators, IEEE Trans. Automat. Control, AC-50, 1, 2–18, 2005.

13. K. Fujimoto and J.M.A. Scherpen, Singular value

analysis and balanced realizations for nonlinear systems, in: Model Order Reduction: Theory, Research Aspects and Applications, Eds. W. Schilders, H. van der Vorst, and J. Rommes, Springer-Verlag, Berlin, 251–272, 2008.

14. K. Fujimoto and J.M.A. Scherpen, Model reduction for nonlinear systems based on the balanced realization, SIAM J. Control Optim., 48 (7), 4591–4623, 2010.

15. F.R. Gantmacher, The Theory of Matrices, Chelsea, New York, 1960.

16. W. Gawronski, Balanced Control of Flexible Structures, Lecture Notes in Contr. Inf. Sc. 211, SpringerVerlag, Berlin, 1996.

17. K. Glover, All optimal Hankel-norm approximations of linear multivariable systems and their L ∞ -error bounds, Int. J. Control, 39, 1115–1193, 1984.

18. W.S. Gray and J.M.A. Scherpen, State dependent matrices in quadratic forms, Systems Control Lett., 44, 3, 219–232, 2001.

19. S. Gugercin and A. C. Antoulas, A survey of model reduction by balanced truncation and some new results, Int. J. Control, 77 (8), 748–766, 2004.

20. J. Hahn and T.F. Edgar, An improved method for nonlinear model reduction using balancing of empirical Gramians, Comp. Chem. Eng., 26, 10, 1379–1397, 2002.

21. P. Heuberger, A family of reduced order models, based on open-loop balancing, Ident. Model. Contr., Delft University Press, 1, 1–10, 1990.

22. J.W.Hoffmann,Normalized coprime factorizations in continuous and discrete time—a joint state-space approach, IMA J. Math. Contr. Info. 13, 359–384, 1996.

23. T.C. Ionescu, K. Fujimoto and J.M.A. Scherpen, Dissipativity preserving balancing for nonlinear systems—a Hankel operator approach, Systems and Control Letters, 59, 180–194, 2010.

24. A. Isidori, Nonlinear Control Systems (3rd edition), Springer-Verlag, Berlin, 1995.

25. E.A. Jonckheere and L.M. Silverman, Singular value analysis of deformable systems, J. Circ., Syst. Sign.

Proc., 1, 447–470, 1982.

26. E.A. Jonckheere and L.M. Silverman, A new set of invariants for linear systems—Applications to reduced order compensator design, IEEE Trans. Automat. Control, AC-28, 953–964, 1983.

27. T. Kailath, Linear Systems, Prentice-Hall, Englewood Cliffs, NJ, 1980. “73648_C004” — #23

28. S. Lall, J.E. Marsden, and S. Glavaski, A subspace approach to balanced truncation for model reduction of nonlinear control systems, Int. J. Robust Nonlinear Control, 12, 6, 519–535, 2002.

29. D.G. Meyer, Fractional balanced reduction—model reduction via a fractional representation, IEEE Trans. Automat. Control, AC-26, 1341–1345, 1990.

30. D.G. Meyer and S. Srinivasan, Balancing and model reduction for second order form linear systems, IEEE Trans. Automat. Control, AC-41, 11, 1632–1644, 1996.

31. J. W. Milnor, Topology from the Differential Viewpoint, Princeton University Press, Princeton, NJ, 1965.

32. B.C. Moore, Principal component analysis in linear systems: Controllability, observability and model reduction, IEEE Trans. Automat. Control, AC-26, 17–32, 1981.

33. D. Mustafa, H ∞ -characteristic values, Proc. 28th IEEE Conf. on Decision and Control, Tampa, FL, 1483–1487, 1989.

34. D.Mustafa and K. Glover,Minimum Entropy H ∞ Control, Lect. Notes Contr. Inf. Sci.No. 146, SpringerVerlag, Berlin, 1990.

35. D. Mustafa and K. Glover, Controller reduction by H ∞ -balanced truncation, IEEE Trans. Automat. Control, AC-36, 668–682, 1991.

36. C.N. Nett, C.A. Jacobson, and M.J. Balas, A connection between state-space and doubly coprime fractional representations, IEEE Trans. Autom. Control, AC-29, 831–832, 1984.

37. A.J. Newman and P.S. Krishnaprasad, Computation for nonlinear balancing, Proc. 37th IEEE Conf. on Decision and Control, Tampa, FL, 4103–4104, 1998.

38. A. Newman and P. S. Krishnaprasad, Computing balanced realizations for nonlinear systems, Proc. Symp. Mathematical Theory of Networks and Systems, 2000.

39. R. Ober andD.McFarlane, Balanced canonical forms forminimal systems: A normalized coprime factor approach, Linear Algebra and its Applications, 122–124, 23–64, 1989.

40. G. Obinata and B.D.O. Anderson,Model Reduction for Control SystemDesign, Springer-Verlag, London, 2001.

41. P.C. Opdenacker and E.A. Jonckheere, LQG balancing and reduced LQG compensation of symmetric passive systems, Int. J. Control, 41, 73–109, 1985.

42. P. C. Opdenacker and E. A. Jonckheere, A contraction mapping preserving balanced reduction scheme and its infinity norm error bounds, IEEE Trans. Circuits Systems 35 (2), 184–189, 1988.

43. L. Pernebo and L.M. Silverman, Model reduction via balanced state space representations, IEEE Trans. Automat. Control, AC-27, 382–387, 1982.

44. H. Sandberg and A. Rantzer, Balanced truncation of linear time-varying systems, IEEE Trans. Automat. Control, 49, 2, 217–229, 2004.

45. J.M.A. Scherpen, Balancing for nonlinear systems, Systems Control Lett., 21, 143–153, 1993.

46. J.M.A. Scherpen, H ∞ balancing for nonlinear systems, Int. J. Robust Nonlinear Control, 6, 645–668, 1996.

47. J.M.A. Scherpen and W.S. Gray, Minimality and local state decompositions of a nonlinear state space realization using energy functions, IEEE Trans. Automat. Control, AC-45, 11, 2079–2086, 2000.

48. J.M.A. Scherpen and W.S. Gray, Nonlinear Hilbert adjoints: Properties and applications to Hankel singular value analysis, Nonlinear Anal., Theory, Methods Appl., 51, 5, 883–901, 2002.

49. J.M.A. Scherpen and A.J. van der Schaft, Normalized coprime factorizations and balancing for unstable nonlinear systems, Int. J. Control, 60, 6, 1193–1222, 1994.

50. W.H.A. Schilder, H.A. van der Vorst, and J. Rommes,

Eds., Model Order Reduction: Theory, Research Aspects and Applications, Mathematics in Industry, Vol. 13, Springer, Berlin, 2008.

51. G.W. Stewart, On the early history of the singular value decomposition, SIAMRev., 35, 4, 551–566, 1993.

52. H.L. Trentelman, Bounded real and positivey real balanced truncation using Σ normalized coprime factor, System Control Lett., 58, 12, 871–879, 2009.

53. H.L. Trentelman and P. Rapisarda, A behavioral approach to passivity and bounded realness preserving balanced truncation with error bounds, Proc. 47th IEEE Conf. on Decision and Control, Shanghai, China, 2009.

54. A.J. van der Schaft, Balancing of lossless and passive systems, IEEE Trans. Automat. Control, AC-53, 2153–2157, 2008.

55. A.J. van der Schaft and J.E. Oeloff, Model reduction of linear conservative mechanical systems, IEEE Trans. Automat. Control, AC-35, 729–733, 1990.

56. A.J. van der Schaft and R. Polyuga, Structure-preserving model reduction of complex physical systems, Proc. 48th IEEE Conf. on Decision and Control, Shanghai, China, December 16–18, 2009.

57. E.Verriest, Low sensitivity design andoptimal order reduction for the LQG-problem,Proc. 24thMidwest Symp. Circ. Syst. Albuquerque, NM, 1981, 365–369.

58. E.I. Verriest and W.S. Gray, Flow balancing nonlinear systems, Proc. 2000 Int. Symp. Math. Th. Netw. Syst. (MTNS), 2000. “73648_C004” — #24

59. E.I. Verriest and W.S. Gray, Balanced nonlinear realizations, Proc. 43rd IEEE Conf. on Decision and Control, The Bahamas, 1164–1169, 2004.

60. M. Vidyasagar, Control System Synthesis: A Factorization Approach, MIT Press, Cambridge, 1985.

61. S. Weiland, Theory of Approximation and Disturbance Attenuation for Linear Systems, Doctoral dissertation, University of Groningen, 1991.

62. P. Wortelboer, Frequency-Weighted Balanced Reduction of Closed Loop Mechanical Servo Systems: Theory and Tools,

doctoral dissertation, Delft University of Technology, 1994.

63. K. Zhou, J. C. Doyle, and K. Glover, Robust and Optimal Control, Prentice-Hall, Inc., Upper Saddle River, NJ, 1996.

64. A. J. Krener, Reduced order modeling of nonlinear control systems, in Analysis and Design of Nonlinear Control Systems, A. Astolfi and L. Marconi, eds., Springer-Verlag, 41–62, 2008.

Akashi, H. and Imai, H. 1979. Disturbance localization and output deadbeat control through an observer in discrete-time linear multivariable systems. IEEE Trans. Automat. Control, AC-24: 621-627.

Barbagli, F., Marro, G.,Mercorelli, P., and Prattichizzo, D. 1998. Some results on output algebraic feedback with applications to mechanical systems. Proc. 37 th IEEE Conf. Decision Control (CDC 1998), Tampa, Florida, USA, 3545-3550.

Basile, G. andMarro, G. 1969. Controlled and conditioned invariant subspaces in linear system theory. J. Optim. Theory Appl., 3(5): 305-315.

Basile, G. andMarro,G. 1982. Self-bounded controlled invariant subspaces. J. Optim. TheoryAppl., 38(1):71-81.

Basile, G. and Marro, G. 1992. Controlled and Conditioned Invariants in Linear System Theory, Prentice-Hall, Englewood Cliffs, NJ.

Basile, G., Marro, G., and Piazzi, A. 1986. Stability without eigenspaces in the geometric approach: some new results. In Frequency Domain and State Space Methods for Linear Systems, C. A. Byrnes and A. Lindquist (Eds). North-Holland (Elsevier), Amsterdam, pp. 441-450.

Conte, G. and Perdon, A. M. 1988. A geometric approach to the theory of 2-D systems. IEEE Trans. Automat. Control, 33: 946-950.

Conte, G. and Perdon, A.M. 1998. The block decoupling problem for systems over a ring. IEEE Trans. Automat. Control, AC-43: 1600-1604.

Hamano, F. and Basile, G. 1983. Unknown-input present-state observability of discrete-time linear systems. J. Optim. Theory Appl., 40(2): 293-307.

Hamano, F. and Furuta, K. 1975. Localization of disturbances and output decomposition of linearmultivariable systems. Int. J. Control, 22(4): 551-562.

Heymann, M. 1968. Comments on Pole assignment in multi-input controllable linear systems. IEEE Trans. Automat. Control, AC-13: 748-749.

Isidori, A. 1995. Nonlinear Control Systems (3rd edition), Springer-Verlag, New York.

Kalman, R. E., Falb, P. L., and Arbib, M. A. 1969. Topics in Mathematical System Theory, McGraw-Hill, New York. “73648_C005” — #26

Longhi, S. and Monteriù, A. 2007. A geometric approach to fault detection of periodic systems. Proc. 46th IEEE Conf. Decision Control, New Orleans, USA.

Marro, G. 2008. The geometric approach to control: a light presentation of theory and applications. In Control Science Evolution, S. Bittanti (Ed.), CNR Publications, Rome, pp. 157–204.

Marro, G. and Zattoni, E. 2004. Detection of incipient failures by using geometric approach techniques: An application to railway switching points. Proceedings of the 2nd IFAC Symposium on System, Structure and Control, Oaxaca, Mexico. Marro, G. and Zattoni, E. 2006. Signal decoupling with preview in the geometric context: Exact solution for nonminimum-phase systems. J. Optim. Theory Appl., 129(1): 165–183.

Massoumnia, M. A., Verghese, G. C., and Willski, A. S. 1989. Failure detection and identification. IEEE Trans. Automat. Control, 34: 316–321.

Mattone, R. and De Luca, A. 2006. Nonlinear fault detection and isolation in a three-tank heating system. IEEE Trans. Control Systems Tech.,14: 1158–1166.

Morse, A. S. 1973. Structural invariants of linearmultivariable systems. SIAM J. Control Optim., 11(3): 446–465.

Nijmeijer, H. and van der Schaft, A. J. 1990. Nonlinear Dynamical Control Systems, Springer-Verlag, New York.

Prattichizzott, D., Mercorellit, P., Bicchit, A., and Vicinol, A. 1997. On the geometric control of internal forces in power grasps. Proceedings of the 36th Conf Decision Control, San Diego, USA, pp. 1942–1947.

Schumacher, J.M. 1983. On a conjecture of Basile and Marro. J. Optim. Theory Appl., 41(2): 371–376.

Takamatsu, T., Hashimoto, I., andNakai, Y. 1979. A

geometric approach tomultivariable control system design of a distillation column. Automatica, 15: 387–402.

Trentelman, H. L., Stoorvogel, A. A., and Hautus, M. 2002. Control Theory for Linear Systems. Springer-Verlag, London.

Willems, J. C. 1981. Almost invariant subspaces: An approach to high gain feedback design—Part I: Almost controlled invariant subspaces. IEEE Trans. Automat. Control AC-26: 235–252.

Willems, J. C. 1982. Almost invariant subspaces: An approach to high gain feedback design—Part II: Almost conditionally invariant subspaces. IEEE Trans. Automat. Control AC-27: 1071–1085.

Willems, J. C. and Commault, C. 1981. Disturbance decoupling by measurement feedback with stability or pole placement. SIAM J. Control Optim., 19(4): 490–504.

Wonham, W. M. 1985. Linear Multivariable Control, A Geometric Approach (3rd edition), Springer-Verlag, New York.

Wonham, W. M. and Morse, A. S. 1970. Decoupling and pole assignment in linear multivariable systems: A geometric approach. SIAM J. Control Optim., 8(1): 1–18.

6 Polynomial and Matrix Fraction Descriptions

1. Boyd, S. P. and Barratt, C., Linear Controller Design: Limits of Performance, Prentice Hall, Englewood Cliffs, NJ, 1991.

2. Delchamps, D. F., State Space and Input-Output Linear Systems, Springer, New York, 1988.

3. Doyle, J. C., Francis, B.A., and Tannenbaum, A., Feedback Control Theory, MacMillan, New York, 1992.

4. Forney, G. D., Minimal bases of rational vector spaces with applications to multivariable linear systems, SIAM J. Control, 13, 493–520, 1975.

5. Francis, B., A Course in H ∞ Control Theory, Volume 88 In Lecture Notes in Control and Information Sciences, Springer, 1987.

6. Jacobson, N., Basic Algebra I, W. H. Freeman, San Francisco, 1974.

7. Kailath, T., Linear Systems, Prentice Hall, Englewood Cliffs, NJ, 1980.

8. Kalman, R. E., Irreducible realizations and the degree of a rational matrix, J. SIAM, 13, 520–544, 1965.

9. MacMillan, B., An introduction to formal realizability theory parts I and II, Bell Syst. Tech. J., 31, 217–279, 591-600, 1952.

10. Rugh, W. J., Linear System Theory, Prentice Hall, Englewood Cliffs, NJ, 1993.

11. Sontag, E., Mathematical Control Theory, Springer, New York, 1990.

12. Strang, G., Linear Algebra and Its Applications, Academic, New York, 1976.

13. Vidyasagar, M., Control System Synthesis: A Factorization Approach, MIT, Cambridge, MA, 1985.

14. Vidyasagar, M., On the use of right-coprime factorizations in distributed feedback systems containing unstable subsystems, IEEE Trans. Circuits Syst. CAS-25, 916–921, 1978. “73648_C006” #22

7 Robustness Analysis with Real Parametric Uncertainty

Abate, M., Barmish, B.R., Murillo-Sanchez, C., and Tempo, R. 1994. Application of some new tools to robust stability analysis of spark ignition engines: A case study. IEEE Transactions on Control Systems Technology, CST-2:22–30.

Ackermann, J.E. 1980. Parameter space design of robust control systems. IEEE Transactions on Automatic Control, AC-25:1058–1072.

Ackermann, J. in cooperation with Bartlett, A., Kaesbauer, D., Sienel, W., and Steinhauser, R. 1993. Robust Control, Systems with Uncertain Physical Parameters, Springer-Verlag, London.

Ackermann, J., Curtain, R. F., Dorato, P., Francis, B. A., Kimura, H., and Kwakernaak, H. (Editors). 1993. Special Issue on robust control. Automatíca, 29:1–252.

Barmish, B.R. 1994. New Tools for Robustness of Linear Systems, Macmillan, New York.

Barmish, B.R., Hollot, C.V., Kraus, F., and Tempo, R. 1992. Extreme point results for robust stabilization of interval plants with first-order compensators. IEEE Transactions on Automatic Control, AC-37:707–714.

Barmish, B.R. and Shi, Z. 1990. Robust stability of perturbed systems with time delays.Automatica, 25:371–381.

Barmish, B.R. and Tempo, R. 1990. The robust root locus. Automatica, 26:283–292.

Barmish, B.R. and Tempo, R. 1991. On the spectral set for a family of polynomials. IEEE Transactions on Automatic Control, AC-36:111–115.

Bartlett, A.C., Hollot, C.V., and Huang, L. 1988. Root locations of an entire polytope of polynomials: It suffices to check the edges. Mathematics of Control, Signals and Systems, 1:61–71.

Bhattacharyya, S.P., Chapellat,H., andKeel, L.H. 1995.Robust Control: The ParametricApproach, Prentice-Hall, Englewood Cliffs, NJ.

Blanchini, F. and Miani, S. 2008. Set-Theoretic Methods in Control, Birkhäuser, Boston.

Chockalingam, G. and Dasgupta, S. 1993. Minimality, stabilizability and strong stabilizability of uncertain plants. IEEE Transactions on Automatic Control. AC-38:1651–1661.

Dasgupta, S. 1988. Kharitonov's theorem revisited. Systems and Control Letters. 11:381–384.

deGaston,R.R.E. andSafonov,M.G. 1988. Exact calculationof themultiloop stabilitymargin. IEEETransactions on Automatic Control, AC-33:156–171.

Djaferis, T.E. 1995. Robust Control Design: A Polynomial Approach, Kluwer Academic Publishers, Boston.

Doyle, J.C., Francis, B.A., and Tannenbaum, A.R. 1992. Feedback Control Theory, Macmillan, New York.

Frazer, R.A. and Duncan, W.J. 1929. On the criteria for the stability of small motion. Proceedings of the Royal Society, A, 124:642–654.

Fu, M. 1990. Computing the frequency response of linear systems with parametric perturbations. Systems and Control Letters, 15:45–52.

Ho, M.-T., Datta, A., and Bhattacharyya, S.P. 2000, Structure and Synthesis of PID Controllers, Springer-Verlag, London.

Horowitz, I.M. 1963. Synthesis of Feedback Systems, Academic Press, New York.

Horowitz, I. 1991. Survey of quantitative feedback theory (QFT). International Journal of Control, 53:255–291.

Kharitonov,V.L. 1978.Asymptotic stability of an equilibriumposition of a family of systems of linear differential equations. Differential'nye Uraveniya, 14:1483–1485.

Kogan, J. 1995. Robust Stability and Convexity. Lecture Notes in Control and Information Sciences, SpringerVerlag, London.

Marden, M. 1966. Geometry of Polynomials, Mathematical Surveys, No. 3, American Mathematical Society, Providence, RI.

Mikhailov, A.W. 1938. Methods of harmonic analysis in control theory. Avtomatika i Telemekhanika, 3:27–81.
“73648_C007” #18

Minnichelli, R.J., Anagnost, J.J., and Desoer, C.A. 1989. An elementary proof of Kharitonov’s theorem with extensions. IEEE Transactions on Automatic Control. AC-34:995–998.

Rantzer, A. 1992. Stability conditions for polytopes of polynomials, IEEE Transactions on Automatic Control, AC-37:79–89.

Sanchez Pena, R. and Sznaier, M. 1998. Robust Systems, Theory and Applications, John Wiley and Sons, New York.

Sideris, A. and Sanchez Pena, R.S. 1989. Fast calculation of themultivariable stabilitymargin for real interrelated uncertain parameters. IEEE Transactions on Automatic Control, AC-34:1272–1276.

Tempo, R., Calafiore, G., and Dabbene, F. 2005. Randomized Algorithms for Analysis and Control of Uncertain Systems, Springer-Verlag, London.

Vicino, A., Tesi, A., and Milanese, M. 1990. Computation of nonconservative stability perturbation bounds for systems with nonlinearly correlated uncertainties. IEEE Transactions on Automatic Control, AC35:835–841.

Zadeh, L.A. and Desoer, C.A., 1963. Linear System Theory, McGraw-Hill, New York.

Zhou, K., Doyle, J.C., and Glover, K. 1996. Robust and Optimal Control, Prentice-Hall, Upper Saddle River, NJ.

8 MIMO Frequency Response Analysis and the Singular Value Decomposition

1. Athans, M., Lecture Notes for Multivariable Control Systems I and II, Massachusetts Institute of Technology, 1994. (This reference may not be generally available.)

2. Freudenberg, J.S. and Looze, D.P., Frequency Domain Properties of Scalar and Multivariable Feedback Systems, Springer, Berlin, 1987.

3. Kailath, T., Linear Systems, Prentice Hall, Englewood Cliffs, NJ, 1980.

4. Maciejowski, J.M., Multivariable Feedback Design, Addison-Wesley, Wokingham, 1989.

5. Strang, G., Linear Algebra and Its Applications, Harcourt Brace Jovanovich, San Diego, 1988.

10 10.4 Constraints Imposed by Stability

1. A.P. Aguiar, J.P. Hespanha, and P.V. Kokotovic. Path-following for non-minimum phase systems removes performance limitations. IEEE Transactions on Automatic Control, 50(2):234–239, 2005.

2. P. Antsaklis and J. Baillieul (Eds). Special issue on networked control systems. IEEE Transactions on Automatic Control, 49(9), 2004.

3. K. J. ÅströmandR.M.Murray. Feedback Systems: An Introduction for Scientists andEngineers. Princeton University Press, Princeton, NJ, 2008.

4. K. J. Åström, R. E. Klein, and A. Lennartsson. Bicycle dynamics and control. IEEE Control Systems Magazine, 25(4):26–47, 2005.

5. A. Banaszuk, P. G. Mehta, C. A. Jacobson, and A. I. Khibnik. Limits of achievable performance of controlled combustion processes. IEEE Transactions on Control Systems Technology, 14(5):881–895, 2006.

6. O. Beker, C. V. Hollot, and Y. Chait. Plant with integrator: An example of reset control overcoming limitations of linear feedback. IEEE Transactions on Automatic Control, 46(11):1797–1799, 2001.

7. H. W. Bode. Network Analysis and Feedback Amplifier Design. D. van Nostrand, New York, 1945.

8. S. Boyd and C. Desoer. Subharmonic functions and performance bounds on linear time invariant feedback systems. IMA Journal of Math. Control and Information, 2(2): 153–170, 1995.

9. J. H. Braslavsky, M.M. Seron, D.Q.Mayne, and P.V. Kokotovic´. Limiting performance of optimal linear filters. Automatica, 35(2):189–199, 1999.

10. J. H. Braslavsky, R. H. Middleton, and J. S. Freudenberg. Cheap control performance of a class of non-right-invertible nonlinear systems. IEEE Transactions on Automatic Control, 47(8):1314–1319, 2002.

11. J. H. Braslavsky, R. H. Middleton, and J. S. Freudenberg. Feedback stabilization over signal to noise ratio constrained channels. IEEE Transactions on Automatic Control, 52(8):1391–1403, 2007.

12. T. S. Brinsmead and G. C. Goodwin. Cheap decoupled control. Automatica, 37(9):1465–1471, 2001.

13. F. A. Chandra, G. Buzi, and J. C. Doyle. Linear control analysis of the autocatalytic glycolysis system. In Proceedings of the 2009 American Control Conference, St. Louis, MI, 319–324, June 2009.

14. D. Chen and B. Paden. Stable inversion of nonlinear nonminimumphase systems. International Journal of Control, 64(1):81–97, 1996.

15. J. Chen. Sensitivity integral relations and design tradeoffs in linearmultivariable feedback systems. IEEE Transactions on Automatic Control, 40(10):1700–1716, 1995.

16. J. Chen and R.H.Middleton (Eds). Special issue on ‘new developments and applications in performance limitation of feedback control’. IEEE Transactions on Automatic Control, 48(8):1297–1393, 2003.

17. J. Chen, L. Qiu, and O. Toker. Limitations on maximal tracking accuracy. IEEE Transactions on Automatic Control, 45(2):326–331, 2000.

18. J. Chen, R. Zhang, S. Hara, and L. Qiu. Optimal tracking performance: Preview control and exponential signals. IEEE Transactions on Automatic Control, 46(10):1647–1653, 2001.

19. J. C. Clegg. A nonlinear integrator for servomechanisms. Transactions of the AIEE (Part II), 77:41–42, 1958.

20. J. B. Cruz and W. R. Perkins. A new approach to the sensitivity problem in multivariable feedback design. IEEE Transactions on Automatic Control, 9:216–223, 1964.

21. R. C. Dorf. Modern Control Systems. Addison-Wesley, Reading, MA, 1991.

22. J. C. Doyle and G. Stein. Multivariable feedback design: Concepts for a classical/modern synthesis. IEEE Transactions on Automatic Control, 26(1):4–16, 1981.

23. J. C. Doyle, B. A. Francis, and A. R. Tannenbaum. Feedback Control Theory. Macmillan Publishing Company, New York, 1992.

24. A. Feuer and G.C. Goodwin. Sampling in Digital Signal Processing and Control. Birkhauser, Boston, MA, 1996.

25. A. Feuer, G. C. Goodwin, and M. Salgado. Potential benefits of hybrid control for linear time invariant plants. In Proceedings of the American Control Conference, Albuquerque, NM, 5:2790–2794, 1997. “73648_C010” — #35

26. B.A. Francis and G. Zames. On H ∞ -optimal sensitivity theory for SISO feedback systems. IEEE Transactions on Automatic Control, 29(1):9–16, 1984.

27. G.F. Franklin, J.D. Powell, and A. Emami-Naeini. Feedback Control of Dynamic Systems. Prentice-Hall, (4th ed.) NJ, Englewood Cliffs, 2002.

28. J. S. Freudenberg and D. P. Looze. Frequency Domain Properties of Scalar and Multivariable Feedback Systems, Vol. 104, Lecture Notes in Control and Information Sciences. Springer-Verlag, Berlin, 1988.

29. J. S. Freudenberg, R. H. Middleton, and J. H. Braslavsky. Robustness of zero-shifting via generalized sampled-data hold functions. IEEE Transactions on Automatic Control, 42(12):1681–1692, 1997.

30. J. S. Freudenberg, C. V. Hollot, R. H. Middleton, and V. Toochinda. Fundamental design limitations of the general control configuration. IEEE Transactions on Automatic Control, 48(8):1355–1370, 2003.

31. J.S. Freudenberg and D.P. Looze. Right half plane poles and zeros and design tradeoffs in feedback systems. IEEE Transactions on Automatic Control, 30(6):555–565, 1985.

32. T.Glad andL. Ljung.ControlTheory,Multivariable andNonlinearMethods. Taylor&Francis,NewYork, 2000.

33. G. C. Goodwin, A. R. Woodyatt, R. H. Middleton, and J. Shim. Fundamental limitations due to jω-axis zeros in SISO systems. Automatica, 35(5):857–863, 1999.

34. G. C. Goodwin, S. F. Graebe, and M. E. Salgado. Control System Design. Prentice-Hall, Englewood Cliffs, NJ, 2001.

35. P. G. Griffiths, R. B. Gillespie, and J. S. Freudenberg. A fundamental tradeoff between performance and sensitivity within haptic rendering. IEEE Transactions on Robotics, 24(3):537–548, 2008.

36. I. M. Horowitz. Synthesis of Feedback Systems. Academic Press, New York, 1963.

37. I. M. Horowitz and Y.-K. Liao. Limitations of non-minimum-phase feedback systems. International Journal of Control, 40(5):1003–1013, 1984.

38. P. A. Iglesias. Tradeoffs in linear time-varying systems: An analogue of Bode's sensitivity integral. Automatica, 37(10):1541–1550, 2001.

39. L. B. Jemaa and E. J. Davison. Limiting performance of optimal linear discrete filters. Automatica, 39(7): 1221–1226, 2003.

40. K.H. Johansson. Interaction bounds in multivariable control systems. Automatica, 38(6):1045–1051, 2002.

41. S. Liu andA.G. Stefanopoulou. Effects of control structure onperformance for an automotive powertrain with a continuously variable transmission. IEEE Transactions on Control Systems Technology, 10(5): 701–708, 2002.

42. N. C. Martins and M. A. Dahleh. Feedback control in the presence of noisy channels: "Bode-like" fundamental limitations of performance. IEEE Transactions on Automatic Control, 53(7):1604–1615, 2008.

43. R. H. Middleton. Tradeoffs in linear control system design. Automatica, 27(2):281–292, 1991.

44. R. H. Middleton and J. H. Braslavsky. On the relationship between logarithmic sensitivity integrals and limiting optimal control problems. In Proceedings of the IEEE Conference on Decision and Control, Sydney, Australia, 5:4990–4995, 2000.

45. R.H. Middleton and G.C. Goodwin. Digital Control and Estimation. A Unified Approach. Prentice-Hall, Inc., Englewood Cliffs, NJ, 1990.

46. R.H. Middleton and S.F. Graebe. Slow stable open loop poles: To cancel or not to cancel. Automatica, 35(5):877–886, 1999.

47. R.H. Middleton, J. Chen, and J.S. Freudenberg. Tracking sensitivity and achievable hinfinity performance in preview control. Automatica, 40(8):1297–1306, 2004.

48. D.E. Miller and R.H. Middleton. On limitations to the

achievable path following performance for linear multivariable plants. IEEE Transactions on Automatic Control, 53(11):2586–2601, 2008.

49. D. A. Mindell. Between Human and Machine: Feedback, Control, and Computing before Cybernetics. Princeton University Press, Princeton, NJ, 2002.

50. G.N. Nair and R.J. Evans. Exponential stabilisability of finite-dimensional linear systems with limited data rates. Automatica, 39(4):585–593, 2003.

51. T. Perez. Ship Motion Control: Course Keeping and Roll Stabilisation Using Rudder and Fins. SpringerVerlag, London, UK, 2005.

52. L.Qui andE.J. Davison. Performance limitations of nonminimumphase systems in the servomechanism problem. Automatica, 29(2):337–349, 1993.

53. H. H. Rosenbrock. Computer-Aided Control System Design. Academic Press, London, 1974.

54. R. S. Sánchez Peña, M. A. Cugueró, A. Masip, J. Quevedo, and V. Puig. Robust identification and feedback design: An active noise control case study. Control Engineering Practice, 16(11):1265–1274, 2008. “73648_C010” — #36

55. P. Seiler, A. Pant, and K. Hedrick. Disturbance propagation in vehicle strings. IEEE Transactions on Automatic Control, 49(10):1835–1841, 2004.

56. M. M. Seron, J. H. Braslavsky, and G. C. Goodwin. Fundamental Limitations in Filtering and Control. Springer, Berlin, 1997.

57. M. M. Seron, J. H. Braslavsky, P.V. Kokotovic´, and D.Q. Mayne. Feedback limitations in nonlinear systems: From Bode integrals to cheap control. IEEE Transactions on Automatic Control, 44(4):829– 833, 1999.

58. M.M. Seron and G.C. Goodwin. Sensitivity limitations in nonlinear feedback control. Systems and Control Letters, 27(4):249–254, 1996.

59. J.S. Shamma. Performance limitations in sensitivity reduction for nonlinear plants. Systems and Control Letters, 17(1):43–47, 1991.

60. C. E. Shannon. A mathematical theory of communication. Bell Systems Technical Journal, 27:379–423, 623–656, 1948.

61. S. Skogestad and I. Postlethwaite. Multivariable Feedback Control: Analysis and Design. Wiley, New York, 1996.

62. G. Stein. Respect the unstable. Control Systems Magazine, 23(4):12–25, 2003.

63. K.-W. Suh and A. G. Stefanopoulou. Performance limitations of air flow control in power-autonomous fuel cell systems. IEEE Transactions on Control Systems Technology, 15(3):465–473, 2007.

64. H.-K. Sung and S.Hara. Properties of sensitivity and complementary sensitivity functions in single-input single-output digital control systems. International Journal of Control, 48(6):2429–2439, 1988.

65. H.-K. Sung and S. Hara. Properties of complementary sensitivity function in SISO digital control systems. International Journal of Control, 50(4):1283–1295, 1989.

66. S. Tatikonda and S. M. Mitter. Control under communication constraints. IEEE Transactions on Automatic Control, 49(7):1056–1068, 2004.

67. N. M. Thibeault and R. S. Smith. Magnetic bearing measurement configurations and associated robustness and performance limitations. ASME Journal of Dynamic Systems, Measurement, and Control, 124 (4):589–597, 2002.

68. J.G. Truxal. Automatic Feedback Control System Synthesis. McGraw-Hill, New York, 1955.

69. G. Zang and P.A. Iglesias. Nonlinear extension of bode's integral based on an information-theoretic interpretation. Systems and Control Letters, 50(1):11–19, 2003.

70. K. Zhou, J. C. Doyle, and K. Glover. Robust and Optimal Control. Prentice-Hall, Englewood Cliffs, NJ, 1996.

Balas, G. J., A. K. Packard, and P. J. Seiler. Uncertain model set calculation from frequency domain data. In P. M. J. Hof, C. Scherer, and P. S. C. Heuberger (Eds). Model-Based Control. Springer, Berlin, 2009.

Boulet, B. and B. Francis. Consistency of open-loop experimental frequency-response data with coprime factor plant models. IEEE Transactions on Automatic Control, 43, 1680–1691, 1998.

Cruz, J., J. S. Freudenberg, and D. P. Looze. A relationship between sensitivity and stability of multivariable feedback systems. IEEE Transactions Automatic Control, 26, 66–74, 1981.

Doyle, J. C. Analysis of feedback systems with structured uncertainties. Proceedings of IEE Part D, 129, 242–250, 1982.

Doyle, J. C. Structured uncertainty in control system design. In 24th IEEE Conference on Decision and Control, 260–265, 1985.

Doyle, J. C., B. A. Francis, and A. R. Tannenbaum. Feedback Control Theory. Macmillan, 1992. Republished by Dover, New York, 2008.

Doyle, J. C., and G. Stein. Multivariable feedback design: Concepts for a classical/modern synthesis. IEEE Transactions Automatic Control, 26, 4–16, 1981.

Glad, T. and L. Ljung. Control Theory: Multivariable and Nonlinear Methods. Taylor & Francis, New York, 2000.

Hindi, H., C.-Y. Seong, and S. Boyd. Computing optimal uncertainty models from frequency domain data. In 41st IEEE Conference on Decision and Control, 2898–2905, 2002.

Special issue on system identification for robust control design. IEEE Transactions on Automatic Control, Vol. 37, July 1992.

Lunze, J. Robustness tests for feedback control systems using multidimensional uncertainty bounds. System and Control Letters, 4, 85–89, 1984.

Lunze, J. Robust Multivariable Control. Prentice-Hall, Englewood Cliffs, NJ, 1989.

Maciejowski, J. M. Multivariable Feedback Design. Addison-Wesley, Reading, MA, 1989.

McFarlane, D. and K. Glover. Robust Controller Design using Normalized Coprime Factor Plant Descriptions, Vol. 138 Lecture Notes in Control and Information Sciences. Springer-Verlag, Berlin, 1990.

Owens, D. H. and A. Chotai. On eigenvalues, eigenvectors, and singular values in robust stability analysis. International Journal Control, 40, 285–296, 1984.

Raisch, J. Mehrgrößenregelung im Frequenzbereich. R. Oldenbourg-Verlag, München/Wien, 1994.

Skogestad, S. and I. Postlethwaite. Multivariable Feedback Control: Analysis and Design (2nd edition), Wiley, New York, 2005.

Vidyasagar, M. Control System Synthesis. MIT Press, Cambridge, Mass, 1985.

Vidyasagar, M. Normalized comprime factorizations for non-strictly proper systems. IEEE Trans. Automatic Control, 33, 300–301, 1988.

Volker, M. and S. Engell. Computation of tight uncertainty bounds from time-domain data with application to a reactive distillation column. In 41st IEEE Conference on Decision and Control, 2939–2944, 2002. “73648_S002” #1 II

KalmanFilter

andObservers II-1 “73648_S002” #2

13 Kalman Filtering

1. Wiener, N., Extrapolation, Interpolation and Smoothing of Stationary Time Series, MIT Press and John Wiley & Sons, New York, 1950 (reprinted from a publication restricted for security reasons in 1942).

2. Kalman, R.E., A new approach to linear filtering and prediction problems, ASME J. Basic Eng., Ser. D, 82, 34-45, 1960. “73648_C013” — #11

3. Kalman, R.E., Newmethods and results in linear prediction and filtering theory, Proc. Symp. Engineering Applications of Random Function Theory and Probability, John Wiley & Sons, New York, 1961.

4. Kalman, R.E. and Bucy, R.S., New results in linear filtering and prediction theory, ASME J. Basic Eng., Ser. D, 83, 95-108, 1961.

5. Kalman, R.E., New methods in Wiener filtering, Proc. First Symp. Engineering Applications of Random Function Theory and Probability, John Wiley & Sons, New York, 1963, chap. 9.

6. Van Trees, H.L., Detection Estimation, and Modulation Theory, J. Wiley & Sons, New York, 1968.

7. Kwakernaak, H. and Sivan, R., Linear Optimal Control Systems, John Wiley & Sons, New York, 1972.

8. Doyle, J.C. and Stein, G., Multivariable feedback design: concepts for a classical/modern synthesis, IEEE Trans. Autom. Control, 1981.

9. Kwakernaak, H., Optimal low sensitivity linear feedback systems, Automatica, 5, 279-286, 1969.

Further Reading

There is a vast literature on Kalman filtering and its applications. A reasonable starting point is the

recent textbook Kalman Filtering: Theory and Practice by M.S. Grewal and A.P. Andrews, Prentice Hall,

Englewood Cliffs, NJ, 1993.

primarily in applications for some time.

1. Gelb, A., Kasper, J.F., Jr., Nash, R.A., Jr., Price, C.F., and Sutherland, A.A., Jr., Applied Optimal Estimation, MIT Press, Cambridge, MA, 1974.

2. Jazwinski, A.H., Stochastic Processes and Filtering, Academic Press, New York, 1970.

3. Maybeck, P.S., Stochastic Models, Estimation, and Control, Vol. 1, Academic Press, San Diego, 1979.

4. Maybeck, P.S., Stochastic Models, Estimation, and Control, Vol. 2, Academic Press, San Diego, 1982.

The reprint volume edited by H. Sorenson, Kalman Filtering: Theory and Application, IEEE Press, New

York, 1985 contains reprints of many of the early theoretical papers on Kalman filtering, as well as a

collection of applications.

A very readable introduction to the theory is

5. Davis, M.H.A., Linear Estimation and Stochastic Control, Halsted Press, New York, 1977. “73648_C013” — #12

14 14.4 Riccati Algebraic Equation

Historical documents:

1. Riccati, J. F., Animadversationes in aequationes differentiales secundi gradus, Acta Eruditorum Lipsiae, 8, 67–73, 1724.

2. Boyer, C. B., The History of Mathematics, Wiley, New York, NY, 1974.

Tutorial textbooks:

3. Reid, W. T., Riccati Differential Equations, Academic Press, New York, NY, 1972.

4. Bittanti, S., Laub, A. J., and Willems, J. C., Eds., The Riccati Equation, Springer-Verlag, Berlin, 1991.

Survey paper:

5. Kuˇcera, V., A review of the matrix Riccati equation, Kybernetika, 9, 42–61, 1973.

Original sources on optimal control and filtering:

6. Kalman, R. E., Contributions to the theory of optimal control, Bol. Soc. Mat. Mexicana, 5, 102–119, 1960.

7. Kalman, R. E. and Bucy, R. S., New results in linear filtering and prediction theory, J. Basic Eng. (ASME Trans.), 83D, 95–108, 1961.

Original sources on the algebraic equation:

8. Willems, J. C., Least squares stationary optimal control and the algebraic Riccati equation, IEEE Trans. Autom. Control, 16, 612–634, 1971.

9. Kuˇcera, V., A contribution to matrix quadratic equations, IEEE Trans. Autom. Control, 17, 344–347, 1972.

10. Kuˇcera, V., On nonnegative definite solutions to matrix quadratic solutions, Automatica, 8, 413–423, 1972.

Original sources on the limiting behavior:

11. Callier, F. M. and Willems, J. L., Criterion for the convergence of the solution of the Riccati differential equation, IEEE Trans. Autom. Control, 26, 1232–1242, 1981.

12. Willems, J. L. and Callier, F. M., Large finite horizon and infinite horizon LQ-optimal control problems, Optimal Control Appl. Methods, 4, 31-45, 1983.

Original sources on the numerical methods:

13. Laub, A. J., A Schur method for solving algebraic Riccati equations, IEEE Trans. Autom. Control, 24, 913-921, 1979.

14. Roberts, J. D., Linear model reduction and solution of the algebraic Riccati equation by use of the sign function, Int. J. Control, 32, 677-687, 1980. “73648_C014” #22

1. Luenberger, D., Observers for multivariable systems, IEEE Trans. Automat. Control, 11, 190-197, 1966.

2. Doyle, J.C. and Stein, G., Robustness with observers, IEEE Trans. Automat. Control, 24, 607-611, 1979.

3. Bass, R.W. and Gura, I., High-order system design via state-space considerations, Proc. Joint. Automat. Control Conf., Troy, NY, 311-318, 1965.

4. Friedland, B., On the properties of reduced-order Kalman filters, IEEE Trans. Automat. Control, 34 (3), 321-324, 1989. “73648_C015” — #23

5. Rynaski, E.G., Flight control synthesis using robust observers, Proc. AIAA Guidance Control Conf., San Diego, CA, 825-831, 1982.

6. Friedland, B., Control System Design: An Introduction to State-Space Methods, McGraw-Hill, New York, 1986.

7. Madiwale, A.N. and Williams, D.E., Some extensions of loop transfer recovery, Proc. Am. Control Conf., Boston, MA, 790-795, 1985.

8. Thau, F.E., Observing the state of nonlinear dynamic systems, Int. J. Control, 17, 471-479, 1973.

9. Kou, S.R., Elliot, D.L., and Tarn, T.J., Exponential observers for nonlinear dynamic systems, Inf. Control, 29 (3), 204-216.

10. Ciccarella, G., DallaMora, M., and Germani, A., A Luenberger-like observer for nonlinear systems, Int. J. Control, 57 (3), 537-556, 1993. “73648_C015” — #24 “73648_S003” #1 III

DesignMethods

forMIMOLTI

Systems III-1 “73648_S003” #2

16 16.4 Eigenstructure Assignment Using Dynamic Compensation

1. Andry, A.N., Shapiro, E.Y., and Chung, J.C., Eigenstructure assignment for linear systems, IEEE Trans. Aerospace Electron. Systems, 19, 711–729, 1983.

2. Ashari, A.E., Sedigh, A.K., and Yazdanpanah, M.J., Reconfigurable control system design using eigenstructure assignment: static, dynamic, and robust approaches, Int. J. Control, 78, 1005-1016, 2005. “73648_C016” — #18

3. Bruyere, L., Tsourdos, A., and White, B.A., Quasilinear parameter varying autopilot design using polynomial eigenstructure assignment with actuator constraints, J. Guidance, Control, Dyn., 29, 1282–1294, 2006.

4. Burrows, S.P., Patton, R.J., and Szymanski, J.E., Robust eigenstructure assignment with a control design package, IEEE Control Systems Mag., 9, 29–32, 1989.

5. Calvo-Ramon, J.R., Eigenstructure assignment by output feedback and residue analysis, IEEE Trans. Automat. Control, 31, 247–249, 1986.

6. Clarke, T., Griffin, S.J., and Ensor, J., A polynomial approach to eigenstructure assignment using projection with eigenvalue tradeoff, Int. J. Control, 76, 403–413,2003.

7. Doyle, J.C. and Stein, G.,Multivariable feedback design: Concepts for a classical/modern synthesis, IEEE Trans. Automat. Control, 26, 4–16, 1981.

8. Gavito, V.F. and Collins, D.J., Application of eigenstructure assignment to design of robust decoupling controllers in mimo systems, Proc. AIAA Guidance, Navig, and Control Conf., 1986, AIAA Paper No. 86-2246, 828–834.

9. Jiang, J., Design of reconfigurable control systems using eigenstructure assignments, Int. J. Control, 59, 395–410, 1994.

10. Johnson, T.L. and Athans, M., On the design of optimal constrained dynamic compensation for linear constant systems, IEEE Trans. Automat. Control, 15, 658–660, 1970.

11. Kautsky, J., Nichols, N.K., and Van Dooren, P., Robust pole assignment in linear state feedback, Int. J. Control, 41, 1129–1155, 1985.

12. Konstantopoulos, I., Optimal controller design and reconfigurable control under stability robustness, PhD Thesis, University of Notre Dame, Notre Dame, IN, 1996.

13. Lehtomaki, N.A., Practical robustness measures in multivariable control system analysis, PhD Thesis, Massachusetts Institute of Technology, Cambridge, MA, 1981.

14. Liebst, B.S. andGarrard,W.L., Application of eigenspace techniques to design of aircraft control systems, Proc. Am. Control Conf., 475–480, 1985.

15. Liebst, B.S.,Garrard,W.L., andAdams,W.M.,Designof an active flutter suppression system, J. Guidance, Control, Dyn., 9, 64–71, 1986.

16. Lu, J., Chiang, H.D., and Thorp, J.S., Eigenstructure assignment by decentralized feedback control, IEEE Trans. Automat. Control, 38(4), 587–594, 1993.

17. Middleton, R.H. and Goodwin, Digital Control and Estimation: A Unified Approach. Prentice-Hall, Englewood Cliffs, NJ, 1990.

18. Anon., Military Specification—Flying Qualities of Piloted Airplanes, US Department of Defense, Washington, DC, 1980, MIL-F8785C.

19. Nieto-Wire, C. and Sobel, K., Eigenstructure assignment for a tailless aircraft, Proc. AIAA Guidance, Navig. Control Conf., AIAA Paper No. 2007-6417, 2007.

20. Nieto-Wire, C. and Sobel, K., Reconfigurable delta operator eigenstructure assignment for a tailless aircraft, Proc. AIAA Guidance, Navig. Control Conf., AIAA Paper No. 2009-6306, 2009.

21. O'Brien, M.J. and Broussard, J.R., Feedforward control to track the output of a forced model, Proc. 17th IEEE Conf. Decision Control, San Diego, 1149–1155, 1978.

22. Piou, J.E. and Sobel, K.M., Robust sampled data eigenstructure assignment using the delta operator, Proc. Guidance, Navig. Control Conf., Hilton Head, SC, 1992.

23. Piou, J.E., Sobel, K.M., and Shapiro, E.Y., Robust Lyapunov constrained sampled data eigenstructure assignment using the delta operator with application to flight control design, Proc. First IEEE Conf. Control Appl., Dayton, Ohio, 1000–1005, 1992.

24. Press,W.H., Flannery, B.P., Teukolsky, S.A., andVetterling,W.T.,NumericalRecipes: TheArt of Scientific Computing, Cambridge University Press, New York, 1987.

25. Satoh, A. and Sugimoto, K., Loose eigenstructure assignment via rank one LMI approach with application to transient response shaping in H-infinity control, Int. J. Control, 82, 497–507, 2009.

26. Schulz, M.J. and Inman, D.J., Eigenstructure assignment and controller optimization for mechanical systems, IEEE Trans. Control Systems Tech., 2(2), 88–100, 1994.

27. Sobel, K.M., Shapiro, E.Y., andAndry, A.N., Jr., Eigenstructure Assignment, Int. J. Control, 59(1), 13–37, 1994.

28. Sobel, K.M. and Shapiro, E.Y., Application of eigensystem assignment to lateral translation and yaw pointing flight control, Proc. 23rd IEEE Conf. Decision Control, Las Vegas, 1423–1428, 1984.

29. Sobel, K.M. and Shapiro, E.Y., A design methodology for pitch pointing flight control systems, J. Guidance, Control, Dyn., 8, 181–187, 1985.

30. Sobel, K.M. and Shapiro, E.Y., Application of eigenstructure assignment to flight control design: Some extensions, J. Guidance, Control, Dyn., 10, 73–81, 1987. “73648_C016” — #19

31. Sobel, K.M. and Shapiro, E.Y., An extension to the pseudocontrol strategy with application to an eigenstructure assignment yaw pointing/lateral translation control law, Proc. 30th IEEE Conf. Decision Control, Brighton, UK, 515–516, 1991.

32. Sobel, K.M. and Lallman, F.J., Eigenstructure assignment for the control of highly augmented aircraft, J. Guidance, Control, Dyn., 12, 318–324, 1989.

33. Sobel,K.M., Yu,W., andLallman, F.J., Eigenstructure assignmentwith gain suppressionusing eigenvalue and eigenvector derivatives, J. Guidance, Control, Dyn., 13, 1008–1013, 1990.

34. Srinathkumar, S., Eigenvalue/eigenvector assignment using output feedback, IEEE Trans. Automat Control, AC-23(1), 79–81, 1978.

35. Yedavalli, R.K., Perturbation bounds for robust stability in linear state space models, Int. J. Control, 42, 1507–1517, 1985.

36. Yu, W., Piou, J.E., and Sobel, K.M., Robust eigenstructure assignment for the extended medium range air to air missile, Automatica, 29(4), 889–898, 1993.

Further Reading

Kautsky et al. [11] suggested that eigenstructure assignment can be used to obtain a design with eigen

values, which are least sensitive to parameter variation, by reducing one of several sensitivity measures.

Among these measures are the quadratic norm condition number of the closed-loop modal matrix and

the sum of the squares of the quadratic norms of the left eigenvectors. Burrows et al. [4] have proposed a

stability augmentation system for a well-behaved light aircraft by using eigenstructure assignment with an

optimization which minimizes the condition number of the closed-loop modal matrix. Such an approach

may sometimes produce an acceptable controller. In contrast to eigensensitivity, Doyle and Stein [7]

have characterized the stability robustness of a multi-input, multi-output system by the minimum of the

smallest singular value of the return difference matrix at the plant input or output. Gavito and Collins [8]

have proposed an eigenstructure assignment design in which a constraint is placed on the minimum of

the smallest singular value of the return difference matrix at the inputs of both an L-1011 aircraft and

a CH-47 helicopter. Several authors have proposed different approaches to eigenstructure assignment.

Clarke et al. [6] present a method to trade exact closed-loop eigenvalue location against an improvement

in the associated eigenvector match. Bruyere et al. [3] use eigenstructure assignment in a polynomial

framework. Satoh and Sugimoto [25] present a regional eigenstructure assignment using a linear matrix

inequalities (LMI) approach. Nieto-Wire and Sobel [19] applied eigenstructure assignment to the design

of a flight control system for a tailless aircraft. Other applications of eigenstructure assignment include

air-to-airmissiles [36],mechanical systems [26], and power systems [16]. Jiang [9], Konstantopoulos [12],

and Ashari et al. [2] have proposed methods for reconfiguration using eigenstructure assignment. Nieto

Wire and Sobel [20] used eigenstructure assignment for the accomodation of symmetric lock in place

actuator failures for a tailless aircraft.

Appendix

Data for the F-18 HARV Lateral Directional Dynamics at M = 0.38, H = 5000 ft., and α = 5 ∘ A = ⎡ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎣ -30.0000 0 0 0 0 0 0 0 0 -30.0000 0 0 0 0 0 0 0 0 -30.0000 0 0 0 0 0 -0.0070 -0.0140 0.0412 -0.1727 0.0873 -0.9946 0.0760 0 15.3225 12.0601 2.2022 -11.0723 -2.1912 0.7096 0 0 -0.3264 0.2041 -1.3524 2.1137 -0.0086 -0.1399 0 0 0 0 0 0 1.0000 0.0875 0 0 0 0 0 0 0 0.5000 0 -0.5 ⎤ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎦ , “73648_C016” — #20 B = ⎡ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎢ ⎣ 30.0000 0 0 0 30.0000 0 0 0 30.0000 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ⎤ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎥ ⎦ , C = ⎡ ⎢ ⎢ ⎢ ⎣ 0 0 0 0 0 1 0 -1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 ⎤ ⎥ ⎥ ⎥ ⎦ ,

Data for the FPCC Lateral Directional Dynamics A = ⎡ ⎢ ⎢ ⎢ ⎢ ⎣ -0.340 0.0517 0.001 -0.997 0 0 0 1 0 0 -2.69 0 -1.15 0.738 0 5.91 0 0.138 -0.506 0 -0.340 0.0517 0.001 0.0031 0 ⎤ ⎥ ⎥ ⎥ ⎥ ⎦ , B = ⎡ ⎢ ⎢ ⎢ ⎢ ⎣ 0.0755 0 0.0246 0 0 0 4.48 5.22 -0.742 -5.03 0.0998 0.984 0.0755 0 0.0246 ⎤ ⎥ ⎥ ⎥ ⎥ ⎦ .

1. Barabanov, A. and Sokolov, A., The geometrical approach to l 1 optimal control, Proc. 33rd IEEE Conf. Decision Control, 1994.

2. Dahleh, M.A. and Diaz-Bobillo, I., Control of Uncertain Systems: A Linear Programming Approach, Prentice Hall, Englewood Cliffs, NJ, 1995.

3. Dahleh, M.A. and Khammash, M.H., Controller design for plants with structured uncertainty, Automatica, 29(1), 1993.

4. Dahleh, M.A. and Ohta, Y., A necessary and sufficient condition for robust BIBO stability, Syst. Contr. Lett., 11, 1988.

5. Dahleh, M.A. and Pearson, J.B., L 1 optimal feedback compensators for continuous time systems, IEEE Trans. Automat. Control, 32, October 1987.

6. Dahleh, M.A. and Pearson, J.B., 1 optimal feedback controllers for mimo discrete-time systems, IEEE Trans. Automat. Control, April 1987.

7. Dahleh, M.A. and Pearson, J.B., Optimal rejection of persistent disturbances, robust stability and mixed sensitivity minimization, IEEE Trans. Automat. Control, 33, August 1988.

8. Diaz-Bobillo, I.J. and Dahleh, M.A., Minimization of the maximum peak-to-peak gain: The general multiblock problem, IEEE Trans. Automat. Control, October 1993.

9. Khammash, M. and Pearson, J.B., Performance robustness of discrete-time systems with structured uncertainty, IEEE Trans. Automat. Control, 36, 1991.

10. Khammash, M. and Pearson, J.B., Robust disturbance rejection in 1 -optimal control systems, Syst. Control Lett., 14, 1990.

11. McDonald, J.S. and Pearson, J.B., 1 -optimal control of multivariable systems with output norm constraints, Automatica, 27, 1991.

12. Shamma, J.S., Nonlinear state feedback for 1 optimal control, Syst. Control Lett., to appear.

13. Vidyasagar, M., Optimal rejection of persistent bounded disturbances, IEEE Trans. A-C, 31(6), 1986. “73648_C019” — #14

1. Balas, G.J. and Doyle, J.C., Control of lightly damped, flexible modes in the controller crossover region, AIAA J. Guidance, Dyn. Control, 17(2), 370-377, 1994.

2. Balas, G.J., Doyle, J.C., Glover, K., Packard, A., and Smith, R., The μ analysis and synthesis toolbox, mathworks, Natick, MA, 1991.

3. Boyd, S., El Ghaoui, L., Feron, E., Balakrishnan, V., Linear Matrix Inequalities in System and Control Theory, SIAM Studies in Applied Mathematics, SIAM, Philadelphia, 1994.

4. Chiang, R. and Safonov, M., Robust control toolbox, MathWorks, Natick, MA, 1988.

5. Doyle, J.C., Analysis of feedback systems with structured uncertainties, IEEE Proc., 129 Part D(6), 242-250, 1982.

6. Doyle, J.C., Lenz, K., and Packard, A., Design examples using μ synthesis: Space shuttle lateral axis FCS during reentry, IEEE CDC, 1986, 2218-2223; also Modelling, Robustness and Sensitivity Reduction in Control Systems, NATO ASI Series, Curtain, R.F., Ed., Springer, 1987, vol. F34, 128-154.

7. Fan, M., Tits, A., and Doyle, J.C., Robustness in the presence of joint parametric uncertainty and unmodeled dynamics, IEEE Trans. Automat. Control, 36, 25-38, 1991.

8. Packard, A., Doyle, J.C., and Balas, G.J., Linear, multivariable robust control with aμ perspective,ASME J. Dyn., Meas. Control, Special Ed. on Control, 115(2b), 426-438, 1993.

9. Packard, A. and Doyle, J.C., The complex structured singular value, Automatica, 29(1), 71-109, 1993.

10. Packard, A. and Pandey, P., Continuity properties of the real/complex structured singular value, IEEE Trans. Automat. Control, 38(3), 415-428, 1993.

11. Redheffer, On a certain linear fractional transformation, J. Math. Phys., 39, 269-286, 1960.

12. Safonov, M.G., Stability margins of diagonally perturbed multivariable feedback systems, Proc. IEEE, 129

Part D(6), 251-256, 1982.

13. Skogestad, S., Morari, M., and Doyle, J.C., Robust control of ill-conditioned plants: high-purity distillation, IEEE Trans. Automat. Control, 33(12), 1092-1105, 1988.

14. Stein, G. and Doyle, J.C., Beyond singular values and loopshapes, AIAA J. Guidance and Control, 14(1), 5-16, 1991.

15. Young, P.M., Newlin, M.P., and Doyle, J.C., μ Analysis with real parametric uncertainty, In Proc. 30 th IEEE Conf. Decision and Control, Hawaii, 1991, 1251-1256.

1. D’Azzo, J.J., and Houpis, C.H., Linear Control System Analysis and Design, 4th Ed., McGraw-Hill, New York, 1995.

2. Houpis, C. H., Quantitative feedback theory (QFT) for the engineer: A paradigm for the design of control systems for uncertain nonlinear plants, WL-TR-95-3061, AF Wright Aeronautical Laboratory, Wright-Patterson AFB, OH, 1987. (Available from National Technical Information Service, 5285 Port Royal Road, Springfield, VA 22151, document number AD-A297574.)

3. Horowitz, I. M. and Sidi, M., Synthesis of feedback systems with large plant ignorance for prescribed time domain tolerances, Int. J. Control, 16, 287–309, 1972.

4. Horowitz, I. M. and Loecher, C., Design of a 3× 3 multivariable feedback system with large plant uncertainty, Int. J. Control, 33, 677–699, 1981.

5. Horowitz, I. M., Optimum loop transfer function in single-loop minimum phase feedback systems, Int. J. Control, 22, 97–113, 1973.

6. Horowitz, I. M., Synthesis of feedback systems with non-linear time uncertain plants to satisfy quantitative performance specifications, IEEE Proc., 64, 123–130, 1976.

7. Horowitz, I. M., Quantitative synthesis of uncertain multiple-input multiple-output feedback systems, Int. J. Control, 30, 81–106, 1979.

8. Thompson, D. F. and Nwokah, O.D.I., Optimal loop synthesis in quantitative feedback theory, Proc. Am. Control Conf., San Diego, CA, 626–631, 1990. “73648_C022” — #27

9. Houpis, C. H. and Chandler, P.R., Eds., Quantitative Feedback Theory Symposium Proceedings, WL-TR-92-3063, Wright Laboratories, Wright-Patterson AFB, OH, 1992.

10. Keating, M. S., Pachter, M., and Houpis, C.H., Damaged aircraft control system design using QFT, Proc. Nat. Aerosp. Electron. Conf. (NAECON), Vol. 1, Ohio, 621–628, May 1994.

11. Reynolds, O.R., Pachter, M., and Houpis, C.H., Design of a subsonic flight control system for the Vista F-16 using quantitative feedback theory, Proc. Am. Control

Conf., 350–354, 1994.

12. Trosen, D. W., Pachter, M., and Houpis, C.H., Formation flight control automation, Proc. Am. Inst. Aeronaut. Astronaut. (AIAA) Conf., Scottsdale, AZ, 1379–1404, 1994.

13. Houpis, C. H. and Lamont, G., Digital Control Systems: Theory, Hardware, Software, 2nd ed., McGrawHill, NY, 1992.

14. Horowitz, I.M. and Liao, Y.K., Quantitative feedback design for sampled-data system, Int. J. Control, 44, 665–675, 1986.

15. Sating, R.R.,Horowitz, I.M., andHoupis, C.H.,Development of aMIMOQFTCADpackage (Version 2), paper presented at the 1993Am. Control Conf., Air Force Institute of Technolgy,Wright-Patterson AFB, Ohio.

16. Boyum,K. E., Pachter,M., andHoupis, C.H., High angle of attack velocity rolls, 13th IFAC Symp. Autom. Control Aerosp., 51–57, Palo Alto, CA, September 1994.

17. Schneider, D. L., QFT digital flight control design as applied to the AFTI/F-16, M.S. thesis, AFIT/GE/ENG/86D-4, School of Engineering, Air Force Institute of Technolgy, Ohio, 1986.

18. Arnold, P. B., Horowitz, I.M., andHoupis, C.H., YF-16CCV flight control system reconfiguration design using quantitative feedback theory,Proc. Nat. Aerosp. Electron. Conf. (NAECON), Vol. 1, 578–585, 1985.

23 23.4 Applications and Example Calculations

1. Davison, E.J., The feedforward control of linear time invariant multivariable systems, Automatica, 9(5), 561–573, 1973.

2. Davison, E.J. and Wang, S.H., Properties of linear time invariant multivariable systems subject to arbitrary output and state feedback, IEEE Trans. Autom. Control, 18, 24–32, 1973.

3. Davison, E.J. and Wang, S.H., Properties and calculation of transmission zeros of linear multivariable time-invariant systems, Automatica, 10, 643–658, 1974.

4. Goldenberg, A. and Davison, E.J., The feedforward and robust control of a general servomechanism problem with time lag, 8th Annu. Princeton Conf. Inf. Sci. Syst., 80–84, 1974.

5. Davison, E.J. and Goldenberg, A., The robust control of a general servomechanism problem: The servo compensator, Automatica, 11, 461–471, 1975.

6. Davison, E.J., The robust control of a servomechanism problem for linear time-invariant multivariable systems, IEEE Trans. Autom. Control, 21, 25–34, 1976.

7. Davison, E.J., Multivariable tuning regulators: The feedforward and robust control of a general servomechanism problem, IEEE Trans. Autom. Control, 21, 35–47, 1976.

8. Davison, E.J., The robust decentralized control of a general servomechanism problem, IEEE Trans. Autom. Control, 21, 14–24, 1976.

9. Davison, E.J. and Ferguson, I.J., The design of controllers for the multivariable robust servomechanism problem using parameter optimization methods, IEEE Trans. Autom. Control, 26, 93–110, 1981.

10. Davison, E.J. and Copeland, B., Gain margin and time lag tolerance constraints applied to the stabilization problem and robust servomechanism problem, IEEE Trans. Autom. Control, 30, 229–239, 1985.

11. Davison, E.J. and Chang, T., Decentralized controller design using parameter optimization methods, Control: Theor. Adv. Technol. 2, 131–154, 1986.

12. Davison, E.J. and Scherzinger, B., Perfect control of the robust servomechanism problem, IEEE Trans. Autom. Control, 32(8), 689–702, 1987.

13. Qiu, Li and Davison, E.J., Performance limitations of non-minimum phase systems in the servomechanism problem, Automatica, 29(2), 337–349, 1993.

14. Zhang, H. and Davison, E.J., A uniform high gain compensator for multivariable systems, 1994 IEEE Control Decision Conf., 892–897, 1994.

Appendix: Convex Sets and Convex Functions

1. P. Apkarian and R. J. Adams. Advanced gain-scheduling techniques for uncertain systems. IEEEControl System Magazine, 6(1):21–32, 1998.

2. V. Balakrishan and L. Vandenberghe. Semidefinite programming duality and linear time-invariant systems. IEEE Transactions on Autmatic Control, 48(1):30–41, 2003.

3. S.P. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishan. Linear Matrix Inequalities in System and Control Theory. SIAM Studies in Applied Mathematics 15. SIAM, Philadelphia, 1994.

4. M. Chilali and P. Gahinet. H ∞ design with pole placement constraints: An LMI approach. IEEE Transactions on Automatic Control, 41(3):358–367, 1996.

5. M.C. De Oliveira, J.C. Geromel, and J. Bernussou. Extended H 2 and H ∞ -norm characterizations and controller parametrizations for discrete-time systems. International Journal of Control, 75(9):666–679, 2002.

6. C.A. Desoer and M. Vidyasagar. Feedback Systems: Input–Output Approach. Academic Press, London, 1975.

7. L. El Ghaoui and S.I. Niculescu, Eds. Advances in Linear Matrix Inequality Methods in Control. SIAM, Philadelphia, 2000. “73648_C024” — #30

8. P. Gahinet and P. Apkarian. A linear matrix inequality approach to H ∞ control. International Journal of Robust and Nonlinear Control, 4:421–448, 1994.

9. J.C. Geromel. Optimal linear filtering under parametric uncertainty. IEEE Transactions on Signal Processing, 47(1):168–175, 1999.

10. S.V.Gusev andA.L. Likhtarnikov. Kalman–Popov–Yakubovich lemmaand the S-procedure:Ahistorical essay. Automation and Remote Control, 67(11):1768–1810, 2006.

11. T. Iwasaki and R.E. Skelton. All controllers for the general H ∞ control problem: LMI existence conditions and state space formulas. Automatica, 30:1307–1317, 1994.

12. J. Löfberg. YALMIP: A toolbox for modeling and optimization in MATLAB. In Proceedings of the CACSD Conference, Taipei, Taiwan, 2004.

13. I. Masubuchi, A. Ohara, and N. Suda. LMI-based controller synthesis: A unified formulation and solution. International Journal of Robust and Nonlinear Control, 8:669–686, 1998.

14. A. Megretski and A. Rantzer. System analysis via integral quadratic constraints. IEEE Transactions on Automatic Control, 42:819–830, 1997.

15. C.W. Scherer. An efficient solution to multi-objective control problems with LMI objectives. System & Control Letters, 40(1):43–57, 2000.

16. C.W. Scherer. Robust mixed control and LPV control with full block scalings. In L. El Ghaoui and S.I. Niculescu, Eds, Advances in Linear Matrix Inequality Methods in Control, pp. 187–207. SIAM, Philadelphia, 2000.

17. C.W. Scherer, P.Gahinet, andM.Chilali. Multi-objective output-feedback control via LMI optimization. IEEE Transactions on Automatic Control, 42:896–911, 1997.

18. S. Skogestad and I. Postlethwaite. Multivariable Feedback Control, Analysis and Design. John Wiley & Sons, New York, 1996.

19. J.C.Willems. Dissipative dynamical systems, part II: Linear systems with quadratic supply rates. Archive for Rational Mechanics and Analysis, 45:352–393, 1972.

20. K. Zhou, J. Doyle, and K. Glover. Robust and Optimal Control. Prentice-Hall, Upper Saddle River, NJ, 1996.

1. Anderson, B.D.O, and Moore, J.B., Optimal Control, Prentice-Hall, Englewood Cliffs, NJ, 1990.

2. Athans, M, and Falb, P., Optimal Control, McGraw-Hill, New York, 1966.

3. Bryson, A.E., Jr. and Ho, Y.-C., Applied Optimal Control, Hemisphere, New York, 1975.

4. Grimble, M.J. and Johnson, M.A., Optimal Control and Stochastic Estimation: Theory and Applications, John Wiley & Sons, New York, Vol. 1, 1988.

5. Kirk, D.E., Optimal Control Theory, Prentice-Hall, Englewood Cliffs, NJ, 1970.

6. Kwakernaak, H. and Sivan, R., Linear Optimal Control Systems, John Wiley & Sons, New York, 1972.

7. Lewis, F.L. and Syrmos, V., Optimal Control, 2nd edition, John Wiley and Sons, New York, 1995.

8. Lewis, F.L., Applied Optimal Control and Estimation, Prentice-Hall, Englewood Cliffs, NJ, 1992.

9. MATRIXx, Integrated Systems, Inc., Santa Clara, CA, 1989.

10. Moler, C., Little, J., and Bangert, S., PC-MATLAB, The Mathworks, Inc., Sherborn, MA, 1987.

11. Stevens, B.L. and Lewis, F.L., Aircraft Modelling, Dynamics, and Control, John Wiley & Sons, New York, 1992.

12. Verriest, E.I. and Lewis, F.L., On the linear quadratic minimum-time problem, IEEE Transactions on Automatic Control, 859-863, 1991.

Further Reading

1. Anderson, B. D. O. and Clements, D. J., Algebraic characterization of fixed modes in decentralized control, Automatica, 17, 703–712, 1981.

2. Brusin, V. A. and Ugrinovskaya, E. Ya., Decentralized adaptive control with a reference model, Avtomatika i Telemekhanika, 10, 29–36, 1992.

3. Chae, S. and Bien, Z., Techniques for decentralized control for interconnected systems, in Control and Dynamic Systems, C. T. Leondes, Ed., Academic Press, Boston, 41, 273–315, 1991.

4. Chen, Y. H., Decentralized robust control design for large-scale uncertain systems: The uncertainty is time-varying, J. Franklin Institute, 329, 25–36, 1992.

5. Chen, Y. H. and Han, M. C., Decentralized control design for interconnected uncertain systems, in Control and Dynamic Systems, C. T. Leondes, Ed., Academic Press, Orlando, FL, 56, 219–266, 1993.

6. Cheng, C. F., Wang, W. J., and Lin, Y. P., Decentralized robust control of decomposed uncertain interconnected systems, Trans. ASME, 115, 592–599, 1993.

7. Cho, Y. J. and Bien, Z., Reliable control via an additive redundant controller, Int. J. Control, 50, 385–398, 1989.

8. Date, R. A. and Chow, J. H., A parametrization approach to optimal H 2 and H ∞ decentralized control problems, Automatica, 29, 457–463, 1993.

9. Datta, A., Performance improvement in decentralized adaptive control: A modified model reference scheme, IEEE Trans. Automatic Control, 38, 1717–1722, 1993.

10. Gajic´, Z. and Shen, X., Parallel Algorithms for Optimal Control of Large Scale Linear Systems, Springer-Verlag, Berlin, Germany, 1993. “73648_C026” — #23

11. Geromel, J. C., Bernussou, J., and Peres, P. L. D., Decentralized control through parameter space optimization, Automatica, 30, 1565–1578, 1994.

12. Gündes, A. N. and Kabuli, M. G., Reliable decentralized control, Proc. Am. Control Conf., Baltimore, MD, pp.

3359–3363, 1994.

13. Iftar, A., Decentralized estimation and control with overlapping input, state, and output decomposition, Automatica, 29, 511–516, 1993.

14. Ikeda,M., Decentralized control of large scale systems, inThree Decades ofMathematical SystemTheory, H. Nijmeiyer and J. M. Schumacher, Eds., Springer-Verlag, New York, 219–242, 1989.

15. Jamshidi, M., Large-Scale Systems. Modeling and Control, North-Holland, New York, 1983.

16. Lakshmikantham, V. and Liu, X. Z., Stability Analysis in Terms of Two Measures, World Scientific, Singapore, 1993.

17. Lakshmikantham, V., Matrosov, V. M., and Sivasundaram, S., Vector Lyapunov Functions and Stability Analysis of Nonlinear Systems, Kluwer, The Netherlands, 1991.

18. Leitmann, G., One approach to the control of uncertain systems,ASME J. Dyn. Syst., Meas., and Control, 115, 373–380, 1993.

19. Leondes, C. T., Ed., Control and Dynamic Systems, Vols. 22–24, Decentralized/Distributed Control and Dynamic Systems, Academic Press, Orlando, FL, 1985.

20. Lin, C. T., Structural controllability, IEEE Trans. Auto. Control, AC–19, 201–208, 1974.

21. Lyon, J., Note on decentralized adaptive controller design, IEEE Trans. Auto. Control, 40, 89–91, 1995.

22. Michel, A. N., On the status of stability of interconnected systems, IEEE Trans. Circuits Syst., CAS–30, 326–340, 1983.

23. Mills, J. K., Stability of robotic manipulators during transition to and from compliant motion, Automatica, 26, 861–874, 1990.

24. Sandell, N. R., Jr., Varaiya, P., Athans, M., and Safonov, M. G., Survey of decentralized control methods for large scale systems, IEEE Trans. Auto. Control, AC–23, 108–128, 1978.

25. Savastuk, S. V. and Šiljak, D. D., Optimal

decentralized control, Proc. Am. Control Conf., Baltimore, MD, pp. 3369–3373, 1994.

26. Sezer, M.E. and Šiljak, D. D., Structurally fixed modes, Syst. Control Lett., 1, 60–64, 1981.

27. Sezer, M. E. and Šiljak, D. D., Nested Ω-decomposition and clustering of complex systems, Automatica, 22, 321–331, 1986.

28. Shi, L. and Singh, S. K., Decentralized adaptive controller design for large-scale systems with higher order interconnections, IEEE Trans. Auto. Control, 37, 1106–1118, 1992.

29. Šiljak, D. D., Large-Scale Dynamic Systems: Stability and Structure, North-Holland, New York, 1978.

30. Šiljak, D. D., Decentralized Control of Complex Systems, Academic Press, Cambridge, MA, 1991.

31. Tamura, H. and Yoshikawa, T., Large-Scale Systems Control and Decision Making, Marcel Dekker, New York, 1990.

32. Ünyeliog˘lu, K. A. and Özgüler, A. B., Reliable decentralized stabilization of feed-forward and feedback interconnected systems, IEEE Trans. Auto. Control, 37, 1119–1132, 1992.

33. Voronov, A. A., Present state and problems of stability theory, Automatika i Telemekhanika, 5, 5–28, 1982.

34. Wang, S. H. and Davison, E. J., On the stabilization of decentralized control systems, IEEE Trans. Auto. Control, AC-18, 473–478, 1973.

35. Wu, W. and Lin, C., Optimal reliable control system design for steam generators in pressurized water reactors, Nuclear Technology, 106, 216–224, 1994.

36. Yu, R., Ocali, O., and Sezer, M. E., Adaptive robust sampled-data control of a class of systems under structured perturbations, IEEE Trans. Auto. Control, 38, 1707–1713, 1993.

Further Reading

There is a number of survey papers on decentralized control and large scale systems [3,14,24]. The

books on the subject are [10,15,19,29,31]. For a comprehensive treatment of decentralized control theory,

For further information on vector Lyapunov functions and stability analysis of large scale intercon

nected systems, see the survey papers [22,33], and books [16,17].

Adaptive decentralized control has been of widespread recent interest, see [2,9,21,23,28,30,36]. “73648_C026” — #24

Robustness of decentralized control to both structured and unstructured perturbations has been one of

the central issues in the control of large scale systems. For the background of robustness issues in control,

which are relevant to decentralized control, see [18,30]. For new and interesting results on the subject,

see [4–6,8].

There is a number of papers devoted to design of decentralized control via parameter space optimiza

Overlapping decentralized control and the inclusion principle are surveyed in [30]. Useful extensions

were presented in [13]. The concept of overlapping is basic to reliable control under controller failures

using multiple decentralized controllers [30]. For more information about this area, see [7,12,32,35].

In a recent development [25], it has been shown how optimal decentralized control of large scale inter

connected systems can be obtained in the classical optimization framework of Lagrange. Both sufficient

and necessary conditions for optimality are derived in the context of Hamilton–Jacobi equations and

Pontryagin’s maximum principle.

27 Decoupling

1. Antsaklis, P.J., Stable proper n-th order inverses, IEEE Trans. Autom. Control, 23, 1104–1106, 1978.

2. Antsaklis, P.J., Maximal order reduction and supremal (A,B)-invariant and controllability subspaces, IEEE Trans. Autom. Control, 25, 44–49, 1980.

3. Falb, P.L. andWolovich,W.A., Decoupling in the design and synthesis of multivariable control systems, IEEE Trans. Autom. Control, 12, 651–659, 1967.

4. Furuta, K. and Kamiyama, S., State feedback and inverse system, Intern. J. Control, 25(2), 229–241, 1977.

5. Gilbert, E.G., The decoupling of multivariable systems by state feedback, SIAM J. Control, 50–63, 1969.

6. Kamiyama, S. and Furuta, K., Integral invertibility of linear time-invariant systems, Intern. J. Control, 25(3), 403–412, 1977.

7. Moore, B.C. and Silverman, L.M., A new characterization of feedforward, delay-free inverses, IEEE Trans. Inf. Theory, 19, 126–129, 1973.

8. Morse, A.S. and Wonham, W.M., Decoupling and pole assignment by dynamic compensation, SIAM J. Control, 317–337, 1970.

9. Morse, A.S. and Wonham, W.M., Triangular decoupling of linear multivariable systems, IEEE Trans. Autom. Control, 447–449, 1970. “73648_C027” #15

10. Rekasius, Z.V., Decoupling of multivariable systems by means of state variable feedback, Proc. 3rd Allerton Conf., 439–448, 1965.

11. Silverman, L.M., Decoupling with state feedback and precompensation, IEEE Trans. Autom. Control, 15, 487–489, 1970.

12. Silverman, L.M. and Payne, H.J., Input–output structure of linear systems with application to the decoupling problem, SIAM J. Control, 9, 188–233, 1971.

13. Williams, T., Inverse and Decoupling Problems in Linear Systems, Ph.D. thesis, Imperial College, London, 1981.

14. Williams, T. and Antsaklis, P.J., A unifying approach to the decoupling of linear multivariable systems, Intern. J. Control, 44(1), 181–201, 1986.

15. Wolovich, W.A. and Falb, P.L., On the structure of multivariable systems, SIAM J. Control, 437–451, 1969.

16. Wolovich, W.A. and Falb, P.L., Invariants and canonical forms under dynamic compensation, SIAM J. Control, 996–1008, 1976.

17. Wolovich, W.A., Linear Multivariable Systems, Springer-Verlag, New York, 1974.

18. Wolovich, W.A., Output feedback decoupling, IEEE Trans. Autom. Control, 148–149, 1975.

19. Wonham, W.M., Linear Multivariable Control: A Geometric Approach, Springer-Verlag, New York, 1979.

Further Reading

Making the systemdiagonally dominant is a powerful design approach.Details on how to achieve diagonal

dominance using Rosenbrock’s Inverse Nyquist Array method can be found in Rosenbrock, H.H. 1974.

Computer-Aided Control System Design, Academic Press, New York.

A good introduction to the geometric approach and to the decoupling problemusing that approach can

be found inWohnam,W.M. 1985. Linear Multivariable Control: A Geometric Approach, Springer-Verlag,

New York. The problem of disturbance decoupling or disturbance rejection, where a disturbance in the

state equations must become unobservable from the output, is also studied there using the geometric

approach.

A geometric approach has also been used to study non-interacting control in nonlinear systems; see, for

example, Battilotti, S. 1994. Noninteracting Control with Stability for Nonlinear Systems, Springer-Verlag,

New York.

For the decoupling of singular systems see, for example Paraskevopoulos, P.N. and Koumboulis, F.N.

1992. The decoupling of generalized state-space systems with state feedback, IEEE Trans. Autom. Control,

pp. 148-152, vol.37.

The following journals report advances in all areas of decoupling including diagonal, block and triangu

lar decoupling: IEEETransactions onAutomatic Control, International Journal of Control andAutomatica.

1. C. R. Cutler and B. L. Ramaker. Dynamic matrix control—A computer control algorithm. In Proceedings of the Joint Automatic Control Conference, San Francisco, CA, 1980.

2. J.H. Lee andB.Cooley. Recent advances inmodel predictive control and related areas. In 5th International Conference on Chemical Process Control, AIChE Symposium Series, Vol. 93, pp. 201-216, New York, NY, 1997.

3. J. H. Lee, M.Morari, and C. E. Garcia. State space interpretation of model predictive control. Automatica, 30:707-717, 1994. “73648_C028” — #25

4. J. H. Lee andZ.H. Yu. Tuning ofmodel predictive control for robust performance.Computers&Chemical Engineering, 18:15-37, 1994.

5. W. Li and J. H. Lee. Control relevant identification of ill-conditioned systems. Computers & Chemical Engineering, 20:1023-1042, 1996.

6. J. Richalet, A. Rault, Testud J. L., and J. Papon. Model predictive heuristic control: Applications to industrial processes. Automatica, 14(5):413-428, 1978.

7. E. Y. Tse and M. Athans. Adaptive stochastic control for a class of systems. IEEE Transactions on Automatic Control, 17:38-52, 1972.

8. P. Van Overschee and B. De Moor. N4SID: Subspace algorithms for the identification of combined deterministic-stochastic systems. Automatica, 30(1):75-94, 1994. “73648_C028” — #26 “73648_S004” #1 IV

Analysis and

Design ofHybrid

Systems IV-1 “73648_S004” #2

29 Computation of Reach Sets for Dynamical Systems

1. CDD/CDD+ homepage. www.cs.mcgill.ca/~fukuda/soft/cdd_home/cdd.html.

2. CheckMate homepage. www.ece.cmu.edu/~webk/checkmate.

3. d/dt homepage. www-verimag.imag.fr/~tdang/ddt.html.

4. Ellipsoidal Toolbox homepage. code.google.com/p/ellipsoids.

5. Level Set Toolbox homepage. www.cs.ubc.ca/~mitchell/ToolboxLS.

6. MATISSE homepage. www.seas.upenn.edu/~agirard/Software/MATISSE.

7. MPT homepage. control.ee.ethz.ch/~mpt.

8. PHAVer homepage. www-verimag.imag.fr/~frehse/phaver_web.

9. PPL homepage. www.cs.unipr.it/ppl.

10. Requiem homepage. www.seas.upenn.edu/~hybrid/requiem/requiem.html.

11. Zonotope methods on Wolfgang Kühn homepage. www.decatur.de.

12. Z. Artstein and S. Rakovic´. Feedback and invariance under uncertainty via set-iterates. Automatica, 44(2):520–525, 2008.

13. E. Asarin, O. Bournez, T. Dang, and O. Maler. Approximate reachability analysis of piecewise linear dynamical systems. In N. Lynch and B. H. Krogh, Eds, Hybrid Systems: Computation and Control, Lecture Notes in Computer Science, Vol. 1790, pp. 21–31. Springer, Berlin, 2000. “73648_C029” — #31

14. D. Avis, D. Bremner, and R. Seidel. How good are convex hull algorithms? Computational Geometry: Theory and Applications, 7:265–301, 1997.

15. F. Blanchini. Set-invariance in control. Automatica, 35(11):1747–1767, 1999.

16. A. N. Daryin, A. B. Kurzhanskiy, and I. V. Vostrikov.

Reachability approaches and ellipsoidal techniques for closed-loop control of oscillating systems under uncertainty. In IEEE Conference on Decision and Control, Seville, Spain, 2006.

17. A.Girard. Reachability of uncertain linear systems using zonotopes. InM.Morari, L. Thiele, and F. Rossi, editors, Hybrid Systems: Computation and Control, Lecture Notes in Computer Science, Vol. 3414, pp. 291–305. Springer, Berlin, 2005.

18. A. Girard, C. Le Guernic, and O. Maler. Efficient computation of reachable sets of linear time-invariant systems with inputs. In J. Hespanha and A. Tiwari, Eds, Hybrid Systems: Computation and Control, Lecture Notes in Computer Science, Vol. 3927, pp. 257–271. Springer, Berlin, 2006.

19. E. K. Kostousova. Control synthesis via parallelotopes: Optimization and parallel computations. Optimization Methods and Software, 14(4):267–310, 2001.

20. N. N. Krasovski and A. I. Subbotin. Positional Differential Games. Springer-Verlag, New York, 1988.

21. A. B. Kurzhanski. Set-valued calculus and dynamic programming in problems of feedback control. Vol. 124, International Series of Numerical Mathematics (ISNM), pp. 163–174. Birkhäuser, Basel, 1998.

22. A. B. Kurzhanski and I. Vályi. Ellipsoidal Calculus for Estimation and Control. Ser. SCFA. Birkhäuser, Boston, 1997.

23. A. B. Kurzhanski and P. Varaiya. On ellipsoidal techniques for reachability analysis. Optimization Methods and Software, 17:177–237, 2000.

24. A. B. Kurzhanski and P. Varaiya. Dynamic optimization for reachability problems.Optimization Theory and Applications, 108(2):227–251, 2001.

25. A. A. Kurzhanskiy and P. Varaiya. Ellipsoidal techniques for reachability analysis of discrete-time linear systems. IEEE Transactions on Automatic Control, 52(1):26–38, 2007.

26. M. Kvasnica, P. Grieder, M. Baotic´, and M. Morari. Multi-parametric toolbox (MPT). In R. Alur and G. J. Pappas, Eds, Hybrid Systems: Computation and Control,

Lecture Notes in Computer Science, Vol. 2993, pp. 448–462. Springer, Berlin, 2004.

27. G. Lafferriere, G. J. Pappas, and S. Yovine. Symbolic reachability computation for families of linear vector fields. Journal of Symbolic Computation, 32:231–253, 2001.

28. E. B. Lee and L. Markus. Foundations of Optimal Control Theory. Wiley & Sons, New York, 1961.

29. G. Leitmann. Optimality and reachability with feedback controls. In A. Blaquiere and G. Leitmann, Eds, Dynamical Systems and Microphysics, Vol. 1790, pp. 119–141. Academic Press, New York, NY, 1982.

30. J. Lygeros, C. Tomlin, and S. Sastry. Controllers for reachability specifications for hybrid systems. Automatica, 35:349–370, 1999.

31. I. Mitchell and C. Tomlin. Level set methods for computation in hybrid systems. In N. Lynch and B. H. Krogh, Eds, Hybrid Systems: Computation and Control, Lecture Notes in Computer Science, Vol. 1790, pp. 21–31. Springer, Berlin, 2000.

32. T. S. Motzkin, H. Raiffa, G. L. Thompson, and R. M. Thrall. The double description method. In H. W. Kuhn and A. W. Tucker, Eds, Contributions to Theory of Games, Vol. 2. Princeton University Press, Princeton, NJ, 1953.

33. L. Muñoz, X. Sun, R. Horowitz, and L. Alvarez. Traffic density estimation with the cell transmission model. In American Control Conference, pp. 3750–3755, 2003.

34. S. Prajna. Barrier certificates for nonlinear model validation. Automatica, 42(1):117–126, 2006.

35. A. Puri and P. Varaiya. Decidability of hybrid systems with rectangular differential inclusion. In Proceedings of the 6th International Conference on Computer Aided Verification, Lecture Notes in Computer Science, Vol. 818, pp. 95–104. Springer, Berlin, 1994.

36. O. Stursberg and B. H. Krogh. Efficient representation and computation of reachable sets for hybrid systems. In O. Maler and A. Pnueli, Eds, Hybrid Systems: Computation and Control, Lecture Notes in Computer Science, Vol. 2623, pp. 482–497. Springer, Berlin, 2003.

37. P. Varaiya. Reach set computation using optimal

control. Proceedings of KIT Workshop on Verification on Hybrid Systems . Verimag, Grenoble., 1998. “73648_C029” — #32

30 30.5 Conclusion

1. P. J. Antsaklis and A. Nerode. Hybrid control systems: An introductory discussion to the special issue. IEEE Trans. Automat. Control, 43(4):457–460, 1998.

2. P. J. Antsaklis, Ed., Proceedings of the IEEE, Special Issue on Hybrid Systems: Theory and Applications, Vol. 88. IEEE Press, Piscataway, NJ, July 2000.

3. R. A. Decarlo, M. S. Branicky, S. Pettersson, and B. Lennartson. Perspectives and results on the stability and stabilizability of hybrid systems. In P. J. Antsaklis, Ed., Proceedings of the IEEE: Special Issue on Hybrid Systems, Vol. 88, pp. 1069–1082. IEEE Press, Piscataway, NJ, July 2000.

4. J. P. Hespanha. Stabilization through hybrid control. In H. Unbehauen, Ed., Encyclopedia of Life Support Systems (EOLSS), Vol. Control Systems, Robotics, and Automation. Developed under the auspices of the UNESCO, Eolss Publishers, Oxford, UK, 2004.

5. M. Johansson. Piecewise Linear Control Systems—A Computational Approach, Lecture Notes in Control and Information Sciences, Vol. 284, Springer-Verlag, Berlin, 2002.

6. D. Liberzon and A. S. Morse. Basic problems in stability and design of switched systems. IEEE Control System Magazine, 19(5):59–70, 1999.

7. H. Lin and P. J. Antsaklis. Switching stabilizability for continuous-time uncertain switched linear systems. IEEE Transactions on Automatic Control, 52(4):633–646, 2007.

8. H. Lin and P. J. Antsaklis. Stability and stabilizability of switched linear systems: A survey of recent results. IEEE Transactions on Automatic Control, 54(2):308–322, 2009.

9. A. N. Michel. Recent trends in the stability analysis of hybrid dynamical systems. IEEE Transactions on Circuits and System I, 46(1):120–134, 1999.

10. A. P. Molchanov and E. Pyatnitskiy. Criteria of asymptotic stability of differential and difference inclusions encountered in control theory. Systems & Control Letters, 13:59–64, 1989.

11. S. Pettersson. Synthesis of switched linear systems. In Proceedings of 42nd IEEE Conference Decision Control, pp. 5283–5288, Maui, HI, 9-12, December 2003.

12. E. Skafidas, R. J. Evans, A. V. Savkin, and I. R. Petersen. Stability results for switched controller systems. Automatica, 35(4):553–564, 1999.

13. Z. Sun and S. S. Ge. Switched Linear Systems: Control and Design. Springer-Verlag, Berlin, 2005. “73648_C030” — #20

1. O'Dwyer, A., Handbook of PI and PID Controller Tuning Rules. Imperial College Press, London, 2009.

2. Åström, K.J. and Hägglund, T., Advanced PID Control, Instrumentation, Systems, and Automation Society, Research Triangle Park, NC, 2005.

3. Ziegler, J.G. andNichols, N.B., Optimum settings for automatic controllers,Trans. ASME, 64, 759, 1942.

4. Chien, K.L., Hrones, J.A., and Reswick, J.B., On the automatic control of generalized passive systems, Trans. ASME, 74, 175, 1952.

5. Cohen, G.H. andCoon, G.A., Theoretical consideration of retarded control,Trans. ASME, 75, 827, 1953.

6. Kaya, A. and Scheib, T.J., Tuning of PID controls of different structures, Control Eng., July, 62, 1988.

7. Åström, K.J. and Murray, R.M., Feedback Systems and Introduction for Scientists and Engineers. Princeton University Press, Princeton, NJ, 2008.

8. Dahlin, E.B., Designing and tuning digital controllers, Instr. Control Syst., 42, 77, 1968.

9. Morari, M. and Zafiriou, E., Robust Process Control, Prentice-Hall, Englewood Cliffs, NJ, 1989. "73648_C032" — #20

10. Messner,W.C., Classical control revisited: Variations on a theme. Proc. 10th Int. Workshop on Advanced Motion Control, AMC'08. Trento, 2008, pp. 15–20.

11. Garpinger, O. andHägglund, T. A., software tool for robust PIDdesign.Proc. 17th IFACWorld Congress, Seoul, Korea, 2008.

12. Hägglund, T. and Åström, K.J., Industrial adaptive controllers based on frequency response techniques, Automatica, 27, 599, 1991.

13. Åström, K.J., Hägglund, T., Hang, C.C., and Ho, W.K., Automatic tuning and adaptation for PID controllers—A survey, Control Eng. Prac., 1(4), 699, 1993.

14. Kraus, T.W. and Myron, T.J., Self-tuning PID controller

uses pattern recognition approach, Control Eng., June, 106, 1984.

1. Åström, K.J. and Wittenmark, B., Adaptive Control, Addison-Wesley, Reading, MA, 1989.

2. Bitmead, R.R., Gevers, M., and Wertz, V., Adaptive Optimal Control, Prentice Hall, Englewood Cliffs, NJ, 1990.

3. Clarke, D.W., Ed., Advances in Model-Based Predictive Control, Oxford University Press, UK, 1994.

4. Gawthrop, P.J., Continuous-Time Self-Tuning Control, Research Studies Press, Letchworth, UK, 1987.

5. Goodwin, G.C. and Sin, K.S.,Adaptive Filtering, Prediction and Control, PrenticeHall, EnglewoodCliffs, NJ, 1984.

6. Hang, C.C., Lee, T.H., and Ho, W.K., Adaptive Control, Instrument Society of America, Research Triangle Park, NC, 1993.

7. Harris, C.J. and Billings, S.A., Eds., Self-Tuning and Adaptive Control: Theory and Applications, Peter Perigrinus Ltd., Stevenage, UK, 1981.

8. Isermann, R., Lachmann, K.-H., and Drago, D., Adaptive Control Systems, Prentice Hall, Englewood Cliffs, NJ, 1992.

9. Mosca, E., Optimal Predictive and Adaptive Control, Prentice Hall, Englewood Cliffs, NJ, 1995.

10. Soeterboek, R., Predictive Control: A Unified Approach, Prentice Hall, Englewood Cliffs, NJ, 1992.

11. Wellstead, P.E. and Zarrop, M.B., Self-Tuning Systems, Wiley, New York, 1991. “73648_C033” — #36

1. Åström, K.J., Theory and applications of adaptive control—a survey, Automatica, 19(5), 471–486, 1983.

2. Ioannou, P.A. andDatta, A., Robust adaptive control: a unified approach,Proc. IEEE, 79(12), 1735–1768, 1991.

3. Kanellakopoulos, I., Kokotovic, P.V., and Morse, A.S., Systematic design of adaptive controllers for feedback linearizable systems, IEEE Trans. Autom. Control, 36, 1241–1253, 1991.

4. Morse, A.S., Global stability of parameter adaptive control systems, IEEE Trans. Autom. Control, 25, 433–439, 1980.

5. Narendra, K.S., Lin, Y.H., and Valavani, L.S., Stable adaptive controller design, II. Proof of stability, IEEE Trans. Autom. Control, 25(3), 440–448, 1980.

6. Parks, P.C., Lyapunov redesign of model reference adaptive control systems, IEEE Trans. Autom. Control, 11, 362–367, 1966. “73648_C034” #19

7. Rohrs, C.E., Valavani, L., Athans, M., and Stein, G., Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics, IEEE Trans. Autom. Control, 30(9), 881–889, 1985.

8. Taylor, L.W. and Adkins, E.J., Adaptive control and the X-15, Proc. Princeton University Conference on Aircraft Flying Qualities, Princeton University, 1965.

9. Whitaker, H.P., Yamron, J., and Kezer, A., Design of Model Reference Adaptive Control Systems for Aircraft, Report R-164, InstrumentationLaboratory,Massachusetts Institute of Technology, Cambridge, 1958.

10. Åström, K.J. and Eykhoff, P., System identification a survey, Automatica, 20(1), 123, 1971.

11. Bellman, R.E., Dynamic Control Processes—A Guided Tour, Princeton University Press, Princeton, NJ, 1961.

12. Egardt, B., Stability of Adaptive Controllers, Lecture Notes in Control and Information Sciences, Vol. 20, Springer-Verlag, Berlin, 1979.

13. Fel’dbaum, A.A., Optimal Control of Systems, Academic

Press, New York, 1965.

14. Goodwin, G.C, and Sin, K.C.,Adaptive Filtering Prediction and Control, Prentice Hall, EnglewoodCliffs, NJ, 1984.

15. Harris, C.J. and Billings, S.A., Eds., Self-Tuning and Adaptive Control: Theory and Applications, Peter Peregrinus, London, 1981.

16. Ioannou, P.A. and Kokotovic, P.V., Adaptive Systems with Reduced Models, Lecture Notes in Control and Information Sciences, Vol. 47, Springer-Verlag, New York, 1983.

17. Ioannou, P.A. and Sun, J., Robust Adaptive Control, Prentice Hall, Englewood Cliffs, NJ, 1996.

18. Landau, I.D., Adaptive Control: The Model Reference Approach, Marcel Dekker, New York, 1979.

19. Tsakalis, K.S. and Ioannou, P.A., Linear Time Varying Systems: Control and Adaptation, Prentice Hall, Englewood Cliffs, NJ, 1993.

Further Reading

Further details on MRAC for continuous-time plants can be found in the following textbooks:

1. Robust Adaptive Control, by P. Ioannou and J. Sun, Prentice Hall, Englewood Cliffs, NJ, 1996.

2. Stable Adaptive Systems, by K.S. Narendra and A. M. Annaswamy, Prentice Hall, Englewood Cliffs, NJ, 1989.

3. Adaptive Control: Stability, Convergence and Robustness, by S. Sastry and M. Bodson, Prentice Hall, Englewood Cliffs, NJ, 1989.

4. Adaptive Control: The Model Reference Approach, by I. D. Landau, Marcel Dekker, New York, 1979.

5. Adaptive Control, by K. J. Åström and B. Wittenmark, Addison-Wesley, Reading, MA, 1989.

More information on the design and analysis of MRAC for linear, time-varying plants can be found in the following monograph:

6. Linear Time-Varying Systems: Control and Adaptation, by K. Tsakalis and P. Ioannou, Prentice Hall, Englewood Cliffs, NJ, 1993.

More information on the design and analysis of robust MRAC can be found in the books 1, 2, 3, and 6 given above.

Details on MRAC for discrete-time plants can be found in the book 4 by Landau given above and in:

7. Adaptive Filtering Prediction and Control by G. Goodwin and K. Sin, Prentice Hall, Englewood Cliffs, NJ, 1984.

For applications of MRAC the reader is referred to the following books.

8. Self-Tuning and Adaptive Control: Theory and Applications, Edited by C. J. Harris and S. A. Billings, Peter Peregrinus, London, 1981.

9. Adaptive and Learning Systems: Theory and Applications, Edited by K. S. Narendra, Plenum Press, New York, 1986.

1. K.S. Tsakalis and P.A. Ioannou. Linear Time Varying Systems: Control and Adaptation. Prentice-Hall, Englewood Cliffs, NJ, 1993.

2. H.P.Whitaker, J. Yamron, andA.Kezer. Design ofModel ReferenceAdaptive Control Systems forAircraft. Report R-164, Instrumentation Laboratory, MIT, Cambridge, MA, 1958.

3. R.E. Kalman. Design of a self-optimizing control system. Trans. ASME, 80:468–478, 1958.

4. R.E. Bellman. Dynamic Programming. Princeton University Press, Princeton, NJ, 1957.

5. A.A. Feldbaum. Optimal Control Systems. Academic Press, New York, 1965.

6. P.C. Parks. Lyapunov redesign of model reference adaptive control systems. IEEE Trans. Automat. Control, 11:362–367, 1966.

7. A.S. Morse. Global stability of parameter adaptive control systems. IEEE Trans. Automat. Control, 25:433–439, 1980.

8. K.S. Narendra, Y.H. Lin, and L.S. Valavani. Stable adaptive controller design, Part II: Proof of stability. IEEE Trans. Automat. Control, 25:440–448, 1980.

9. I.D. Landau. Adaptive Control: The Model Reference Approach. Marcel Dekker, New York, 1979.

10. G.C. Goodwin, P.J. Ramadge, and P.E. Caines. Discrete-time multi-variable adaptive control. IEEE Trans. Automat. Control, 25:449–456, 1980.

11. B. Egardt. Stability ofAdaptiveControllers, LectureNotes inControl and Information Sciences, SpringerVerlag, Berlin, 1979.

12. P.A. Ioannou and P.V. Kokotovic. Instability analysis and improvement of robustness of adaptive control. Automatica, 20:583–594, 1984.

13. C.E. Rohrs, L. Valavani, M. Athans, and G. Stein. Robustness of continuous-time adaptive control algorithms

in the presence of unmodeled dynamics. IEEE Trans. Automat. Control, 30:881–889, 1985.

14. P.A. Ioannou and J. Sun. Theory and design of robust direct and indirect adaptive control schemes. Int. J. Control, 47:775–813, 1988.

15. L. Praly. Robust model reference adaptive controllers, Part I: Stability analysis. Proc. of the 22nd IEEE Conference on Decision and Control, San Antonio, Texas, 1984.

16. P.A. Ioannou and K.S. Tsakalis. A robust direct adaptive controller. IEEE Trans. Automat. Control, 31:1033–1043, 1986.

17. M. Krstic, I. Kanellakopoulos, and P. Kokotovic. Nonlinear and Adaptive Control Design. John Willey & Sons Inc, New York, 1995.

18. Y. Zhang, B. Fidan, and P.A. Ioannou. Backstepping control of linear time-varying systems with known and unknown parameters. IEEE Trans. Automat. Control, 48:1908–1925, 2003.

19. B. Fidan, Y. Zhang, and P.A. Ioannou. Adaptive control of a class of slowly time-varying systems with modeling uncertainties. IEEE Trans. Automat. Control, 50:915–920, 2005.

20. M.M. Polycarpou. Stable adaptive neural control scheme for nonlinear systems. IEEE Trans. Automat. Control, 41:447–451, 1996.

21. H. Xu and P.A. Ioannou. Robust adaptive control of linearizable nonlinear single input systems with guaranteed error bounds. Automatica, 40:1905–1911, 2003.

22. K.S. Narendra and J. Balakrishnan. Adaptive control using multiple models. IEEE Trans. Automat. Control, 42:171–187, 1997.

23. B.D.O. Anderson, T.S. Brinsmead, D. Liberzon, and A.S. Morse. Multiple model adaptive control with safe switching. Int. J. Adapt. Control Signal Process, 15:445–470, 2001.

24. A.S.Morse, D.Q.Mayne, andG.C. Goodwin. Applications of hysteresis switching in parameter adaptive control. IEEE Trans. Automat. Control, 37:1343–1354, 1992.

25. P.V. Zhivoglyadov, R.H. Middleton, and M. Fu. Localization based switching adaptive control for time-varying discrete-time systems. IEEE Trans. Automat. Control, 45:752–755, 2000.

26. M. Stefanovic and M.G. Safonov. Safe adaptive switching control: Stability and convergence. IEEE Trans. Automat. Control, 53:2012–2021, 2008. “73648_C035” — #22

27. S. Baldi, G. Battistelli, E. Mosca, and P. Tesi. Multi-model unfalsified adaptive switching supervisory control. Automatica, 46:249–259, 2010.

28. P.A. Ioannou and J. Sun. Robust Adaptive Control. Prentice-Hall, Englewood cliffs, NJ, 1996.

29. P.A. Ioannou and B. Fidan. Adaptive Control Tutorial. Advances in design and control, SIAM, Philadelphia, PA, 2006.

30. P.A. Ioannou and A. Datta. Robust adaptive control: Design, analysis and robustness bounds, in P.V. Kokotovic Ed., Grainger Lectures: Foundations of Adaptive Control, Springer-Verlag, New York, 1991.

31. J.P. Hespanha, D. Liberzon, and A.S. Morse. Hysteresis-based switching algorithms for supervisory control of uncertain systems. Automatica, 39:263–272, 2003.

32. A.S. Morse. Supervisory control of families of linear set-point controllers, Part 1: Exact matching. IEEE Trans. Automat. Control, 41:1413–1431, 1996.

33. S. Fekri, M. Athans, and A. Pascoal. Issues, progress and new results in robust adaptive control. Int. J. Adapt. Control Signal Process., 20:519–579, 2006.

34. S. Fekri, M. Athans, and A. Pascoal. Robust multiple model adaptive control (rmmac): A case study. Int. J. Adapt. Control Signal Process., 21:1–30, 2007.

35. P. Rosa, J.S. Shamma, C.J. Silvestre, and M. Athans. Integration of the stability overlay (so) with the robust multiple-model adaptive control (rmmac). Proc. of the 17th Mediterranean Conference on Automation and Control, Thessaloniki, Greece, 2009.

36. M. Stefanovic, R. Wang, A. Paul, and M.G. Safonov. Cost detectability and stability of adaptive control systems. Int. J. Robust Nonlinear Control, 17:549–561, 2007.

37. M.G. Safonov and T.C. Tsao. The unfalsified control concept and learning. IEEE Trans. Automat. Control, 42:843–847, 1997.

38. C. Manuelli, S.G. Cheong, E. Mosca, and M.G. Safonov. Stability of unfalsified adaptive control with non-scli controllers and related performance under different prior knowledge. Proc. of the European Control Conference, Kos, Greece, 2007.

39. A. Dehghani, B.D.O. Anderson, and A. Lanzon. Unfalsified adaptive control: A new controller implementation and some remarks. Proc. of the European Control Conference, Kos, Greece, 2007.

40. S. Baldi, P. Ioannou, and E. Mosca. Evaluation of identifier based and non-identifier based adaptive supervisory control using a benchmark example. 4th International Symposium on Communications Control and Signal Processing, Limassol, Cyprus, 2010.

41. M. Kuipers and P.A. Ioannou. Multiple model adaptive control with mixing. IEEE Trans. Automat. Control, to appear in 2010.

1. Amann, N., Owens, D. H., and Rogers, E., Iterative learning control for discrete-time systems with exponential rate of convergence, IEE Proceedings: Control Theory and Applications, 143(2), 217–224, 1996.

2. Arimoto, S., Kawamura, S., andMiyazaki, F., Bettering pperation of robots by learning, Journal of Robotic Systems, 1, 123–140, 1984.

3. Ayn,H.-S., Chen, Y.Q., andMoore, K. L., Iterative learning control: Brief survey and categorization, IEEE Transactions on Systems, Man, and Cybernetics—Part C: Applications and Reviews, 37(6), 1099–1121, 2007.

4. Bien, Z. and Xu, J.-X., Iterative Learning Control: Analysis, Design, Integration and Applications, Boston: Kluwer Academic Publishers, 1998.

5. Bristow, D. A., Tharayil, M., and Alleyne, A. G., A survey of iterative learning control, Control Systems Magazine, 26(3), 96–114, 2006.

6. Chen, Y. and Wen, C., Iterative Learning Control: Convergence, Robustness, and Applications, London: Springer, 1999.

7. de Roover, D. and Bosgra, O. H., Synthesis of robust multivariable iterative learning controllers with application to a wafer stage motion system, International Journal of Control, 73(10), 968–979, 2000.

8. Gunnarsson, S. and Norrlof, M., On the design of ILC algorithms using optimization, Automatica, 37(12), 2011–2016, 2001.

9. Horowitz, R., Learning control of robot manipulators, Transactions of the ASME Journal of Dynamic Systems, Measurement and Control, 115(2B), 402–411, 1993.

10. Lee, J. H., Lee, K. S., and Kim, W. C., Model-based iterative learning control with a quadratic criterion for time-varying linear systems, Automatica, 36(5), 641–657, 2000.

11. Longman, R. W., Iterative learning control and repetitive control for engineering practice, International Journal of Control, 73(10), 930–954, 2000.

12. Moore, K. L., Iterative Learning Control for
Deterministic Systems, London: Springer-Verlag, 1993.

13. Moore, K. L., Dahleh, M., and Bhattacharyya, S. P.,
Iterative learning control: A survey and new results,
Journal of Robotic Systems, 9(5), 563–594, 1992.

14. Norrlof,M. andGunnarsson, S., Timeand frequencydomain
convergenceproperties in iterative learning control,
International Journal of Control, 75(14), 1114–1126, 2002.

15. Saab, S. S., On a discrete-time stochastic learning
control algorithm, IEEE Transactions on Automatic Control,
46(8), 1333–1336, 2001.

16. Steinbuch, M., Bosgra, O., and Dynamics and Control
Technology Group, Technical University of Eindhoven,
Eindhoven, The Netherlands, personal communication, 2007.

17. Tomizuka, M., Tsao, T.-C., and Chew, K.-K., Analysis
and synthesis of discrete-time repetitive controllers,
Journal of Dynamic Systems, Measurement and Control,
Transactions ASME, 111(3), 353–358, 1989.

18. Xu, J.-X. and Tan, Y., Linear and Nonlinear Iterative
Learning Control, Berlin: Springer, 2003. “73648_C036” —
#20 “73648_S006” #1 VI

Analysis andDesign

ofNonlinear

Systems VI-1 “73648_S006” #2

1. Khalil, H.K. 2002. Nonlinear Systems, 3rd Edn., Prentice-Hall, Upper Saddle River, NJ.

2. Kokotovic, P., Khalil, H.K. and O'Reilly, J. 1999. Singular Perturbation Methods in Control: Analysis and Design, Classic Edition, SIAM, Pennsylvania, PA.

For Further Information

Ourpresentationof the asymptoticmethods is based on the textbookbyKhalil [1]. For further information

on this topic, the reader is referred to Chapters 10 and 11 of Khalil's book. Chapter 10 covers the

perturbation method and averaging, and Chapter 11 covers the singular perturbation method. Proofs of

the results stated here are given in the book.

The discussion of model order reduction is based on Chapter 1 of Kokotovic et al. [2]. This book gives a

broader view of the use of singular perturbation methods in systems and control. Modeling two timescale

systems in the singularly perturbed form is discussed in Chapters 1 and 2 of Kokotovic [2] and Chapter 11

of Khalil [1].

For a broader view of the averaging method, the reader may consult Sanders, J.A., Verhulst, F. and

Murdock, J. 2007. Averaging Methods in Nonlinear Dynamical Systems, 2nd Edn., Springer, Berlin. A

unified treatment of singular perturbation and averaging is given in Teel, A.R., Moreau, L. and Nesic, D.

2003. A unified framework for input-to-state stability in systems with two timescales, IEEE Transactions

on Automatic Control, vol. 48, no. 9, 1526–1544.

40 Volterra and Fliess Series Expansions for Nonlinear Systems

1. d’Alessandro, P., Isidori, A., and Ruberti, A., Realizations and structure theory of bilinear dynamical systems, SIAM J. Control, 12, 517–535, 1974.

2. Baccar, S., Lamnabhi-Lagarrigue, F., and Salembier, G., Utilisation du calcul formel pour lamodélisation et la simulation des circuits électroniques faiblement non linéaires, Annales des Télécommunications, 46, 282–288, 1991.

3. Bianchini, R.M. and Stefani, G., A high order maximum principle and controllability, 1991.

4. Brezinski, C., Padé-type approximants and general orthogonal polynomials, INSM, 50, Birkhaüser, 1980.

5. Brockett, R.W., Nonlinear systems and differential geometry, Proceedings IEEE, 64, 61–72, 1976.

6. Brockett, R.W., Volterra series and geometric control theory, Automatica, 12, 167–176, 1976; Brockett, R.W. and Gilbert, E.G., An addendum to Volterra series and geometric control theory, Automatica, 12, 635, 1976.

7. Chen, K.T., Integration of paths, geometric invariants and a generalized Baker-Hausdorff formula, Ann. Math., 65, 163–178, 1957.

8. Crouch, P.E., Dynamical realizations of finite Volterra series, SIAM J. Control and Optimization, 19, 177–202, 1981.

9. Crouch, P.E., Solvable approximations to control systems, SIAM J. Control and Optimization, 22(1), 40–54, 1984.

10. Crouch, P.E. and Lamnabhi-Lagarrigue, F., Algebraic and multiple integral identities, Acta Applicanda Mathematicae, 15, 235–274, 1989. “73648_C040” #17

11. Feynman, R.P., An operator calculus having applications in quantum electrodynamics, Physical Review, 84, 108–128, 1951.

12. Fliess, M., Fonctionnelles causales non linéaires et indéterminées non commutatives, Bull. Soc. Math. France, 109, 3–40, 1981.

13. Fliess, M., Lamnabhi, M., and Lamnabhi-Lagarrigue, F., Algebraic approach to nonlinear functional expansions, IEEE CS, 30, 550–570, 1983.

14. Fliess, M., and Lamnabhi-Lagarrigue, F., Application of a new functional expansion to the cubic anharmonic oscillator, J. Math. Physics, 23, 495–502, 1982.

15. Gilbert, E.G., Functional expansions for the response of nonlinear differential systems, IEEE AC, 22, 909–921, 1977.

16. Gröbner, W., Die Lie-Reihen und ihre Anwendungen (2 e édition), VEB Deutscher Verlag der Wissenschaften, Berlin, 1967.

17. Grunenfelder, L., Algebraic aspects of control systems and realizations, J. Algebra, 165, 446–464, 1994.

18. Hassine, L., Toffano, Z., Lamnabhi-Lagarrigue, F. Destrez, A., and Birocheau, C., Volterra functional series expansions for semiconductor lasers under modulation, IEEE J. Quantum Electron., 30, 918–928, 1994; Hassine, L., Toffano, Z., Lamnabhi-Lagarrigue, F., Joindot, I., and Destrez, A., Volterra functional series for noise in semiconductor lasers, IEEE J. Quantum Electron., 30, 2534–2546, 1994.

19. Hoang Ngoc Minh, V. and Jacob, G., Evaluation transform and its implementation in MACSYMA, in New Trends in Systems Theory, G. Conte, A.M. Perdon, B. Wyman, Eds., Progress in Systems and Control Theory, Birkhäuser, Boston, 1991.

20. Jacob, G., Algebraic methods and computer algebra for nonlinear systems' study, Proc. IMACS Symposium MCTS, 1991.

21. Jacob, G., Motion planning by piecewise constant or polynomial inputs, Proc. NOLCOS'92, 1992.

22. Jacob, G. and Lamnabhi-Lagarrigue, F., Algebraic Computing in Control, Lect. Notes Contr. Inform. Sc., Springer-Verlag, 165, 1991.

23. Hespel, C. and Jacob, G., Truncated bilinear approximants: Carleman, finite Volterra, Padé-type, geometric and structural automata, in Algebraic Computing in Control, Lect. Note Contr. Inform. Sc., G. Jacob and F. Lamnabhi-Lagarrigue, Eds., Springer-Verlag, 165, 1991.

24. Jakubczyk, B. and Kaskosz, B., Realizability of Volterra series with constant kernels, Nonlinear Analysis, Theory, Methods and Applications, 5, 167–183, 1981.

25. Krener, A.J., Local approximations of control systems, J. Differential Equations, 19, 125–133, 1975.

26. Lafferriere, G. and Sussmann,H.J.,Motion planning for controllable systemswithout drift: a preliminary report, Report SYCON-90-04, Rutgers Center for Systems and Control, June 1990.

27. Lamnabhi, M., A new symbolic response of nonlinear systems, Systems and Control Letters, 2, 154–162, 1982.

28. Lamnabhi, M., Functional analysis of nonlinear circuits: a generating power series approach, IEE Proceedings, 133, 375–384, 1986.

29. Lamnabhi-Lagarrigue, F., Séries de Volterra et commande optimale singulière, Thèse d'état, Université Paris-Sud, Orsay, 1985.

30. Lamnabhi-Lagarrigue, F., Analyse des systèmes non linéaires, Hermes Ed., 1994.

31. Lamnabhi-Lagarrigue F. and Lamnabhi, M., Détermination algébrique des noyaux de Volterra associés à certains systèmes non linéaires. Ricerche di Automatica, 10, 17–26, 1979.

32. Lamnabhi-Lagarrigue, F. and Stefani, G., Singular optimal control problems: on the necessary conditions for optimality, SIAM J. Control Optim., 28, 823–840, 1990.

33. Lamnabhi-Lagarrigue, F., Leroux, P., and Viennot, X.G., Combinatorial approximations of Volterra series by bilinear systems, in Analyse des Systèmes Contrôlés, B. Bonnard, B. Bride, J.P. Gautier, and I. Kupka, Eds., Progress in Systems and Control Theory, Birkhäuser, 304–315, 1991.

34. Leroux, P. and Viennot, X.G., A combinatorial approach to nonlinear functional expansions,Theoritical Computer Science, 79, 179–183, 1991.

35. Leroux, P., Martin A., and Viennot, X.G., Computing iterated derivatives along trajectories of nonlinear systems, NOLCOS'92, M. Fliess, Ed., Bordeaux, 1992.

36. Lesiak, C. and Krener, A.J., The existence and

uniqueness of Volterra series for nonlinear systems, IEEE AC, 23, 1090–1095, 1978.

37. Magnus, W., On the exponential solution of differential equations for a linear operator, Comm. Pure Appl. Math., 7, 649–673, 1954.

38. Martin, A., Calcul d'approximations de la solution d'un système non linéaire utilisant le logiciel SCRATCHPAD, in Algebraic Computing in Control, Lect. Note Contr. Inform. Sc., G. Jacob and F. Lamnabhi-Lagarrigue, Eds., Springer-Verlag, 165, 1991.

39. Rugh, W.J., Nonlinear System Theory, The John Hopkins University Press, Baltimore, MD, 1981. "73648_C040" #18

40. Schetzen, I.W., The Volterra and Wiener Theories of Nonlinear Systems, John Wiley & Sons, New York, 1980.

41. Schutzenberger, M.P., On the definition of a family of automata, Information and Control, 4, 245–270, 1961.

42. Stefani, G., Volterra approximations, NOLCOS'90, Nantes, 1990.

43. Stefani, G. and Zezza, P., A new type of sufficient optimality conditions for a nonlinear constrained optimal control problem, NOLCOS'92, Bordeaux, 1992.

44. Sontag, E.D., Bilinear realizability is equivalent to existence of a singular affine differential input/output equation, Syst. Control Lett., 11, 181–187, 1988.

45. Sussmann, H.J., Lie brackets and local controllability: A sufficient condition for scalar-input systems, SIAM J. Control Optimiz., 21, 686–713, 1983.

46. Sussmann, H.J., Local controllability and motion planning for some classes of systems with drift, Proc. 30th IEEE CDC, 1110–1113, 1991.

47. Tang, Y.–S., Mees, A.I., and Chua, L.O., Hopf bifurcation via Volterra series, IEEE AC, 28(1), 42–53, 1983.

48. Volterra, V., Theory of Functionals (translated from Spanish), Blackie, London, 1930 (reprinted by Dover, New York, 1959).

49. Wang, Y. and Sontag, E.D., Generating series and

nonlinear systems: Analytic aspects, local realizability, and input-output representations, Forum Math., 4, 299-322, 1992; Algebraic differential equations and rational control systems, SIAM J. Control Optimiz.,

50. Wei, J. and Norman, E., On global representations of the solutions of linear differential equations as a product of exponentials, Proc. Am. Math. Soc., 15, 327-334, 1964.

51. Wiener, N., Nonlinear Problems in Random Theory, John Wiley & Sons, New York, 1958.

1. Bianchini, R. M. and G. Stefani. Controllability along a trajectory: A variational approach, SIAM Journal on Control and Optimization, 31(4):900–927, 1993.

2. Bloch, A. M., with J. Baillieul, P. E. Crouch and J. E. Marsden. Nonholonomic Mechanics and Control, Springer-Verlag, New York, 2003.

3. Bloch, A. M. and S. V. Drakunov. Stabilization and tracking in the nonholonomic integrator via sliding modes, Systems and Control Letters, 29:91–99, 1996.

4. Bloch,A.M., S.V.Drakunov, andM.K.Kinyon. Stabilization of nonholonomic systemsusing isospectral flows, SIAM Journal on Control and Optimization, 38(3):855–874, 2000.

5. Bloch, A.M., P. S. Krishnaprasad, J. E. Marsden, and R. Murray. Nonholonomic mechanical systems with symmetry, Archive for Rational Mechanical Analysis., 136:21–99, 1996.

6. Bloch, A. M., J. E. Marsden, and D. V. Zenkov. Quasivelocities and stabilization of relative equilibria of underactuated nonholonomic systems, Proceedings of the IEEE Conference on Decision and Control, Shanghai, China, vol. 48, pp. 3335–3340, 2009.

7. Bloch, A.M.,M. Reyhanoglu, andH.McClamroch. Control and stabilization of nonholonomic dynamic systems, IEEE Transations on Automatic Control, 37(11):1746–1757, 1992.

8. Brockett, R.W. Control theory and singular Riemannian geometry, in New Directions in Applied Mathematics, P. J. Hilton and G. S. Young (Eds), Springer-Verlag, New York, pp. 11–27, 1981.

9. Brockett, R.W. Nonlinear control theory and differential geometry, in Proceedings of International Congress of Mathematicians, Warsaw, Poland, 1357–1368, 1983.

10. Brockett, R.W.Asymptotic stability and feedback stabilization, inDifferential Geometric Control Theory, R. Brockett, R. Millman, and H. Sussmann (Eds), Birkhauser, Boston, pp. 181–191, 1983.

11. Bullo, F. and A. D. Lewis. Geometric Control of Mechanical Systems, Texts in Applied Mathematics, Vol. 49, Springer-Verlag, New York, 2005.

12. Bullo, F. and K.M. Lynch. Kinematic controllability for decoupled trajectory planning of underactuated mechanical systems, IEEE Transactions on Robotics and Automation, 17(4):402-412, 2001.

13. Choset, H., K. M. Lynch, S. Hutchinson, G. Kantor, W. Burgard, L. Kavraki, and S. Thrun, Principles of Robot Motion, MIT Press, Cambridge, MA, 2005.

14. Coron, J.M. A necessary condition for feedback stabilization, Systems and Control Letters, 14(3): 227-232, 1990.

15. Coron, J.M. Global asymptotic stabilization for controllable systems without drift, Mathematics of Controls, Signals and Systems, 5(3):295-312, 1992.

16. Drakunov, S. V., T. Floquet, and W. Perruquetti. Stabilization and tracking for an extended Heisenberg system with a drift, Systems and Control Letters, 54(5):435-445, 2005.

17. LaValle, S. M. Planning Algorithms, Cambridge University Press, Cambridge, UK, 2006.

18. Murray, R. M., Z. Li, and S. Sastry. A Mathematical Introduction to Robotic Manipulation, CRC Press, Boca Raton, FL, 1994.

19. Neimark J. I and N. A. Fufaev. Dynamics of Nonholonomic Systems, Series Translations of Mathematical Monographs. AMS, Providence, Rhode Island: vol. 33, 1972.

20. Nijmeijer, H. and A.J. van der Schaft. Nonlinear Dynamical Control Systems, Springer-Verlag, New York, 1990.
“73648_C042” — #36

21. Reyhanoglu, M., A. J. van der Schaft, N.H. McClamroch, and I. Kolmanovsky. Dynamics and control of a class of underactuated mechanical systems, IEEE Transactions on Automatic Control, 44(9): 1663-1671, 1999.

22. Reyhanoglu, M., S. Cho, and N.H. McClamroch. Discontinuous feedback control of a class of underactuated mechanical systems, International Journal of Robust and Nonlinear Control, 10(4): 265-281, 2000.

23. Sira-Ramirez, H. and S. K. Agrawal. Differentially Flat Systems, CRC Press, Boca Raton, FL, 2004.

24. Sussmann, H. J. Lie brackets, real analyticity and geometric control, in Differential Geometric Control Theory, R. Brockett, R. Millman, and H. Sussmann (Eds), Birkhauser, Boston, 1983.

25. Sussmann, H. J. A general theorem on local controllability, SIAM Journal on Control and Optimization, 25(1):158-194, 1987.

26. Zenkov, D. V. Linear conservation laws of nonholonomic systems with symmetry, Discrete and Continuous Dynamical Systems, Extended volume: 963-972, 2003. “73648_S007” #1 VII

1. Georgiou, T.T. and Smith, M.C., Optimal robustness in the gap metric, IEEE Trans. Auto. Control, 35, 673-686, 1990.

2. Jiang, Z.P., Teel, A.R., and Praly, L., Small-gain theorem for ISS systems and applications,Math. Control, Sign., Syst., 7(2), 95-120, 1995.

3. Mareels, I.M.Y. and Hill, D.J., Monotone stability of nonlinear feedback systems, J. Math. Syst., Est. Control, 2(3), 275-291, 1992.

4. McFarlane, D.C. and Glover, K., Robust Controller Design Using Normalized Coprime Factor Plant Descriptions, Lecture Notes in Control and Information Sciences, Springer-Verlag, vol. 138, 1989.

5. Popov, V.M., Absolute stability of nonlinear systems of automatic control, Auto. Remote Control, 22, 857-875, 1961.

6. Safonov, M., Stability and Robustness of Multivariable Feedback Systems, The MIT Press, Cambridge, MA, 1980.

7. Sandberg, I.W., On theL 2 -boundedness of solutions of nonlinear functional equations, Bell Sys. Tech. J., 43, 1581-1599, 1964.

8. Sontag, E., Smooth stabilization implies coprime factorization, IEEE Trans. Auto. Control, 34, 435-443, 1989.

9. Zames, G., On the input-output stability of time-varying nonlinear feedback systems, Part I: Conditions using concepts of loop gain, conicity, and positivity, IEEE Trans. Auto. Control, 11, 228-238, 1969.

10. Zames,G., On the input-output stability of time-varying nonlinear feedback systems, Part II: Conditions involving circles in the frequency plane and sector nonlinearities, IEEE Trans. Auto. Control, 11, 465- 476, 1966.

1. D. Angeli, B. Ingalls, E.D. Sontag, and Y. Wang. Separation principles for input-output and integralinput-to-state stability. SIAM J. Control Optim., 43(1):256–276, 2004.

2. D. Angeli, E.D. Sontag, and Y. Wang. A characterization of integral input-to-state stability. IEEE Trans. Automat. Control, 45(6):1082–1097, 2000.

3. D. Angeli, E.D. Sontag, and Y. Wang. Further equivalences and semiglobal versions of integral input to state stability. Dynam. Control, 10(2):127–149, 2000.

4. J.C. Doyle, B. Francis, and A. Tannenbaum. Feedback Control Theory. MacMillan Publishing Co., London, 1990.

5. R.A. Freeman and P.V. Kokotovic´. Robust Nonlinear Control Design, State-Space and Lyapunov Techniques. Birkhauser, Boston, 1996.

6. B. Ingalls, E.D. Sontag, and Y. Wang. An infinite-time relaxation theorem for differential inclusions. Proc. Am. Math. Soc., 131(2):487–499, 2003.

7. A. Isidori. Nonlinear Control Systems II. Springer-Verlag, London, 1999.

8. A. Isidori, L. Marconi, and A. Serrani. Robust Autonomous Guidance: An Internal Model-Based Approach. Springer-Verlag, London, 2003.

9. Z.-P. Jiang and L. Praly. Preliminary results about robust lagrange stability in adaptive nonlinear regulation. Intern. J. Control, 6:285–307, 1992.

10. Z.-P. Jiang, A. Teel, and L. Praly. Small-gain theorem for ISS systems and applications. Math. Control, Signals, and Systems, 7:95–120, 1994.

11. G.Kaliora,A.Astolfi, andL. Praly.Normestimators andglobal output feedback stabilizationof nonlinear systems with ISS inverse dynamics. In Proc. 43rd IEEE Conference on Decision and Control, Paradise Island, Bahamas, p. ThC07.2, 2004.

12. H.K. Khalil. Nonlinear Systems, 2nd edn. Prentice-Hall, Upper Saddle River, NJ, 1996.

13. M. Krichman, E.D. Sontag, and Y. Wang.
Input-output-to-state stability. SIAM J. Control Optim.,
39(6):1874–1928, 2001.

14. M. Krstic´ and H. Deng. Stabilization of Uncertain
Nonlinear Systems. Springer-Verlag, London, 1998.

15. M. Krstic´, I. Kanellakopoulos, and P.V. Kokotovic´.
Nonlinear and Adaptive Control Design. John Wiley & Sons,
New York, 1995.

16. Y. Lin, E.D. Sontag, and Y. Wang. A smooth converse
Lyapunov theorem for robust stability. SIAM J. Control
Optim., 34(1):124–160, 1996.

17. M. Malisoff, L. Rifford, and E.D. Sontag. Global
asymptotic controllability implies input-to-state
stabilization. SIAM J. Control Optim., 42(6):2221–2238,
2004.

18. J.L. Massera. Contributions to stability theory. Ann.
Math., 64:182–206, 1956.

19. L. Praly and Y. Wang. Stabilization in spite of matched
unmodelled dynamics and an equivalent definition of
input-to-state stability. Math. Control, Signals, and
Systems, 9:1–33, 1996.

20. R. Sepulchre, M. Jankovic, and P.V. Kokotovic´.
Constructive Nonlinear Control. Springer-Verlag, New York,
1997. “73648_C045” — #21

21. E.D. Sontag. Smooth stabilization implies coprime
factorization. IEEE Trans. Automat. Control, 34(4):435–443,
1989.

22. E.D. Sontag. Further facts about input to state
stabilization. IEEE Trans. Automat. Control, 35(4):
473–476, 1990.

23. E.D. Sontag. Comments on integral variants of ISS.
Systems Control Lett., 34(1–2):93–100, 1998.

24. E.D. Sontag. Mathematical Control Theory. Deterministic
Finite-Dimensional Systems, Vol. 6 Texts in Applied
Mathematics. 2nd edn., Springer-Verlag, New York, 1998.

25. E.D. Sontag. Stability and stabilization:
Discontinuities and the effect of disturbances. In
Nonlinear analysis, differential equations and control

(Montreal, QC, 1998), Vol. 528 NATO Sci. Ser. C Math. Phys. Sci., pp. 551–598. Kluwer Acad. Publ., Dordrecht, 1999.

26. E.D. Sontag. Input to state stability: Basic concepts and results. In P. Nistri and G. Stefani, editors, Nonlinear and Optimal Control Theory, pp. 163–220. Springer-Verlag, Berlin, 2006.

27. E.D. Sontag and A.R. Teel. Changing supply functions in input/state stable systems. IEEE Trans. Automat. Control, 40(8):1476–1478, 1995.

28. E.D. Sontag and Y. Wang. On characterizations of the input-to-state stability property. Systems Control Lett., 24(5):351–359, 1995.

29. E.D. Sontag and Y. Wang. New characterizations of input-to-state stability. IEEE Trans. Automat. Control, 41(9):1283–1294, 1996.

30. E.D. Sontag and Y. Wang. Lyapunov characterizations of input to output stability. SIAM J. Control Optim., 39(1):226–249, 2000.

31. J.C. Willems. Mechanisms for the stability and instability in feedback systems. Proc. IEEE, 64:24–35, 1976. “73648_C045” — #22 “73648_S008” #1 VIII

Design VIII-1 “73648_S008” #2

46 Feedback Linearization of Nonlinear Systems

1. Jakubczyk, B. and Respondek, W., On linearization of control systems, Bull. Acad. Polonaise Sci. Ser. Sci. Math., 28, 517–522, 1980.

2. Su, R., On the linear equivalents of nonlinear systems, Syst. Control Lett., 2, 48–52, 1982.

3. Isidori, A., Krener, A.J., Gori-Giorgi, C., and Monaco, S., Nonlinear decoupling via feedback: a differential geometric approach, IEEE Trans. Autom. Control, 26, 331–345, 1981.

4. Hunt, L.R., Su, R., and Meyer, G., Design for multi-input systems, in Differential Geometric Control Theory, Brockett, R.W., Millman, R.S., and Sussmann, H.J., Eds., Birkhauser, 268–298, 1983.

5. Isidori, A., Nonlinear Control Systems, 2nd Ed., Springer-Verlag, 1989. “73648_C046” — #16

47 The Steady-State Behavior of a Nonlinear System

1. Birkhoff, G.D., Dynamical Systems, American Mathematical Society, 1927.

2. Isidori, A. and Byrnes, C.I., Steady-state behaviors in nonlinear systems, with an application to robust disturbance rejection, Annual Reviews in Control, 32, 1–16, 2008.

3. Hale J.K., Magalhães L.T., and Oliva W.M., Dynamics in Infinite Dimensions, Springer-Verlag, New York, NY, 2002.

* For a definition of input-to-state stability, see Chapter 45.

48 Nonlinear Output Regulation

1. Francis, B.A., The linear multivariable regulator problem, SIAM J. Control Optimiz., 14, 486–505, 1976.

2. Francis, B.A. and Wonham, W.M., The internal model principle of control theory, Automatica, 12, 457–465, 1977.

3. Isidori, A. and Byrnes, C.I., Output regulation of nonlinear systems, IEEE Trans. Automat. Control, 35, 131–140, 1990. “73648_C048” — #17

4. Marconi, L., Praly, L., and Isidori, A., Output stabilization via nonlinear Luenberger observers, SIAM J. Control Optimiz., 45, 2277–2298, 2006.

5. Gauthier, J.P. and Kupka,I., Deterministic Observation Theory and Applications, Cambridge University Press, Cambridge, UK 2001.

6. Byrnes, C.I. and Isidori, A., Nonlinear internal models for output regulation, IEEE Trans. Automat. Control, 49, 2244–2247, 2004.

7. Marconi, L. and Praly, L., Uniform practical nonlinear output regulation, IEEE Trans. Automat. Control, 53, 1184–1202, 2008.

8. Esfandiari, F, and Khalil, H., Output feedback stabilization of fully linearizable systems, Int. J. Control, 56, 1007–1037, 1992.

9. Byrnes, C.I. and Isidori, A., Asymptotic stabilization of minimum phase nonlinear systems, IEEE Trans. Automat. Control, 36, 1122–1137, 1991.

10. Isidori, A., Marconi, L., and Serrani, A., Robust Autonomous Guidance: An Internal-Model Approach, Springer-Verlag, London, 2003.

11. Huang, J., Nonlinear Output Regulation: Theory and Applications, PA, SIAM, Philadephia, 2004.

12. Pavlov, A., van de Wouw, N., and Nijmeijer, H., Uniform Output Regulation of Nonlinear Systems: A Convergent Dynamics Approach, Birkhauser, Boston, 2006. “73648_C048” — #18

1. Sira-Ramirez, H., Nonlinear P-I controller design for switchmode dc-to-dc power converters, IEEE Trans. Circuits Systems, 38(4), 410–417, 1991. “73648_C050” — #22

2. DeCarlo, R.A., Z˙ak, S.H., and Matthews, G.P., Variable structure control of nonlinear multivariable systems: A tutorial, Proc. IEEE, 76(3), 212–232, 1988.

3. Hui, S. and Z˙ak, S.H., Robust control synthesis for uncertain/nonlinear dynamical systems, Automatica, 28(2), 289–298, 1992.

4. Matthews,G.P. andDeCarlo,R.A.,Decentralized tracking for a class of interconnectednonlinear systems using variable structure control, Automatica, 24(2), 187–193, 1988.

5. Utkin, V.I., Sliding Modes and Their Application in Variable Structure Control, Mir, Moscow, 1978.

6. Utkin, V.I., Sliding Modes in Control and Optimization, Springer, Berlin, 1992.

7. Filippov, A.F.,Differential Equations withDiscontinuous Righthand Sides, Kluwer Academic, Dordrecht, The Netherlands, 1988.

8. Hunt, L.R., Su, R., and Meyer, G., Global transformations of nonlinear systems, IEEE Trans. Automat. Control, AC-28(1), 24–31, 1983.

9. Young, K.-K.D., Kokotovic´, P.V., andUtkin, V.I., A singular perturbation analysis of high-gain feedback systems, IEEE Trans. Automat. Control, AC-22(6), 931–938, 1977.

10. DeCarlo, R.A., Drakunov, S., and Li, Xiaoqui, A unifying characterization of sliding mode control: A Lyapunov approach, ASME J. Dynamic Systems, Measurement, Control, special issue on Variable Structure Systems, 122(4), 708–718, 2000.

11. Corless, M.J. and Leitmann, G., Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems, IEEE Trans. Automat. Control, AC-26(5), 1139–1144, 1981.

12. Edwards, C. and Spurgeon, S.K., On the development of discontinuous observers, Int. J. Control, 59(5), 1211–1229,

1994.

13. Hui, S. and Z˙ak, S.H., Observer design for systems with unknown inputs, Int. J. Appl. Math. Comput. Sci., 15(4), 431–446, 2005.

14. Perruquetti, W. and Barbot, J.P., Sliding Mode Control in Engineering, Marcel Dekker, New York, 2002.

15. Edwards, C. and Spurgeon, S.K., Sliding Mode Control: Theory and Applications, Taylor & Francis, London, UK, 1998.

16. Sabanovic, A., Fridman, L., and Spurgeon S. (Eds), Variable Structure Systems: From Principles to Implementation, IEE Control Engineering Series, The Institute of Electrical Engineers, UK, 2004.

17. Drakunov, S.V., Izosimov, D.B., Lukyanov, A. G., Utkin, V. A., and Utkin, V.I., The block control principle, Part II, Autom. Remote Control, 51(6), 737–746, 1990.

18. Drakunov, S. V., Adaptive quasioptimal filters with discontinuous parameters, Autom. Remote Control, 44(9), 1167–1175, 1984.

19. Rundell, A., Drakunov, S., and DeCarlo, R., A sliding mode observer and controller for stabilization of rotational motion of a vertical shaft magnetic bearing, IEEE Trans. Control Systems Technol., 4(5), 598–608, 1996.

20. Drakunov, S.V., Sliding-mode observers based on equivalent control method, Proc. 31st IEEE Conf. Decision Control, Tucson, AZ, pp. 2368–2369, 1992.

21. Barbot, J.P., Boukhobza, T., and Djemai, M., Slidingmode observer for triangular input form. Proc. 35th IEEE CDC, Kobe, Japan, 1996.

22. Fridman, L., Levant, A., andDavila, J., Observation of linear systemswith unknown inputs via high-order sliding modes, Int. J. Syst. Sci., 38(10), 773–791, 2007.

23. Utkin, V., Guldner, J., and Shi, J., Sliding Model Control in Electromechanical Systems, Taylor & Francis, London, 1999.

24. El-Khazali, R. and DeCarlo, R.A., Output feedback variable structure control design, Automatica, 31(5), 805–816, 1995.

25. Matthews, G. and DeCarlo, R.A., Decentralized variable structure control of interconnected multiinput/multi-output nonlinear systems, Circuits Syst. Signal Process., 6(3), 363–387, 1987.

26. Emelyanov, S.V., Korovin, S.K., and Levantovsky, L.V., Second-order sliding modes in controlling uncertain systems, Sov. J. Comput. Syst. Sci., 24(4), 63–68, 1986.

27. Li, X. and DeCarlo, R.A., Robust sliding mode control of uncertain time-delay systems, Int. J. Control, 76(13), 1296–1305, 2003.

28. Walcott, B.L. and Z˙ak, S.H., State observation of nonlinear/uncertain dynamical systems, IEEE Trans. Automat. Control, AC-32(2), 166–170, 1987.

29. Bengea, S., Li, X., and DeCarlo, R.A., Combined controller–observer design for time delay systems with application to engine idle speed control, ASME J. Dyn. Sys., Meas. Control, 126, 2004.

53 Adaptive Nonlinear Control

1. A. Astolfi, D. Karagiannis, and R. Ortega, Nonlinear and Adaptive Control with Applications, Springer, Berlin, 2008.

2. D. M. Dawson, J. Hu, and T. C. Burg, Nonlinear Control of Electric Machinery, Marcel Dekker, 1998.

3. M. French, C. Szepesvari, and E. Rogers, Performance of Nonlinear Approximate Adaptive Controllers, Wiley, New York, 2003.

4. Z. P. Jiang and J. B. Pomet, Combining backstepping and time-varying techniques for a new set of adaptive controllers, Proceedings of the 33rd IEEE Conference on Decision and Control, Lake Buena Vista, FL, December 1994, pp. 2207–2212.

5. I. Kanellakopoulos, P. V. Kokotovic´, and A. S. Morse, Systematic design of adaptive controllers for feedback linearizable systems, IEEE Transactions on Automatic Control, 36, 1241–1253, 1991.

6. I. Kanellakopoulos, P. V. Kokotovic´, and A. S. Morse, Adaptive nonlinear control with incomplete state information, International Journal of Adaptive Control and Signal Processing, 6, 367–394, 1992. “73648_C053” — #25

7. H. Khalil, Adaptive output-feedback control of nonlinear systems represented by input–output models, Proceedings of the 33rd IEEE Conference on Decision and Control, Lake Buena Vista, FL, December 1994, pp. 199–204.

8. M. Krstic´, I. Kanellakopoulos, and P. V. Kokotovic´, Adaptive nonlinear control without overparametrization, Systems and Control Letters, 19, 177–185, 1992.

9. M. Krstic´, I. Kanellakopoulos, and P. V. Kokotovic´, Nonlinear and Adaptive Control Design, Wiley, New York, NY, 1995.

10. M. Krstic´ and P. V. Kokotovic´, Adaptive nonlinear design with controller-identifier separation and swapping, IEEE Transactions on Automatic Control, 40, 426–441, 1995.

11. R. Marino and P. Tomei, Global adaptive output-feedback control of nonlinear systems, Part I: Linear parametrization, IEEE Transactions on Automatic Control, 38, 17–32, 1993.

12. R. Marino and P. Tomei, Nonlinear Control Design: Geometric, Adaptive, and Robust, Prentice-Hall, Englewood Cliffs, NJ, 1995.

13. J. B. Pomet and L. Praly, Adaptive nonlinear regulation: Estimation from the Lyapunov equation, IEEE Transactions on Automatic Control, 37, 729–740, 1992.

14. L. Praly, G. Bastin, J.-B. Pomet, and Z. P. Jiang, Adaptive stabilization of nonlinear systems, in Foundations of Adaptive Control, P. V. Kokotovic´, Ed., Springer-Verlag, Berlin, pp. 347–434, 1991.

15. Z. Qu, Robust Control of Nonlinear Uncertain Systems, Wiley, New York, 1998.

16. S. S. Sastry and A. Isidori, Adaptive control of linearizable systems, IEEE Transactions on Automatic Control, 34, 1123–1131, 1989.

17. D. Seto, A. M. Annaswamy, and J. Baillieul, Adaptive control of a class of nonlinear systems with a triangular structure, IEEE Transactions on Automatic Control, 39, 1411–1428, 1994.

18. J. T. Spooner, M. Maggiore, R. Ordonez, and K. M. Passino, Stable Adaptive Control and Estimation for Nonlinear Systems: Neural and Fuzzy Approximator Techniques, Wiley, New York, 2002. “73648_C053” — #26

Antsaklis, P.J., Passino, K.M., eds., An Introduction to Intelligent and Autonomous Control, Kluwer Academic Publishers, Norwell, MA, 1993.

Astrom, K.J., Anton, J.J., Arzen, K.E., Expert control, Automatica, Vol. 22, pp. 277–286, 1986.

Dean, T., Wellman, M.P., Planning and Control, Morgan Kaufman, CA, 1991. “73648_C054” #12

Farrell, J.A., Polycarpou,M.M.,AdaptiveApproximationBasedControl: UnifyingNeural, Fuzzy, andTraditional Approximation Based Approaches, John Wiley, NJ, 2006.

Gazi, V., Moore, M.L., Passino, K.M., Shackleford, W., Proctor, F., and Albus, J.S., The RCS Handbook: Tools for Real Time Control Systems Software Development, John Wiley and Sons, New York, NY, 2001.

Goldberg, D.E., Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley, Reading, MA, 1989.

Hunt, K.J., Sbarbaro, D., Zbikowski, R., Gawthrop, P.J., Neural networks for control systems—A survey, in M.M. Gupta and D.H. Rao, eds., Neuro-Control Systems: Theory and Applications, IEEE Press, New York, 1994.

Jenkins, D., Passino, K.M., An introduction to nonlinear analysis of fuzzy control systems, Journal of Intelligent and Fuzzy Systems, Vol. 7, No. 1, pp. 75–103, 1999.

Layne, J.R., Passino, K.M., Fuzzymodel reference learning control for cargo ship steering, IEEE Control Systems Magazine, Vol. 13, No. 6, pp. 23–34, 1993.

Passino, K. M. and Yurkovich, S., Fuzzy Control, Addison-Wesley Longman, Menlo Park, CA, 1998

Passino, K. M., Biomimicry for Optimization, Control, and Automation, Springer-Verlag, London, UK, 2005.

Spooner, J. T., Maggiore, M., Ordonez, R., and Passino, K. M., Stable Adaptive Control and Estimation for Nonlinear Systems: Neural and Fuzzy Approximator Techniques, John Wiley and Sons, New York, NY, 2002.

Valavanis, K. P., Saridis, G. N., Intelligent Robotic

Systems: Theory, Design, and Applications, Kluwer Academic Publishers, Norwell, MA, 1992.

For Further Information

While the books and articles referenced above (particularly, Passino, 2005) should provide the reader with an introduction to the area of intelligent control, there are other sources that may also be useful. For instance, there are many relevant conferences including: (1) IEEE International Symposium on Intelligent Control, (2) American Control Conference, (3) IEEE Conference on Decision and Control, (4) IEEE Confer ence on Control Applications, and (5) IEEE International Conference on Systems, Man, and Cybernetics.

In addition, there are many conferences on fuzzy systems, expert systems, genetic algorithms, and neural networks where applications to control are often studied. There are many journals that cover the topic of intelligent control including: (1) IEEE Control Systems Magazine, (2) IEEE Trans. on Control Systems Technology, (3) IEEE Trans. on Systems, Man, and Cybernetics, (4) IEEE Trans. on Fuzzy Systems, (5) IEEE Trans. on Neural Networks, (6) Engineering Applications of Artificial Intelligence, (7) Journal of Intelligent and Robotic Systems, (8) Applied Artificial Intelligence, and (9) Journal of Intelligent and Fuzzy Systems.

There are many other journals on expert systems, neural networks, genetic algorithms, and fuzzy systems where applications to control can often be found. The professional societies most active in intelligent control are the IEEE Control Systems Society, International Federation on Automatic Control, and the IEEE Systems, Man, and Cybernetics Society.

1. P. J. Antsaklis,W.Kohn, A.Nerode, and S. Sastry,Hybrid Systems II, LectureNotes inComputer Science, vol. 999. New York: Springer-Verlag, 1995.

2. M. Athans, The role and use of the stochastic linear-quadratic-Gaussian problem in control system design, IEEE Transactions on Automatic Control, vol. 16, no. 6, pp. 529–552, 1971.

3. C. G. Atkeson, A. W. Moore, and S. Schaal, Locally weighted learning, Artificial Intelligence Review, vol. 11, pp. 11–73, 1997.

4. A. Barron, Universal approximation bounds for superpositions of a sigmoidal function, IEEE Transactions on Information Theory, vol. 39, no. 3, pp. 930–945, 1993.

5. A. G. Barto, R. S. Sutton, and C. W. Anderson, Neuronlike adaptive elements that can solve difficult learning control problems, IEEE Transactions on Systems, Man, and Cybernetics, vol. 13, no. 5, pp. 834– 846, 1983.

6. R. Beard, G. Saridis, and J. Wen, Galerkin approximations of the generalized Hamilton-Jacobi–Bellman equation, Automatica, vol. 33, no. 12, pp. 2159–2177, 1997.

7. R. Bellman, The theory of dynamic programming, Proceedings of the National Academy of Science, USA, vol. 38, 1952.

8. R. Bellman, Dynamic Programming. Princeton University Press, Princeton, NJ, 1957.

9. D. P. Bertsekas, Dynamic Programming and Optimal Control, 3rd ed. Athena Scientific, Nashua, NH, 2007.

10. G. Box, G. M. Jenkins, and G. Reinsel, Time Series Analysis: Forecasting and Control, 3rd ed. Englewood Cliffs, NJ: Prentice-Hall, 1994.

11. M. Cannon and J.-J. Slotine, Space–frequency localized basis function networks for nonlinear system estimation and control, Neurocomputing, vol. 9, pp. 293–342, 1995.

12. J. A. Farrell and W. Baker, Learning control systems, in Intelligent-Autonomous Control Systems, P. Antsaklis and K. Passino, Eds. Dordrecht: Kluwer Academic, 1993.

13. J. A. Farrell andM.M. Polycarpou, Adaptive Approximation Based Control: Unifying Neural, Fuzzy, and Traditional Adaptive Approximation Approaches. Hoboken, NJ: Wiley-Interscience, 2006.

14. K. Fu, Learning control systems—review and outlook, IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 8, no. 3, pp. 327–342, 1986.

15. G. C. Goodwin and K. S. Sin, Adaptive Filtering Prediction and Control. Englewood Cliffs, NJ: PrenticeHall, 1984.

16. K. Hornik, M. Stinchcombe, and H. White, Multilayer feedforward networks are universal approximators, Neural Networks, vol. 2, pp. 359–366, 1989.

17. P. A. Ioannou and J. Sun, Robust Adaptive Control. Upper Saddle River, NJ: Prentice-Hall, 1996.

18. H. Khalil, Nonlinear Systems. Englewood Cliffs, NJ: Prentice-Hall, 1996.

19. M. Krstic, I. Kanellakopoulos, and P. Kokotovic, Nonlinear and Adaptive Control Design. New York: Wiley, 1995.

20. R. J. Leake and R.-W. Liu, Construction of suboptimal control sequences, Journal of SIAM Control, vol. 5, no. 1, pp. 54–63, 1967. “73648_C056” #20

21. L. Ljung, System Identification: Theory for the User, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1999.

22. G. Lorentz, Approximation of Functions. New York: Holt, Rinehart, and Winston, 1966.

23. D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, Constrained model predictive control: Stability and optimality, Automatica, vol. 36, pp. 789–814, 2000.

24. W. T. Miller, R. S. Sutton, and P. J. Werbos, Neural Networks for Control. Cambridge, MA: MIT Press, 1990.

25. P. Millington, Associative reinforcement learning for optimal control, Master’s thesis, Department of Aeronautics and Astronautics, MIT, Cambridge, MA, 1991.

26. J. Park and I. Sandberg, Universal approximation using radial basis function networks, Neural Computation, vol. 3,

no. 2, pp. 246–257, 1991.

27. M. M. Polycarpou, Stable adaptive neural control scheme for nonlinear systems, IEEE Transactions on Automatic Control, vol. 41, no. 3, pp. 447–451, 1996.

28. L. S. Pontryagin, Optimal control processes, Uspehi Mat. Nauk (in Russian), vol. 14, pp. 3–20, 1959.

29. M. Powell, Approximation Theory and Methods. Cambridge, UK: Cambridge University Press, 1981.

30. W. B. Powell, Approximate Dynamic Programming: Solving the Curses of Dimensionality. Hoboken, NJ: Wiley & Sons Inc., 2007.

31. D. V. Prokhorov and D. C. Wunsch, Adaptive critic designs, IEEE Transactions on Neural Networks, vol. 8, no. 5, pp. 997–1007, 1997.

32. R. Sanner and J. Slotine, Gaussian networks for direct adaptive ccontrol, IEEE Transactions on Neural Networks, vol. 3, no. 6, pp. 837–863, 1992.

33. G. N. Saridis and C.-S. Lee, An approximation theory of optimal control for trainable manipulators, IEEE Transactions on Systems, Man, and Cybernetics, vol. 9, no. 3, pp. 152–158, 1979.

34. J. S. Shamma andM. Athans, Analysis of gain scheduled control for nonlinear plants, IEEE Transactions on Automatic Control, vol. 35, no. 8, pp. 898–907, 1990.

35. J. Si, A. Barto, W. Powell, and D. Wunsch, Eds., Handbook of Learning and Approximate Dynamic Programming. Hoboken, NJ: Wiley-Interscience, 2004.

36. Y. Tsypkin, Foundations of the Theory of Learning Systems. New York, NY: Academic Press, 1971.

37. D. Vrabie and F. Lewis, Adaptive optimal control algorithm for continuous-time nonlinear systems based on policy iteration, Proc. of the 47th IEEE Conf. on Decision and Control, pp. 73–79, 2008.

38. Y. Zhao and J. A. Farrell, Self-organizing approximation based control for higher order systems, IEEE Transactions on Neural Networks, vol. 18, no. 4, pp. 1220–1231, 2007. “73648_S009” #1 IX

System

Identification IX-1 “73648_S009” #2

Akaike, H. 1974a. A new look at the statistical model identification. IEEE Transactions on Automatic Control, AC-19:716–723.

Akaike, H. 1974b. Stochastic theory of minimal realization. IEEE Transactions on Automatic Control, AC19:667–674.

Åström, K. J. and Bohlin, T. 1965. Numerical identification of linear dynamic systems from normal operating records. In IFAC Symposium on Self-Adaptive Systems, Teddington, England. “73648_C057” — #39

Bohlin, T. 2006. Practical Grey-Box Process Identification. Springer-Verlag, London.

Box, G. E. P. and Jenkins, D. R. 1970. Time Series Analysis, Forcasting and Control. Holden-Day, San Francisco.

Breiman, L. 1993. Hinging hyperplanes for regression, classification and function approximation. IEEE Trans. Inf. Theory, 39:999–1013.

Brillinger, D. 1981. Time Series: Data Analysis and Theory. Holden-Day, San Francisco.

Dennis, J. E. and Schnabel, R. B. 1983. Numerical Methods for Unconstrained Optimization and Nonlinear Equations. Prentice-Hall, Englewood Cliffs, NJ.

Draper, N. and Smith, H. 1981. Applied Regression Analysis, Wiley, New York, NY.

Garnier, H. andMensler, M. 2000. The CONTSID toolbox: AMATLAB toolbox for CONtinuous-Time System IDentification. In Proceedings of 12th IFAC Symposium on Identification, Santa Barbara, CA, USA.

Hjalmarsson, H. 2005. From experiment design to closed loop control. Automatica, 41(3):393–438.

Juditsky, A., Hjalmarsson, H., Benveniste, A., Delyon, B., Ljung, L., Sjöberg, J., and Zhang, Q. 1995. Nonlinear black-box modeling in system identification: Mathematical foundations. Automatica, 31(12):1724–1750.

Kushner, H. J. and Yang, J. 1993. Stochastic approximation with averaging of the iterates: Optimal asymptotic rate of

convergence for general processes. SIAM Journal of Control and Optimization, 31(4):1045–1062.

Landau, I. D. 1990. System Identification and Control Design Using P.I.M. + Software. Prentice-Hall, Englewood Cliffs, NJ.

Larimore, W. E. 1983. System identification, reduced order filtering and modelling via canonical variate analysis. In Proceedings of 1983 American Control Conference, San Francisco, CA.

Ljung, L. 1999. System Identification—Theory for the User, 2nd edn. Prentice-Hall, Upper Saddle River, NJ.

Ljung, L. 2003a. Educational aspects of identification software user interfaces. In P. van der Hof, B. Wahlberg, S. W., Ed. Proceedings of 13th IFAC Symposium on System Identification, pp. 1590–1594, Rotterdam, The Netherlands.

Ljung, L. 2003b. Linear system identification as curve fitting. In New Directions in Mathematical Systems Theory and Optimization, Springer Lecture Notes In Control and Information, volume 286, pp. 203–215. Springer-Verlag, Berlin.

Ljung, L. 2004. State of the art in linear system identification: Time and frequency domain methods. In Proceedings of American Control Conference, Boston, MA.

Ljung, L. 2007. System Identification Toolbox for Use with MATLAB. Version 7th edn., The MathWorks, Inc, Natick, MA.

Ljung, L. 2010. Pespectives on system identification. IFAC Annual Reviews, Spring Issue, 34(1): 1–12.

Ljung, L. and Glad, T. 1994. Modeling of Dynamic Systems. Prentice Hall, Englewood Cliffs, NJ.

Ljung, L. and Söderström, T. 1983. Theory and Practice of Recursive Identification. MIT Press, Cambridge, MA.

MATRIXx 1991. MATRIXx Users Guide. Integrated Systems Inc., Santa Clara, CA.

McKelvey, T. 2000. Frequency domain identification. In Smith, R. and Seborg, D., Ed., Proceedings of 12th IFAC Symposium on System Identification, Plenary Paper, Santa Barbara, CA, USA.

Ninness, B. and Wills, A. 2006. An identification toolbox for profiling novel techniques. In 14th IFAC Symposium on System Identification, Newcastle, Australia. http://sigpromu.org/idtoolbox/.

Pintelon, R. and Schoukens, J. 2001. System Identification—A Frequency Domain Approach. IEEE Press, New York, NY.

Poggio, T. and Girosi, F. 1990. Networks for approximation and learning. Proceedings of IEEE, 78:1481–1497.

Polyak, B. T. and Juditsky, A. B. 1992. Acceleration of stochastic approximation by averaging. SIAM Journal of Control and Optimization, 30:838–855.

Rissanen, J. 1974. Basis of invariants and canonical forms for linear dynamic systems. Automatica, 10:175–182.

Rissanen, J. 1978. Modelling by shortest data description. Automatica, 14:465–471.

Schoukens, J. and Pintelon, R. 1991. Identification of Linear Systems: A Practical Guideline to AccurateModeling. Pergamon Press, London, UK.

Schoukens, J., Pintelon, R., and Rolain, Y. 2004. Time domain identification, frequency domain identification, equivalences! differences? In Proceedings of American Control Conference, Boston, MA.

Sjöberg, J., Zhang, Q., Ljung, L., Benveniste, A., Delyon, B., Glorennec, P., Hjalmarsson, H., and Juditsky, A. 1995. Nonlinear black-box modeling in system identification: A unified overview. Automatica, 31(12):1691–1724.

Söderström, T. and Stoica, P. 1989. System Identification. Prentice-Hall Int., London.

Van Overschee, P. and DeMoor, B. 1996. Subspace Identification of Linear Systems: Theory, Implementation, Applications. Kluwer Academic Publishers, Dordrecht.

Verhaegen, M. 1994. Identification of the deterministic part of MIMO state space models, given in innovations form from input–output data. Automatica, 30(1):61–74.

Zhang, Q. and Benveniste, A. 1992. Wavelet networks. IEEE Trans. Neural Networks, 3:889–898. “73648_C057” — #40 “73648_S010” #1 X

StochasticControl X-1 “73648_S010” #2

Çinlar, E., Introduction to Stochastic Processes,
Prentice-Hall, Englewood Cliffs, NJ, 1975.

A precise, elegant and accessible book, covering the basics.

Kemeny, J.G., Snell, J., and Knapp, A.W., Denumerable
Markov Chains, Van Nostrand, Princeton, NJ, 1966.

The most elementary in this list, but fairly thorough, not
only in coverage of Markov chains, but also as

introduction to Markov processes.

Nummelin, E., General Irreducible Markov Chains and
Non-Negative Operators, Cambridge University Press,

Cambridge, 1984.

More general than [3], treats general state spaces.
Introduced many new techniques; perhaps less encyclo

pedic than [2], but fairly mathematical.

Orey, S., Limit Theorems for Markov Chain Transition
Probabilities, Van Nostrand Reinhold, London, 1971.

A thin gem on limit theorems, fairly accessible.

Revuz, D., Markov Chains, North-Holland, Amsterdam, 1984.

A mathematical, thorough treatment.

1. Chung, K.L., Markov Chains with Stationary Transition
Probabilities, Springer-Verlag, New York, 1967. A classic
on discrete space Markov chains, both discrete and
continuous time. Very thorough and detailed, but
mathematically not elementary.

2. Meyn, S.P. andTweedie, R.L.,MarkovChains and Stochastic
Stability, 2nd edition.CambridgeUniversity Press,
Cambridge, 2009. Deals with general discrete-timeMarkov
chains, and covers the state of the art. It is therefore
demanding mathematically, although not much measure theory
is required. The most comprehensive book if you can handle
it.

3. Seneta, E., Non-Negative Matrices and Markov Chains,
Springer-Verlag, New York, 1981. Gives the algebraic point

of view on Markov chains.

4. Tijms, H.C., Stochastic Modelling and Analysis: A Computational Approach, John Wiley, New York, 1986. Contains a basic introduction to the subject of Markov chains, both discrete and continuous time, with a wealth of examples for applications and computations.

59 59.4 Stochastic Differential Equations and Itô's Rule

1. Arnold, L., Stochastic Differential Equations: Theory and Applications, Wiley, New York, 1974, 48–56, 91, 98, 105, 113, 128, 137, 168.

2. Billingsley, P., Probability andMeasure, 2nd ed.,Wiley, NewYork, 1986, sect. 37, pp 469–470 (Equations 34.2, 34.5, 34.6).

3. Coddington, E. A. and Levinson, N., Theory of Ordinary Differential Equations, McGraw-Hill, New York, 1955, chap. 1.

4. Davis, M. H. A., Linear Estimation and Stochastic Control, Chapman and Hall, London, 1977, chap. 3, 52–53.

5. Elliott, R. J., Stochastic Calculus, Springer-Verlag, New York, 1982.

6. Ethier, S. N. and Kurtz, T. G., Markov Processes: Characterization and Convergence, Wiley, New York, 1986.

7. Karatzas, I. and Shreve, S. E., Brownian Motion and Stochastic Calculus, 2nd ed., Springer-Verlag, New York, 1991, 397.

8. Protter, P., Stochastic Integration and Differential Equations, Springer-Verlag, Berlin, 1990.

9. Segall, A., Stochastic processes in estimation theory, IEEE Trans. Inf. Theory, 22(3), 275–286, 1976.

10. Wong, E. and Hajek, B., Stochastic Processes in Engineering Systems, Springer-Verlag, New York, 1985, chap. 4, pp 173–174.

Further Reading

For background on probability theory, especially conditional expectation, we recommend [2].

For linear SDEs driven by orthogonal-increments processes, we recommend the very readable text by

Davis [4].

For SDEs driven by continuous martingales, there is the more advanced book by Karatzas and

Shreve [7].

For SDEs driven by right-continuousmartingales, the theory becomes considerably more complicated.

However, the tutorial paper by Segall [9], which compares discrete-time and continuous-time results, is

very readable. Also, Chapter 6 of [10] is accessible.

Highly technical books on SDEs driven by right-continuous martingales include [5] and [8].

For the reader interested in Markov processes there is the advanced text of Ethier and Kurtz [6].

60 60.2 ARMA and ARMAX Models

1. K. J. Åström, Introduction to Stochastic Control. Academic Press, New York, 1970.

2. K. J. Åström and B. Wittenmark, Computer Controlled Systems, Prentice-Hall, Englewood Cliffs, NJ, (1984), 1990.

3. M. J. Grimble and M. A. Johnson, Optimal Control and Stochastic Estimation. John Wiley & Sons, Chichester, UK, 1988.

4. K. J. Hunt, Ed. Polynomial Methods in Optimal Control and Filtering. Peter Peregrinus Ltd, Stevenage, UK, 1993 (in particular, Chapter 6: A. Ahlén and M. Sternad: Optimal filtering problems.).

5. V. Kuc˘era, Discrete Linear Control. John Wiley & Sons, Chichester, UK, 1979.

6. T. Söderström, Discrete-Time Stochastic Systems: Estimation and Control, Prentice-Hall International, Hemel Hempstead, UK, 1994.

7. T. Söderström, Discrete-Time Stochastic Systems: Estimation and Control, 2nd edition, Springer-Verlag, London, UK, 2002.

1. Al-Tamimi A., Lewis F. L., and Abu-Khalaf M., Model-free Q-learning designs for discrete-time zerosum games with application to H-infinity control, Automatica, 43, 473–481, 2007.

2. Al-Tamimi A. and Lewis, F. Discrete-time nonlinear HJB solution using approximate dynamic programming: Convergence proof, Proc. of IEEE Symposium on Approximate Dynamic Programming and Reinforcement Learning ADPRL'07, 38–43, Hawaii, 2007.

3. Al-Tamimi A., Vrabie D., Lewis F. L., and Abu-Khalaf M., Model-free approximate dynamic programming schemes for linear systems, Proc. of IJCNN'07, 371–378, Orlando, 2007.

4. Balakrishnan S.N., Ding J., and Lewis F.L., Issues on stability of ADP feedback controllers for dynamical systems, Trans. Systems, Man, Cybernetics, Part B: Special issue on ADP/RL, 38(4), 913–917, 2008.

5. Barto A. G., Sutton R. S., and Anderson C., Neuron-like adaptive elements that can solve difficult learning control problems, IEEE Trans. on Systems, Man, and Cybernetics, SMC-13, 834–846, 1983.

6. Bellman R., Dynamic Programming, Princeton University Press, Princeton, NJ, 1957.

7. Bertsekas D. P. and J. N. Tsitsiklis D. P., Neuro-Dynamic Programming, Athena Scientific, MA, 1996.

8. S. Bradtke S., B. Ydstie B., and A. Barto A., Adaptive linear quadratic control using policy iteration, Proc. of American Control Conference, 3, 3475–3479, 1994.

9. T. Hanselmann T., L. Noakes L., and Zaknich A., Continuous-time adaptive critics, IEEE Trans. on Neural Networks, 18(3), 631–647, 2007.

10. D. Kleinman D., On an iterative technique for Riccati equation computations, IEEE Trans. on Automatic Control, 13, 114–115, 1968.

11. T. Landelius T., Reinforcement Learning and Distributed Local Model Synthesis, PhD Dissertation, Linkoping University, Sweden, 1997.

12. Mehta P. and Meyn S., Q-learning and Pontryagin's

minimum principle, Proc. of IEEE Conference on CDC/CCC, 3598-3605, 2009.

13. Murray J. J., Cox C. J., Lendaris G. G., and Saeks R., Adaptive dynamic programming, IEEE Trans. on Systems, Man and Cybernetics, 32(2), 140-153, 2002.

14. Pareigis S., Numerical schemes for the continuous Q-function of reinforcement learning, Technical report, Bericht 97-15, Lehrstuhl Praktische Mathematik, Kiel University, 1997. “73648_C062” — #33

15. Prokhorov D. andWunsch D., Adaptive critic designs, IEEE Trans. on Neural Networks, 8(5), 997-1007, 1997.

16. Si J., Barto A., Powell W., and Wunsch D., Handbook of Learning and Approximate Dynamic Programming, John Wiley, New Jersey, NJ, 2004.

17. Sutton R. S. and Barto A. G., Reinforcement Learning—An Introduction, MIT Press, Cambridge, MT, 1998.

18. Vrabie D. and Lewis F., Generalized policy iteration for continuous-time systems, Proc. of IJCNN’09, Atlanta, 2009.

19. Vrabie D. and Lewis F.L., Neural network approach to continuous-time direct adaptive optimal control for partially-unknownnonlinear systems,Neural Networks—special issue: Goal-DirectedNeural Systems, 22(3), 237-246, 2009.

20. Vrabie D. Abu-Khalaf M., Lewis F.L., and Wang Y., Continuous-time ADP for linear systems with partially unknown dynamics, Proc. of Symposium on Approximate Dynamic Programming and Reinforcement Learning (ADPRL), 247-253, Hawaii, 2007.

21. WangY., ZhouR., andWenC., Robust load-frequency controller design for power systems, IEE Proc.-C, 140(I), 1993.

22. Wang F. Y., Zhang H., and Liu D., Adaptive dynamic programming: An introduction, IEEE Comp. Intelligence Magazine, 39-47, May 2009.

23. Watkins C. J. C. H. and Dayan P., Q-learning, Machine Learning, 8, 279-292, 1992.

24. Werbos P. J., Approximate dynamic programming for real-time control and neuralmodeling,Handbook of

Intelligent Control, eds. D.A. White and D.A. Sofge, Van
Nostrand Reinhold, New York, NY, 1992. “73648_C062” — #34

1. Arnold, L., A formula connecting sample and moment stability of linear stochastic systems, SIAM J. Appl. Math., 44, 793–802, 1984.

2. Arnold, L., Crauel, H., and Wihstutz, V., Stabilization of linear system by noise. SIAM J. Control Optim., 21(3), 451–461, 1983.

3. Arnold, L., Kliemann, W., and Oeljeklaus, E., Lyapunov exponents of linear stochastic systems. In Lecture Notes in Math., No. 1186, Springer, Berlin, 1985.

4. Arnold, L., Papanicolaou, G., and Wihstutz, V., Asymptotic analysis of the Lyapunov exponent and rotation number of the random oscillator and application. SIAM J. App. Math., 46(3), 427–449, 1986.

5. Arnold, L. and Wihstutz, V., Eds., Lyapunov Exponents, Lecture Notes in Math., No. 1186, Springer, Berlin, 1985.

6. Auslender, E.I. and Mil'shtein, G.N., Asymptotic expansion of Lyapunov index for linear stochastic system with small noise. Prob. Math. Mech. USSR, 46, 277–283, 1983.

7. Bellman, R., Limit theorem for noncommutative operations-I. Duke Math. J., 21, 491–500, 1954.
"73648_C063" — #35

8. Bertram, J.E. and Sarachik, P.E., Stability of circuits with randomly time-varying parameters. Trans. IRE, PGIT-5, Special Supplement, p. 260, 1959.

9. Bitsris, G., On the stability in the quadratic mean of stochastic dynamical systems. Int. J. Control, 41(4), 1061–1075, 1985.

10. Brockett, R.W., Finite Dimensional Linear Systems, John Wiley & Sons, New York, 1970.

11. Brockett, R.W. and Willems, J.C., Average value criteria for stochastic stability. In Stability of Stochastic Dynamical Systems, Lecture Notes in Math., No. 294, Curtain, R.F., Ed., Springer, New York, 1972.

12. Caughey, T.K. and Gray, A.H., Jr., On the almost sure stability of linear dynamic systems with stochastic coefficients. J. Appl. Mech., 32, 365, 1965.

13. Feng, X. and Loparo, K.A., Almost sure instability of the random harmonic oscillator. SIAM J. Appl. Math., 50(3), 744–759, 1990.

14. Feng, X. and Loparo, K.A., A nonrandom spectrum for Lyapunov exponents of linear stochastic systems. Stochastic Anal. Appl., 9(1), 25–40, 1991.

15. Feng, X. and Loparo, K.A., A nonrandom spectrum theorem for products of randommatrices and linear stochastic systems. J. Math. Syst., Estimation Control, 2(3), 323–338, 1992.

16. Furstenberg, H. and Kesten, H., Products of random matrices. Ann. Math. Statist., 31, 457–469, 1960.

17. Furstenberg, H., Noncommuting random products. Trans. Am. Math. Soc., 108, 377–428, 1963.

18. Furstenberg, H., A Poisson formula for semi-simple Lie group. Ann. Math., 77, 335–386, 1963.

19. Has'minskii, R.Z., A limit theorem for solutions of differential equations with random right-hand sides. Theory Probl. Appl., 11, 390–406, 1966.

20. Has'minskii, R.Z., Necessary and sufficient condition for the asymptotic stability of linear stochastic systems. Theory Prob. Appl., 12, 144–147, 1967.

21. Has'minskii, R.Z., Stability of Systems of Differential Equations Under Random Perturbation of Their Parameters, (in Russian), Nauka Moscow, 1969.

22. Has'minskii, R.Z., Stochastic Stability of Differential Equations, Sijthoff and Noordhoff, Ma., 330–332, 1980.

23. Infante, E.F., On the stability of some linear non-autonomous random system. J. Appl. Mech., 35, 7–12, 1968.

24. Kats, I.I. and Krasovskii, N.N., On the stability of systems with random parameters. Prkil. Met. Mek., 24, 809, 1960.

25. Kleinman, D.L., On the stability of linear stochastic systems. IEEE Trans. Automat. Control, AC-14, 429–430, 1969.

26. Kozin, F., On almost sure stability of linear systems with random coefficients. M.I.T. Math. Phys., 43, 59, 1963.

27. Kozin, F., A survey of stability of stochastic systems. Automatica, 5, 95-112, 1969.

28. Kozin, F. and Prodromou, S., Necessary and sufficient condition for almost sure sample stability of linear Ito equations. SIAM J. Appl. Math., 21(3), 413-424, 1971.

29. Kozin, F. and Wu, C.M., On the stability of linear stochastic differential equations. J. Appl. Mech., 40, 87-92, 1973.

30. Kushner, H.J., Stochastic Stability and Control, Academic Press, New York, 1967.

31. Kushner, H.J., Introduction to Stochastic Control Theory, Holt, Rinehart and Winston, New York, 1971.

32. Kushner, H.J., Stochastic stability. In Stability of Stochastic Dynamical Systems, Lecture Notes in Math., No. 249, Curtain, R.F., Ed., Springer, New York, pp. 97-124, 1972.

33. Ladde, G.S. and Siljak, D.D., Connective stability of large scale systems. Int. J. Syst. Sci., 6(8), 713-721, 1975.

34. Li, C.W. and Blankenship, G.L., Almost sure stability of linear stochastic system with Poisson process coefficients. SIAM J. Appl. Math., 46(5), 875-911, 1986.

35. Loparo, K.A. and Blankenship, G.L., Almost sure instability of a class of linear stochastic system with jump parameter coefficients. In Lyapunov Exponents, Lecture Notes in Math., No. 1186, Springer, Berlin, 1985.

36. Loparo, K.A. and Feng, X., Lyapunov exponent and rotation number of two-dimensional linear stochastic systems with telegraphic noise. SIAM J. Appl. Math., 53(1), 283-300, 1992.

37. Lyapunov, A.M., Probléme générale de la stabilité du muvement. Comm. Soc. Math. Kharkov, 2, 1892,3, 1893. Reprint Ann. Math. Studies, 17, Princeton University Press, Princeton, 1949.

38. Mahalanabis, A.K. and Parkayastha, S., Frequency domain criteria for stability of a class of nonlinear stochastic

systems. IEEE Trans. Automat. Control, AC-18(3), 266-270, 1973.

39. Man, F.T., On the almost sure stability of linear stochastic systems. J. Appl. Mech., 37(2), 541, 1970.

40. Michel, A.N., Stability analysis of stochastic large scale systems. Z. Angew. Math. Mech., 55, 93-105, 1975. "73648_C063" — #36

41. Michel, A.N. and Rasmussen, R.D., Stability of stochastic composite systems. IEEE Trans. Automat. Control, AC-21, 89-94, 1976.

42. Mitchell, R.R., Sample stability of second order stochastic differential equation with nonsingular phase diffusion. IEEE Trans. Automat. Control, AC-17, 706-707, 1972.

43. Mitchell, R.R. and Kozin, F., Sample stability of second order linear differential equation with wide band noise coefficients. SIAM J. Appl. Math., 27, 571-605, 1974.

44. Molchanov, S.A., The structure of eigenfunctions of one-dimensional unordered structures.Math. USSR Izvestija, 12, 69-101, 1978.

45. Nishioka, K., On the stability of two-dimensional linear stochastic systems. Kodai Math. Sem. Rep., 27, 221-230, 1976.

46. Oseledec, V.Z., A multiplicative ergodic theorem Lyapunov characteristic number for dynamical systems. Trans. Moscow Math. Soc., 19, 197-231, 1969.

47. Pardoux, E. and Wihstutz, V., Two-dimensional linear stochastic systems with small diffusion. SIAM J. Appl. Math., 48, 442-457, 1988.

48. Parthasarathy, A. and Evan-Zwanowskii, R.M., On the almost sure stability of linear stochastic systems. SIAM J. Appl. Math., 34(4), 643-656, 1978.

49. Pinsky, M.A., Stochastic stability and the Dirichlet problem. Common. Pure. Appl. Math., 27, 311-350, 1974.

50. Pinsky,M.A., Instability of the harmonic oscillatorwith small noise. SIAMJ.Appl.Math., 46(3), 451-463, 1986.

51. Rasmussen, R.D. andMichel, A.N., On vector Lyapunov

functions for stochastic dynamic systems. IEEE Trans. Automat. Control, AC-21, 250-254, 1976.

52. Siljak, D.D., Large-Scale Dynamic Systems. North-Holland, Amsterdam, 1978.

53. Socha, L., Application of Yakubovich criteria for stability of nonlinear stochastic systems. IEEE Trans. Automat. Control, AC-25(2), 330-332, 1980.

54. Socha, L., The asymptotic stochastic stability in large of the composite systems. Automatica, 22(5), 605-610, 1986.

55. Walker, J.A.,On the applicationof Lyapunov's directmethod to linear lumped-parameter elastic systems. J. Appl. Mech., 41, 278-284, 1974.

56. Wiens, G.J. and Sinha, S.C., On the application of Lyapunov direct method to discrete dynamic systems with stochastic parameters. J. Sound. Vib., 94(1), 19-31, 1984.

57. V.Wihstutz, V. Parameter dependence of the Lyapunov exponent for linear stochastic system. A survey. In Lyapunov Exponents, Lecture Notes in Math., No. 1186, Springer, Berlin, 1985.

58. Willems, J.L., Lyapunov functions and global frequency domain stability criteria for a class of stochastic feedback systems. In Stability of Stochastic Dynamical Systems, LectureNotes inMath., No. 294, Curtain, R.F., Ed., Springer, New York, 1972.

59. Willems, J.L., Mean square stability criteria for stochastic feedback systems. Int. J. Syst. Sci., 4(4), 545-564, 1973.

65 Probabilistic and Randomized Tools for Control Design

1. G. Calafiore and M.C. Campi. The scenario approach to robust control design. IEEE Transactions on Automatic Control, 51(5):742–753, 2006.

2. G. Calafiore, F. Dabbene, and R. Tempo. Randomized algorithms for probabilistic robustness with real and complex structured uncertainty. IEEE Transactions on Automatic Control, 45:2218–2235, 2000.

3. G. Calafiore and B.T. Polyak. Stochastic algorithms for exact and approximate feasibility of robust LMIs. IEEE Transactions on Automatic Control, 46:1755–1759, 2001.

4. G.C. Calafiore and F. Dabbene. A probabilistic analytic center cutting plane method for feasibility of uncertain LMIs. Automatica, 43:2022–2033, 2007.

5. H. Chernoff. A measure of asymptotic efficiency for tests of a hypothesis based on the sum of observations. Annals of Mathematical Statistics, 23:493–507, 1952.

6. Y. Fujisaki, F. Dabbene, and R. Tempo. Probabilistic robust design of LPV control systems. Automatica, 39:1323–1337, 2003.

7. S. Kanev, B. De Schutter, and M. Verhaegen. An ellipsoid algorithm for probabilistic robust controller design. Systems and Control Letters, 49:365–375, 2003.

8. Y. Oishi. Polynomial-time algorithms for probabilistic solutions of parameter-dependent linear matrix inequalities. Automatica, 43(3):538–545, 2007.

9. B.T. Polyak and R. Tempo. Probabilistic robust design with linear quadratic regulators. Systems and Control Letters, 43:343–353, 2001.

10. R. Tempo, E.-W. Bai, and F. Dabbene. Probabilistic robustness analysis: Explicit bounds for the minimum number of samples. Systems and Control Letters, 30:237–242, 1997.

11. R. Tempo, G. Calafiore, and F. Dabbene. Randomized Algorithms for Analysis and Control of Uncertain Systems. Communications and Control Engineering Series. Springer-Verlag, London, 2005.

12. M. Vidyasagar. Statistical learning theory and

randomized algorithms for control. IEEE Control Systems Magazine, 18:69–85, 1998.

13. M.Vidyasagar.Learning andGeneralization:WithApplications toNeuralNetworks (2nd edn.). SpringerVerlag, New York, NY, 2002.

14. T. Alamo, R. Tempo, and E.F. Camacho. Statistical learning theory: A pack-based strategy for uncertain feasibility and optimization problems. In V.D. Blondel, S.P. Boyd, and H. Kimura, Eds, Recent Advances in Learning and Control. Springer-Verlag, London, 2008. “73648_C065” #23

15. C. Alippi. A probably approximately correct framework to estimate performance degradation in embedded systems. IEEETransactions onComputer-AidedDesign of IntegratedCircuits andSystems, 21(7):749–762, 2002.

16. C. Alippi. Randomized algorithms: A system-level, poly-time analysis. IEEE Transactions on Computers, 51(7):740–749, 2002.

17. T. Alpcan, T. Bas¸ar, and R. Tempo. Randomized algorithms for stability and robustness analysis of high speed communication networks. IEEE Transactions on Neural Networks, 16:1229–1241, 2005.

18. B.D.O.Anderson and J.B.Moore.OptimalControl: LinearQuadraticMethods. Prentice-Hall, Englewood Cliffs, NJ, 1990.

19. E.-W. Bai, R. Tempo, andM. Fu.Worst-case properties of the uniformdistribution and randomized algorithms for robustness analysis. Mathematics of Control, Signals, and Systems, 11:183–196, 1998.

20. B.R. Barmish. New Tools for Robustness of Linear Systems. MacMillan, New York, NY, 1994.

21. B.R. Barmish and C.M. Lagoa. The uniform distribution: A rigorous justification for its use in robustness analysis. Mathematics of Control, Signals, and Systems, 10:203–222, 1997.

22. S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. Linear Matrix Inequalities in System and Control Theory. SIAM, Philadelphia, 1994.

23. G. Calafiore, F. Dabbene, and R. Tempo. Radial and

uniform distributions in vector and matrix spaces for probabilistic robustness. In D.E. Miller and L. Qiu, Eds, Topics in Control and its Applications, p. 17–31. Springer-Verlag, New York, NY, 1999.

24. G.C. Calafiore and F. Dabbene. Optimization under uncertainty with applications to design of truss structures. Structural and Multidisciplinary Optimization, 35(3):189–200, 2008.

25. M.C. Campi and M. Prandini. Randomized algorithms for the synthesis of cautious adaptive controllers. Systems and Control Letters, 49:21–36, 2003.

26. M.C.Campi and S.Garatti. The exact feasibility of randomized solutions of robust convex programs. SIAM Journal on Optimization, 19(3):1211–1230, 2008.

27. Y. Fujisaki and Y. Oishi. Guaranteed cost regulator design: A probabilistic solution and a randomized algorithm. Automatica, 43:317–324, 2007.

28. O.J. Gietelink, B. De Schutter, andM. Verhaegen. Probabilistic approach for validation of advanced driver assistance systems. Transportation Research Record, (1910):20–28, 2005.

29. H. Ishii, T. Basar, and R. Tempo. Randomized algorithms for synthesis of switching rules for multimodal systems. IEEE Transactions on Automatic Control, 50:754–767, 2005.

30. S. Kanev and M. Verhaegen. Robustly asymptotically stable finite-horizon MPC. Automatica, 42(12):2189–2194, 2006.

31. C. Lagoa, F. Dabbene, and R. Tempo. Hard bounds on the probability of performance with application to circuit analysis. IEEE Transactions on Circuits and Systems I, 55:3178–3187, 2008.

32. B. Lu and F. Wu. Probabilistic robust linear parameter-varying control of an F-16 aircraft. Journal of Guidance, Control, and Dynamics, 29(6):1454–1460, 2006.

33. W.Ma,M. Sznaier, andC.M. Lagoa. A risk adjusted approach to robust simultaneous fault detection and isolation. Automatica, 43(3):499–504, 2007.

34. C.I. Marrison and R.F. Stengel. Design of robust control systems for a hypersonic aircraft. Journal of

Guidance, Control, and Dynamics, 21(1):58–63, 1998.

35. M. Sznaier, C.M. Lagoa, andM.C.Mazzaro. An algorithm for sampling subsets of H ∞ with applications to risk-adjusted performance analysis and model (in)validation. IEEE Transactions on Automatic Control, 50(3):410–416, 2005.

36. A. Tremba, G. Calafiore, F. Dabbene, E. Gryazina, B.T. Polyak, P.S. Shcherbakov, and R. Tempo. RACT: Randomized algorithms control toolbox for MATLAB. In Proceedings of the IFAC World Congress, Seoul, 2008. “73648_C065” #24

1. H. Deng andM. Krstic´, Stochastic nonlinear stabilization-I: A backstepping design, Systems and Control Letters, vol. 32, pp. 143–150, 1997.

2. H. Deng and M. Krstic´, Stochastic nonlinear stabilization-II: Inverse optimality, Systems and Control Letters, vol. 32, pp. 151–159, 1997.

3. H. Deng, M. Krstic´, and R. J. Williams, Stabilization of stochastic nonlinear systems driven by noise of unknown covariance, IEEE Transactions on Automatic Control, vol. 8, pp. 1237–1253, 2001.

4. P. Florchinger, A universal formula for the stabilization of control stochastic differential equations, Stochastic Analysis and Its Applications, vol. 11, pp. 155–162, 1993.

5. P. Florchinger, Lyapunov-like techniques for stochastic stability, SIAM Journal of Control and Optimization, vol. 33, pp. 1151–1169, 1995.

6. W. Hahn, Stability of Motion, New York, NY: Springer-Verlag, 1967.

7. H. Z. Khalil, Nonlinear Systems (2nd edition) Upper Saddle River, NJ: Prentice-Hall, 1996.

8. R. Z. Khas'minskiĭ, Stochastic Stability of Differential Equations, Rockville, MD; S & N International Publisher, 1980.

9. M. Krstic´ and H. Deng, Stabilization of Nonlinear Uncertain Systems, London: Springer-Verlag, 1998.

10. Z. G. Pan, Canonical forms for stochastic nonlinear systems, Automatica, vol. 38, pp. 1163–1170, 2002.

11. Z. G. Pan and T. Bas¸ar, Backstepping controller design for nonlinear stochastic systems under a risksensitive cost criterion, SIAM Journal on Control and Optimization, vol. 37, pp. 957–995, 1999.

12. Z. G. Pan, Y. G. Liu, and S. J. Shi, Output feedback stabilization for stochastic nonlinear systems in observer canonical form with stable zero-dynamics, Science in China (Series F), vol. 44, pp. 292–308, 2001.

13. Y. G. Liu and J. F. Zhang, Practical output-feedback risk-sensitive control for stochatic nonlinear systems with stable zero-dynamics, SIAM Journal on Control and Optimization, vol. 45, pp. 885–926, 2006.

14. S. J. Liu, J. F. Zhang, and Z. P. Jiang, Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems, Automatica, vol. 34, pp. 238–251, 2007.

15. E. D. Sontag, Smooth stabilization implies coprime factorization, IEEE Transactions on Automatic Control, vol. 34, pp. 435–443, 1989.

16. J. Tsinias, The concept of Exponential input to state stability for stochastic systems and applications to feedback stabilization, Systems and Control Letters, vol. 36, pp. 221–229, 1999.

17. Z. J. Wu, X. J. Xie, and S. Y. Zhang, Adaptive backstepping controller design using stochastic small-gain theorem, Automatica, vol. 43(4), pp. 608–620, 2007.

67 Control of Systems Governed by Partial Differential Equations

1. M. J. Balas. Active control of flexible systems. J. Optim. Theory Appl., 23(3):415–436, 1976.

2. H. T. Banks and K. Ito. Approximation in LQR problems for infinite-dimensional systems with unbounded input operators. J. Math. Systems Estim. Control, 7(1):1–34, 1997.

3. H. T. Banks and K. Kunisch. The linear regulator problem for parabolic systems. SIAM J. Control Optim., 22(5):684–698, 1984.

4. A. Bensoussan and P. Bernhard. On the standard problem of H ∞ -optimal control for infinite dimensional systems. In Identification and Control in Systems Governed by Partial Differential Equations, pp. 117–140. SIAM, Philadelphia, PA, 1993.

5. J. A. Burns, K. Ito, and G. Propst. On non-convergence of adjoint semigroups for control systems with delays. SIAM J. Control Optim., 26(6):1442–1454, 1988. “73648_C067” — #36

6. J. A. Burns, E. W. Sachs, and L. Zietsman. Mesh independence of Kleinman-Newton iterations for Riccati equations in Hilbert space. SIAM J. Control Optim., 47(5):2663–2692, 2008.

7. R. F. Curtain and K. A Morris. Transfer functions of distributed parameter systems: A tutorial. Automatica, 45(5):1101–1116, 2009.

8. R. F. Curtain and H. Zwart. An Introduction to Infinite-Dimensional Linear Systems Theory. SpringerVerlag, Berlin, 1995.

9. C. A. Desoer and M. Vidyasagar. Feedback Systems: Input-Output properties. Academic Press, New York, 1975.

10. F. Feitzinger, T. Hylla, and E.W. Sachs. Inexact Kleinman-Newtonmethod for Riccati equations. SIAM J. Matrix Anal. Appl., 21(2):272–288, 2009.

11. C. Foias, H. H. Ozbay, and A. Tannenbaum. Robust Control of Infinite Dimensional Systems: Frequency Domain Methods LNCIS vol. 209. Springer-Verlag, Berlin, 1995.

12. J. S. Gibson. The Riccati integral equations for optimal control problems on Hilbert spaces. SIAM J. Control Optim., 17(4):637–665, 1979.

13. J. S. Gibson. A note on stabilization of infinite-dimensional linear oscillators by compact linear feedback. SIAM J. Control Optim., 18(3):311–316, 1980.

14. R. B. Guenther and J. W. Lee. Partial Differential Equations of Mathematical Physics and Integral Equations. Prentice-Hall, Englewood Cliffs, NJ, 1988.

15. K. Ito. Strong convergence and convergence rates of approximating solutions for algebraic Riccati equations in Hilbert spaces. In W. Schappacher F. Kappel, and K. Kunisch (Eds), Distributed Parameter Systems. Springer-Verlag, Berlin, 1987.

16. K. Ito and K. A. Morris. An approximation theory for solutions to operator Riccati equations for H ∞ control. SIAM J. Control Optim., 36(1):82–99, 1998.

17. B. Jacob and K. A. Morris. Second-order systems with acceleration measurements. Preprint, 2009.

18. C. A. Jacobson and C. N. Nett. Linear state-space systems in infinite-dimensional space: The role and characterization of joint stabilizability/detectability. IEEE Trans. Automat. Control, 33(6):541–549, 1988.

19. K. Kashima and Y. Yamamoto. On standard H ∞ control problems for systems with infinitely many unstable poles. Systems Control Lett., 57(4):309–314, 2008.

20. B.V. Keulen. H ∞ -Control for Distributed Parameter Systems: A State-Space Approach. Birkhauser, Boston, 1993.

21. D. Kleinman. On an iterative technique for Riccati equation computations. IEEE Trans. Automat. Control, 13:114–115, 1968.

22. I. Lasiecka and R. Triggiani. Control Theory for Partial Differential Equations: Continuous and Approximation Theories, Vol. I. Cambridge University Press, Cambridge, UK, 2000.

23. I. Lasiecka and R. Triggiani. Control Theory for Partial Differential Equations: Continuous and Approximation Theories, Vol. II. Cambridge University Press, Cambridge, UK, 2000.

24. H. Logemann. Stabilization and regulation of infinite-dimensional systems using coprime factorizations. In R. F. Curtain (Ed.), Analysis and Optimization of Systems: State and Frequency Domain Approaches for Infinite-Dimensional Systems, pp. 102–139. Springer-Verlag, Berlin, 1992.

25. H. L. Logemann. Circle criteria, small-gain conditions and internal stability for infinite-dimensional systems. Automatica, 27:677–690, 1991.

26. H. L. Logemann, E. P. Ryan, and S. Townley. Integral control of linear systems with actuator nonlinearities: Lower bounds for the maximal regulating gain. IEEE Trans. Automat. Control, 44:1315–1319, 1999.

27. K. A. Morris. H ∞ output feedback control of infinite-dimensional systems via approximation. Systems Control Lett., 44(3):211–217, 2001.

28. K. A. Morris. Convergence of controllers designed using state-space methods. IEEE Trans. Automat. Control, 39:2100–2104, 1994.

29. K. A. Morris. Design of finite-dimensional controllers for infinite-dimensional systems by approximation. J. Math. Systems, Estim. Control, 4:1–30, 1994.

30. K. A. Morris. An Introduction to Feedback Controller Design. Harcourt-Brace Ltd., New York, 2001.

31. K. A.Morris. Convergence of LQ-optimal actuator locations for systems governed by partial differential equations. IEEE Trans. Automat. Control, 46:93–111, 2010.

32. K. A.Morris and C. Navasca. Approximation of low rank solutions for linear quadratic feedback control of partial differential equations. Comp. Opt. App., 46:93–111, 2010.

33. N. Ottosen and H. Petersson. Introduction to the Finite Element Method. Prentice-Hall, Englewood Cliffs, NJ, 1992. “73648_C067” — #37

34. A. Pazy. Semigroups of Linear Operators and Applications to Partial Differential Equations. SpringerVerlag, New York, 1983.

35. R. R. Rebarber and G. Weiss. Internal model-based tracking and disturbance rejection for stable wellposed

systems. Automatica, 39(9):1555–1569, 2003.

36. D. L. Russell. Linear stabilization of the linear oscillator in Hilbert space. J. Math. Anal. App., 25(3): 663–675, 1969.

37. R. E. Showalter. Hilbert Space Methods for Partial Differential Equations. Pitman Publishing Ltd., London, 1977.

38. D. H. Towne. Wave Phenomena. Dover Publications, New York, 1988.

39. M. Vidyasagar. Control System Synthesis: A Factorization Approach. MIT Press, Cambridge, USA, 1985.

40. G. Zames and A. El-Sakkary. Unstable systems and feedback: The gap metric. In Proc. Allerton Conf., pp. 380–385, 1980.

41. K. Zhou, J. C. Doyle, and K. Glover. Robust and Optimal Control. Prentice-Hall, Englewood Cliffs, NJ, 1996.

42. S. Q. Zhu. Graph topology and gap topology for unstable systems. IEEE Trans. Automat. Control, 34:848–855, 1989.

43. B. J. Zimmer, S. P. Lipshitz, K.A. Morris, J. Vanderkooy, and E. E. Obasi. An improved acoustic model for active noise control in a duct. ASME J. Dynamic Systems, Measurement Control, 125(3):382–395, 2003. “73648_C067” — #38

1. Adams, R.A., Sobolev Spaces, Academic, New York, 1975.

2. Balakrishnan, A.V., Applied Functional Analysis, 2nd ed., Springer, New York, 1981.

3. Banks, H.T. and Smith, R.C., Models for control in smart material structures, in Identification and Control in Systems Governed by Partial Differential Equations, Banks, H.T., Fabiano, R.H., and Ito K., Eds., SIAM, 1992, pp 27-44. “73648_C068” #31

4. Bradley, M. and Lasiecka, I., Local exponential stabilization for a nonlinearly perturbed von Kármán plate, Nonlinear Analysis: Theory, Methods and Appl., 18, 333-343, 1992.

5. Bradley,M. andLasiecka, I., Global decay rates for the solutions to a vonKármánplatewithout geometric constraints, J. Math. Anal. Appl., 181, 254-276, 1994.

6. Bresse, J. A. C., Cours de mechanique applique, Mallet Bachellier, 1859.

7. Hirschhorn, M. and Reiss, E., Dynamic buckling of a nonlinear Timoshenko beam, SIAM J. Appl.Math., 37, 290-305, 1979.

8. Horn, M. A. and Lasiecka, I., Nonlinear boundary stabilization of a von Kármán plate equation, Differential Equations, Dynamical Systems and Control Science: A Festschrift in Honor of Lawrence Markus, Elworthy, K. D., Everit, W. N., and Lee, E. B., Eds., Marcel Dekker, New York, 1993, pp 581-604.

9. Kane, T.R., Ryan, R.R., and Barnerjee, A.K., Dynamics of a beam attached to a moving base, AIAA J. Guidance, Control Dyn., 10, 139-151, 1987.

10. Komornik, V., A new method of exact controllability in short time and applications, Ann. Fac. Sci. Toulouse, 10, 415-464, 1989.

11. Komornik, V., Exact controllability and stabilization, in The Multiplier Method, Masson John Wiley & Sons, Paris, 1994.

12. Krabs, W., On Moment Theory and Controllability of

One-Dimensional Vibrating Systems and Heating Processes, in Lecture Notes in Control and Information Sciences, Vol. 173, Springer, New York, 1992.

13. Lagnese, J. E., Boundary Stabilization of Thin Plates, Studies in Applied Mathematics, SIAM, Philadelphia, 1989, Vol. 10.

14. Lagnese, J. E., TheHilbert uniquenessmethod: A retrospective, inOptimal Control of Partial Differential Equations, Hoffmann, K. H. and Krabs, W., Eds., Springer, Berlin, 1991, pp 158–181.

15. Lagnese, J. E., Recent progress in exact boundary controllability and uniform stabilizability of thin beams and plates, in Distributed Parameter Systems: New Trends and Applications Chen, G., Lee, E. B., Littmann, W., and Markus, L., Eds., Marcel Dekker, New York, 1991, pp 61–112.

16. Lagnese, J. E. and Leugering, G., Uniform stabilization of a nonlinear beam by nonlinear boundary feedback, J. Diff. Eqns., 91, 355–388, 1991.

17. Lagnese, J. E., Leugering, G., and Schmidt, E.J.P.G., Modeling, Analysis and Control of Dynamic Elastic Multi-Link Structures, Birkhäuser, Boston-Basel-Berlin, 1994.

18. Lagnese, J. E. and Lions, J. L., Modelling, Analysis and Control of Thin Plates, Collection RMA, Masson, Paris, 1988, Vol. 6.

19. Lasiecka, I. and Triggiani, R., Exact controllability of semilinear abstract systems with application to waves and plates boundary control problems, Appl. Math. Opt., 23, 109–154, 1991.

20. Lasiecka, I. and Triggiani, R., Sharp trace estimates of solutions to Kirchhoff and Euler-Bernoulli equations, Appl. Math. Opt., 28, 277–306, 1993.

21. LeDret, H., Problèmes variationnels dans les multi-domains: Modélisation des jonctions et applications, Collection RMA, Masson, Paris, 1991, vol. 19.

22. Lions, J. L., Contrôlabilité exacte, perturbations et stabilisation de systèmes distribués, Tome I: Contrôlabilté exacte, Collection RMA, Masson, Paris, 1988, Vol. 8.

23. Lions, J. L., Exact controllability, stabilization and

perturbations for distributed systems, SIAM Review, 30, 1–68, 1988.

24. Russell, D. L., On mathematical models for the elastic beam with frequency-proportional damping, in Control and Estimation in Distributed Parameter Systems, Banks, H.T., Ed., SIAM, 1992, pp 125–169.

25. Triggiani, R., Lack of exact controllability for wave and plate equations with finitely many boundary controls, Diff. Int. Equations, 4, 683–705, 1991.

26. Washizu, K., Variational Methods in Elasticity and Plasticity, 3rd ed., Pergamon, Oxford, 1982.

27. Wempner, G., Mechanics of Solids with Applications to Thin Bodies, Sijthoff and Noordhoff, Rockville, MD, 1981.

28. Zuazua, E., Contrôlabilité exacte d’un modèle de plaques vibrantes en un temps arbitrairement petit, C. R. Acad. Sci., 304, 173–176, 1987. “73648_C068” #32

1. A. Bensoussan, G. Da Prato, M. C. Delfour, S. K. Mitter, Representation and Control of Infinite Dimensional Systems, 2nd edition, Birkhäuser, Boston, 2007.

2. R.F. Curtain and A.J. Pritchard, Functional Analysis in Modern Applied Mathematics, Academic Press, New York, 1977.

3. R.F. Curtain and H. Zwart, An Introduction to Infinite-Dimensional Linear Systems Theory, SpringerVerlag, New York, 1995.

4. A.V. Fursikov and O.Yu. Imanuvilov, Controllability of Evolution Equations, Lecture Notes Series 34, Seoul National University, Research Institute of Mathematics, Seoul, 1996.

5. W. Krabs, On Moment Theory and Controllability of One-Dimensional Vibrating Systems and Heating Processes, Lecture Notes in Control and Inf. Sci. # 173, Springer-Verlag, Berlin, 1992.

6. I. Lasiecka and R. Triggiani, Differential and Algebraic Riccati Equations with Application to Boundary/Point Control Problems: Continuous Theory and Approximation Theory, Lecture Notes in Control and Inf. Sci. # 164, Springer-Verlag, Berlin, 1991.

7. J.-L. Lions, Exact controllability, stabilization, and perturbations, SIAM Review 30, pp. 1-68, 1988.

8. D.L. Russell, A unified boundary controllability theory for hyperbolic and parabolic partial differential equations, Stud. Appl. Math. LII, pp. 189-211, 1973.

9. T.I. Seidman, Boundary observation and control for the heat equation, in Calculus of Variations and Control Theory D.L. Russell, (Ed.), Academic Press, New York, NY, pp. 321-351, 1976.

10. T.I. Seidman, Exact boundary controllability for some evolution equations, SIAM J. Control Opt. 16, pp. 979-999, 1978. “73648_C069” — #20

1. Anderson, B.D.O. and Moore, J.B., Linear Optimal Control, Elect. Eng. Series, Prentice Hall, Englewood Cliffs, NJ, 1971, chap. 8.

2. Bardos, C., LeBeau, G., and Rauch, J., Contrôle et stabilisation dans des problèmes hyperboliques, in Controlabilité Exacte, Perturbations et Stabilisation de Systèmes Distribués, Lions, J.-L., Masson, Paris, 1988, appendix 2.

3. Beck, J.V. and Arnold, K.J., Parameter Estimation in Engineering and Science, John Wiley & Sons, New York, 1977.

4. Courant, R. and Hilbert, D., Methods of Mathematical Physics; Vol. 2: Partial Differential Equations, Interscience Publishers, New York, 1962.

5. Dolecki, S. and Russell, D.L., A general theory of observation and control, SIAM J. Control Opt., 15, 185–220, 1977.

6. Dunford, N. and Schwartz, J.T., Linear Operators; Vol. 1: General Theory, Interscience, New York, 1958.

7. Fattorini, H.O. and Russell, D.L., Exact controllability theorems for linear parabolic equations in one space dimension, Arch. Ration. Mech. Anal., 4, 272–292, 1971.

8. Graham, K.D. and Russell, D.L., Boundary value control of the wave equation in a spherical region, SIAM J. Control, 13, 174–196, 1975.

9. Ho, L.F., Observabilité frontière de l'équation des ondes, Cah. R. Acad. Sci., Paris, 302, 1986.

10. Ingham, A.E., Some trigonometrical inequalities with applications to the theory of series, Math. Z., 41, 367–379, 1936.

11. Lagnese, J., Controllability and stabilizability of thin beams and plates, in Distributed Parameter Control Systems, Chen, G., Lee, E.B., Littman, W., Markus, L., Eds., Lecture Notes in Pure and Applied Math., Marcel Dekker, New York, 1991, 128.

12. LaSalle, J.P. and Lefschetz, S., Stability by Lyapunov's Direct Method, with Applications, Academic

Press, New York, 1961.

13. Lions, J.L., Optimal Control of Systems Governed by Partial Differential Equations, Grund. Math. Wiss. Einz., Springer-Verlag, New York, 1971, 170.

14. Lions, J.L., Controlabilité Exacte, Perturbations et Stabilisation de Systèmes Distribués, Tomes 1,2, Recherches en Mathematiques Appliquées, Masson, Paris, 1988, 8, 9.

15. Luenberger, D.G., An introduction to observers, IEEE Trans. Autom. Control, 22, 596-602, 1971.

16. Paley, R.E.A.C. and Wiener, N., Fourier Transforms in the Complex Domain, Colloq. Pub. 19, American Mathematical Society, Providence, RI, 1934.

17. Pazy, A., Semigroups of Linear Operators and Applications to Partial Differential Equations, Applied Mathematical Sciences 44, Springer-Verlag, New York, 1983.

18. Russell, D.L., Mathematics of Finite Dimensional Control Systems: Theory and Design, Marcel Dekker, Inc., New York, 1979.

19. Russell, D.L., Controllability and stabilizability theory for linear partial differential equations: Recent progress and open questions, SIAM Rev., 20, 639-739, 1978. “73648_C070” — #12

20. Russell, D.L., Decay rates for weakly damped systems in Hilbert space obtained with control-theoretic methods, J. Diff. Eq., 19, 344-370, 1975.

21. Russell, D.L., A unified boundary controllability theory for hyperbolic and parabolic partial differential equations, Stud. Appl. Math., LII, 189-211, 1973.

22. Showalter, R.E., Hilbert Space Methods for Partial Differential Equations, Pitman Publishing Ltd., San Francisco, 1977.

1. O.M. Aamo, A. Smyshlyaev, andM. Krstic. Boundary control of the linearizedGinzburg-Landaumodel of vortex shedding. SIAM Journal of Control and Optimization, 43:1953-1971, 2005. “73648_C071” — #26

2. A. Bensoussan, G. Da Prato, M. C. Delfour, and S. K. Mitter. Representation and Control of InfiniteDimensional Systems. Birkhäuser, Boston, 2006.

3. J. Bentsman and Y. Orlov. Reduced spatial order model reference adaptive control of spatially varying distributed parameter systems of parabolic and hyperbolic types. International Journal of Adaptive Control Signal Process., 15:679-696, 2001.

4. M. Bohm, M. A. Demetriou, S. Reich, and I. G. Rosen. Model reference adaptive control of distributed parameter systems. SIAM Journal of Control and Optimization, 36:33-81, 1998.

5. D. M. Boskovic and M. Krstic. Nonlinear stabilization of a thermal convection loop by state feedback. Automatica, 37:2033-2040, 2001.

6. D. M. Boskovic and M. Krstic. Stabilization of a solid propellant rocket instability by state feedback. International Journal of Robust and Nonlinear Control, 13:483-495, 2003.

7. P. Christofides. Nonlinear and Robust Control of Partial Differential Equation Systems: Methods and Applications to Transport-Reaction Processes. Birkhäuser, Boston, 2001.

8. R. F. Curtain and H. J. Zwart. An Introduction to Infinite Dimensional Linear Systems Theory. SpringerVerlag, 1995.

9. K. S. Hong and J. Bentsman. Direct adaptive control of parabolic systems: algorithm synthesis and convergence and stability analysis. IEEE Transactions on Automatic Control, 39:2018-2033, 1994.

10. P. Ioannou and J. Sun. Robust Adaptive Control. Prentice-Hall, Englewood cliffs, NJ, 1996.

11. M. Krstic. Systematization of approaches to adaptive boundary stabilization of PDEs. International Journal of Robust and Nonlinear Control, 16:812-818, 2006.

12. M. Krstic, I. Kanellakopoulos, and P. Kokotovic. Nonlinear and Adaptive Control Design. Wiley, New York, 1995.

13. M.Krstic andA. Smyshlyaev. Adaptive boundary control for unstable parabolic PDEs—Part I: Lyapunov design. IEEE Transactions on Automatic Control, 53:1575–1591, 2008.

14. M. Krstic and A. Smyshlyaev. Backstepping boundary control for first order hyperbolic PDEs and application to systems with actuator and sensor delays. Systems and Control Letters, 57:750–758, 2008.

15. M. Krstic and A. Smyshlyaev. Boundary Control of PDEs: A Course on Backstepping Designs. SIAM, 2008.

16. M. Krstic, A. Smyshlyaev, and A.A. Siranosian. Backstepping boundary controllers and observers for the slender Timoshenko beam: Part I—Design. In Proceedings of the 2006 American Control Conferencel, 2006.

17. I. Lasiecka and R. Triggiani. Control Theory for Partial Differential Equations: Continuous and Approximation Theories. Cambridge Univ. Press, Cambridge, UK, 2000.

18. Z. H. Luo, B. Z. Guo, and O. Morgul. Stability and Stabilization of Infinite Dimensional Systems with Applications. Springer-Verlag, 1999.

19. L. Praly. Adaptive regulation: Lyapunov design with a growth condition. International Journal of Adaptive Control Signal Processing, 6:329–351, 1992.

20. A. Smyshlyaev, B. Z. Guo, and M. Krstic. Boundary controllers for Euler-Bernoulli beam with arbitrary decay rate. In Proceedings of the 2008 IEEE Conference on Decision and Control, 2008.

21. A. Smyshlyaev and M. Krstic. Closed form boundary state feedbacks for a class of 1D partial integrodifferential equations. IEEE Transactions on Automatic Control, 49:2185–2202, 2004.

22. A. Smyshlyaev and M. Krstic. Backstepping observers for a class of parabolic PDEs. Systems and Control Letters, 54:613–625, 2005.

23. A. Smyshlyaev and M. Krstic. Lyapunov adaptive boundary

control for parabolic PDEs with spatially varying coefficients. In Proceedings of 2006 American Control Conference, 2006.

24. A. Smyshlyaev and M. Krstic. Output-feedback adaptive control for parabolic PDEs with spatially varying coefficients. In Proceedings of 2006 IEEE Conference on Decision and Control, 2006.

25. A. Smyshlyaev and M. Krstic. Adaptive boundary control for unstable parabolic PDEs—Part II: Estimation-based designs. Automatica, 43:1543–1556, 2007.

26. A. Smyshlyaev andM. Krstic. Adaptive boundary control for unstable parabolic PDEs—Part III: Output feedback examples with swapping identifiers. Automatica , 43:1557–1564, 2007.

27. R. Vazquez andM. Krstic. Explicit integral operator feedback for local stabilization of nonlinear thermal convection loop PDEs. Systems and Control Letters, 55:624–632, 2006.

28. R. Vazquez and M. Krstic. Control of Turbulent and Magnetohydrodynamic Channel Flows. Birkhäuser, Boston, 2007.

29. R. Vazquez and M. Krstic. Control of 1-D parabolic PDEs with Volterra nonlinearities—Part I: Design. Automatica, 44:2778–2790, 2008.

30. R.Vazquez andM.Krstic. Control of 1-Dparabolic PDEswithVolterra nonlinearities—Part II: Analysis. Automatica, 44:2791–2803, 2008.

V. Anantharam and C. A. Desoer. On the stabilization of nonlinear systems. IEEE Transactions on Automatic Control, 29(6):569–572, 1984.

C. A. Desoer and R. W. Liu. Global parametrization of feedback systems with nonlinear plants. Systems and Control Letters, 1(4):249–251, 1982.

V. Manousiouthakis. On the parametrization of all stabilizing decentralized controllers. Systems and Control Letters, 21(5):397–403, 1993.

M. Rotkowitz. Information structures preserved under nonlinear time-varying feedback. In Proc. American Control Conference, Minneapolis, Minnesota, pp. 4207–4212, June 2006.

M. Rotkowitz. Parametrization of all stabilizing controllers subject to any structural constraint. In Proc. IEEE Conference on Decision and Control, Atlanta, Georgia, December 2010.

M. Rotkowitz, R. Cogill, and S. Lall. A simple condition for the convexity of optimal control over networks with delays. In Proc. IEEE Conference on Decision and Control, Seville, Spain, pp. 6686–6691, December 2005.

M. Rotkowitz, R. Cogill, and S. Lall. Convexity of optimal control over networks with delays and arbitrary topology. International Journal of Systems, Control, and Communications, 2:30–54, 2010. “73648_C074” — #9

M. Rotkowitz and S. Lall. Decentralized control information structures preserved under feedback. In Proc. IEEE Conference on Decision and Control, Las Vegas, Nevada, pp. 569–575, December 2002.

M. Rotkowitz and S. Lall. Affine controller parameterization for decentralized control over Banach spaces. IEEE Transactions on Automatic Control, 51(9):1497–1500, 2006a.

M. Rotkowitz and S. Lall. A characterization of convex problems in decentralized control. IEEE Transactions on Automatic Control, 51(2):274–286, 2006b.

H. S. Witsenhausen. A counterexample in stochastic optimum

control. SIAM Journal of Control, 6(1):131–147, 1968.

D.C. Youla, H.A. Jabr, and J.J. Bonjiorno Jr. Modern Wiener–Hopf design of optimal controllers: Part II. IEEE Transactions on Automatic Control, 21(3):319–338, 1976.

References

1. Dahleh, M. A. and Diaz-Bobillo, I. J., *Control of Uncertain Systems: A Linear Programming Approach*, Englewood Cliffs, NJ: Prentice Hall, 1995.
2. Hildebrand, F. B., *Advanced Calculus for Applications*, 2nd Ed., Englewood Cliffs, NJ: Prentice Hall, 1976.
3. Desoer, C. A. and Vidyasagar, M., *Feedback Systems: Input–Output Properties*, New York: Academic Press, 1975.
4. Freudenburg, J. S. and Looze, D. P., *Frequency Domain Properties of Scalar and Multivariable Feedback Systems*, Berlin: Springer-Verlag, 1987.
5. Doyle, J.C. and Stein, G., Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis, *IEEE Trans. Autom. Control*, 26(1), 4–16, 1981.
6. Maciejowski, J. M., *Multivariable Feedback Design*, Reading, MA: Addison-Wesley, 1989.
7. Doyle J. C., Francis, B. A., and Tannenbaum, A. R., *Feedback Control Theory*, New York: Macmillan, 1992.
8. Green, M. and Limebeer, D. J. N., *Linear Robust Control*, Englewood Cliffs, NJ: Prentice-Hall, 1995.
9. Morari, M. and Zafiriou, E., *Robust Process Control*, Englewood Cliffs, NJ: Prentice Hall, 1989.

References

1. Leon-Garcia, A., *Probability and Random Processes for Electrical Engineering*, Addison-Wesley, Reading, MA, 1989.
2. Wong, E., *Introduction to Random Processes*, Springer-Verlag, Amsterdam, 1983.
3. Breiman, L., *Probability*, SIAM (republication of a 1968 book published by Addison-Wesley), Philadelphia, PA, 1992.
4. Wong, E., *Stochastic Processes in Information and Dynamical Systems*, McGraw-Hill, New York, 1971.
5. Oppenheim, A.V. and Willsky, A.S., with S. H. Nawab, *Signals and Systems*. 2nd edition, Prentice-Hall, Upper Saddle River, NJ, 1997.
6. Davis, M.H.A., *Linear Estimation and Stochastic Control*, Chapman & Hall, London, 1977.
7. Astrom, K.J., *Introduction to Stochastic Control Theory*, Academic Press, New York, 1970.
8. Caines, P.E., *Linear Stochastic Systems*, John Wiley & Sons, New York, 1988.
9. Grimmett, G. and Stirzaker, D., *Probability and Random Processes*, 3rd Ed., Oxford University Press, New York, 2001.

References

1. Anderson, B.D.O. and Moore, J.B., *Optimal Control: Linear Quadratic Methods,* Prentice Hall, Englewood Cliffs, NJ, 1990.
2. Kwakernaak, H. and Sivan, R., *Linear Optimal Control Systems,* John Wiley & Sons, New York, 1972.
3. Emami-Naeini, A. and Rock, S.M., On asymptotic behavior of non-square linear optimal regulators, in *Proc. 23rd Conf. Decision and Control,* Las Vegas, NV, December 1984, pp.1762–1763.
4. Weinmann, A., *Uncertain Models and Robust Control,* Springer-Verlag, New York, 1991.
5. Grocott, S.C.O., Comparision of Control Techniques for Robust Performance on Uncertain Structural Systems, Master's thesis, Massachusetts Institute of Technology, 1994. MIT SERC report No. 2-94.
6. Sesak, J.R., Sensitivity Constrained Linear Optimal Control Analysis and Synthesis, Ph.D. thesis, University of Wisconsin, 1974.
7. Grocott, S.C.O., How, J.P., and Miller, D.W., A comparison of robust control techniques for uncertain structural systems, *Proc. AIAA Guidance Navigation and Control Conference,* Scottsdale, AZ, August 1994, pp. 261–271.
8. Gupta, N., Frequency-shaped cost functionals: extension of linear-quadratic-Gaussian methods, *J. Guidance Control Dynam.,* 3(6), 529–535, 1980.
9. Doyle, J., Francis, B., and Tannenbaum, A., *Feedback Control Theory,* Macmillan, New York, 1992.
10. Kwakernaak, H., Robust control and $\mathcal{H}_\infty$ optimization-tutorial paper, *Automatica,* 29(2), 255–273, 1993.

18

$\mathcal{H}_2$ (LQG) and $\mathcal{H}_\infty$ Control

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18.1 Introduction

The fundamentals of output feedback $\mathcal{H}_2$ (linear quadratic Gaussian or LQG) and $\mathcal{H}_\infty$ controllers, which are the primary synthesis tools available for linear time-invariant systems, are presented in an analogous and tutorial fashion without rigorous mathematics. Since $\mathcal{H}_2$ and $\mathcal{H}_\infty$ syntheses are carried out in the modern control design paradigm, a review of the paradigm is presented, along with the definitions of the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms and the methods used to compute them. The state-space formulae for the optimal controllers, under less restrictive assumptions than are usually found in the literature, are provided in an analogous fashion to emphasize the similarities between them. Rather than emphasizing the derivation of the controllers, we elaborate on the physical interpretation of the results and how one uses frequency weights to design $\mathcal{H}_\infty$ and $\mathcal{H}_2$ controllers. Finally, a simple disturbance rejection design for the longitudinal motion of an aircraft is provided to illustrate the similarities and differences between $\mathcal{H}_\infty$ and $\mathcal{H}_2$ controller synthesis.

18.2 The Modern Paradigm

$\mathcal{H}_2$ and $\mathcal{H}_\infty$ syntheses are carried out in the modern control paradigm. In this paradigm both performance and robustness specifications can be incorporated in a common framework along with the controller synthesis. In the modern paradigm, all of the information about a system is cast into the generalized block diagram shown in Figure 18.1 [1–3]. The generalized plant, P, which is assumed to be linear and time-invariant throughout this article contains all the information a designer would like to incorporate into the synthesis of the controller, K. System dynamics, models of the uncertainty in the system's dynamics, frequency weights to influence the controller synthesis, actuator dynamics, sensor dynamics, and implementation hardware dynamics from amplifiers, and analog-to-digital and digital-to-analog converters are all included in P. The inputs and outputs of P are, in general, vector valued signals. The sensor measurements that are used by the feedback controller are denoted y, and the inputs generated

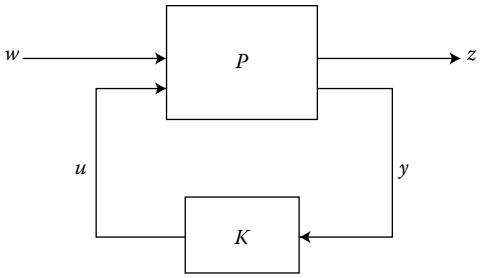

FIGURE 18.1 Generalized block diagram of the modern paradigm.

by the controller are denoted u. The components of w are all the exogenous inputs to the system. Typically these consist of disturbances, sensor noise, reference commands, and fictitious signals that drive frequency weights and models of the uncertainty in the dynamics of the system. The components of z are all the variables we wish to control. These include the performance variables of interest, tracking errors between reference signals and plant outputs, and the actuator signals which cannot be arbitrarily large and fast.

The general control problem in this framework is to synthesize a controller that will keep the size of the performance variables, z, small in the presence of the exogenous signals, w. For a classical disturbance rejection problem, z would contain the performance variables we wish to keep small in the presence of the disturbances contained in w that would tend to drive z away from zero. Hence, the disturbance rejection performance would depend on the "size" of the closed-loop transfer function from w to z, which we shall denote as $T_{zw}(s)$. This is also true for a command following control problem in which z would contain the tracking error that we would like to keep small in the presence of the commands in w that drive the tracking error away from zero.

Clearly then, the "size" of $T_{zw}(s)$ influences the effect that the exogenous signals in w have on z. Thus, in this framework, we seek controllers that minimize the "size" of the closed-loop transfer function $T_{zw}(s)$. Given that $T_{zw}(s)$ is a transfer function matrix, it is necessary to use appropriate norms to quantify its size. The two most common and physically meaningful norms that are used to classify the "size" of $T_{zw}(s)$ are the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms. As such, we seek controllers that minimize either the $\mathcal{H}_2$ or $\mathcal{H}_\infty$ norm of $T_{zw}(s)$ in the modern control paradigm.

18.2.1 System Norms

Here we define and discuss the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of the linear, time-invariant, stable system with transfer function matrix

$$G(s) = C(sI - A)^{-1}B$$

This notation is meant to be general, and the reader should not think of $G(s)$ as only the actuator to sensor transfer function of a system. Realize that $G(s)$ is a system and thus requires an appropriate norm to classify its size. By a norm, we mean a positive, scalar number that is a measure of the size of $G(s)$ over all points in the complex s-plane. This is quite different from, for example, the maximum singular value of a matrix, $\sigma_{\max}[A]$, which is a norm that classifies the size of the matrix A.

The $\mathcal{H}_2$ Norm

Definition 18.1: $\mathcal{H}_2$ Norm

The $\mathcal{H}_2$ norm of $G(s)$, denoted $\|G\|_2$, is defined as

$$\|G\|_2 = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} \text{trace}\left[G(\jmath\omega)G^*(\jmath\omega)\right] \mathrm{d}\omega\right)^{\frac{1}{2}} = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} \sum_{i=1}^{r} \sigma_i^2[G(\jmath\omega)]\mathrm{d}\omega\right)^{\frac{1}{2}}$$

where σ_i denotes the ith singular value, $G^*(\jmath\omega)$ is the complex conjugate transpose of $G(\jmath\omega)$, and r is the rank of $G(\jmath\omega)$.

The $\mathcal{H}_2$ norm has an attractive, physically meaningful interpretation. If we consider $G(s)$ to be the transfer function matrix of a system driven by independent, zero mean, unit intensity white noise, u, then the sum of the variances of the outputs y is exactly the square of the $\mathcal{H}_2$ norm of $G(s)$. That is

$$\mathrm{E}\left[y^T(t)y(t)\right] = \|G(s)\|_2^2 \tag{18.1}$$

The $\mathcal{H}_2$ norm of $G(s)$ thus gives a precise measure of the "power" or signal strength of the output of a system driven with unit intensity white noise. Note that in the scalar case $\sqrt{\mathrm{E}[y^T(t)y(t)]}$ is the RMS or root mean squared value for $y(t)$ so the $\mathcal{H}_2$ norm specifies the RMS value of $y(t)$. A well-known fact for stochastic systems is that the mean squared value of the outputs can be computed by solving the appropriate Lyapunov equation [4]. As such, a state space procedure for computing the $\mathcal{H}_2$ norm of $G(s)$ is as follows [2].

Computing the $\mathcal{H}_2$ Norm

If L_c denotes the controllability Gramian of (A, B) and L_o the observability Gramian of (A, C), then

$$AL_c + L_cA^T + BB^T = 0 \quad A^TL_o + L_oA + C^TC = 0$$

and

$$\|G\|_2 = \left[\text{trace}(CL_cC^T)\right]^{\frac{1}{2}} = \left[\text{trace}(B^TL_oB)\right]^{\frac{1}{2}}$$

Note that this procedure for computing the $\mathcal{H}_2$ norm involves the solution of linear Lyapunov equations and can be done without iteration.

The $\mathcal{H}_\infty$ Norm

Definition 18.2: $\mathcal{H}_\infty$ Norm

The $\mathcal{H}_\infty$ norm of $G(s)$, denoted $\|G\|_\infty$, is defined as

$$\|G\|_\infty = \sup_\omega \sigma_{\max}[G(\jmath\omega)]$$

In this definition "sup" denotes the supremum or least upper bound of the function $\sigma_{\max}[G(\jmath\omega)]$, and thus the $\mathcal{H}_\infty$ norm of $G(s)$ is nothing more than the maximum value of $\sigma_{\max}[G(\jmath\omega)]$ over all frequencies ω. The supremum must be used in the definition since, strictly speaking, the maximum of $\sigma_{\max}[G(\jmath\omega)]$ may not exist even though $\sigma_{\max}[G(\jmath\omega)]$ is bounded from above.

$\mathcal{H}_\infty$ norms also have a physically meaningful interpretation when considering the system $y(s) = G(s)u(s)$. Recall that when the system is driven with a unit magnitude sinusoidal input at a specific frequency, $\sigma_{\max}[G(\jmath\omega)]$ is the largest possible output size for the corresponding sinusoidal output. Thus, the $\mathcal{H}_\infty$ norm is the largest possible amplification over all frequencies of a unit sinusoidal input. That is, it classifies the greatest increase in energy that can occur between the input and output of a given system. A state space procedure for calculating the $\mathcal{H}_\infty$ norm is as follows.

Computing the $\mathcal{H}_\infty$ Norm

Let $\|G\|_\infty = \gamma_{\min}$. For the transfer function $G(s) = C(sI - A)^{-1}B$ with A stable and $\gamma > 0$, $\|G\|_\infty < \gamma$ if and only if the Hamiltonian matrix

$$H = \begin{bmatrix} A & \frac{1}{\gamma^2}BB^T \\ -C^TC & -A^T \end{bmatrix}$$

has no eigenvalues on the $\jmath\omega$-axis. This fact lets us compute a bound, γ, on $\|G\|_\infty$ such that $\|G\|_\infty < \gamma$. So to find $\gamma_{\min}$, select a $\gamma > 0$ and test if H has eigenvalues on the $\jmath\omega$-axis. If it does, increase γ. If it does not, decrease γ and recompute the eigenvalues of H. Continue until $\gamma_{\min}$ is calculated to within the desired tolerance.

The iterative computation of the $\mathcal{H}_\infty$ norm, which can be carried out efficiently using a bisection search over γ, is to be expected given that by definition we must search for the largest value of $\sigma_{\max}[G(\jmath\omega)]$ over all frequencies.

Note, the $\mathcal{H}_2$ norm is not an induced norm, whereas the $\mathcal{H}_\infty$ norm is. Thus, the $\mathcal{H}_2$ norm does not obey the submultiplicative property of induced norms. That is, the $\mathcal{H}_\infty$ norm satisfies

$$\|G_1G_2\|_\infty \leq \|G_1\|_\infty \|G_2\|_\infty$$

but the $\mathcal{H}_2$ norm does not have the analogous property. This fact makes synthesizing controllers that minimize $\|T_{zw}(s)\|_\infty$ attractive when one is interested in directly shaping loops to satisfy norm bounded robustness tests*. On the other hand, given the aforementioned properties of the $\mathcal{H}_2$ norm, synthesizing controllers that minimize $\|T_{zw}(s)\|_2$ is attractive when the disturbances, w, are stochastic in nature. In fact, $\mathcal{H}_2$ controllers are nothing more than linear quadratic Gaussian (LQG) controllers so the vast amount of insight into the well-understood LQG problem can be readily applied to $\mathcal{H}_2$ synthesis.

Example 18.1:

In this example the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms are calculated for the simple four-spring, four-mass system shown in Figure 18.2. The equations of motion for this system can be found in Example 17.2 in Chapter 17. The system has force inputs on the second and fourth masses along with two sensors that provide a measure of the displacement of these masses. The singular values of the transfer function from the inputs to outputs, which we denote by $G(s)$, are shown in Figure 18.3. The $\mathcal{H}_\infty$ norm of the system is equal to the peak of $\sigma_1 = 260.4$, and the $\mathcal{H}_2$ norm of the system is equal to the square root of the sum of the areas under the square of each of the singular values, 14.5. Note that when considering the $\mathcal{H}_2$ norm, observing the log log plot of the transfer function can be very deceiving, since the integral is of σ_i, not $\log(\sigma_i)$, over ω, not $\log \omega$.

As pointed out in the example, the differences between the $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms for a system $G(s)$ are best viewed in the frequency domain from a plot of the singular values of $G(\jmath\omega)$. Specifically, the $\mathcal{H}_\infty$ norm is the peak value of $\sigma_{\max}[G(\jmath\omega)]$ while the $\mathcal{H}_2$ norm is related to the area underneath the singular values of $G(\jmath\omega)$. For a more in-depth treatment of these norms the reader is referred to [1,2,5,6].

* See Chapter 20 for a detailed exposition of this concept.

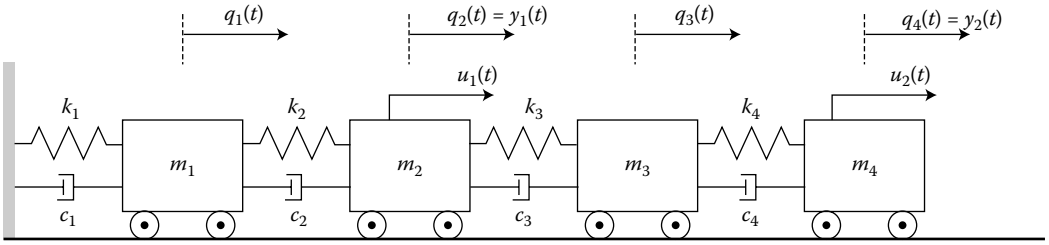

FIGURE 18.2 Mass, spring, dashpot system from Example 18.1. For the example $k_i = m_i = 1\ \forall i$, and $c_i = 0.05\ \forall i$.

18.3 Output Feedback $\mathcal{H}_\infty$ and $\mathcal{H}_2$ Controllers

Given that all the information a designer would like to include in the controller synthesis is incorporated into the system P, the synthesis of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ controllers is quite straightforward. In this respect all of the design effort is focused on defining P. Below, we discuss how to define P using frequency weights to meet typical control system specifications. Here we simply present the formulas for the controllers.

All the formulas will be based on the following state-space realization of P,

$$P := \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

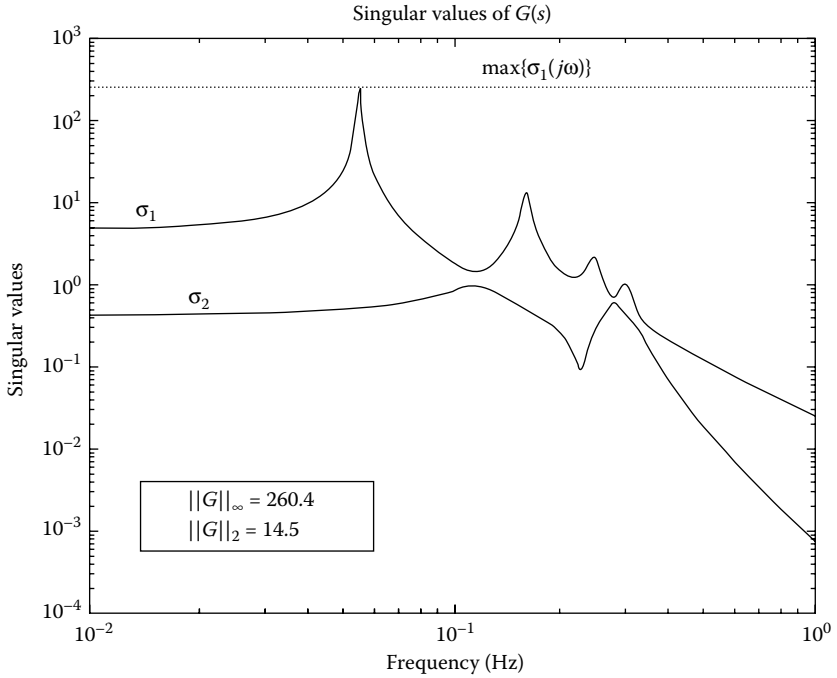

FIGURE 18.3 Singular values of the transfer function between the inputs and outputs of the mass, spring system shown in Figure 18.2.

This notation is a shorthand representation for the system of equations

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \tag{18.2}$$
$$z(t) = C_1 x(t) + D_{11} w(t) + D_{12} u(t) \tag{18.3}$$
$$y(t) = C_2 x(t) + D_{21} w(t) + D_{22} u(t) \tag{18.4}$$

Additionally, the following assumptions concerning the allowable values for the elements of P are made.

Assumptions on P

1. $D_{11} = 0$ (A.1)
2. $[A \quad B_2]$ is stabilizable (A.2)
3. $[A \quad C_2]$ is detectable (A.3)
4. $V = \begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} \begin{bmatrix} B_1^T & D_{21}^T \end{bmatrix} := \begin{bmatrix} V_{xx} & V_{xy} \\ V_{xy}^T & V_{yy} \end{bmatrix} \geq 0 \quad \text{with } V_{yy} > 0$ (A.4)
5. $R = \begin{bmatrix} C_1^T \\ D_{12}^T \end{bmatrix} \begin{bmatrix} C_1 & D_{12} \end{bmatrix} := \begin{bmatrix} R_{xx} & R_{xu} \\ R_{xu}^T & R_{uu} \end{bmatrix} \geq 0 \quad \text{with } R_{uu} > 0$ (A.5)

Assumption A.1 ensures that none of the disturbances feed through to the performance variables which is necessary for $\mathcal{H}_2$ synthesis but may be removed for $\mathcal{H}_\infty$ synthesis (see [7] for details.) Assumptions A.2 and A.3 are needed to guarantee the existence of a stabilizing controller while the remaining assumptions are needed to guarantee the existence of positive semidefinite solutions to the Riccati equations associated with the optimal controllers.

Theorem 18.1: $\mathcal{H}_2$ Output Feedback

Assuming that $w(t)$ is a unit intensity white noise signal, $E[w(t)w^T(\tau)] = I\delta(t-\tau)$, the unique, stabilizing, optimal controller which minimizes the $\mathcal{H}_2$ norm of $T_{zw}(s)$ is

$$K_2 := \left[\begin{array}{c|c} A + B_2F_2 + L_2C_2 + L_2D_{22}F_2 & -L_2 \\ \hline F_2 & 0 \end{array}\right] \tag{18.5}$$

where

$$F_2 = -R_{uu}^{-1}\left(R_{xu}^T + B_2^T X_2\right)$$
$$L_2 = -\left(Y_2 C_2^T + V_{xy}\right) V_{yy}^{-1} \tag{18.6}$$

and X_2 and Y_2 are the unique, positive semidefinite solutions to the following Riccati equations

$$0 = X_2 A_r + A_r^T X_2 + R_{xx} - R_{xu} R_{uu}^{-1} R_{xu}^T - X_2 B_2 R_{uu}^{-1} B_2^T X_2 \tag{18.7}$$
$$0 = A_e Y_2 + Y_2 A_e^T + V_{xx} - V_{xy} V_{yy}^{-1} V_{xy}^T - Y_2 C_2^T V_{yy}^{-1} C_2 Y_2 \tag{18.8}$$

where

$$A_r = \left(A - B_2 R_{uu}^{-1} R_{xu}^T\right) \quad \text{and} \quad A_e = \left(A - V_{xy} V_{yy}^{-1} C_2\right)$$

Theorem 18.2: $\mathcal{H}_\infty$ Output Feedback [8]

Assuming that $w(t)$ is a bounded $\mathcal{L}_2$ signal, $\int_{-\infty}^{\infty} w^T(t)w(t)\,\mathrm{d}t < \infty$, a stabilizing controller which satisfies $\|T_{zw}(\jmath\omega)\|_\infty < \gamma$ is

$$K_\infty := \left[\begin{array}{c|c} A & -Z_\infty L_\infty \\ \hline F_\infty & 0 \end{array}\right] \tag{18.9}$$

where

$$A_\infty = A + \left(B_1 + L_\infty D_{21}\right) W_\infty + B_2 F_\infty + Z_\infty L_\infty C_2 + Z_\infty L_\infty D_{22} F_\infty$$

where

$$F_\infty = -R_{uu}^{-1}\left(R_{xu}^T + B_2^T X_\infty\right) \quad W_\infty = \frac{1}{\gamma^2} B_1^T X_\infty$$

$$L_\infty = -\left(Y_\infty C_2^T + V_{xy}\right) V_{yy}^{-1} \quad Z_\infty = \left(I - \frac{1}{\gamma^2} Y_\infty X_\infty\right)^{-1}$$

and X_∞ and Y_∞ are the solutions to the following Riccati equations

$$0 = X_\infty A_r + A_r^T X_\infty + R_{xx} - R_{xu} R_{uu}^{-1} R_{xu}^T - X_\infty \left(B_2 R_{uu}^{-1} B_2^T - \frac{1}{\gamma^2} B_1 B_1^T\right) X_\infty \tag{18.10}$$

$$0 = A_e Y_\infty + Y_\infty A_e^T + V_{xx} - V_{xy} V_{yy}^{-1} V_{xy}^T - Y_\infty \left(C_2^T V_{yy}^{-1} C_2 - \frac{1}{\gamma^2} C_1^T C_1\right) Y_\infty \tag{18.11}$$

that satisfy the following conditions

1. $X_\infty \geq 0$
2. *The Hamiltonian matrix for Equation 18.10,*

$$\begin{bmatrix} A - B_2 R_{uu}^{-1} R_{xu}^T & -B_2 R_{uu}^{-1} B_2^T + \frac{1}{\gamma^2} B_1 B_1^T \\ -R_{xx} + R_{xu} R_{uu}^{-1} R_{xu}^T & -\left(A - B_2 R_{uu}^{-1} R_{xu}^T\right)^T \end{bmatrix}$$

   has no $\jmath\omega$-axis eigenvalues, or equivalently $A + B_1 W_\infty + B_2 F_\infty$ is stable
3. $Y_\infty \geq 0$
4. *The Hamiltonian matrix for Equation 18.11,*

$$\begin{bmatrix} \left(A - V_{xy} V_{yy}^{-1} C_2\right)^T & -C_2^T V_{yy}^{-1} C_2 + \frac{1}{\gamma^2} C_1^T C_1 \\ -V_{xx} + V_{xy} V_{yy}^{-1} V_{xy}^T & -A + V_{xy} V_{yy}^{-1} C_2 \end{bmatrix}$$

   has no $\jmath\omega$-axis eigenvalues, or equivalently $A + L_\infty C_2 + \frac{1}{\gamma^2} Y_\infty C_1^T C_1$ is stable
5. $\rho(Y_\infty X_\infty) < \gamma^2$, *where $\rho(\cdot) = \max_i |\lambda_i(\cdot)|$ is the spectral radius*

The (sub)optimal central $\mathcal{H}_\infty$ controller which minimizes $\|T_{zw}\|_\infty$ to within the desired tolerance is K_∞ with γ equal to the smallest value of $\gamma > 0$ that satisfies conditions 1 to 5.

Unlike the $\mathcal{H}_2$ controller, the $\mathcal{H}_\infty$ controller presented here is not truly optimal. Since there is no closed-form, state-space solution to the problem of minimizing the infinity norm of a multiple-input,

multiple-output (MIMO) transfer function matrix $T_{zw}(s)$, the connections between the mini-max optimization problem

$$\inf_{u}\sup_{w}\int_0^\infty \left[z^T(t)z(t)-\gamma^2 w^T(t)w(t)\right]\mathrm{d}t \tag{18.12}$$

and $\mathcal{H}_\infty$ optimization are used to arrive at the constructive approach for synthesizing suboptimal $\mathcal{H}_\infty$ controllers given in Theorem 18.2. In fact, satisfying the conditions 1 to 5 of Theorem 18.2 is analogous to finding a saddle point of the optimization problem (Equation 18.12), and the search for $\gamma_{\min}$ is analogous to finding the global minimum over all the possible saddle points. As such, any value of $\gamma > \gamma_{\min}$ will also satisfy conditions 1 to 5 of Theorem 18.2, and thus produce a stabilizing controller. Such controllers are neither $\mathcal{H}_2$ nor $\mathcal{H}_\infty$ optimal. Since in the limit as $\gamma \to \infty$ the equations from Theorem 18.2 reduce to the equations for the $\mathcal{H}_2$ optimal controller, controllers with values of γ between $\gamma_{\min}$ and infinity provide a trade off between $\mathcal{H}_\infty$ and $\mathcal{H}_2$ performance. Along these lines, it is also worth noting that there is a rich theory for mixed $\mathcal{H}_2/\mathcal{H}_\infty$ controllers that minimize the $\mathcal{H}_2$ norm of $T_{zw}(s)$ subject to additional $\mathcal{H}_\infty$ constraints. See [9–11] for details.

The value of $w(t)$ that maximizes the cost in Equation 18.12 is known as the worst case disturbance, as it seeks to maximize the detrimental effect the disturbances have on the system. In this regard, $\mathcal{H}_\infty$ controllers provide optimal disturbance rejection to worst case disturbance, whereas the $\mathcal{H}_2$ controllers provide optimal disturbance rejection to stochastic disturbances.

Both $\mathcal{H}_2$ and $\mathcal{H}_\infty$ controllers are observer-based compensators [2], which can be seen from their block diagrams, shown in Figures 18.4 and 18.5. The regulator gains F_2 and F_∞ arise from synthesizing the full-state feedback controller, which minimizes the appropriate size of $z^T(t)z(t)$ constrained by the system dynamics Equation 18.2. Then the control law is formed by applying these regulator gains to an estimate of the states $x(t)$. The states, $x(t)$, are estimated using the noisy measurements of $y(t)$ from Equation 18.4, and L_2 and $Z_\infty L_\infty$ are the corresponding filter gains of the estimators.

In particular, F_2 is the full-state feedback LQR gain that minimizes the quadratic cost

$$J_{\mathrm{LQ}} = \mathrm{E}\left\{\lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau \left[z^T(t)z(t)\right]\mathrm{d}t\right\}$$

constrained by the dynamics of Equation 18.2, and L_2 is the Kalman filter gain from estimating the states x based on the measurements $y(t)$. Under the assumption that $z(t)$ is an ergodic process*

$$J_{\mathrm{LQ}} = \lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau z^T(t)z(t)\mathrm{d}t = \mathrm{E}\left[z^T(t)z(t)\right] = \|T_{zw}\|_2^2 \tag{18.13}$$

and this is exactly why $\mathcal{H}_2$ synthesis is nothing more than LQG control.

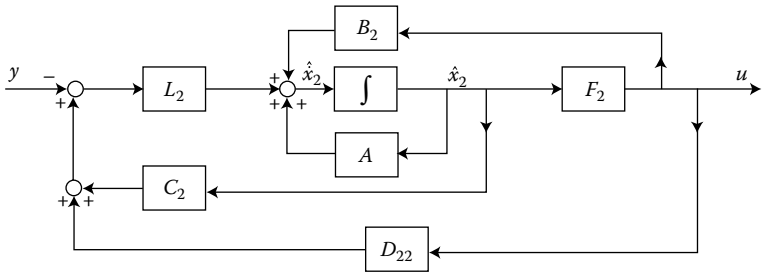

FIGURE 18.4 Block diagram of K_2 from Equation 18.9. Note, the Kalman Filter estimate of the states $x(t)$ from Equation 18.2, $\hat{x}_2(t)$, are the states of K_2.

* Assuming $z(t)$ is ergodic implies that its mean can be computed from the time average of a measurement of $z(t)$ as $t \to \infty$ [4].

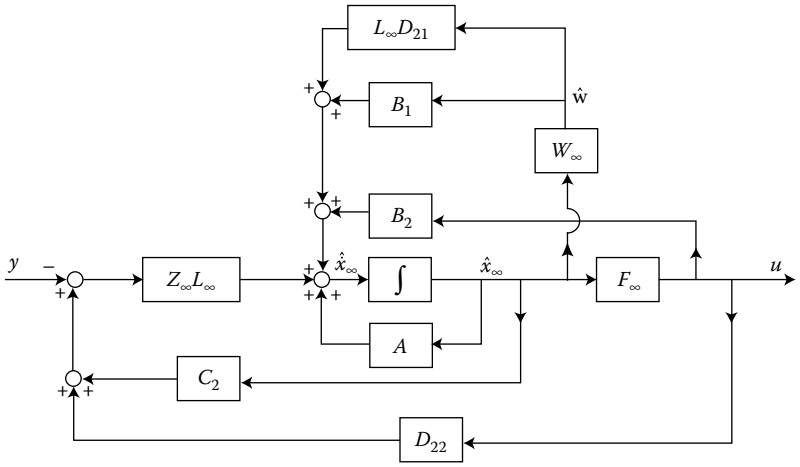

FIGURE 18.5 Block diagram of K_∞ from Equation 18.9. Note, the $\mathcal{H}_\infty$ optimal estimate of the states $x(t)$ from Equation 18.2, $\hat{x}_\infty(t)$, are the states of K_∞, and $\hat{w}(t)$ is an estimate of the worst case disturbance.

Analogously, F_∞ is the full-state feedback $\mathcal{H}_\infty$ control gain that results from optimizing the mini-max cost of Equation 18.12, and W_∞ is the full-state feedback gain that produces the worst case disturbance which maximizes the cost of Equation 18.12*. Unlike the Kalman filter in the $\mathcal{H}_2$ controller, the $\mathcal{H}_\infty$ optimal estimator must estimate the states of P in the presence of the worst case disturbance which is evident from the block diagram of K_∞ shown in Figure 18.5 [12]. This is why the filter gain of the $\mathcal{H}_\infty$ optimal estimator, $Z_\infty L_\infty$, is coupled to the regulator portion of the problem through X_∞ from Equation 18.10.

Since the $\mathcal{H}_2$ controller is an LQG controller, the closed-loop poles of $T_{zw}(s)$ separate into the closed-loop poles of the regulator, eig$(A - B_2F_2)$, and estimator, eig$(A - L_2C_2)$. A consequence of this separation property is that the $\mathcal{H}_2$ Riccati equations (Equations 18.7 and 18.8) can be solved directly without iteration. Since the worst case disturbance must be taken into consideration when synthesizing the $\mathcal{H}_\infty$ optimal estimator, the regulator and estimator problems in the $\mathcal{H}_\infty$ synthesis are coupled. Thus, the $\mathcal{H}_\infty$ controller does not have a separation structure that is analogous to that of the $\mathcal{H}_2$ controller. In addition, the $\mathcal{H}_\infty$ Riccati equations (Equations 18.10 and 18.11) are further coupled through the γ parameter, and we must iterate over the value of γ to find solutions of the $\mathcal{H}_\infty$ Riccati equations that satisfy conditions 1 to 5 of Theorem 18.2.

Note that in the literature the following set of additional, simplifying assumptions on the values of the elements of P are often made to arrive at less complicated sets of equations for the optimal $\mathcal{H}_\infty$ and $\mathcal{H}_2$ controllers [6,13,14].

Additional Assumptions on P

1.	$D_{22} = 0$	(No control feed-through term)
2.	$C_1^T D_{12} = 0$	(No cross penalty on control and state)
3.	$B_1 D_{21}^T = 0$	(Uncorrelated process and sensor noise)
4.	$D_{12}^T D_{12} = I$	(Unity penalty on every control)
5.	$D_{21}^T D_{21} = I$	(Unit intensity sensor noise on every measurement)

* See the section on $\mathcal{H}_\infty$ Full State Feedback in Chapter 17 for details.

References

1. Doyle, J., Francis, B., and Tannenbaum, A., *Feedback Control Theory*, Macmillan, New York, 1992.
2. Boyd, S. P. and Barrat, C. H., *Linear Controler Design: Limits of Performance*, Prentice Hall, Englewood Cliffs, NJ, 1991.
3. Zhou, K. Doyle, J., and Glover, K., *Robust and Optimal Control*, Prentice Hall, Englewood Cliffs, NJ, 1995.
4. Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., McGraw Hill, New York, 1991.
5. Francis, B. A., *A Course in $\mathcal{H}_\infty$ Control Theory*, Springer-Verlag, Berlin, 1987.
6. Doyle,J., Glover,K., Khargonekar, P., and Francis, B., State space solutions to standard $\mathcal{H}_2/\mathcal{H}_\infty$ control problems, *IEEE Trans. Autom. Control,* 34(8), 831–847, 1989.
7. Maciejowski, J., *Multivariable Feedback Design,* Addison-Wesley, Wokingham, England, 1989.
8. Glover, K. and Doyle, J., State-space formulae for all stabilizing controllers that satisfy an $\mathcal{H}_\infty$-norm bound and relations to risk sensitivity, *Syst. Control Lett.,* 11, 167–172, 1988.
9. Bernstein, D.S. and Haddad, W.M., LQG control with an $\mathcal{H}_\infty$ performance bound: A Riccati equation approach, *IEEE Trans. Autom. Control,* 34(3), 293–305, 1989.
10. Doyle, J., Zhou, K., Glover, K., and Bodenheimer, B., Mixed $\mathcal{H}_2$ and $\mathcal{H}_\infty$ performance objectives, II. Optimal control, *IEEE Trans. Autom. Control,* 39(8), 1575–1587, 1994.
11. Scherer, C.W., Multiobjective $\mathcal{H}_2/\mathcal{H}_\infty$ control, *IEEE Trans. Autom. Control,* 40(6), 1054–1062, 1995.
12. Nagpal, K. and Khargonekar, P., Filtering and smoothing in a $\mathcal{H}_\infty$ setting, *IEEE Trans. Autom. Control,* 36(2), 152–166, 1991.
13. Kwakernaak, H. and Sivan, R., *Linear Optimal Systems,* John Wiley & Sons, New York, 1972.
14. Kwakernaak, H., Robust control and $\mathcal{H}_\infty$ optimization—tutorial paper, *Automatica,* 29(2), 255–273, 1993.
15. Anderson, B.D.O. and Moore, J.B., *Optimal Control: Linear Quadratic Methods,* Prentice Hall, Englewood Cliffs, NJ, 1990.
16. Lublin, L. and Athans, M., An experimental comparison of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ designs for an interferometer testbed, in *Lecture Notes in Control and Information Sciences: Feedback Control, Nonlinear Systems, and Complexity,* Francis, B. and Tannenbaum, A., Eds., Springer-Verlag, Amsterdam, 1995, 150–172.

References

1. Hedlund, S. and Rantzer, A. Optimal control of hybrid systems. *Proceedings of the IEEE Conference on Decision and Control*, Phoenix, AZ, USA, 3972–3977, 1999.
2. Wei, S., Žefran, M., Uthaichana, K., and DeCarlo, R.A. Hybrid model predictive control for stabilization of wheeled mobile robots subject to wheel slippage. *Proceedings of the IEEE International Conference on Robotics and Automation*, Rome, 2373–2378, 2007.
3. Uthaichana, K., Bengea, S., DeCarlo, R., Pekarek, S., and Žefran, M. Hybrid model predictive control tracking of a sawtooth driving profile for an HEV. *Proceedings of the American Control Conference*, Seattle, WA, 967–974, 2008.
4. Moerdyk, B., De Carlo, R., Birdwell, D., Žefran, M., and Chiasson, J. Hybrid optimal control for load balancing in a cluster of computer nodes. *Proceedings of the IEEE International Conference on Control Applications*, Munich, 1713–1718, 2008.
5. Domiìnguez-Navarro, J.A., Bernal-Agustiìn, J.L., Diìaz, A., Requena, D., and Vargas, E.P. Optimal parameters of FACTS devices in electric power systems applying evolutionary strategies, *International Journal of Electrical Power and Energy Systems*, 29(1): 83–90, 2007.
6. Ho, C.Y.F., Ling, B.W.K., Liu, Y.Q., Tam, P.K.S., and Teo, K.L., Optimal PWM control of switched-capacitor DC–DC power converters via model transformation and enhancing control techniques, *IEEE Transactions on Circuits and Systems I: Regular Papers*, 55(5): 1382–1391, 2008.
7. Loxton, R.C., Teo, K.L., Rehbock, V., and Ling, W.K. Optimal switching instants for a switched-capacitor DC/DC power converter, *Automatica*, 45(4): 973–980, 2009.
8. Oettmeier, F.M., Neely, J., Pekarek, S., DeCarlo, R., and Uthaichana, K. MPC of switching in a boost converter using a hybrid state model with a sliding mode observer, *IEEE Transactions on Industrial Electronics*, 56(9): 3453–3466, 2009.
9. Neely, J., Pekarek, S., and DeCarlo, R. Hybrid optimal-based control of a boost converter. *IEEE Applied Power Electronics Conference and Exposition*, Washington, DC, pp. 1129–1137, February 15–19, 2009.
10. Neely, J., Pekarek, S., DeCarlo, R., and Vaks, N. Real-time hybrid model predictive control of a boost converter with constant power load. *25th Annual IEEE Applied Power Electronics Conference and Exposition*, Palm Springs, CA, pp. 480–490, February 23–28, 2010.
11. Giua, A., Seatzu, C., and Van Der Mee, C. Optimal control of switched autonomous linear systems. *Proceedings of the IEEE Conference on Decision and Control*, Orlando, FL, 3: 2472–2477, 2001.
12. Xu, X. and Antsaklis, P.J. Optimal control of switched systems: New results and open problems. *Proceedings of the American Control Conference*, Chicago, IL, 4: 2683–2687, June 28–30, 2000.
13. Bemporad, A. and Morari, M. Control of systems integrating logic, dynamics, and constraints, *Automatica*, 35(3): 407–427, 1999.
14. Sussmann, H.J. Maximum principle for hybrid optimal control problems. *Proceedings of the IEEE Conference on Decision and Control*, Phoenix, AZ, 1: 425–430, 1999.
15. Shaikh, M.S. and Caines, P.E. On the hybrid optimal control problem: Theory and algorithms, *IEEE Transactions on Automatic Control*, 52(9): 1587–1603, 2007.
16. Riedinger, P., Kratz, F., Iung, C., and Zanne, C. Linear quadratic optimization for hybrid systems. *Proceedings of the IEEE Conference on Decision and Control,* Phoenix, AZ, USA, 3: 3059–3064, 1999.
17. Xu, X. and Antsaklis, P.J. Results and perspectives on computational methods for optimal control of switched systems. *Proceedings of the 6th International Conference on Hybrid Systems: Computation and Control*, Lecture Notes in Computer Science, 2623: 540–555, 2003.
18. Borrelli, F., Baotic, M., Bemporad, A., and Morari, M. Dynamic programming for constrained optimal control of discrete-time linear hybrid systems, *Automatica*, 41(10): 1709–1721, 2005.
19. Gapaillard, M. Continuous representation and control of hybrid systems, *International Journal of Control*, 81(1): 1–20, 2008.
20. Bengea, S.C. and DeCarlo, R.A. Optimal control of switching systems, *Automatica*, 41(1): 11–27, 2005.
21. Wei, S., Uthaichana, K., Žefran, M., DeCarlo, R.A., and Bengea, S. Applications of numerical optimal control to nonlinear hybrid systems, *Nonlinear Analysis: Hybrid Systems*, 1(2): 264–279, 2007.
22. Ripaccioli, G., Bemporad, A., Assadian, F., Dextreit, C., Cairano, S.D., and Kolmanovsky, I.V. Hybrid modeling, identification, and predictive control: An application to hybrid electric vehicle energy management, *Hybrid Systems: Computation and Control*, Lecture Notes in Computer Science, 5469: 321–335, 2009.
23. Borhan, H.A., Vahidi, A., Phillips, A.M., Kuang, M.L., and Kolmanovsky, I.V. Predictive energy management of a power-split hybrid electric vehicle. *Proceedings of the American Control Conference*, St. Louis, MO, 3970–3976, 2009.

24. Pekarek, S., Uthaichana, K., Bengea, S., DeCarlo, R., and Žefran, M. Modeling of an electric drive for a HEV supervisory level power flow control problem. *IEEE Vehicle Power and Propulsion Conference,* Chicago, IL, 396–401, 2005.
25. Arce, A., Del Real, A.J., and Bordons, C. MPC for battery/fuel cell hybrid vehicles including fuel cell dynamics and battery performance improvement, *Journal of Process Control*, 19(8): 1289–1304, 2009.
26. Tate, E.D., Grizzle, J.W., and Peng, H. SP-SDP for fuel consumption and tailpipe emissions minimization in an EVT hybrid, *IEEE Transactions on Control Systems Technology*, 18(3): 673–687, May 2010.
27. Filippov, A.F. Differential Equations with discontinuous right-hand sides, *American Mathematical Society Translations*, Series 2, 42: 199–231, 1964.
28. Antsaklis, P., Kohn, W., Nerode, A., and Sastry, S. (Eds) *Hybrid Systems IV*, Lecture Notes in Computer Science, Vol. 1273, Springer, Berlin, 1997.
29. Vaandrager, F.W. and van Schuppen, J.H. (Eds) *Hybrid Systems: Computation and Control*, Lecture Notes in Computer Science, Springer-Verlag, Berlin, 1999.
30. Antsaklis, P.J. (Ed) Hybrid systems: Theory and applications a brief introduction to the theory and applications of hybrid systems, *Proceedings of the IEEE*, 88, 879–887, 2000.
31. Morse, A., Pantelides, C., Sastry, S., and Schumacher, J. (Eds) Hybrid Systems, *Automatica*, 35(3): 347–348, 1999.
32. Imura, J.I. and Van Der Schaft, A. Characterization of well-posedness of piecewise-linear systems, *IEEE Transactions on Automatic Control*, 45(9): 1600–1619, 2000.
33. Lygeros, J., Johansson, K.H., Simić, S.N., Zhang, J., and Sastry, S.S. Dynamical properties of hybrid automata, *IEEE Transactions on Automatic Control*, 48(1): 2–17, 2003.
34. Berkovitz, L.D. *Optimal Control Theory*, Springer, Berlin, 1974.
35. Bai, X. and Yang, X.S. A new proof of a theorem on optimal control of switched systems, *Journal of Mathematical Analysis and Applications*, 331(2): 895–901, 2007.
36. Bengea, S.C. Optimal control of switched hybrid systems with applications to the control of hybrid electric vehicles. PhD Thesis, School of ECE, Purdue University, 2005.
37. Bengea, S.C., DeCarlo, R.A., Hokayem, P.F., and Abdallah, C.T. Suboptimal control techniques for hybrid systems operating via networks, in *Advanced in Communication Control Networks*, Tarbouriech, S., Abdallah, C.T. and Chiasson, J. (Eds), Springer-Verlag, Berlin, 281–302, 2005.
38. Camacho, E.F. and Bordons, C. *Model Predictive Control*, Springer-Verlag, Berlin, 2004.
39. Von Stryk, O. Numerical solution of optimal control problems by direct collocation in optimal control—Calculus of variations, optimal control theory, and numerical methods, *International Series of Numerical Mathematics*, 111: 129–143, 1993.
40. Zhang, W. Controller synthesis for switched systems using approximate dynamic programming, PhD dissertation, School of Electrical and Computer Engineering, Purdue University, December 2009.
41. Zhang, W. and J. Hu, On the value functions of the discrete-time switched LQR problem, *IEEE Transactions on Automatic Control*, 54(11): 2669–2674, 2009.
42. Zhang, W., Abate, A., and Hu, J. Efficient suboptimal solutions of switched LQR problems, *Proceedings of American Control Conference*, St Louis, MO, 1084–1091, 2009.
43. Zhang, A. Abate, Hu, J., and Vitus, M. P. Exponential stabilization of discrete-time switched linear systems, *Automatica*, 45(11): 2526–2536, 2009.
44. Gregory, J. and Lin, C. *Constrained Optimization in the Calculus of Variations and Optimal Control Theory*, 1st edn. Van Nostrand Reinhold, London, 1992.
45. Žefran, M. Continuous methods for motion planning. PhD Thesis, University of Pennsylvania, 1996.
46. Pytlak, R. and Vinter, R.B. Second-order method for optimal control problems with state constraints and piecewise-constant controls. *Proceedings of the IEEE Conference on Decision and Control*, New Orleans, LA, 1995.
47. Neuman, C.P. and Sen, A. A suboptimal control algorithm for constrained problems using cubic splines, *Automatica*, 9: 601–613, 1973.

V

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Bibliography

1. Byrnes, C.I. and Isidori, A., New results and examples in nonlinear feedback stabilization, *Syst. Control Lett.*, 12, 437–442, 1989.
2. Tsinias, J., Sufficient Lyapunov-like conditions for stabilization, *Math. Control Signals Syst.*, 2, 424–440, 1989.
3. Byrnes, C.I. and Isidori, A., Asymptotic stabilization of minimum phase nonlinear systems, *IEEE Trans. Autom. Control,* 36, 1122–1137, 1991.
4. Byrnes, C.I. and Isidori, A., On the attitude stabilization of a rigid spacecraft, *Automatica*, 27, 87–96, 1991.
5. Isidori, A., *Nonlinear Control Systems,* 2nd ed., Springer-Verlag, Berlin, 1989.

References

1. R. Brockett and J. Willems. Frequency domain stability criteria — Part I. *IEEE Transactions on Automatic Control*, 10(3):255–261, 1965.
2. C.A. Desoer and M. Vidyasagar. *Feedback Systems: Input–Output Properties*. Academic Press, New York, 1975.
3. U. Jönsson. Lecture notes on integral quadratic constraints. Technical Report TRITA/MAT-00-OS12, Department of Mathematics, Royal Institute of Technology, September 2000. http://www.math.kth.se/ulfj/5B5744/Lecturenotes.ps.
4. C.-Y. Kao and A. Megretski. On the interior point method for a class of semi-definite programs. *SIAM Journal on Control and Optimization*, 46(2):468–495, 2007.
5. C.-Y. Kao, A. Megretski, and U. Jönsson. Specialized fast algorithms for IQC feasibility and optimization problems. *Automatica*, 40(2):239–252, 2004.
6. C.-Y. Kao, A. Megretski, U. Jönsson, and A. Rantzer. A MATLAB toolbox for robustness analysis. In *IEEE Conference on Computer Aided Control Systems Design*, pp. 297–302, Taipei, Taiwan, 2004.
7. A. Megretski. KYP lemma for non-strict inequalities and the associated minimax theorem. arXiv:1008.2552 [math,oc], 2010.
8. A. Megretski and A. Rantzer. System analysis via integral quadratic constraints. *IEEE Transactions on Automatic Control*, 42(6):819–830, 1997.
9. A. Megretski and A. Rantzer. System analysis via integral quadratic constraints. Part I. Technical Report ISRN LUTFD2/TFRT–7531–SE, Department of Automatic Control, Lund University, Sweden, April 1995.
10. A. Packard and J.C. Doyle. The complex structured singular value. *Automatica*, 29(1):71–109, 1993.
11. A. Rantzer and A. Megretski. System analysis via integral quadratic constraints. Part II. Technical Report ISRN LUTFD2/TFRT–7559–SE, Department of Automatic Control, Lund University, Sweden, September 1997.
12. R. Wallin, A. Hansson, and J.H. Johansson. A structure exploiting preprocessor for semidefinite programs derived from the Kalman–Yakubovich–Popov lemma. *IEEE Transactions on Automatic Control*, 54(4):697–704, April 2009.
13. J.C. Willems. *The Analysis of Feedback Systems*. MIT Press, Cambridge, MA, 1971.
14. J.C. Willems. Dissipative dynamical systems. *European Journal of Control*, 13:134–151, 2007.
15. V.A. Yakubovich. Popov's method and its subsequent development. *European Journal of Control*, 2:2000–2008, 2002.
16. G. Zames and P.L. Falb. Stability conditions for systems with monotone and slope-restricted nonlinearities. *SIAM Journal of Control*, 6(1):89–108, 1968.

References

1. Artstein, Z., Stabilization with relaxed controls, *Nonlinear Anal.* 7(11), 1163–1173, 1983.
2. Barmish, B. R., Corless, M. J., and Leitmann, G., A new class of stabilizing controllers for uncertain dynamical systems, *SIAM J. Cont. Optimiz.*, 21, 246–255, 1983.
3. Corless, M. J., Robust stability analysis and controller design with quadratic Lyapunov functions, in *Variable Structure and Lyapunov Control,* Zinober, A., Ed., Springer, Berlin, 1993.
4. Corless, M. J. and Leitmann, G., Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems, *IEEE Trans. Automat. Control,* 26(5), 1139–1144, 1981.
5. Freeman, R. A. and Kokotović, P. V., Design of 'softer' robust nonlinear control laws, *Automatica* 29(6), 1425–1437, 1993.
6. Freeman, R. A. and Kokotović, P. V., Inverse optimality in robust stabilization, *SIAM J. Control Optimiz.,* to appear revised November 1994.
7. Glad, S. T., Robustness of nonlinear state feedback — A survey, *Automatica,* 23(4), 425–435, 1987.
8. Gutman, S., Uncertain dynamical systems — Lyapunov min-max approach, *IEEE Trans. Automat. Control,* 24, 437–443, 1979.
9. Jacobson, D. H., *Extensions of Linear-Quadratic Control, Optimization and Matrix Theory,* Academic, London, 1977.
10. Jurdjevic, V. and Quinn, J. P., Controllability and stability, *J. Diff. Eq.,* 28, 381–389, 1978.
11. Kanellakopoulos, I., Kokotović, P. V., and Morse, A. S., Systematic design of adaptive controllers for feedback linearizable systems, *IEEE Trans. Automat. Control,* 36(11), 1241–1253, 1991.
12. Khalil, H. K., *Nonlinear Systems,* Third Edition, Prentice-Hall, Upper Saddle River, 2002.
13. Krasovsky, A. A., A new solution to the problem of a control system analytical design, *Automatica,* 7, 45–50, 1971.
14. Marino, R. and Tomei, P., Robust stabilization of feedback linearizable time-varying uncertain nonlinear systems, *Automatica,* 29(1), 181–189, 1993.
15. Praly, L., d'Andréa Novel, B., and Coron, J.-M., Lyapunov design of stabilizing controllers for cascaded systems, *IEEE Trans. Automat. Control,* 36(10), 1177–1181, 1991.
16. Qu, Z., Robust control of nonlinear uncertain systems under generalized matching conditions, *Automatica,* 29(4), 985–998, 1993.
17. Slotine, J. J. E. and Hedrick, K., Robust input–output feedback linearization, *Int. J. Control,* 57, 1133–1139, 1993.
18. Sontag, E. D., A Lyapunov-like characterization of asymptotic controllability, *SIAM J. Control Optimiz.,* 21(3), 462–471, 1983.

Further Reading

Contributions to the development of the Lyapunov design methodology for systems with no uncertainties include [1,9,10,13,18]. A good introduction to Lyapunov redesign can be found [12]. Corless [3] has recently surveyed various methods for the design of robust controllers for nonlinear systems using *quadratic* Lyapunov functions. Details of the recursive design presented in Section 49.4 can be found in [6, 14,16]. The state-space techniques discussed in this chapter can be combined with nonlinear input/output techniques to obtain more advanced designs (see Chapter 44). Finally, when the uncertain nonlinearities are given by *constant* parameters multiplying *known* nonlinearities, adaptive control techniques apply (see Chapter 53).

References

1. Abed, E.H. and Fu, J.-H., Local feedback stabilization and bifurcation control, I. Hopf bifurcation, *Syst. Control Lett.*, 7, 11–17, 1986.
2. Abed E.H. and Fu J.-H., Local feedback stabilization and bifurcation control, II. Stationary bifurcation, *Syst. Control Lett.*, 8, 467–473, 1987.
3. Abed E.H. and Wang H.O., Feedback control of bifurcation and chaos in dynamical systems, in *Nonlinear Dynamics and Stochastic Mechanics*, W. Kliemann and N. Sri Namachchivaya, Eds., CRC Press, Boca Raton, 153–173, l995.
4. Abed E.H., Wang H.O., and Chen R.C., Stabilization of period doubling bifurcations and implications for control of chaos, *Physica D*, 70(1–2), 154–164, 1994.
5. Anderson P.M. and Fouad A.A., *Power System Control and Stability*, Iowa State University Press, Ames, 1977.
6. Andronov A.A., Vitt, A.A., and Khaikin S.E., *Theory of Oscillators*, Pergamon Press, Oxford, 1966 (reprinted by Dover, New York, 1987), English translation of Second Russian Edition; Original Russian edition published in 1937.
7. Blakelock J.H., *Automatic Control of Aircraft and Missiles*, 2nd ed., John Wiley & Sons, New York, 1991.
8. Chapman, G.T., Yates, L.A., and Szady, M.J., Atmospheric flight dynamics and chaos: Some issues in modeling and dimensionality, in *Applied Chaos,* J.H. Kim and J. Stringer, Eds., John Wiley & Sons, New York, 87–141, 1992.
9. Chen, G. and Dong, X., From chaos to order—Perspectives and methodologies in controlling chaotic nonlinear dynamical systems, *Internat. J. Bifurcation Chaos*, 3, 1363–1409, 1993.
10. Chow, S.N. and Hale, J.K., *Methods of Bifurcation Theory*, Springer-Verlag, New York, 1982.
11. Devaney, R.L., *An Introduction to Chaotic Dynamical Systems*, 2nd ed., Addison-Wesley, Redwood City, CA, 1989.
12. Dobson, I., Glavitsch, H., Liu, C.-C., Tamura, Y., and Vu, K., Voltage collapse in power systems, *IEEE Circuits and Devices Magazine*, 8(3), 40–45, 1992.
13. Dole, C.E., *Flight Theory and Aerodynamics: A Practical Guide for Operational Safety*, John Wiley & Sons, New York, 1981.
14. Dorning, J.J. and Kim, J.H., Bridging the gap between the science of chaos and its technological applications, in *Applied Chaos,* J.H. Kim and J. Stringer, Eds., John Wiley & Sons, New York, 3–30, 1992.
15. Etkin, B., *Dynamics of Atmospheric Flight*, John Wiley & Sons, New York, 1972.
16. Eveker, K.M., Gysling, D.L., Nett, C.N., and Sharma, O.P., Integrated control of rotating stall and surge in aeroengines, in *Sensing, Actuation, and Control in Aeropropulsion*, J.D. Paduano, Ed., Proc. SPIE 2494, 21–35, 1995.
17. Gills, Z., Iwata, C., Roy, R., Schwartz, I.B., and Triandaf, I., Tracking unstable steady states: Extending the stability regime of a multimode laser system, *Phys. Rev. Lett.*, 69, 3169–3172, 1992.
18. Hassard, B.D., Kazarinoff, N.D., and Wan, Y.H., *Theory and Applications of Hopf Bifurcation*, Cambridge University Press, Cambridge, 1981.
19. Jackson, E.A., *Perspectives of Nonlinear Dynamics*, Vols. 1 and 2, Cambridge University Press, Cambridge, 1991.
20. Kerrebrock, J.L., *Aircraft Engines and Gas Turbines*, 2nd ed., MIT Press, Cambridge, MA, 1992.
21. Kim, J.H. and Stringer, J., Eds., *Applied Chaos*, John Wiley & Sons, New York, 1992.
22. Lee, H.C., *Robust Control of Bifurcating Nonlinear Systems with Applications*, Ph.D. Dissertation, Department of Electrical Engineering, University of Maryland, College Park, 1991.
23. Lee, H.-C. and Abed, E.H., Washout filters in the bifurcation control of high alpha flight dynamics, *Proc. 1991 Am. Control Conf.*, Boston, pp. 206–211, 1991.
24. Liaw, C.Y. and Bishop, S.R., Nonlinear heave-roll coupling and ship rolling, *Nonlinear Dynamics*, 8, 197–211, 1995.
25. Liaw, D.-C. and Abed, E.H., Analysis and control of rotating stall, *Proc. NOLCOS'92: Nonlinear Control System Design Symposium*, (M. Fliess, Ed.), June 1992, Bordeaux, France, pp. 88–93, Published by the International Federation of Automatic Control; See also: Active control of compressor stall inception: A bifurcation-theoretic approach, *Automatica*, 32, 1996 (to appear).
26. Lin, C.-F., *Advanced Control Systems Design*, Prentice Hall Series in Advanced Navigation, Guidance, and Control, and their Applications, Prentice Hall, Englewood Cliffs, NJ, 1994.
27. Lorenz, E.N., Deterministic nonperiodic flow, *J. Atmosph. Sci.*, 20, 130–141, 1963.
28. Marsden, J.E. and McCracken, M., *The Hopf Bifurcation and Its Applications*, Springer-Verlag, New York, 1976.

29. McRuer, D., Ashkenas, I., and Graham, D., *Aircraft Dynamics and Automatic Control*, Princeton University Press, Princeton, 1973.
30. Mees, A.I., *Dynamics of Feedback Systems*, John Wiley & Sons, New York, 1981.
31. Murray, J.D., *Mathematical Biology*, Springer-Verlag, New York, 1990.
32. Nayfeh, A.H. and Balachandran, B., *Applied Nonlinear Dynamics: Analytical, Computational, and Experimental Methods*, Wiley Series in Nonlinear Science, John Wiley & Sons, New York, 1995.
33. Ott, E., Grebogi, C., and Yorke, J.A., Controlling chaos, *Phys. Rev. Lett.*, 64, 1196–1199, 1990.
34. Ott, E., *Chaos in Dynamical Systems*, Cambridge University Press, Cambridge, 1993.
35. Ott, E., Sauer, T., and Yorke, J.A., Eds., *Coping with Chaos: Analysis of Chaotic Data and the Exploitation of Chaotic Systems*, Wiley Series in Nonlinear Science, John Wiley & Sons, New York, 1994.
36. Pecora, L.M. and T.L. Carroll, Synchronization in chaotic systems, *Phys. Rev. Lett.*, 64, 821–824, 1990.
37. Poincaré, H., *New Methods of Celestial Mechanics*, Parts 1, 2, and 3 (Edited and introduced by D.L. Goroff), Vol. 13 of the History of Modern Physics and Astronomy Series, American Institute of Physics, U.S., 1993; English translation of the French edition *Les Méthodes nouvelles de la Mécanique céleste*, originally published during 1892–1899.
38. Romeiras, F.J., Grebogi, C., Ott, E., and Dayawansa, W.P., Controlling chaotic dynamical systems, *Physica D*, 58, 165–192, 1992.
39. Roskam, J., *Airplane Flight Dynamics and Automatic Flight Controls (Part II)*, Roskam Aviation and Engineering Corp., Lawrence, Kansas, 1979.
40. Sanchez, N.E. and Nayfeh, A.H., Nonlinear rolling motions of ships in longitudinal waves, *Internat. Shipbuilding Progress*, 37, 247–272, 1990.
41. Sauer, P.W. and Pai, M.A., *Power System Dynamics and Stability*, Draft manuscript, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, 1995.
42. Scott, S.K., *Chemical Chaos*, International Series of Monographs on Chemistry, Oxford University Press, Oxford, 1991.
43. Seydel, R., *Practical Bifurcation and Stability Analysis: From Equilibrium to Chaos*, 2nd ed., Springer-Verlag, Berlin, 1994.
44. Shinbrot, T., Grebogi, C., Ott, E., and Yorke, J.A., Using small perturbations to control chaos, *Nature*, 363, 411–417, 1993.
45. Stevens, B.L. and Lewis, F.L., *Aircraft Control and Simulation*, John Wiley & Sons, New York, 1992.
46. Strogatz, S.H., *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*, Addison-Wesley, Reading, MA, 1994.
47. Tesi, A., Abed, E.H., Genesio, R., and Wang, H.O., Harmonic balance analysis of period doubling bifurcations with implications for control of nonlinear dynamics, Report No. RT 11/95, Dipartimento di Sistemi e Informatica, Università di Firenze, Firenze, Italy, 1995 (submitted for publication).
48. Thompson, J.M.T. and Stewart, H.B., *Nonlinear Dynamics and Chaos*, John Wiley & Sons, Chichester, U.K., 1986.
49. Thompson, J.M.T., Stewart, H.B., and Ueda, Y., Safe, explosive, and dangerous bifurcations in dissipative dynamical systems, *Phys. Rev. E*, 49, 1019–1027, 1994.
50. Wang, H.O. and Abed, E.H., Bifurcation control of a chaotic system, *Automatica*, 31, 1995, in press.
51. Wegener, P.P., *What Makes Airplanes Fly?: History, Science, and Applications of Aerodynamics*, Springer-Verlag, New York, 1991.
52. Ziegler, F., *Mechanics of Solids and Fluids*, Springer-Verlag, New York, 1991.

Further Reading

Detailed discussions of bifurcation and chaos are available in many excellent books (e.g., [6,10,11,18,19, 28,30,32,34,35,43,46,48]). These books also discuss a variety of interesting applications. Many examples of bifurcations in mechanical systems are given in [52]. There are also several journals devoted to bifurcations and chaos. Of particular relevance to engineers are *Nonlinear Dynamics,* the *International Journal of Bifurcation and Chaos,* and the journal *Chaos, Solitons and Fractals.*

The book [45] discusses feedforward control in the context of "trimming" an aircraft using its nonlinear equations of motion and the available controls. Bifurcation and chaos in flight dynamics are discussed in [8]. Lucid explanations on specific uses of washout filters in aircraft control systems are given in [15, pp. 474–475], [7, pp. 144–146], [39, pp. 946–948 and pp. 1087–1095], and [45, pp. 243–246 and p. 276]. The book [31] discussed applications of nonlinear dynamics in biology and population dynamics.

Computational issues related to bifurcation analysis are addressed in [43]. Classification of bifurcations as safe or dangerous is discussed in [32,43,48,49].

The edited book [14] contains interesting articles on research needs in applications of bifurcations and chaos.

The article [3] contains a large number of references on bifurcation control, related work on stabilization, and applications of these techniques. The review papers [9,44] address control of chaos methods. In particular, [44] includes a discussion of use of sensitive dependence on initial conditions to direct trajectories to targets. The book [35] includes articles on control of chaos, detection of chaos in time series, chaotic data analysis, and potential applications of chaos in communication systems. The book [32] also contains discussions of control of bifurcations and chaos, and of analysis of chaotic data.

References

1. Baillieul, J., Multilinear optimal control, *Proc. Conf. Geom. Control Eng.,* (NASA-Ames, Summer 1976), Brookline, MA: Math. Sci. Press, 337–359, 1977.
2. Baillieul, J., Geometric methods for nonlinear optimal control problems, *J. Optimiz. Theory Appl.,* 25(4), 519–548, 1978.
3. Baillieul, J., The behavior of super-articulated mechanisms subject to periodic forcing, in *Analysis of Controlled Dynamical Systems*, B. Bonnard, B. Bride, J.P. Gauthier, and I. Kupka, Eds., Birkhaüser, 35–50, 1991.
4. Baillieul, J. and Levi, M., Constrained relative motions in rotational mechanics, *Arch. Rational Mech. Anal.,* 115/2, 101–135, 1991.
5. Baillieul, J., The behavior of single-input super-articulated mechanisms, *Proc. 1991 Am. Control Conf.*, Boston, June 26–28, pp. 1622–1626, 1991.
6. Baillieul, J., Stable average motions of mechanical systems subject to periodic forcing, *Dynamics and Control of Mechanical Systems: The Falling Cat and Related Problems*, Fields Institute Communications, **1**, AMS, Providence, RI, 1–23, 1993.
7. Baillieul, J., Energy methods for stability of bilinear systems with oscillatory inputs, *Int. J. Robust Nonlinear Control*, Special Issue on the "Control of Nonlinear Mechanical Systems," H. Nijmeijer and A.J. van der Schaft, Guest Eds., July, 285–301, 1995.
8. Bellman, R., Bentsman, J., and Meerkov, S.M., Vibrational control of nonlinear systems: Vibrational stabilizability, *IEEE Trans. Automatic Control*, AC-31, 710–716, 1986.
9. Bellman, R., Bentsman, J., and Meerkov, S.M., Vibrational control of nonlinear systems: Vibrational controllability and transient behavior, *IEEE Trans. Automatic Control*, AC-31, 717–724, 1986.
10. Bishop, R.L. and Crittenden, R.J., *Geometry of Manifolds*, Academic Press, New York, 1964.
11. Bogolyubov, N.N., Perturbation theory in nonlinear mechanics, *Sb. Stroit. Mekh. Akad. Nauk Ukr*. SSR *14*, 9–34, 1950.
12. Bogolyubov, N.N. and Mitropolsky, Y.A., *Asymptotic Methods in the Theory of Nonlinear Oscillations*, 2nd ed., Gordon & Breach Publishers, New York, 1961.
13. Brockett, R.W., System theory on group manifolds and coset spaces, *SIAM J. Control*, 10(2), 265–284, 1972.
14. Brockett, R.W., Control Theory and Analytical Mechanics, in *Geometric Control Theory*, Vol. VII of Lie Groups: History, Frontiers, and Applications, C. Martin and R. Hermann, Eds., Math Sci Press, Brookline, MA, 1–46, 1977.
15. Brockett, R.W., Asymptotic stability and feedback stabilization, in *Differential Geometric Control Theory*, R.W. Brockett, R.S. Millman, and H.J. Sussmann, Eds., Birkhaüser, Basel, 1983.
16. Brockett, R.W., Control theory and singular Riemannian geometry, in *New Directions in Applied Mathematics*, Springer-Verlag, New York, 13–27, 1982.
17. Brockett, R.W., On the rectification of vibratory motion, *Sens. Actuat.,* 20, 91–96, 1989.
18. Brockett, R.W. and Dai, L., Nonholonomic kinematics and the role of elliptic functions in constructive controllability, in *Nonholonomic Motion Planning*, Kluwer Academic Publishers, 1–21, 1993.
19. Chow, W.L., Über Systeme von Linearen Partiellen Differentialgleichungen erster Ordnung, *Math. Ann.*, 117, 98–105, 1939.
20. Coron, J.M., Global asymptotic stabilization for controllable systems without drift, *Math. Control, Sig., Syst.,* 5, 295–312, 1992.
21. Haynes, G.W. and Hermes, H., Nonlinear controllability via Lie theory, *SIAM J. Control*, 8(4), 450–460, 1970.
22. Lafferriere, G. and Sussmann, H.J., Motion planning for controllable systems without drift, *Proc. IEEE Intl. Conf. Robot. Automat.,* 1148–1153, 1991.
23. Lafferriere, G. and Sussmann, H.J., A differential geometric approach to motion planning, in *Nonholonomic Motion Planning*, Kluwer Academic Publishers, 235–270, 1993.
24. Lehman, B., Bentsman, J., Lunel, S.V., and Verriest, E.I., Vibrational control of nonlinear time lag systems with bounded delay: Averaging theory, stabilizability, and transient behavior, *IEEE Trans. Auto. Control*, AC-39, 898–912, 1994.
25. Lehman, B., Vibrational control of time delay systems, in *Ordinary and Delay Equations*, J. Wiener and J. Hale, Eds., Pitman Research Notes in Mathematical Series (272), 111–115, 1992.
26. Kapitsa, P.L., Dynamic stability of a pendulum with a vibrating point of suspension, *Zh. Ehksp. Teor. Fiz.,* 21(5), 588–598, 1951.

27. Leonard, N.E. and Krishnaprasad, P.S., Control of switched electrical networks using averaging on Lie groups, *The 33rd IEEE Conference on Decision and Control*, Orlando, FL, Dec. 14–16, pp. 1919–1924, 1994.
28. Leonard, N.E. and Krishnaprasad, P.S., Motion control of drift-free, left-invariant systems on Lie groups, *IEEE Trans. Automat. Control*, AC40(9), 1539–1554, 1995.
29. Murray, R.M. and Sastry, S.S., Nonholonomic motion planning: steering using sinusoids, *IEEE Trans. Auto. Control*, 38(5), 700–716, 1993.
30. Nijmeijer, H. and van der Schaft, A.J., *Nonlinear Dynamical Control Systems*, Springer-Verlag, New York, 1990.
31. Sanders, J.A. and Verhulst, F., *Averaging Methods in Nonlinear Dynamical Systems*, Springer-Verlag, Applied Mathematical Sciences, 59, New York, 1985.
32. Speyer, J.L., Nonoptimality of steady-state cruise for aircraft, *AIAA J.,* 14(11), 1604–1610, 1976.
33. Speyer, J.L. and Evans, R.T., A second variational theory for optimal periodic processes, *IEEE Trans. Auto. Control*, AC-29(2), 138–148, 1984.
34. Stoker, J.J., *Nonlinear Vibrations in Mechanical and Electrical Systems*, J. Wiley & Sons, New York, 1950. Republished 1992 in Wiley Classics Library Edition.
35. Sussmann, H. and Jurdjevic, V., Controllability of nonlinear systems, *J. Diff. Eqs.*, 12, 95–116, 1972.
36. Sussmann, H.J. and Liu, W., Limits of highly oscillatory controls and approximation of general paths by admissible trajectories, *30th IEEE Conf. Decision Control*, Brighton, England, 1991.
37. Sussmann, H.J. and Liu, W., An approximation algorithm for nonholonomic systems, Rutgers University, Department of Mathematics preprint, SYCON-93-11. To appear in *SIAM J. Optimiz. Control.*
38. Sussmann, H.J. and Liu, W., Lie bracket extension and averaging: The single bracket case, in *Nonholonomic Motion Planning*, Kluwer Academic Publishers, 109–147, 1994.
39. Tilbury, D., Murray, R. and Sastry, S., Trajectory generation for the *N*-trailer problem using Goursat normal form, *30th IEEE Conf. Decision Control*, San Antonio, Texas, 971–977, 1993.
40. Tilbury, D., Murray, R. and Sastry, S., Trajectory generation for the *N*-trailer problem using Goursat normal form, *IEEE Trans. Auto. Control*, AC40(5), 802–819, 1995.
41. Weibel, S., Baillieul, J., and Kaper, T., Small-amplitude periodic motions of rapidly forced mechanical systems, *34th IEEE Conf. on Decision and Control,* New Orleans, 1995.

References

1. D. Jenkins and K. Passino, An introduction to nonlinear analysis of fuzzy control systems, *Journal of Intelligent and Fuzzy Systems*, vol. 7, no. 1, pp. 75–103, 1999.
2. K. Passino and S. Yurkovich, *Fuzzy Control*. Menlo Park, CA: Addison Wesley Longman, 1998. This book available for free download at author's Web site.
3. L.-X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1994.
4. D. Driankov, H. Hellendoorn, and M. Reinfrank, *An Introduction to Fuzzy Control*. New York, NY: Springer-Verlag, 1993.
5. W. Pedrycz, *Fuzzy Control and Fuzzy Systems*. New York, NY: Wiley, 2nd ed., 1993.
6. G. Klir and T. Folger, *Fuzzy Sets, Uncertainty and Information*. Englewood Cliffs, NJ: Prentice-Hall, 1988.
7. L. Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 3, no. 1, pp. 28–44, 1973.
8. M. Widjaja and S. Yurkovich, Intelligent control for swing up and balancing of an inverted pendulum system, in *Proceedings of the IEEE Conference on Control Applications* (Albany, NY), pp. 534–542, September 1995.
9. M. Spong, Swing up control of the acrobot, in *Proceedings of the IEEE Conference on Robotics and Automation* (San Diego, CA), pp. 2356–2361, May 1994.
10. W. Kwong, K. Passino, E. Laukonen, and S. Yurkovich, Expert supervision of fuzzy learning systems for fault tolerant aircraft control, in *Proceedings of the IEEE, Special Issue on Fuzzy Logic in Engineering Applications*, vol. 83, pp. 466–483, March 1995.
11. J. Layne and K. Passino, Fuzzy model reference learning control for cargo ship steering, *IEEE Control Systems Magazine*, vol. 13, pp. 23–34, 1993.
12. J. Layne, K. Passino, and S. Yurkovich, Fuzzy learning control for anti-skid braking systems, *IEEE Transactions on Control Systems Technology*, vol. 1, pp. 122–129, 1993.
13. V. Moudgal, W. Kwong, K. Passino, and S. Yurkovich, Fuzzy learning control for a flexible-link robot, *IEEE Transactions on Fuzzy Systems*, vol. 3, pp. 199–210, 1995.
14. W. Kwong and K. Passino, Dynamically focused fuzzy learning control, *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 26, pp. 53–74, 1996.
15. A. Angsana and K. Passino, Distributed fuzzy control of flexible manufacturing systems, *IEEE Transactions on Control Systems Technology*, vol. 2, pp. 423–435, 1994.
16. E. Garcia-Benitez, S. Yurkovich, and K. Passino, Rule-based supervisory control of a two-link flexible manipulator, *Journal of Intelligent and Robotic Systems*, vol. 7, no. 2, pp. 195–213, 1993.

Further Reading

Other introductions to fuzzy control are given in [2–5]. The book [2] is available for a free download (as a .pdf file) from http://www.ece.osu.edu/˜passino/. To study other applications of fuzzy control the reader should consult the references in the papers and books cited above. There are several conferences that have sessions or papers that study various aspects of fuzzy control. For instance, see the *Proceedings of FuzzIEEE* (in the World Congress on Computational Intelligence), *IEEE International Symposium on Intelligent Control, Proceedings of the IEEE Conference on Decision and Control*, *the American Control Conference*, and many others. The best journals to consult are the *IEEE Transactions on Fuzzy Systems*, *Fuzzy Sets and Systems*, and the *IEEE Transactions on Systems, Man, and Cybernetics*. Also, there are many short courses that are given on the topic of fuzzy control.

References

1. Blackwell, D., Discounted dynamic programming. *Ann. of Math. Statist.* 36, 226–335, 1965.
2. Strauch, R., Negative dynamic programming. *Ann. Math. Statist.*, 37, 871–890, 1966.
3. Blackwell, D., Positive dynamic programming. *Proceedings of the 5th Berkeley Symposium on Mathematical Statistics and Probability*, 415–418, 1965.
4. Blackwell, D., On stationary policies. *J. Roy. Statist. Soc. Ser. A*, 133, 33–38, 1970.
5. Ornstein, D., On the existence of stationary optimal strategies. *Proc. Amer. Math. Soc.*, 20, 563–569, 1969.
6. Blackwell, D., Discrete dynamic programming. *Ann. of Math. Statist.*, 33, 719–726, 1962.
7. Hernandez-Lerma, O. and Lasserre, J. B., Weak conditions for average optimality in Markov control processes. *Syst. and Contr. Lett.*, 22, 287–291, 1994.
8. Bertsekas, D. P. and Shreve, S. E., *Stochastic Optimal Control: The Discrete Time Case*, Academic, New York, 1978.
9. Lions, P. L., Optimal control of diffusion processes and Hamilton-Jacobi equations, Part I–the dynamic programming principle and applications. *Comm. Partial Differential Equations*, 10, 1101–1174, 1983.
10. Crandall, M., Ishii, H. and Lions, P. L., User's guide to viscosity solutions of second order partial differential equations. *Bull. Amer. Math. Soc.* 27, 1–67, 1990.
11. Bellman, R., *Dynamic Programming*. Princeton University Press, Princeton, NJ, 1957.
12. Bertsekas, D. P., *Dynamic Programming: Deterministic and Stochastic Models*. Prentice Hall, Englewood Cliffs, NJ, 1987.
13. Kumar, P. R. and Varaiya, P. P., *Stochastic Systems: Estimation, Identification and Adaptive Control*. Prentice Hall, Englewood Cliffs, NJ, 1986.
14. Ross, S. M., *Applied Probability Models with Optimization Applications*. Holden-Day, San Francisco, 1970.

Further Reading

For discounted dynamic programming, the classic reference is Blackwell [1]. For the positive cost case, we refer the reader to Strauch [2]. For the negative cost case, we refer the reader to Blackwell [3,4] and Ornstein [5]. For the average cost case, we refer the reader to Blackwell [6] for early fundamental work, and to Hernandez-Lerma and Lasserre [7] and the references contained there for recent developments. For a study of the measurability issues which arise when considering uncountable sets, we refer the reader to Bertsekas and Shreve [8]. For continuous time stochastic control of diffusions we refer the reader to Lions [9], and to Crandall, Ishii and Lions [10] for a guide to viscosity solutions.

For the several ways in which dynamic programming can be employed, see Bellman [11].

Some recent textbooks which cover dynamic programming are Bertsekas [12], Kumar and Varaiya [13], and Ross [14].

References

1. B. Bercu, Weighted estimation and tracking for ARMAX models, *SIAM J. Control Optim.* 33, 1995, 89–106.
2. H. F. Chen, T. E. Duncan, and B. Pasik-Duncan, Stochastic adaptive control for continuous-time linear systems with quadratic cost, *Appl. Math. Optim.*, 34, 1996, 113–138.
3. T. E. Duncan, Prediction for some processes related to a fractional Brownian motion, *Stat.Prob.Lett.* 76, 2006, 128–134.
4. T. E. Duncan, L. Guo, and B. Pasik-Duncan, Adaptive continuous-time linear quadratic Gaussian control, *IEEE Trans. Automat. Control* 44, 1999, 1653–1662.
5. T. E. Duncan, J. Jakubowski and B. Pasik-Duncan, Stochastic integration for fractional Brownian motions in Hilbert spaces, *Stochastics Dynamics*, 6, 2006, 53–75.
6. T. E. Duncan and B. Pasik-Duncan, Adaptive control of continuous-time linear stochastic systems, *Math. Control Signals Systems*, 3, 1990, 45–60.
7. T. E. Duncan and B. Pasik-Duncan, Parameter identification for some linear systems with fractional Brownian motion, *Proceedings of the Fifteenth Triennial World Congress IFAC 2002*, Barcelona, 2002.
8. T. E. Duncan and B. Pasik-Duncan, Adaptive control of a scalar linear stochastic system with a fractional Brownian motion, *Proc. IFAC World Congress*, Seoul 2002, 4096-4101.
9. W. H. Fleming and R. W. Rishel, *Deterministic and Stochastic Optimal Control*, Springer, New York, 1975.
10. L. Guo, Self-convergence of weighted least-squares with applications to stochastic adaptive control, *IEEE Trans. Automat. Control* 41, 1996, 79–89.
11. H. E. Hurst, Long-term storage capacity in reservoirs, *Trans. Am. Soc. Civil Eng.*, 116, 1951, 400–410.
12. M. L. Kleptsyna, A. Le Breton, and M. Viot, About the linear quadratic regulator problem under a fractional Brownian perturbation, *ESAIM Probab. Stat.* 9, 2003, 161–170.
13. A. N. Kolmogorov, Wienersche spiralen und einige andere interessante kurven in Hilbertschen Raum, *C.R. (Doklady) Acad. USSS (N.S.)*, 26, 1940, 115–118.
14. A. N. Kolmogorov, The local structure of turbulence in incompressible viscouse fluid for very large Reynolds numbers, *Dokl. Akdad. Nauk SSSR*, 30, 1941, 301–305.
15. A. N. Kolmogorov, Dissipation of energy in the locally isotropic turbulence, *Dokl. Adad. Nauk SSSSR*, 31, 1941, 538–540.
16. B. B. Mandelbrot and J. W. Van Ness, Fractional Brownian motions, fractional noises and applications, *SIAM Rev.*, 10, 1968, 422–437.
17. S. G. Samko, A. A. Kilbas, and O. I. Marichev, *Fractional Integrals and Derviatives*, Gordon and Breach, Yverdon, 1993.

References

1. O. M. Aamo and M. Krstic, *Flow Control by Feedback: Stabilization and Mixing*. Berlin: Springer, 2002.
2. M.-R. Alam, W.-J. Liu and G. Haller, Closed-loop separation control: An analytic approach, *Phys. Fluids*, 18:043601, 2006.
3. A. Balogh, W.-J. Liu and M. Krstic, Stability enhancement by boundary control in 2D channel flow, *IEEE Trans. Automat. Control*, 46:1696–1711, 2001.
4. J. Baker, A. Armaou, and P. D. Christofides, Drag reduction in incompressible channel flow using electromagnetic forcing, *Proc. 2000 ACC*, 4269–4273, 2000.
5. J. Baker, A. Armaou, and P. D. Christofides, Nonlinear control of incompressible fluid flow: Application to Burgers' equation and 2D channel flow, *J. Math. Anal. Appl.*, 252:230–255, 2000.
6. V. Barbu, C. Popa, T. Havarneanu, and S. S. Sritharan, Exact controllability of magneto-hydrodynamic equations, *Commun Pure App. Math.*, 56(6):732–783, 2003.
7. T. W. Berger, J. Kim, C. Lee, and J. Lim, Turbulent boundary layer control utilizing the Lorentz force, *Phys. Fluids*, 12:631, 2000.
8. T. R. Bewley and B. Protas, Skin friction and pressure: The 'footprints' of turbulence, *Physica D*, 196:28–44, 2004.
9. K. S. Breuer and J. Park, Actuation and control of a turbulent channel flow using Lorentz forces, *Phys. Fluids*, 16(4):897, 2004.
10. M. Chevalier, J. Hoepffner, T. R. Bewley, and D. S. Henningson, State estimation in wall-bounded flow systems. Part 2. Turbulent flows. *J. Fluid Mech.*, 552:167–187, 2006.
11. J. Hoepffner, M. Chevalier, T. R. Bewley, and D. S. Henningson, State estimation in wall-bounded flow systems. Part 1. Perturbed laminar flows. *J. Fluid Mech.*, 534:263–294, 2005.
12. K. Debbagh, P. Cathalifaud, and C. Airiau, Optimal and robust control of small disturbances in a channel flow with a normal magnetic field, *Phys. Fluids*, 19(1):014103–014103-14, 2007.
13. M. Hogberg, T. R. Bewley, and D. S. Henningson, Linear feedback control and estimation of transition in plane channel flow, *J. Fluid Mech.*, 481:149–175, 2003.
14. M. R. Jovanovic and B. Bamieh, Componentwise energy amplification in channel flows, *J. Fluid Mech.*, 543:145–183, 2005.
15. M. R. Jovanovic, Turbulence suppression in channel flows by small amplitude transverse wall oscillations, *Phys. Fluids*, 20(1):014101, 2008.
16. M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*, New York, NY: Wiley, 1995.
17. D. Lee and H. Choi, Magnetohydrodynamic turbulent flow in a channel at low magnetic Reynolds number, *J. Fluid Mech.*, 439:367–394, 2001.
18. J. Pang and K.-S. Choi, Turbulent drag reduction by Lorentz force oscillation, *Phys. Fluids*, 16(5):L35, 2004.
19. P. J. Schmid and D. S. Henningson, *Stability and Transition in Shear Flows*, Springer, Berlin, 2001.
20. E. Schuster, L. Luo, and M. Krstic, MHD channel flow control in 2D: Mixing enhancement by boundary feedback, *Automatica*. 44:2498–2507, 2008.
21. A. Smyshlyaev and M. Krstic, Closed form boundary state feedbacks for a class of partial integro-differential equations, *IEEE Trans. Automat. Control*, 49:2185–2202, 2004.
22. A. Smyshlyaev and M. Krstic, Backstepping observers for parabolic PDEs, *Systems Control Lett.*, 54:1953–1971, 2005.
23. S. S. Sritharan, V. Barbu, T. Havarneanu and C. Popa, Local controllability for the magnetohydrodynamic equations revisited, *Adv. Diff. Eq*, 10(5):481–504, 2005.
24. J.-P. Thibault and L. Rossi, Electromagnetic flow control: Characteristic numbers and flow regimes of a wall-normal actuator, *J. Phys. D: Appl. Phys.*, 36:2559–2568, 2003.
25. R. Triggiani, Stability enhancement of a 2-D linear Navier–Stokes channel flow by a 2-D, wall-normal boundary controller, *Disc. Cont. Dyn. Sys.—B*, 8(2), 2007.
26. R. Vazquez and M. Krstic, *Control of Turbulent and Magnetohydrodynamic Channel Flows*, Boston: Birkhauser, 2007.
27. R. Vazquez and M. Krstic, A closed-form feedback controller for stabilization of the linearized 2D Navier–Stokes Poiseuille flow, *IEEE Trans. Automat. Control*, 52:2298–2312, 2007.

XII

Networks and Networked Controls

References

1. A. Mahajan and D. Teneketzis, Optimal performance of networked control systems with non-classical information structures, *SIAM Journal of Control and Optimization*, 48(3), 1377–1404, 2009.
2. R. M. Murray (Ed.), Control in an information rich world: Report of the panel on future directions in control, dynamics, and systems, SIAM Report, 2003.
3. G. N. Nair, F. Fagnani, S. Zampieri, and R. J. Evans, Feedback control under data rate constraints: An overview, *Proceedings of the IEEE*, 95(1), 108–137, 2007.
4. A. S. Matveev and A. V. Savkin, Shannon zero error capacity in the problems of state estimation and stabilization via noisy communication channels, *International Journal of Control*, 80(2), 241–255, 2007.
5. L. L. Peterson and B. S. Davie, *Computer Networks: A Systems Approach,* 4th ed., San Mateo, CA: Morgan Kaufmann, 2007.
6. D. F. Delchamps, Stabilizing a linear system with quantized state feedback, *IEEE Transactions on Automatic Control*, 35(9), 916–924, 1990.
7. N. C. Martins, M. A. Dahleh, and N. Elia, Feedback stabilization of uncertain systems in the presence of a direct link, *IEEE Transactions on Automatic Control*, 51(3), 438–447, 2006.
8. A. Sahai and S. Mitter, The necessity and sufficiency of anytime capacity for stabilization of a linear system over a noisy communication link; Part I: Scalar systems, *IEEE Transactions on Information Theory*, 52(8), 3369–3395, 2006.
9. D. Tse and R. Yates, Fading broadcast channels with state information at the receivers, arXiv:0904.3165v1 [cs.IT], *IEEE Transactions on Information Theory*, 2009, submitted for publication.
10. W. S. Wong and R. W. Brockett, Systems with finite communication bandwidth constraints II: Stabilization with limited information feedback, *IEEE Transactions on Automatic Control*, 44(5), 1049–1053, 1999.
11. J. Baillieul, Feedback designs in information-based control, in *Stochastic Theory and Control Proceedings of a Workshop*, B. Pasik-Duncan, Ed. Lawrence, KS, New York: Springer-Verlag, pp. 35–57, 2001.
12. V. S. Borkar and S. K. Mitter, LQG control with communication constraints in *Communications, Computation, Control and Signal Processing*, A. Paulraj, V. Roychowdhury, C.D. Schaper, Eds. Dordrecht: Kluwer, 365–373, 1997.
13. G. N. Nair, R. J. Evans, I. M. Y. Mareels, and W. Moran, Topological feedback entropy and nonlinear stabilization, *IEEE Transactions on Automatic Control*, 49(9), 1585–1597, 2004.
14. P. Minero, M. Franceschetti, S. Dey, and G. N. Nair, Data rate theorem for stabilization over time-varying feedback channels, *IEEE Transactions on Automatic Control*, 54(2), 243–255, 2009.
15. R. H. Middleton, A. J. Rojas, J. S. Freudenberg, and J. H. Braslavsky, Feedback stabilization over a first order moving average Gaussian noise channel, *IEEE Transactions on Automatic Control*, 54(1), 163–167, 2009.
16. J. H. Braslavsky, R. H. Middleton, and J. S. Freudenberg, Feedback stabilization over signal-to-noise ratio constrained channels, *IEEE Transactions on Automatic Control*, 52(8), 1391–1403, 2007.
17. S. Tatikonda and S. Mitter, Control under information constraints, *IEEE Transactions on Automatic Control*, 49(7), 1056–1068, 2004.
18. S. Tatikonda and S. Mitter, Control over noisy channels, *IEEE Transactions on Automatic Control*, 49(7), 1196–1201, 2004.
19. G. N. Nair and R. J. Evans, Stabilizability of stochastic linear systems with finite feedback data rates, SIAM Journal on Control and Optimization, *Society for Industrial and Applied Mathematics, USA*, 43(2), 413–436, 2004.
20. V. Gupta, B. Hassibi, and R. M. Murray, Optimal LQG control across packet-dropping links, *System and Control Letters*, 56(6), 439–446, 2007.
21. J. H. Braslavsky, R. H. Middleton, and J. S. Freudenberg, Feedback stabilization over signal-to-noise ratio constrained channels automatic control, *IEEE Transactions on Automatic Control*, 52(8), 1391–1403, 2007.
22. D. Liberzon and D. Nešić, Input to state stabilization of linear systems with quantized state measurements, *IEEE Transactions on Automatic Control*, 52(5), 2007.
23. E. D. Sontag, *Mathematical Control Theory*, 2nd ed., Berlin: Springer, 1998.
24. R. Brockett and D. Liberzon, Quantized feedback stabilization of linear systems, *IEEE Transactions on Automatic Control*, 45(7), 1279–1289, 2000.
25. D. Nešić and D. Liberzon, A unified framework for design and analysis of networked and quantized control systems (with D. Nesic), *IEEE Transactions on Automatic Control*, 54(4), 732–747, 2009.
26. G. N. Nair and R. J. Evans, Cooperative networked stabilizability of linear systems with measurement noise, *In Proc. 15th IEEE Mediterranean Conf. Control and Automation*, Athens, Greece, 2007.

27. M. Rotkowitz and G. Nair, An LP for stabilization over networks with rate constraints, *Proceedings of the International Symposium on Mathematical Theory of Networks and Systems*, Blacksburg, Virginia, 2008.
28. S. Yuksel, Stochastic nestedness and the belief sharing information pattern, *IEEE Trans. Automat. Control*, 55, 2773–2786, 2009.
29. S. Yuksel and T. Basar, Optimal signaling policies for decentralized multi-controller stabilizability over communication channels, *IEEE Trans. Automat. Control*, 52, 1969–1974, 2007.
30. S. Yuksel and T. Basar, Communication constraints for decentralized stabilizability with time-invariant policies, *IEEE Trans. Automat. Control*, 52, 1060–1066, 2007.
31. A. S. Matveev and A. V. Savkin, *Estimation and Control over Communication Networks (Control Engineering)*, Boston: Birkhauser, 2008.
32. N. C. Martins, Finite gain LP stability requires analog control, systems and control letters, 55, 949–954, 2006.
33. S. Tatikonda, A. Sahai, and S. Mitter, Stochastic linear control over a communication channel, *IEEE Trans. Automat. Control*, 49(9), 1549–1561, 2004.

References

1. K. C. Chang, R. K. Saha, and Y. Bar-Shalom, On optimal track-to-track fusion, *IEEE Trans. Aerospace Electronic Systems*, AES-33(4):1271–1276, 1997.
2. O. L. V. Costa, M. D. Fragoso, and R. P. Marques, Discrete-time Markov jump linear systems, Springer, Berlin, 2006.
3. D. Dacic and D. Nesic, Quadratic stabilization of linear networked control systems via simultaneous controller and protocol synthesis, *Automatica*, 43(7):1145–1155, 2007.
4. E. O. Elliott, Estimates of error rates for codes on burst-noise channels, *Bell Systems Tech. Jo.* 42: 1977–1997, 1963.
5. V. Gupta, T. Chung, B. Hassibi, and R. M. Murray, On a stochastic sensor selection algorithm with applications in Sensor scheduling and dynamic sensor coverage, *Automatica*, 42(2):251–260, 2006.
6. V. Gupta, A. F. Dana, J. Hespanha, R. M. Murray, and B. Hassibi, Data transmission over networks for estimation and control, *IEEE Trans. Automat. Control*, 54(8):1807–1819, August 2009.
7. V. Gupta, D. Spanos, B. Hassibi, and R. M. Murray, Optimal LQG control across a packet-dropping link, *Systems and Controls Letters*, 56(6):439–446, 2007.
8. J. Hespanha, P. Naghshtabrizi, and Y. Xu, A Survey of recent results in networked control systems. *Proc. of IEEE, Special Issue on Techno. Networked Control Systems*, 95(1): 138–162, 2007.
9. O.C. Imer, S. Yuksel and T. Basar, Optimal control of LTI systems over communication networks, *Automatica*, 42(9):1429–1440, 2006.
10. T. Kailath, A.H. Sayed and B. Hassibi, *Linear Estimation*, Prentice-Hall, Englewood Cliffs, NJ, 2000.
11. N. C. Martins and M. A. Dahleh, Feedback control in the presence of noisy channels: 'Bode-like' fundamental limitations of performance, *IEEE Transac. Automat. Control*, 52(7):1604–1615, 2008.
12. A. Matveev and A. Savkin, The problem of state estimation via asynchronous communication channels with irregular transmission times, *IEEE Trans. Automat. Control*, 48(4):670–676, 2003.
13. P. Minero, M. Franceschetti, S. Dey, and G. Nair, Data rate theorem for stabilization over time-varying feedback channels, *IEEE Trans. Automat. Control*, 54(2): 243–255, February 2009.
14. L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry, foundations of control and estimation over Lossy networks, *Proc. IEEE*, 95(1):163–187, 2007.
15. P. Seiler and R. Sengupta, An H_∞ approach to networked control, *IEEE Trans. Automat. Control*, 50(3):356–364, 2005.

References

1. P.J. Moylan and D.J. Hill. Stability criteria for large-scale systems. *IEEE Transactions on Automatic Control*, 23(2):143–149, 1978.
2. M. Vidyasagar. *Input–Output Analysis of Large Scale Interconnected Systems*. Springer-Verlag, Berlin, 1981.
3. A. Megretski and A. Rantzer. System analysis via integral quadratic constraints. *IEEE Transactions on Automatic Control*, 42:819–830, 1997.
4. P. Ögren, E. Fiorelli, and N.E. Leonard. Cooperative control of mobile sensor networks: Adaptive gradient climbing in a distributed network. *IEEE Transactions on Automatic Control*, 49(8):1292–1302, 2004.
5. V. Gazi and K.M. Passino. Stability analysis of social foraging swarms. *IEEE Transactions on Systems, Man, and Cybernetics—Part B: Cybernetics*, 34(1):539–557, 2004.
6. H.G. Tanner, A. Jadbabaie, and G.J. Pappas. Flocking in fixed and switching networks. *IEEE Transactions on Automatic Control*, 52(5):863–868, 2007.
7. J.J. Tyson and H.G. Othmer. The dynamics of feedback control circuits in biochemical pathways. In *Progress in Theoretical Biology*, R. Rosen and F.M. Snell, Ed., Vol. 5, pp. 1–62. Academic Press, New York, NY, 1978.
8. C.D. Thron. The secant condition for instability in biochemical feedback control—Parts I and II. *Bulletin of Mathematical Biology*, 53:383–424, 1991.
9. R. Redheffer. Volterra multipliers—Parts I and II. *SIAM Journal on Algebraic and Discrete Methods*, 6(4):592–623, 1985.
10. E. Kaszkurewicz and A. Bhaya. *Matrix Diagonal Stability in Systems and Computation*. Birkhauser, Boston, 2000.
11. J.C. Willems. Dissipative dynamical systems. Part I: General theory; Part II: Linear systems with quadratic supply rates. *Archive for Rational Mechanics and Analysis*, 45:321–393, 1972.
12. G. Zames. On the input–output stability of time-varying nonlinear feedback systems—Parts I and II. *IEEE Transactions on Automatic Control*, 11:228–238 and 465–476, 1966.
13. J.T. Wen and M. Arcak. A unifying passivity framework for network flow control. *IEEE Transactions on Automatic Control*, 49(2):162–174, 2004.
14. F.P. Kelly, A. Maulloo, and D. Tan. Rate control in communication networks: Shadow prices, proportional fairness and stability. *Journal of the Operational Research Society*, 49:237–252, 1998.
15. S.H. Low, F. Paganini, and J.C. Doyle. Internet congestion control. *IEEE Control Systems Magazine*, 22(1):28–43, 2002.
16. R. Srikant. *The Mathematics of Internet Congestion Control*. Birkhauser, Boston, 2004.
17. X. Fan, M. Arcak, and J.T. Wen. Robustness of network flow control against disturbances and time delays. *Systems and Control Letters*, 53(1):13–29, 2004.
18. X. Fan, K. Chandrayana, M. Arcak, S. Kalyanaraman, and J.T. Wen. A two-time-scale design for edge-based detection and rectification of uncooperative flows. *IEEE/ACM Transactions on Networking*, 14(6):1313–1322, 2006.
19. T. Alpcan, T. Başar, R. Srikant, and E. Altman. CDMA uplink power control as a noncooperative game. *Wireless Networks*, 8:659–669, 2002.
20. X. Fan, T. Alpcan, M. Arcak, J.T. Wen, and T. Basar. A passivity approach to game-theoretic CDMA power control. *Automatica*, 42(11):1837–1847, 2006.
21. T. Alpcan, X. Fan, T. Basar, M. Arcak, and J.T. Wen. Power control for multicell CDMA wireless networks: A team optimization approach. *Wireless Networks*, 14(5):647–657, 2008.
22. M. Arcak. Passivity as a design tool for group coordination. *IEEE Transactions on Automatic Control*, 52(8):1380–1390, 2007.
23. F. Lizarralde and J.T. Wen. Attitude control without angular velocity measurement: A passivity approach. *IEEE Transactions on Automatic Control*, 41(3):468–472, 1996.
24. H. Bai, M. Arcak, and J. Wen. Rigid body attitude coordination without inertial frame information. *Automatica*, 44(12):3170–3175, 2008.
25. H. Bai, M. Arcak, and J.T. Wen. Adaptive design for reference velocity recovery in motion coordination. *Systems and Control Letters*, 57(8):602–610, 2008.
26. A. Loría, E. Panteley, D. Popović, and A.R. Teel. A nested Matrosov theorem and persistency of excitation for uniform convergence in stable nonautonomous systems. *IEEE Transactions on Automatic Control*, 50(2):183–198, 2005.
27. E. Biyik and M. Arcak. Gradient climbing in formation via extremum-seeking and passivity-based coordination rules. *Asian Journal of Control*, 10(2):201–211, 2008.

28. M. Arcak and E.D. Sontag. Diagonal stability of a class of cyclic systems and its connection with the secant criterion. *Automatica*, 42(9):1531–1537, 2006.
29. M. Jovanović, M. Arcak, and E. Sontag. A passivity-based approach to stability of spatially distributed systems with a cyclic interconnection structure. *IEEE Transactions on Automatic Control*, 53(1):75–86, 2008.
30. H. Smith. *Monotone Dynamical Systems: An Introduction to the Theory of Competitive and Cooperative Systems*. American Mathematical Society, Providence, RI, 1995.
31. J. Mallet-Paret and H.L. Smith. The Poincaré–Bendixson theorem for monotone cyclic feedback systems. *Journal of Dynamics and Differential Equations*, 2:367–421, 1990.
32. E.D. Sontag. Passivity gains and the "secant condition" for stability. *Systems Control Letters*, 55(3):177–183, 2006.
33. A. J. van der Schaft. *$\mathcal{L}_2$-gain and Passivity Techniques in Nonlinear Control*, 2nd edn. Springer-Verlag, New York, 2000.
34. R. Sepulchre, M. Janković, and P. Kokotović. *Constructive Nonlinear Control*. Springer-Verlag, New York, NY, 1997.
35. M. Arcak and E. Sontag. A passivity-based stability criterion for a class of biochemical reaction networks. *Mathematical Biosciences and Engineering*, 5(1):1–19, 2008.
36. J. Keener and J. Sneyd. *Mathematical Physiology*. Springer, New York, NY, 2004.
37. B.N. Kholodenko. Negative feedback and ultrasensitivity can bring about oscillations in the mitogen-activated protein kinase cascades. *European Journal of Biochemistry*, 267:1583–1588, 2000.
38. S.Y. Shvartsman, M.P. Hagan, A. Yacoub, P. Dent, H.S. Wiley, and D.A. Lauffenburger. Context-dependent signaling in autocrine loops with positive feedback: Modeling and experiments in the EGFR system. *American Journal of Physiology—Cell Physiology*, 282:C545–C559, 2001.
39. E.D. Sontag. Asymptotic amplitudes and Cauchy gains: A small-gain principle and an application to inhibitory biological feedback. *Systems and Control Letters*, 47:167–179, 2002.
40. D.D. Šiljak. *Large-Scale Systems: Stability and Structure*. North-Holland, New York, NY, 1978.
41. A.N. Michel and R.K. Miller. *Qualitative Analysis of Large Scale Dynamical Systems*. Academic Press, New York, 1977.
42. S.D.M. Santos, P.J. Verveer, and P.I.H. Bastiaens. Growth factor induced MAPK network topology shapes Erk response determining PC-12 cell fate. *Nature Cell Biology*, 9:324–330, 2007.
43. A. Turing. The chemical basis of morphogenesis. *Philosophical Transactions of Royal Society of London*, B273:37–72, 1952.
44. J.C. Willems. *The Analysis of Feedback Systems*. MIT Press, Cambridge, MA, 1971.
45. R. Mancera and M. Safonov. Stability multipliers for MIMO monotone nonlinearities. In *Proceedings of the 2003 American Control Conference*, pp. 1861–1866, Denver, CO, 2003.