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"New Stability Theorems for the General Two Degrees of Freedom Control Systems"[1]

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INTRODUCTION

A comprehensive study of internal stability over a desirable region of the complex plane ($\mathbb{R}_g$-*stability*) for a class of general multivariable systems is presented in this paper. The system consists of a general plant S_p, where the measured and the controlled variables are not necessarily the same, and a general linear controller S_C. We are interested in the two degrees of freedom controller, which has received renewed interest in the literature because of its usefulness in addressing control problems with multiple objectives [1-10]. In this paper, we first present a set of necessary and sufficient conditions for the existence of $\mathbb{R}_g$-*stabilizing* controllers for the plant. A novel theorem to determine $\mathbb{R}_g$-*stability* of the compensated system is then introduced, which clarifies the relation between stability of two degrees of freedom and single degree of freedom compensated systems. Internal stability is clearly shown to be an extension of single degree of freedom stability results. This theorem leads directly to parametric

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characterizations of all input-output maps attainable with $\mathbb{R}_g$-stability from r, the vector of request inputs; also to controller configurations that attain these maps. Furthermore, it directly leads to parameterizations of all $\mathbb{R}_g$-*stabilizing* two degrees of freedom controllers. Although some of the conditions presented in this paper have appeared in a more general algebraic setting in [21,22,30], they are derived here using "internal descriptions"; thus, providing additional insight. The main internal stability theorem appears for the first time.

A motivation for this paper is to solve the usual problem in control of placing the controlled system's eigenvalues in a desirable region of the complex plane $\mathbb{C}$ for the general systems considered here. Let S_g denote this region which corresponds to the "good" portion of the complex plane so that S_g is symmetric with respect to the real axis and contains at least one real point. Then a system will be said to be $\mathbb{R}_g$-stable if all its eigenvalues are in S_g, and a controller will be said to be $\mathbb{R}_g$-stabilizing if the compensated system is $\mathbb{R}_g$-stable. So the problem is to find an $\mathbb{R}_g$-stabilizing controller S_C for the plant S_P.

Let $\mathbb{R}_g(s)$ be a nonempty subset of $\mathbb{R}_p(s)$, the ring of proper rational functions with real coefficients, consisting of the proper rational functions which have all their poles in S_g. Then it can be shown that $\mathbb{R}_g(s)$ is a proper Euclidean domain [12,13]. In particular, $\mathbb{R}_g(s)$ is a principal ideal domain [14], giving $\mathbb{R}_g(s)$ the same nice algebraic properties of the polynomial ring $\mathbb{R}[s]$. To develop the theory we will be using factorizations of transfer function matrices over $\mathbb{R}_g(s)$, that is, a given transfer function matrix is modeled as the *ratio* of two rational matrices with entries in $\mathbb{R}_g(s)$. The background to develop the theory can be found in [5,15,16,4]. We will develop the theory in the context of linear time-invariant continuous and discrete systems but the theory can be easily extended to consider linear distributed continuous and discrete systems (for this and other extensions see [5-Chap. 8,4,15,16]).

The analysis of $\mathbb{R}_g$-stability of the compensated system will be clear if a direct relation to an internal description of the systems is maintained. This has been one of the advantages of working with polynomial matrix factorizations, but in this paper we are using factorizations over $\mathbb{R}_g(s)$. For this reason, the results of Antsaklis in [17] relating proper, stable transfer matrix factorizations to internal descriptions have been extended to our setting in [26,27], when S_g is not necessarily the open left half plane $\mathbb{C}^-$. Furthermore, the results in [17] were extended to characterize the relation between a bicoprime representation [5] of a transfer function matrix and an "internal description" of that system.

PROBLEM FORMULATION

Consider the controlled system depicted in Figure 1,

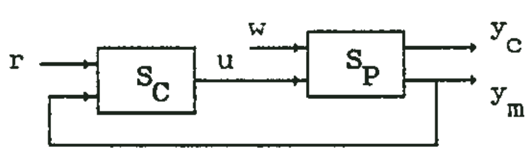

Figure 1. A compensated system $\Sigma(S_P,S_C)$.

where S_P and S_C denote the plant and compensator, respectively. Assume that the plant S_P is controllable and observable, and let an input-output description of S_P be

$$\begin{bmatrix} y_m \\ y_c \end{bmatrix} = P\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} u \\ w \end{bmatrix}, \tag{1}$$

where the vectors y_c, y_m contain the variables of the plant to be controlled and the ones that are measured, respectively; the vector w contains all the variables that affect the plant, but are not manipulated by the compensator (for example, nonmeasurable disturbances and initial conditions); u is the vector of control inputs; and P and P_{ij}, i,j=1,2, are proper transfer function matrices. This general plant model is used because it unifies the study of plants where the controlled and measured variables are not necessarily the same ($y_c \neq y_m$), and where an exogenous signal w is present. Assume that the controller S_C is controllable and observable, and let the control u be given by

$$u = C\begin{bmatrix} y_m \\ r \end{bmatrix} = [-C_y \quad C_r]\begin{bmatrix} y_m \\ r \end{bmatrix}, \tag{2}$$

where $C=[-C_y \quad C_r]$ is the transfer function matrix of S_C, and r is the vector of command inputs. Also, assume that the compensated system $\Sigma(S_P,S_C)$ is well-defined, that is $|I+P_{11}C_y|\neq 0$ and that every input-output map is proper.

An "internal description" of S_P, in so far as $\mathbb{R}_g$-stability is concerned, is

$$\begin{aligned} T'z' &= [U_1' \quad U_2']\begin{bmatrix} u \\ w \end{bmatrix} \\ \begin{bmatrix} y_m \\ y_c \end{bmatrix} &= \begin{bmatrix} V_1' \\ V_2' \end{bmatrix} z', \end{aligned} \tag{3}$$

where T', U_1', U_2', V_1', V_2' $\in M(\mathbb{R}_g(s))$[1], with T' square, nonsingular aand biproper, and (T',$[U_1' \quad U_2']$) left coprime (l.c.) over $\mathbb{R}_g(s)$ and ($[V_1'^t \quad V_2'^t]^t$, T') right coprime (r.c.) over $\mathbb{R}_g(s)$. With the above definitions, the quadruple $\{[V_1'^t \quad V_2'^t]^t, T', [U_1' \quad U_2'], 0\}$ is a bicoprime factorization of P.

The problem is to place all the compensated system's eigenvalues in S_g, then the system is said to be $\mathbb{R}_g$-stable. The compensator S_C is said to be an $\mathbb{R}_g$-stabilizing compensator if $\Sigma(S_P,S_C)$ is $\mathbb{R}_g$-stable.

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[1] $M(\mathbb{R}_g(s))$ denotes the set of all matrices with entries in $\mathbb{R}_g(s)$, regardless of dimension.

MAIN RESULTS

Existence of Stabilizing Controllers

We present some known results and extensions of known results that answer the question: when is the plant S_p *stabilizable over* $\mathbb{R}_g(s)$, that is, when does there exist an $\mathbb{R}_g$-stabilizing controller S_C for the plant S_p? The following definitions are needed. Let $P_{11}=N_1'D_1'^{-1}=\tilde{D}_1'^{-1}\tilde{N}_1'$ be coprime $\mathbb{R}_g$-factorizations, that is, (N_1',D_1') and $(\tilde{D}_1',\tilde{N}_1')$ are r.c. and l.c. in $\mathbb{R}_g(s)$, respectively, with D_1' and $\tilde{D}_1'$ nonsingular and biproper, and let $x_1',x_2' \in M(\mathbb{R}_g(s))$ satisfy the Diophantine equation $x_1'D_1'+x_2'N_1'=I$. Consider the partial fraction expansion $P_{ij}=P_{ij}^{+}+P_{ij}^{-}$, $i,j=1,2$, where all the poles of P_{ij}^{-} (P_{ij}^{+}) are in S_g ($\Omega:=\mathbb{C}\backslash S_g$); and let $\nu^{+}[P_{ij}]$ denote the McMillan degree of P_{ij}^{+}, that is, $\nu[P_{ij}^{+}]$.

A state space description of the plant is given by

$$\dot{x}(t) = Ax(t) + [B_1 \quad B_2]\begin{bmatrix} u \\ w \end{bmatrix}$$
$$\begin{bmatrix} y_m(t) \\ y_c(t) \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(t) + \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}\begin{bmatrix} u(t) \\ w(t) \end{bmatrix}, \tag{4}$$

where A, B, C, and E are nxn, nx(m+s), (q+p)xn and (q+p)x(m+s) real constant matrices, respectively. Stabilizability and detectability in this case can be expressed in terms of rank tests denoted PBH tests in [23]. For example, for continuous systems, we say that the pair (A,B) is stabilizable if and only if $[sI-A \quad B]$ has rank n for all s with $\mathrm{Re}\{s\} \geq 0$. A dual relation holds for dectectability of the pair (C,A). This definitions can be extended to consider stabilizability and detectability over $\mathbb{R}_g(s)$ as follows. A pair (A,B) is *stabilizable over* $\mathbb{R}_g(s)$ if and only if the rank of $[sI-A, B]$ is n for all $s \in \Omega$, and the pair (C,A) is *detectable over* $\mathbb{R}_g(s)$ if and only if the rank of $[C^t, (sI-A)^t]^t$ is n for all $s \in \Omega$.

Theorem 1. Consider the plant S_P, then the following statements are equivalent:

(i) There exists an $\mathbb{R}_g$-stabilizing controller C.

(ii) A1') (T', U_1') are l.c. over $\mathbb{R}_g(s)$, and

A2') (V_1', T') are r.c. over $\mathbb{R}_g(s)$.

(iii) B1') $\nu^+(P_{11}) = \nu^+\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$.

(iv) C1') $P_{21}D_1' \in M(\mathbb{R}_g(s))$,

C2') $\tilde{D}_1'P_{12} \in M(\mathbb{R}_g(s))$, and

C3') $P_{22} - P_{21}D_1'X_2'P_{12} \in M(\mathbb{R}_g(s))$.

(v) D1') $|T'|$ and $|D_1'|$ are associates[2].

(vi) E1') (A, B_1) is stabilizable over $\mathbb{R}_g(s)$, and

E2') (C_1, A) is detectable over $\mathbb{R}_g(s)$.

The conditions in Theorem 1 imply that the plant S_P, has all its uncontrollable eigenvalues from u and unobservable eigenvalues from y_m, if any, in S_g, that is, the existence of a compensator driven by the measurement variables y_m to produce the control signal u requires that S_P be *stabilizable from u and detectable from y_m*. In this way, all the modes of S_P that correspond to an eigenvalue in Ω (the "bad" modes) will be observable from y_m; otherwise, S_C could not be an $\mathbb{R}_g$-stabilizing controller. Conditions (ii) and (iii) extend the results in [24] and [18,19] to the general case considered here. Recently, similar conditions as in (iv) and (v) have appeared in [22]. Condition (vi) extends the stabilizability and detectability properties as have been presented, for example, in [21,22,25]. For a discussion of all these conditions and the proof of Theorem 1 see

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[2] Two elements a, b $\in \mathbb{R}_g(s)$ are associates if they differ by u, a unit in $\mathbb{R}_g(s)$, that is, a = ub.

[26,27]. Two additional sets of conditions to characterize the plants stabilizable over $\mathbb{R}_g(s)$ have been presented in [28, Theorem 4.3.1].

Internal stability, Attainable Maps, and Parameterizations

We now present a novel $\mathbb{R}_g$-stability theorem. Note that any of the conditions in Theorem 1 can be used to characterize the $\mathbb{R}_g$-admissible plants S_P for which there exists an $\mathbb{R}_g$-stabilizing controller S_C.

Theorem 2. If the plant S_P is $\mathbb{R}_g$-admissible then the compensated system $\Sigma(S_P,S_C)$ is $\mathbb{R}_g$-stable if and only if

(a) the compensated system described by $y_m=P_{11}u$, $u=-C_y y_m$ is $\mathbb{R}_g$-stable, and

(b) C_r is such that $M=(I+C_yP_{11})^{-1}C_r$ satisfies $D_1'^{-1}M=X'$, with $X' \in M(\mathbb{R}_g(s))$, where C_y satisfies (i) and $P_{11}=N_1'D_1'^{-1}$ is a r.c. factorization in $\mathbb{R}_g(s)$.

Theorem 2 provides some advantages over other stability theorems presented so far for the two degrees of freedom control systems. These advantages are discussed below. The proof of Theorem 2 is based on an "internal description" of the compensated system and it is given in [26,27].

It separates the role of C_y from C_r in achieving $\mathbb{R}_g$-stability, showing that stability in a two degrees of freedom configuration is based on the stability of a well studied single degree of freedom configuration (condition (a)) while condition (b) represents the condition needed to maintain $\mathbb{R}_g$-stability. Observe that for a single degree of freedom controller condition (b) follows immediately from (a).

From Theorem 2 we can directly characterize the input-output maps attainable from r with $\mathbb{R}_g$-stability. In particular, we consider the characterization of the maps achievable from r to u, y_m and y_c. The first one is given by M (u=Mr) and from (b) we have that $M=D_1'X'$ where $X' \in M(\mathbb{R}_g(s))$.

The next two maps are given by $T_{mr}=P_{11}M$ and $T_{cr}=P_{21}M$ ($y_m=T_{mr}r$ and $y_c=T_{cr}r$), which are characterized starting with the characterization of M, as $T_{mr}=N_1'X'$ and $T_{cr}=P_{21}D_1'X'$, where $X' \in M(\mathbb{R}_g(s))$, and from Theorem 1, $P_{21}D_1' \in M(\mathbb{R}_g(s))$. For control systems design, it is of interest to characterize these input-output maps to see if a desirable T_{cr} can be realized with $\mathbb{R}_g$-stability, while maintaining acceptable T_{mr} and M. This is done in the next theorem, which is shown in [27].

Theorem 3. A triple (T_{cr}, T_{mr}, M) is realized with $\mathbb{R}_g$-stability via a two degrees of freedom configuration if and only if

$$\begin{bmatrix} T_{cr} \\ T_{mr} \\ M \end{bmatrix} = \begin{bmatrix} P_{21}D_1' \\ N_1' \\ D_1' \end{bmatrix} X', \qquad X' \in M(\mathbb{R}_g(s)) \tag{5}$$

If the conditions in Theorem 3 are satisfied, then the triple (T_{cr},T_{mr},M) can be realized via a two degrees of freedom control law since this is the most general linear control law.

Theorems 2 and 3 show a way of choosing C_y and C_r to attain the maps T_{cr}, T_{mr} and M. Consider

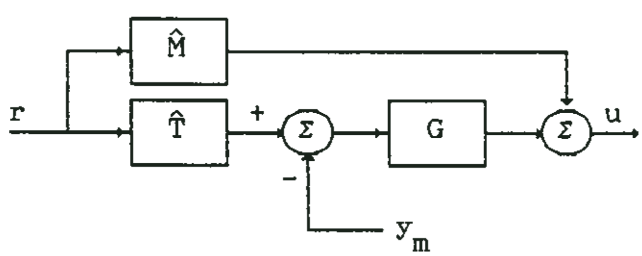

Figure 2. A two degree of freedom compensator, with $[C_y \;\; C_r] = [G \;\; \hat{M}+G\hat{T}]$.

with $\hat{M}$ = M and with $\hat{T} = T_{mr}$. In this way, feedback of the measured variables is used to create an error signal that drives the controller G. Other configurations to attain these maps are, of course, possible [27].

Another useful result from Theorem 2 is that it directly leads to parameterizations of all $\mathbb{R}_g$-stabilizing controllers, starting from any one degree of freedom $\mathbb{R}_g$-stabilizing controller. First, parameterize C_y using a one degree of freedom $\mathbb{R}_g$-stabilizing controller characterization of P_{11}, then C_r is given by $C_r=(I+C_yP_{11})M$. The characterizations of C_y over $\mathbb{R}_g(s)$ can be obtained by extending the characterizations given in [20,15,16,5]; this is done in [26,27]. Three characterizations of all $\mathbb{R}_g$-stabilizing controllers are

$$C = (x_1'-K'\tilde{N}_1')^{-1}[-(x_2'+K'\tilde{D}_1') \quad X'] \tag{6}$$

$$C = (I-Q'P_{11})^{-1}[Q' \quad D_1'X'] \tag{7}$$

$$C = ((I-L'N_1')D_1'^{-1})^{-1}[L' \quad X'], \tag{8}$$

where K', Q', X', L', $(I-L'N_1')D_1' \in M(\mathbb{R}_g(s))$; $|x_1'-K'\tilde{N}_1'|\neq 0$, $|I-Q'P_{11}|\neq 0$ and $|I-L'N_1'|\neq 0$; $(x_1'-K'\tilde{N}_1')$, $(I-Q'P_{11})$, and $(I-L'N_1')$ are biproper. For a description and history of these parameterizations see [20].

From Theorem 2, a necessary condition for $\mathbb{R}_g$-stability is that $|\tilde{D}_{C_y}'|$ and $|\tilde{D}_C'|$ are associates, where $C=\tilde{D}_C^{-1}[\tilde{N}_y \quad \tilde{N}_r]$ and $C_y=\tilde{D}_{C_y}^{-1}\tilde{N}_{C_y}$ are l.c. factorizations in $\mathbb{R}_g(s)$; the proof of this condition follows directly from the proof of Theorem 2. This condition is similar to the plant's $\mathbb{R}_g$-admissibility condition in Theorem 1 (v), and it can be considered to be the $\mathbb{R}_g$-admissibility criterion for the controller S_C. The $\mathbb{R}_g$-admissibility conditions of the plant S_P and of the controller S_C imply that the controlled system is $\mathbb{R}_g$-stable if and only if (a) in Theorem 2 is satisfied.

<u>Theorem 4.</u> If the plant S_P and controller S_C are $\mathbb{R}_g$-admissible, then the compensated system $\Sigma(S_P,S_C)$ is $\mathbb{R}_g$-stable if and only if the single degree of freedom system described by $y_m = P_{11}u$, $u = -C_y y_m$ is $\mathbb{R}_g$-stable as depicted in Figure 3.

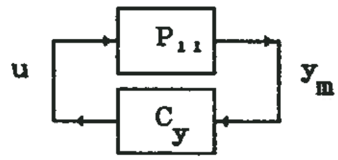

Figure 3. A one degree of freedom system.

Note that a similar result has been independently shown in [21,22,30]; and , recently, for a compensated system that consists of S_P and a one degree of freedom compensator in [28]. A consequence of this theorem is that only four of all the possible input-output maps need to be tested to check $\mathbb{R}_g$-stability; these four maps correspond to the ones presented in [29].

CONCLUSION

We have presented a comprehensive study of $\mathbb{R}_g$-stability of two degrees of freedom control systems. First, several conditions to characterize the set of all plants stabilizable over $\mathbb{R}_g(s)$ were presented. Then a novel $\mathbb{R}_g$-stability theorem was introduced that led directly to:
(i) characterization of all transfer function matrices from r, (ii) a way to realize C_y and C_r to attain these desired maps, and (iii) a direct way to parameterize all $\mathbb{R}_g$-stabilizing two degrees of freedom controllers. These results are expected to be useful in the design of two degrees of freedom control systems.

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