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OBSERVER BASED BLOCK REALIZATIONS OF STABILIZING OUTPUT CONTROLLERS

P. J. Antsaklis

Department of Electrical and Computer Engineering
University of Notre Dame
Notre Dame, IN 46556

I. Introduction

Given a plant $y = Pu$ and a stabilizing controller $u = -Cy$, C can be realized as an observer-based controller. Understanding the exact relation between stabilizing output feedback and state feedback-observer control structures is important as it enhances our understanding of the relationships between design methods which directly produce C and state-space methods which typically design observer-based controllers; further-more, powerful design methods such as LQG/LTR are based on such knowledge.

The fact that stabilizing output feedback can be generated by observer-based control has, of course, been known and can be easily seen by simple block manipulation. The class of all stabilizing C can actually be generated using full order-full state observers with static state feedback augmented by, a design parameter, Q[3]. Here, the starting point is C, and the class of all observer-based realizations of C is characterized. The realizations are based on conventional full or reduced order observers of appropriate linear functions of the state of the plant, instead of on full order-full state observers augmented by Q.

To formally study the observer-based realizations of C, the concept of block realizations of C is introduced. It is a generalization of the conventional realizations using internal descriptions, in the sense that the elementary building blocks are allowed to be more complicated systems than single gains and integrators. Equivalence and minimality of block realizations are defined and an algorithm to generate minimal observer-based realizations of C is presented. The order of such minimal realizations is equal to the order of C; that is, full order-full state observer based controllers of order n can only be minimal realizations of order n stabilizing controllers C.

Proper, stable factorizations of transfer matrices are used and the results presented are based on [1]. Internal descriptions of C and its realizations are derived and used to prove the results.

II. Preliminaries

Let P(s) be the proper transfer matrix of a linear, time-invariant multivariable system. Write

$$y = Pu \quad , \quad P = ND^{-1} \tag{1}$$

where $(N,D) \in M(S)$, that is, matrices with elements in S, the set of all proper and stable rational functions; y and u are the output and input vectors respectively. Let (N,D) be right coprime (rc) in S; that is, there exists $(X,Y) \in M(S)$ such that the Diophantine equation (or Bezout identity)

$$XD + YN = I \tag{2}$$

is satisfied. Note that D^{-1} is also proper, that is, D is biproper. Consider the output controller

$$u = -Cy \quad ; \quad C = X^{-1}Y \tag{3}$$

where C proper, $(X,Y) \in M(S)$ and ℓc. It is known that it internally stabilizes the plant $y = Pu$ iff $XD + YN = U$ where $U, U^{-1} \in M(S)$. Pre or post multiplying by U^{-1} leads to (2) where X, Y or N,D have been relabeled. It can be easily shown that the solutions (X,Y) of the Diophantine which generate stabilizing proper controllers C via (3) are those for which X is biproper. If P is strictly proper, all solutions have X biproper; if, however, P is not strictly proper, care should be exercised when solving the Diophantine to ensure that X will be biproper.

III. Output and Observer-based Controllers

To establish their relation, Theorem 3 of [1] is used:

- $(X,Y) \in M(S)$ are solutions of (2) iff $[I-LX, -LY]$ is an observer of the linear functional of the state Fz.

The matrices F,L define a state feedback (sf) control law $u = Fz + Lr$ applied to the controllable and detectable realization $D_1 z = u$, $y = N_1 z$ of P. They are determined from $P = ND^{-1}$[1] via:

$$\begin{bmatrix} D \\ N \end{bmatrix} = \begin{bmatrix} D_1 \\ N_1 \end{bmatrix} \hat{D}_1^{-1} = \begin{bmatrix} D_1 \\ N_1 \end{bmatrix} D_F^{-1} L \tag{4}$$

a rc polynomial factorization; $D_F := D_1 - F$ and $L := \lim_{s\to\infty} D$.

The solutions of the Diophantine (2) (with X biproper) generate all proper stabilizing controllers C via (3). In view now of [1,Th.3] above, it is rather straightforward to prove the following main theorem:

Suppose a rc factorization (1) is given; using (4) derive a realization $D_1 z = u$, $y = N_1 z$ and a sf pair (F,L).

Theorem All stabilizing proper output controllers $u = -Cy$, $C = R_1^{-1} R_2$ can be realized by observer based controllers C_o

$$u = C_o \begin{bmatrix} u \\ y \end{bmatrix} \quad ; \quad C_o = [I - R_1, \ -R_2] \tag{5}$$

where $R_1, R_2 \in M(S)$, R_1 biproper and C_o is an observer of the linear functional of the state Fz.

Proof Let C_o be an observer of Fz. In view of [1,Th.3], $R_1 D + R_2 N = L$, that is, $C = R_1^{-1} R_2$ is a stabilizing controller. Let now $C = \underline{R_1}^{-1} \underline{R_2}$ be a stabilizing controller. Then $\underline{R_1} D + \underline{R_2} N = U$ from which $R_1 D + R_2 N = L$ if $[R_1, R_2] := LU^{-1}[\underline{R_1}, \underline{R_2}]$; in view of [1,Th.3] C_o is an observer of Fz. QED.

A precise definition of (block) realizations of C is now given.

IV. Block Realizations of $u = -Cy$.

Consider systems S_i $i=1,\dots,k$ completely described by their transfer matrices C_i. Interconnect C_i (in parallel, tandem, feedback configurations) so that the overall transfer matrix (from y to u) is $-C$ (system well posed). Call such configuration of C_i a block realization of the controller C.

A conventional state-space realization of $u = -Cy$ has as its elementary blocks constant gains and integrators. Here we also allow larger blocks C_i to be the elementary blocks of the realization. Clearly, a trivial block realization of C is $u = -Cy$, that is, itself. Other block realizations are: (i) Any state-space realization of $u = -Cy$; (ii) the observer based controller (5); (iii) $b = -R_2 y$, $u = R_1^{-1} b$.

The internal descriptions (state-space or differential operator description) of a block realization of C are determined as follows: A conventional minimal internal description for each C_i is determined; the interconnections are then used to determine the overall internal description between u and y. An internal description of a block realization of C has minimal order iff it is controllable from input y and observable from output u. Such block realizations will be called minimal block realizations. Two block realizations will be called equivalent if their internal descriptions of the controller from y to u are equivalent in the conventional sense. It follows that any block realization of C is equivalent to the realization $u = -Cy$ iff it is a minimal block realization.

In this paper, we are interested in particular block realizations, namely observer-based controller realizations of C and specifically in minimal such realizations.

In the following, the internal descriptions of C and its observer- based controller realizations are studied, leading to minimal such realizations (Lemma) and to the Algorithm.

V. Internal Descriptions

Consider (2) and let

$$\begin{bmatrix} D \\ N \end{bmatrix} = \begin{bmatrix} D_r \\ N_r \end{bmatrix} \Pi \quad ; \quad [X,Y] = \Pi_c[\underline{D}_c, \underline{N}c] \qquad (6)$$

where D_r, N_r are rc polynomial matrices ($P = N_r D_r^{-1}$) and $\underline{D}_c, \underline{N}_c$ are ℓc polynomial matrices ($C = \underline{D}_c^{-1}\underline{N}_c$). Π and Π_c are stable rational matrices. $\Pi = G_r D_F^{-1} L$ (4) where G_r is a greatest crd of N_1, D_1. Π_c is found similarly, below. In view of (6), (2) implies that

$$\underline{D}_c D_r + \underline{N}_c N_r = (\Pi\Pi_c)^{-1} = \underline{D}_k \qquad (7)$$

a polynomial matrix. The cℓ eigenvalues are the zeros of $|\underline{D}_k|$ since $D_r z = u$, $y = N_r z$ and $\underline{D}_c \underline{z}_c = -\underline{N}_c y$, $u = \underline{z}_c$ are minimal realizations of P and C respectively and $\underline{D}_k z = 0$, $y = N_r z$ is the cℓ description of $y = Pu$, $u = -Cy$ [2].

An internal description of the observer-based controller:

$$u = C_o \begin{bmatrix} u \\ y \end{bmatrix} \quad ; \quad C_o = [I - LX, \ -LY] \qquad (8)$$

is now determined. Let $C_o = D_o^{-1}[N_{1o}, N_{2o}]$ be a ℓc polynomial factorization; then $[X,Y] = (D_o L)^{-1}[D_o - N_{1o}, -N_{2o}]$ is a ℓc factorization. If G_ℓ is a greatest cℓd of $D_o - N_{1o}, N_{2o}$ then Π_c in (6) is $\Pi_c = (D_o L)^{-1} G_\ell$. Note that $\underline{D}_k$ is (7) can now be written as $\underline{D}_k = G_\ell^{-1} D_o D_F G_r^{-1}$. The following lemma follows:

Lemma: $(D_o - N_{1o}) z_o = N_{2o} y$, $u = z_o$ is an internal description of the observer-based block realization of C. The closed loop internal description is $G_\ell \underline{D}_k z = 0$, $y = N_r z$.

When C_o (8) is used instead of C, the cℓ eigenvalues contain, in addition, the zeros of $|G_\ell|$. These are uncontrollable from y modes in the description of C_o in the lemma. The following result is now clear:

• C_o in (8) is a minimal block realization of $u = -Cy$ iff G_ℓ is unimodular.

Remark: When it is minimal, the order of C_o is $\partial C_o = \partial C$. This is the case in the state-space approach [1,3] where $C = F[sI-(A+HC+BF+HEF)]^{-1}H$, $\{A,B,C,E\}$ a minimal realization of P, and C_o a full-order full-state observer (see also the LQG/LTR approach to design).

VI. Algorithm

Given stabilizing controller C, determine a minimal block realization in observer-based controller form:

Write $C = R_1^{-1}R_2$ a ℓc proper, stable factorization such that $[R_1, R_2] = D_o^{-1}[\underline{D}_c, \underline{N}_c]$ a ℓc polynomial factorization with $\underline{D}_c, \underline{N}_c$ ℓc polynomial matrices. Then $C_o = [I - R_1, -R_2]$ is such a realization.

To obtain such $[R_1, R_2]$ (i) use ℓc polyn. factorization $\underline{D}_c, \underline{N}_c$ and premultiply by D_o^{-1} stable, or (ii) use a minimal state-space realization of C (Th. 2 in [1]).
To determine Fz, the linear functional estimated by C_o, find $ND^{-1} = P$ such that $R_1 D + R_2 N = I$ and determine F,L from D,N; if the rhs is U instead of I for the particular N,D used, use DU^{-1} NU^{-1} instead.

Exl Let $P = (s-1)/(s+1)(s-2)$ and stabilizing $C = 9(s+1)/(s-5)$. Let $D_o = s+1$ and write $D_o^{-1}[\underline{D}_c, \underline{N}c] = (1/s+1)[s-5, 9(s+1)] = [R_1, R_2]$. A minimal observer-based block realization of C is $C_o = [1-R_1, -R_2] = (1/s+1)[6, -9(s+1)]$.
To determine Fz which the observer C_o estimates, let $P = ND^{-1}$ where $D = (s+1)(s-2)\Pi$, $N = (s-1)\Pi$ with $\Pi = 1/(s+1)^2$; note that $R_1 D + R_2 N = 1$. From $\Pi^{-1} = D_F = D_1 - F = (s+1)^2 = (s+1)(s-2) - F$, $Fz = -(3s+3)z = (-3,-3)x$ is the state functional estimated by C_o. Note that the cℓ eigenvalues are the zeros of $D_o D_F = (s+1)(s+1)^2$ since $G_r = G_\ell = 1$.

Ex2 Let $P = (s+.5)/(s+1)(s-2)$ and stabilizing $C = 5$. Take $[R_1, R_2] = \Pi_c[\underline{D}_c, \underline{N}_c] = ((s+1)^2/(s^2+4s+.5))\,[1,5]$ and $D = (s+1)(s-2)\Pi$, $N = (s+.5)\Pi$ with $\Pi = 1/(s+1)^2$; notice that $R_1 D + R_2 N = 1$. Since $G_\ell = (s+1)^2$ is not a constant, $C_o = [1-R_1, -R_2]$ is not a minimal block realization.
Take $[R_1, R_2] = [1,5]$ ($\Pi_c = 1$) and $\Pi = 1/(s^2+4s+.5)$; then again $R_1 D + R_2 N = 1$. $C_o = [1-R_1, -R_2] = [0,-5]$ is a minimal observer-based block realization of $C = 5$. To find Fz, $\Pi^{-1} = D_1 - F$ from which $Fz = -(5s+2.5)z = (-2.5,-5)x$. In other words, the constant $C = 5$ is seen as an observer of a function of the state of the plant. This last example illustrates the relation between constant output feedback and reduced order observers of a functional of the state. In particular, any constant stabilizing C can be seen as an observer of Fz; Fz is derived from $P = ND^{-1}$ which satisfy $D + CN = I$.

VII. References

[1] P.J. Antsaklis, "Proper Stable Transfer Matrix Factorizations and Internal System Descriptions", IEEE Trans. on Autom. Control, Vol. AC-31, pp 634-638, July 1986.

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[3] J.C. Doyle, Lecture Notes in "Advances in Multivariable Control", ONR/Honeywell Workshop, Minneapolis, MN, 1984.