19th Mediterranean Conference on Control and Automation
Aquis Corfu Holiday Palace, Corfu, Greece
June 20-23, 2011

Robust/Reliable Stabilization of Multi-Channel Systems via Dilated LMIs and Dissipativity-Based Certifications

Getachew K. Befekadu, *Member, IEEE,* Vijay Gupta, *Member, IEEE,* and Panos J. Antsaklis, *Fellow, IEEE*

Abstract— **In this paper, we consider the problem of reliable stabilization for a multi-channel system, where our main objective is to maintain the stability of the perturbed closed-loop system and when there is a single controller failure in any of the control channels. We specifically present a computationally tractable and less-conservative result in terms of a set of dilated LMIs for the reliable state feedback stabilization of the nominal system, while a dissipativity-based certification is used to extend the stability condition under model perturbation in the system. Finally, a numerical example is used to demonstrate the applicability of the proposed technique.**

I. Introduction

Reliable control using multi-controllers, which was originally proposed by Šiljak [1], is used for enhancing robustness against possible component failures that occur in controllers or sensors and actuators. In a multi-channel decentralized control configuration (e.g., see references [2], [3]), the objective is to maintain control performance such as stability of the closed-loop system when all of the controllers work together and when only some of the controllers work. In the past, several major theoretical achievements have been obtained in the context of reliable stabilization via so-called factorization approach [4], [5], [6], [7], performances in the context of reliable control via Riccati and/or Lyapunov equations [8], [9]. For example, in the case of a single input-output channel, a complete characterization of plants that can be reliably stabilized using two controllers, where either of which may fail, was considered in [4]. Alternative characterizations are also derived for a more general situation where a plant with two input-output channels is stabilized by two decentralized controllers in [5], [6], [7]. It should be noted that the problem of reliable stabilization for a general multi-channel system is in general a very difficult problem. This is because reliable stabilization is equivalent to a strong stabilization problem [4] which involves an intractable problem [10], [11].

Recently, the problem of reliable decentralized stabilization for multi-channel systems with a single failure in any of the control channels has been addressed in [12] via dilated LMIs and unknown disturbance observers. In this paper, we present an extension to the problem of reliable stabilization for a perturbed multi-channel system, where our main objective is to maintain the stability for all perturbed closed-loop systems when there is a single controller failure in any of the control channels. We present a dilated LMIs framework which provides a non-conservative reliable state-feedback controller solutions for the general multi-channel system, while a dissipativity-based certification is used for extending the stability condition under model perturbation in the system. Specifically, using a panel of storage functions and a common supply rate, we verify the stability property possessed by all perturbed closed-loop systems (e.g., see references [13], [14], [15]).

This paper is organized as follows. In Section II, we present some preliminary results on the stability condition for a continuous-time linear system in terms of dilated LMIs. Section III presents the main results. A verifiable sufficient condition in terms of a set of dilated LMIs is given for the reliable state-feedback stabilization to the nominal system, while a dissipativity-based certification is used to extend the stability condition under model perturbation in the system. In Section IV, we present a simple numerical example. Finally, Section V provides some concluding remarks.

Notation. We write $\mathrm{He}(A) = A + A^T$, where A^T denotes the transpose of A. We denote an orthogonal complement of $B \in \mathbb{R}^{n\times p}$ by $B^{\perp} \in \mathbb{R}^{(n-r)\times n}$ which is a matrix satisfying $B^{\perp}B = 0$ and $B^{\perp}B^{\perp T} > 0$, where $r = \operatorname{rank} B$. We use $\mathcal{S}_+^n$ to denote the set of strictly positive definite real symmetric matrices.

II. Preliminaries

Consider the following continuous-time linear system

$$\dot{x}(t) = Ax(t) \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ and $x(t) \in \mathbb{R}^n$.

As is well known, the system is stable (or equivalently A is a Hurwitz matrix) if and only if there exits $X \in \mathcal{S}_+^n$ satisfying

$$\mathrm{He}(AX) < 0 \tag{2}$$
$$X > 0. \tag{3}$$

However, since this coupling of A and X leads to several difficulties especially in the context of robust stability analysis for uncertain systems (e.g., see references [16], [17], [18]). To address some of the concerns, stability conditions have recently been proposed based on dilated LMIs framework.

This work was supported in part by the National Science Foundation under Grants No. CCF-0819865 and CNS-1035655; GKB acknowledges support from the Moreau Fellowship of the University of Notre Dame.

G. K. Befekadu is with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN, 46556. gbefekadu1@nd.edu
V. Gupta is with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN, 46556. vgupta2@nd.edu
P. J. Antsaklis is with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN, 46556. antsaklis.1@nd.edu

The following gives an example of dilated LMIs, which is a version of the result given in [12], [19], [20].

Lemma 1: (dilated LMIs): The system (1) is stable if and only if there exist $X \in \mathcal{S}_+^n$, $W \in \mathbb{R}^{n\times n}$, and $\epsilon > 0$ such that

$$\begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix} + \mathrm{He}\left(\begin{bmatrix} A \\ -I \end{bmatrix} W \begin{bmatrix} I & \epsilon I \end{bmatrix}\right) < 0 \quad (4)$$

holds.

Proof: Sufficiency: Note that

$$\begin{bmatrix} A \\ -I \end{bmatrix}^\perp = \begin{bmatrix} I & A \end{bmatrix}, \quad \begin{bmatrix} I \\ \epsilon I \end{bmatrix}^\perp = \begin{bmatrix} \epsilon I & -I \end{bmatrix}. \quad (5)$$

Then, eliminating W of (4) by using these matrices, we have two inequalities

$$\begin{bmatrix} I & A \end{bmatrix}\begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix}\begin{bmatrix} I \\ A^T \end{bmatrix} = AX + XA^T < 0 \quad (6)$$

$$\begin{bmatrix} \epsilon I & -I \end{bmatrix}\begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix}\begin{bmatrix} \epsilon I \\ -I \end{bmatrix} = -2\epsilon X < 0. \quad (7)$$

Thus we see that (2) and (3) actually hold.

Necessity: Suppose that (2) and (3) hold. Then, there exists a sufficiently small $\epsilon > 0$ which satisfies

$$AX + XA^T + \frac{1}{2}\epsilon AXA^T < 0. \quad (8)$$

Since $X > 0$, employing Schur complement, we have

$$\begin{aligned} &\begin{bmatrix} AX + XA^T & \epsilon AX \\ \epsilon XA^T & -2\epsilon X \end{bmatrix} && (9)\\ &= \begin{bmatrix} AX + XA^T & \epsilon AX + X - X \\ \epsilon XA^T + X - X & -2\epsilon X \end{bmatrix} && (10)\\ &= \begin{bmatrix} 0 & X \\ X & 0 \end{bmatrix} + \mathrm{He}\left(\begin{bmatrix} A \\ -I \end{bmatrix} X \begin{bmatrix} I & \epsilon I \end{bmatrix}\right) && (11)\\ &< 0. && (12) \end{aligned}$$

This means that (4) holds with $W = X$. ■

Note that the condition (4) is an LMI with respect to X and W if we fix ϵ. This scalar parameter ϵ can be chosen with a line-search method.

III. Main results

Consider the following continuous-time N-channel system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} B_i u_i(t) \quad (13)$$

where $A \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times r_i}$, $x(t) \in \mathbb{R}^n$ is the state and $u_i(t) \in \mathbb{R}^{r_i}$ is the input of the i-th channel.

For this system, let us also consider the following state-feedback controllers

$$u_i(t) = K_i x(t) \quad (14)$$

where $K_i \in \mathbb{R}^{r_i\times n}$ for $i = 1, 2, \ldots, N$.

Moreover, we model component failures that occur in a controller or a sensor and an actuator by extracting the corresponding controller and setting the control input of the corresponding channel to zero.[1] That is, if the i-th control channel fails, we remove the i-th controller and set

$$u_i(t) = 0 \quad (15)$$

To describe the closed-loop systems with/without failures in compact forms, let us define

$$\begin{aligned} B^{(0)} &= \begin{bmatrix} B_1 & B_2 & \cdots & B_N \end{bmatrix} \\ K^{(0)} &= \begin{bmatrix} K_1^T & K_2^T & \cdots & K_N^T \end{bmatrix}^T \end{aligned}$$

and

$$\begin{aligned} B^{(i)} &= \begin{bmatrix} B_1 & \cdots & B_{i-1} & B_{i+1} & \cdots & B_N \end{bmatrix} \\ K^{(i)} &= \begin{bmatrix} K_1^T & \cdots & K_{i-1}^T & K_{i+1}^T & \cdots & K_N^T \end{bmatrix}^T \end{aligned}$$

where $i = 1, 2, \ldots, N$.

Then, we can write the closed-loop systems for all $j \in \{0, 1, \ldots, N\}$ as

$$\dot{x}(t) = (A + B^{(j)}K^{(j)})x(t) \quad (16)$$

Here we remark that the closed-loop system under normal operation is obtained if $j = 0$, while the closed-loop system under the j-th controller failure is obtained if $j \in \{1, 2, \cdots N\}$.

We now formally state the state-feedback problem.

Problem 1: (reliable state-feedback stabilization problem): Find K_i, $i = 1, 2, \ldots, N$ such that all the closed-loop systems $A + B^{(j)}K^{(j)}$, $j = 0, 1, \ldots, N$ are stable.

Remark 1: Note that Problem 1 is solvable only if the pairs $(A, B^{(j)})$ for all $j \in \{0, 1, \ldots, N\}$ are $stabilizable$ which is assumed in this paper.

If we simply apply standard stability analysis based on the aforementioned equations of (2) and (3) for all instances of closed-loop systems in (16), we have to introduce a common quadratic Lyapunov stability certificate $X \in \mathcal{S}_+^n$ for all $j \in \{0, 1, \ldots, N\}$ which will give us a *sufficient condition* for solving the stabilizing gains K_i for $i = 1, 2, \ldots, N$. However, such coupling of $(A + B^{(j)}K^{(j)})$ and X usually leads either to conservative or infeasible solution set.

In the following, we employ a dilated LMIs technique to check whether all instances in (16) share a common solution set $K_i \in \mathbb{R}^{r_i\times n}$ for $i = 1, 2, \ldots, N$ that maintains stability.

To this end, let us introduce the following notations

$$\begin{aligned} L^{(0)} &= \begin{bmatrix} L_1^T & L_2^T & \cdots & L_N^T \end{bmatrix}^T \\ L^{(i)} &= \begin{bmatrix} L_1^T & \cdots & L_{i-1}^T & L_{i+1}^T & \cdots & L_N^T \end{bmatrix}^T \end{aligned}$$

[1] In this note, we do not discuss *transient* situations in failures. This is justified in the context of stability since stability is defined with behaviors over the *infinite* time interval.

$$A_{WL}^{(0)} = AW + B_1L_1 + B_2L_2 + \cdots + B_NL_N$$

and

$$\begin{aligned} A_{WL}^{(i)} = AW + B_1L_1 + \cdots + B_{i-1}L_{i-1} \\ + B_{i+1}L_{i+1} + \cdots + B_NL_N \end{aligned}$$

where $W \in \mathbb{R}^{n\times n}$ and $L_i \in \mathbb{R}^{r_i\times n}$ for $i = 1, 2, \ldots, N$.

In light of Lemma 1 and the previous discussion, we have the following theorem.

Theorem 1: Problem 1 is solvable if there exist $X_j \in \mathcal{S}_+^n$, $j = 0, 1, \ldots, N$, $W \in \mathbb{R}^{n\times n}$, $L_i \in \mathbb{R}^{r_i\times n}$, $i = 1, 2, \ldots, N$, and $\epsilon_j > 0$, $j = 0, 1, \ldots, N$, which satisfy

$$\begin{bmatrix} 0 & X_j \\ X_j & 0 \end{bmatrix} + \mathrm{He}\left(\begin{bmatrix} A_{WL}^{(j)} \\ -W \end{bmatrix} \begin{bmatrix} I & \epsilon_j I \end{bmatrix}\right) < 0 \quad (17)$$

for all $j \in \{0, 1, \ldots, N\}$.

Once this condition is fulfilled, the state feedback gains in (14) that achieve reliable stabilization are recovered by

$$K_i = L_i W^{-1} \quad (18)$$

with a nonsingular solution W.

Proof: Suppose that the condition (17) is satisfied for all closed-loop systems indexed by $j = 0, 1, \ldots, N$. Note that we can always obtain a nonsingular solution W by introducing a slight perturbation on W if necessary, since the condition (17) is described with a strict inequality. Thus, a candidate of the reliable feedback gains is well-defined as (18). Then, since we have

$$A + B^{(j)}K^{(j)} = A_{WL}^{(j)} W^{-1} \quad (19)$$

the rest of the proof follows Lemma 1. In fact, replacing A in Lemma 1 with $A_{WL}^{(j)} W^{-1}$ where $j = 0, 1, \ldots, N$ immediately gives the condition of Theorem 1. ■

Note that the condition (17) is described by LMIs in terms of W, L_i, $i = 1, 2, \ldots, N$ and X_j, $j = 0, 1, \ldots, N$. This is a desirable property established by employing Lemma 1.

Next, let us assume the multi-channel system in question has an uncertainty term, i.e.,

$$\dot{x}(t) = (A + u_\rho \Delta A)x(t) + \sum_{i=1}^{N} B_i u_i(t) \quad (20)$$

where $u_\rho \in [-\rho, \rho]$, $\rho \geq 0$ is the uncertainty level and $\Delta A \in \mathbb{R}^{n\times n}$ is the basic perturbation term in the system.

Let us also consider the following related problem where we are interested in estimating the effect of perturbation on the stability of system.

Problem 2: (reliable/robust state-feedback stabilization problem): For a given uncertainty set where the perturbed system $(A + u_\rho \Delta A)$ is well defined, find the controller gains K_i for $i = 1, 2, \ldots, N$ and an upper bound on the level of perturbation ρ for which all perturbed closed-loop systems, i.e., $(A + u_\rho \Delta A + B^{(j)}K^{(j)})$ for all $j \in \{0, 1, \ldots, N\}$, are stable.

Solving this problem is not easy in general since it is a non-convex optimization problem. In what follows, we assume there exist state-feedback gains K_i for $i = 1, 2, \ldots, N$ that maintain the stability condition for all instances of Problem 1. We will then estimate an upper bound $\hat{\rho}$ on the uncertainty level for which the state-feedback gains preserve *robust/reliable stability property* for the perturbed multi-channel system.

In the following, we provide a precise statement based on the existence of "*a panel of dissipativity certificates*" that implies reliable stability for the perturbed multi-channel system.

Theorem 2: (a panel of dissipativity certificates): Suppose W and L_i for $i = 1, 2, \cdots, N$ that solve Problem 1 are given. For a given $\alpha > 0$, $\beta \geq 1$ and $Z \in \mathcal{S}_+^n$, if there exist $Y_j \in \mathcal{S}_+^n$ for all $j = \{0, 1, \cdots, N\}$ and an upper bound $\hat{\rho}$ that satisfy

$$\beta^{-1} Z \leq Y_j \leq Z \quad (21)$$

$$\begin{bmatrix} W \\ I \end{bmatrix}^T \begin{bmatrix} u_{\hat{\rho}}\,\mathrm{He}(\Delta A^T Y_j) & Y_j A_{WL}^{(j)} \\ (A_{WL}^{(j)})^T Y_j & 0 \end{bmatrix} \begin{bmatrix} W \\ I \end{bmatrix} \leq -\alpha W^T Z W \quad (22)$$

Then, the system with state-feedback controllers is reliably stable for all instances of perturbation in the system.[2]

Proof: To prove the above theorem, we require the following systems for all $j \in \{0, 1, \cdots N\}$

$$\begin{aligned} \dot{x}(t) &= (A + u_\rho \Delta A + B^{(j)}K^{(j)})x(t) + 0_{n\times 1}\tilde{u}(t) \\ \tilde{y}(t) &= x(t) + 0_{n\times 1}\tilde{u}(t) \end{aligned} \quad (23)$$

to satisfy certain dissipativity property for all instances of perturbation in the system.

Let us define the following supply rate

$$\mathcal{G}(\tilde{y}(t), \tilde{u}(t)) = \begin{bmatrix} \tilde{y}(t) \\ \tilde{u}(t) \end{bmatrix}^T \begin{bmatrix} -\alpha Z & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{y}(t) \\ \tilde{u}(t) \end{bmatrix} \quad (24)$$

where $Z \in \mathcal{S}_+^n$.

Clearly, if the systems in (23) are stable for all instances of perturbation, then the following *dissipation inequalities* will hold

$$V_j(x(0)) + \int_0^t \mathcal{G}(\tilde{y}(t), \tilde{u}(t))dt \geq V_j(x(t)) \quad (25)$$

[2]Note that if there exists a solution set X_j for the Problem 1 that gives a minimum distance between X_j and the set I_Y for all $j \in \{0, 1, \cdots, N\}$, then we have essentially a near optimal solution for the problem we posed in Problem 2. This solution set is also unique since I_Y is a convex and compact set (e.g., see [21]).

for all $t \geq 0$ with non-negative quadratic storage functions $V_j(x(t)) = x(t)^T Y_j x(t)$, $Y_j \in \mathcal{S}_+^n$ for all $j \in \{0, 1, \cdots, N\}$ that satisfy $V_j(0) = 0$.

Condition (25) with (24) further implies the following

$$\mathrm{He}\left((A + u_\rho \Delta A + B^{(j)} K^{(j)})^T Y_j \right) \leq -\alpha Z \tag{26}$$

Therefore, there exists an upper bound $\hat{\rho}$ for which the dissipativity conditions in (26) will hold true for all instances of perturbation in the system.

Using (19), we have the following

$$\begin{aligned}\mathrm{He}\left((A_{WL}^{(j)} W^{-1} + u_{\hat{\rho}} \Delta A)^T Y_j \right) = \\ \begin{bmatrix} W \\ I \end{bmatrix}^T \begin{bmatrix} u_{\hat{\rho}} \mathrm{He}(\Delta A^T Y_j) & Y_j A_{WL}^{(j)} \\ (A_{WL}^{(j)})^T Y_j & 0 \end{bmatrix} \begin{bmatrix} W \\ I \end{bmatrix} \\ \leq -\alpha W^T Z W \end{aligned} \tag{27}$$

where $u_{\hat{\rho}} \in [-\hat{\rho},\ \hat{\rho}]$.

On the other hand, let us define the following matrix interval

$$I_Y = \{ Y : \beta^{-1} Z \leq Y \leq Z \} \tag{28}$$

where Y, $Z \in \mathcal{S}_+^n$; and $\alpha > 0$ and $\beta \geq 1$ are *a-priori* assumed to be known.

Suppose that Y_j satisfies the conditions in (21) and (22), then the trajectory of the perturbed closed-loop system

$$\dot{x}(t) = (A + u_{\hat{\rho}} \Delta A + B^{(j)} K^{(j)}) x(t)$$

for all $j \in \{0, 1, \ldots, N\}$ satisfies

$$\begin{aligned}\frac{d}{dt}(x^T(t) Y_j x(t)) &= x^T(t)\, \mathrm{He}\left((A + u_{\hat{\rho}} \Delta A \right. \\ &\qquad \left. + B^{(j)} K^{(j)})^T Y_j \right) x(t) \\ &\leq -\alpha x^T(t) Z x(t) \\ &\leq -\alpha x^T(t) Y_j x(t)\end{aligned} \tag{29}$$

Note that, for all $t \geq 0$, the condition (29) further implies the following conditions

$$\begin{aligned}x^T(t) Y_j x(t) &\leq exp\{-\alpha t\} x^T(0) Y_j x(0) \\ &\leq exp\{-\alpha t\} x^T(0) Z x(0)\end{aligned} \tag{30}$$

and

$$\begin{aligned}x^T(t) Z x(t) &\leq \beta x^T(t) Y_j x(t) \\ &\leq \beta exp\{-\alpha t\} x^T(0) Z x(0)\end{aligned} \tag{31}$$

Hence, the conditions (29), (30) and (31) stating that the set $\{Y_0, Y_1, \cdots, Y_N\}$ with $Y_j \in I_Y$ consists of a panel of dissipativity certificates, with a common supply rate of (24), for all instances of perturbation in (23).[3] ■

The precise statement behind the result in Theorem 2 comes from the fact that such a panel of certificates, i.e., the set $\{Y_0, Y_1, \cdots, Y_N\}$ with $Y_j \in I_Y$, ensures all perturbed multi-channel systems to possess a dissipativity property. A similar idea has been explored by Barb *et al.* [23] in the context of a common dissipativity certificate for uncertain systems.

Remark 2: Here, we remark that finding an upper bound on $\hat{\rho}$ and a set of solutions Y_j from a convex and compact set I_Y for all $j \in \{0, 1, \cdots, N\}$ is equivalent to solving the verification problem that we posed in Theorem 2 (e.g., see reference [24]).

IV. Numerical Example

Consider the following simple example where the system matrices for the nominal system are given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} B_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

With the following base perturbation in the system,

$$\Delta A = \begin{bmatrix} 0.000 & 0.150 & 0.000 \\ 0.150 & 0.000 & 0.175 \\ 0.000 & 0.125 & 0.000 \end{bmatrix}$$

Note that for this system we cannot design reliable stabilizing feedback controllers based on a common solution of Lyapunov inequalities, i.e., we cannot find a set $\{X, K_1, K_2, K_3\}$ which satisfies the conditions in

$$\begin{aligned}\mathrm{He}\{(A + B_2 K_2 + B_3 K_3) X\} < 0 \\ \mathrm{He}\{(A + B_1 K_1 + B_3 K_3) X\} < 0 \\ \mathrm{He}\{(A + B_1 K_1 + B_2 K_2) X\} < 0\end{aligned}$$

In fact, if we define

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

and eliminate K_1, K_2 and K_3 from the above expressions using

$$\begin{aligned}[B_2\ B_3]^\perp = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \\ [B_1\ B_3]^\perp = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \\ [B_1\ B_2]^\perp = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\end{aligned}$$

Then, we will see that the following necessary conditions $x_{12} < 0$, $-x_{12} + x_{23} < 0$ and $-x_{23} < 0$ cannot hold simultaneously. However, if we employ Theorem 1, the state-feedback controllers that achieve reliable state-feedback stabilization are given by

$$\begin{aligned}K_1 = \begin{bmatrix} -1.3729 & -0.1891 & 0.4599 \end{bmatrix} \\ K_2 = \begin{bmatrix} 0.3228 & -1.2182 & -0.0831 \end{bmatrix} \\ K_3 = \begin{bmatrix} -0.0394 & 0.5235 & -0.8933 \end{bmatrix}\end{aligned}$$

[3] Note that the $exp\{-\alpha t\}$ determines the long-term behavior of the system, whereas $\beta \geq 1$ bounds its short-term or transient behavior. In general, these parameters can be chosen so as to guarantee the reliably stability of the system with acceptable decay and transient behavior [22].

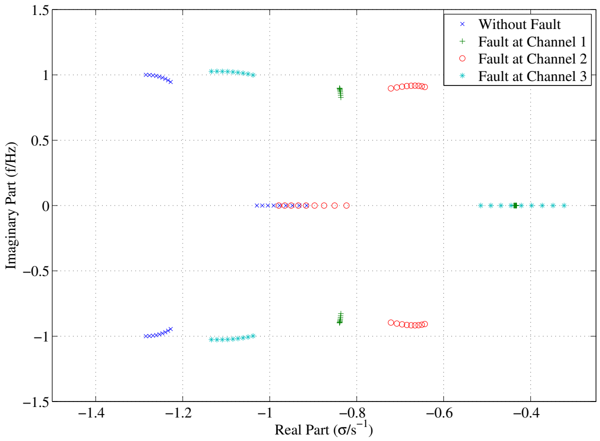


Fig. 1: The eigenvalues of the perturbed closed-loop system

TABLE I: Eigenvalues for the Nominal System

Fault Locations	λ_1	$\lambda_2,\ \lambda_3$
Without Fault	-0.9811	$-1.2516\ \pm\ \mathrm{j}\ 0.9842$
Channel 1	-0.4353	$-0.8381\ \pm\ \mathrm{j}\ 0.8776$
Channel 2	-0.9215	$-0.6723\ \pm\ \mathrm{j}\ 0.9169$
Channel 3	-0.4282	$-1.0814\ \pm\ \mathrm{j}\ 1.0212$

for $\epsilon_0 = 1$, $\epsilon_1 = 1$, $\epsilon_2 = 1$, and $\epsilon_3 = 1$.

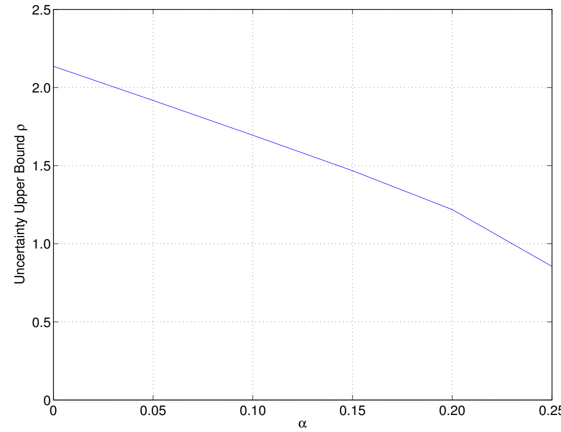
On the other hand, for $\alpha = 0.25$ and $\beta = 2.0$, if we define the matrix interval $I_Y = \{Y : \beta^{-1}Z \leq Y \leq Z\}$ using the following positive definite matrix

$$Z = \begin{bmatrix} 209.1008 & 4.8812 & 4.2082 \\ 4.8812 & 204.1849 & 5.4836 \\ 4.2082 & 5.4836 & 194.2057 \end{bmatrix}$$

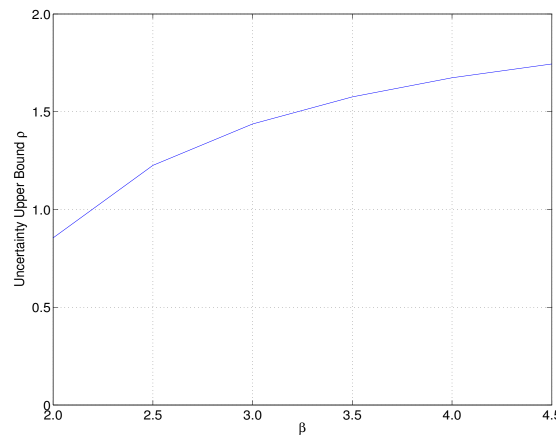
Theorem 2 guarantees reliable stability for all instances of perturbation $u_{\hat{\rho}} \in [-\hat{\rho},\ \hat{\rho}]$ in the system. Here the upper bound on the perturbation level $\hat{\rho}$, which is also computed together with the panel of dissipativity certificates from the set I_Y, is given by

$$\hat{\rho} = 0.8549$$

The eigenvalues of the perturbed closed-loop systems with the controllers in the system, when the perturbations $u_{\hat{\rho}}$ are uniformly sampled from the interval $[-\hat{\rho},\ \hat{\rho}]$, are shown in Fig. 1. As can be seen from this figure, all of the eigenvalues reside in the left half s-plane. The eigenvalues for the nominal closed-loop system with the controllers in the system are given in Table I. Moreover, the upper bounds of the uncertainty level for different values of α and β are shown in Fig. 2. From this figure, we notice that there is a trade-off between the uncertainty upper bound $\hat{\rho}$ and the parameters α (that determines the long-term behavior of the system) and β (which determines the short-term or transient behavior of the system). The exact value of this bound is in general difficult to determine or may differ from the estimated value.



(a) ρ vs α ($\beta = 2.0$)



(b) ρ vs β ($\alpha = 0.25$)

Fig. 2: Uncertainty bound versus the parameters α and β

V. Concluding Remarks

In this paper, we formulated the problem of reliable stabilization for a perturbed multi-channel system. We provided a computationally tractable and less-conservative result in terms of a set of dilated LMIs for the reliable state feedback controllers of the nominal system, while a dissipativity-based certification is employed for extending the stability condition for an additive model perturbation in the system. Moreover, the framework in which we have defined the problem provides a unified treatment for handling the issue of reliable stabilization and model uncertainty.

References

[1] D. D. Šiljak, “Reliable control using multiple control systems,” *Int. J. Contr.,* vol. 31, no. 2, pp. 303-329, 1980.

[2] D. D. Šiljak, *Large-scale dynamic systems, stability and structure*, North Holland, New York, 1978.

[3] D. D. Šiljak, *Decentralized control of complex systems*, Academic Press, 1991.

[4] M. Vidyasagar and N. Viswanadham, “Reliable stabilization using a multi-controller configuration,” *Automatica,* vol. 21, no. 5, pp. 599-602, 1985.

[5] K. A. Ünyelioğlu and A. B. Özgüler, "Reliable decentralized stabilization of feedforward and feedback interconnected systems," *IEEE Trans. Automat. Contr.,* vol. 37, no.8, pp. 1119-1132, 1992.

[6] X.-L. Tan, D. D. Šiljak, and M. Ikeda, "Reliable stabilization via factorization methods," *IEEE Trans. Automat. Contr.,* vol. 37, no.11, pp. 1786-1791, 1992.

[7] A. N. Gündeş, "Reliable decentralized stabilization of linear systems," *IEEE Trans. Automat. Contr.,* vol.43, no. 12, pp. 1733-1739, 1998.

[8] R. J. Veillette, J. V. Medanić, and W. R. Perkins, "Design of reliable control systems," *IEEE Trans. Automat. Contr.,* vol. 37, no.3, pp. 290-304, 1992.

[9] M. H. Shor, W. R. Perkins, and J. V. Medanić, "Design of reliable decentralized controllers: A unified continuous/discrete formulation," *Int. J. Contr.,* vol. 56, no.4, pp. 943-965, 1992.

[10] Y. Ohta, H. Maeda and S. Kodama, "Unit interpolation in H_∞: Bounds of norm and degree of interpolants," *Syst. Contr. Lett.,* vol. 17, no. 4, pp. 251-256, 1991.

[11] V. Blondel and J. Tsitsiklis, "NP-hardness of some linear control design problems," *SIAM J. Control Optim.,* vol. 35, pp. 2118-2127, 1997.

[12] Y. Fujisaki and G. K. Befekadu, "Reliable decentralized stabilization of multi-channel systems: A design method via dilated LMIs and unknown disturbance observers," *Int. J. Contr.,* vol. 82, no.11, pp. 2040-2050, 2009.

[13] J. C. Willems, "Dissipative dynamical systems, part I: General theory," *Arch. Rational Mech. Anal.,* vol. 45, pp. 321-351, 1971.

[14] J. C. Willems, "Dissipative dynamical systems, part II: Linear systems with quadratic supply rates," *Arch. Rational Mech. Anal.,* vol. 45, pp. 352-393, 1971.

[15] S. Weiland and J. C. Willems, "Dissipative dynamical systems in a behavioral context," *Math. Models Meth. Appl. Sci.,* vol. 1, no. 1, pp. 1-26, 1991.

[16] A. Nemirovski, "Several NP-hard problems arising in robust stability analysis," *Math. Contr. Sign. Syst.,* vol. 6, no. 2, pp. 99-105, 1993.

[17] G. Rex and J. Rohn, "Sufficient conditions for regularity and singularity of interval matrices," *SIAM J. Matrix. Anal. Appl.,* vol. 20, no.2, pp. 437-445, 1998.

[18] A. Ben-Tal and A. Nemirovski, "Robust solutions of uncertain quadratic and conic-quadratic problems," *SIAM J. Optim.,* vol. 13, no. 2, pp. 535-560, 2002.

[19] M. C. de Oliveira, J. Bernussou and J. C. Geromel, "A new discrete-time robust stability condition," *Syst. Control Lett.,* vol. 37, pp. 261-265, 1999.

[20] Y. Oishi, "A matrix-dilation approach to robust semidefinite programming and its error bound," in *American Control Conf.,* Minneapolis, MI, 2006, pp. 123-129.

[21] D. G. Luenberger *Optimization by vector space methods,* John Wiley & Sons, 2009.

[22] D. Hinrichsen and A. J. Pritchard "Stability radius for structured perturbations and the algebraic Riccati equation," *Syst. Control Lett.,* vol. 8, no. 2, pp.105-113, 1986.

[23] F. D. Barb, A. Ben-Tal and A. Nemirovski, "Robust dissipativity of interval uncertain systems," *SIAM J. Control Optim.,* vol. 41, no. 6, pp. 1661-1695, 2003.

[24] M. Grötschel, L. Lovàsz and A. Schrijver, "The ellipsoid method and its consequences in combinatorial optimization," *Combinatorica,* vol. 1, no. 2, pp. 169-197, 1981.