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Quantized Output Synchronization of Networked Passive Systems with Event-driven Communication

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Abstract— **In this paper, we study the quantized output synchronization problem of networked passive systems with event-driven communication, in which the data transmissions among networked agents are event-based and quantized measurements are exchanged among neighboring agents. We show that with the event-driven communication strategy proposed in the present paper, output synchronization errors of the networked passive systems are bounded by the quantization errors of the signals transmitted in the communication network.**

I. Introduction

In the active area of consensus, synchronization and coordinated control, there has been an increasing interest in the use of quantized measurements and control. These problems investigate systems or agents which are distributed over a network, and digital communication channels are used for information exchange among agents. Note that quantization is one of the basic limitations induced by the finite bandwidth channels, since measurements must be processed by quantizers to be transmitted over the digital network. Another reason to consider quantized measurements stems from the use of coarse sensors. Although there have been several work on quantized coordination problems for discrete-time systems in the literature (to name a few, see [4]- [7], and the references therein), it is still not very practical to derive the sample-data model of the networked systems and then apply the discrete time results, because this usually requires fast sampling rates and accurate synchronization of all the nodes' clocks in the network. Moreover, the sample-data model may not fully preserve some of the features of the original model.

Recently, several researchers have suggested the idea of *event-driven* control as a promising approach to reduce communication and computation load for the purpose of control in many control applications (see [8]-[12], and [17]-[19]). In a typical event-driven control implementation, the control signals are kept constant until the violation of a *triggering condition* on certain signals triggers the re-computation of the control actions. Compared with *time-driven* control (i.e., sample-data control), where constant and fast sampling rate is applied to guarantee stability in the worst case scenario, the possibility of reducing the number of re-computations, and thus of transmissions, while guaranteeing desired levels of performance in networked control systems, makes event-driven control very appealing.

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Motivated by those problems and ideas just discussed above, we study the quantized output synchronization problem of networked passive systems with event-driven communication in the present paper. The dynamics of the agents are assumed to be passive and evolve in continuous time; each agent is embedded with an *Event-Detector* (ED) and a quantizer; the ED is able to measure the output of the agent with sufficiently fast sampling rate, and whenever it detects that the output of the agent violates a triggering condition, an *event* is issued and a quantized output measurement at that *event time* is transmitted to the neighboring agents through the network. Since quantization effects have to be considered, we derive a triggering condition which leads to practical output synchronization result, namely that the output synchronization error of the networked passive systems is bounded by the quantization errors of the signals transmitted in the communication channels. These results for distributed systems with event-driven communication and signal quantization are new. Some extensions maybe found in the report [20], and related results in [17], [18] and [19] for sensor-actuator networked control systems.

The rest of this paper is organized as follows. We introduce some background on graph theory and passive systems in section II; the problem is stated in section III; triggering condition for output synchronization of networked passive systems is derived in section IV and a case study on the event-driven consensus problems with passive quantizers is provided in section V; triggering condition for quantized output synchronization is derived in section VI; finally, we summarize the main results of this paper in section VII.

II. Background Material

A. Graph Theory

Algebraic graph theory provides straightforward abstractions reflecting the information flow between agents in a network. We consider finite weighted directed graphs $G := (V, E)$ with no self-loops and *adjacency matrix* A, where V denotes the set of all vertices, E denotes the set of all edges, and $A := [a_{ij}]$ with $a_{ij} > 0$ if there is a directed edge from vertex i into vertex j, and $a_{ij} = 0$ otherwise. The *in-degree* and *out-degree* of vertex k are given by $d_i(k) = \sum_j a_{jk}$ and $d_o(k) = \sum_j a_{kj}$ respectively. The *Laplacian* matrix of a directed graph is defined as $L = D - A$, where D is the diagonal matrix of vertex out-degrees.

Definition *1 (strongly connected graph[13]):* A directed graph is strongly connected if for any pair of distinct vertices ν_i and ν_j, there is a directed path from ν_i to ν_j.

Definition 2 (balanced graph[13]): A vertex is balanced if its in-degree is equal to its out-degree. A directed graph is balanced if every vertex is balanced.

Definition 3 (weakly connected[3]): A *path* of length r in a directed graph is a sequence $\nu_0, \dots, \nu_r$ of $r+1$ distinct vertices such that for every $i \in \{0, \dots, r-1\}$, (ν_i, ν_{i+1}) is an edge. A *weak path* is a sequence $\nu_0, \dots, \nu_r$ of $r+1$ distinct vertices such that for each $i \in \{0, \dots, r-1\}$, either (ν_i, ν_{i+1}) or (ν_{i+1}, ν_i) is an edge. A directed graph is *weakly connected* if any two vertices can be joined by a weak path.

Lemma 1 ([13]): Let G be a directed graph and suppose it is balanced. Then G is strongly connected if and only if it is weakly connected.

B. Passivity

Consider the following dynamic system which can be used to describe both linear and nonlinear systems:

$$H : \begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases} \tag{1}$$

where $x \in \mathbb{X} \subset \mathbb{R}^n$, $u \in \mathbb{U} \subset \mathbb{R}^m$ and $y \in \mathbb{Y} \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and $\mathbb{X}$, $\mathbb{U}$ and $\mathbb{Y}$ are the state, input and output spaces, respectively. The representation $\phi(t, t_0, x_0, u)$ is used to denote the state at time t reached from the initial state x_0 at t_0 under control action u.

Definition 4 (supply rate[1]): The supply rate $\omega(t) = \omega(u(t), y(t))$ is a real valued function defined on $\mathbb{U} \times \mathbb{Y}$, such that for any $u(t) \in \mathbb{U}$ and $x_0 \in \mathbb{X}$ and $y(t) = h(\phi(t, t_0, x_0, u), u)$, $\omega(t)$ satisfies

$$\int_{t_0}^{t_1} |\omega(\tau)| d\tau < \infty. \tag{2}$$

Definition 5 (Dissipative System[1]): System H with supply rate $\omega(t)$ is *dissipative* if there exists a nonnegative real function $V : \mathbb{X} \to \mathbb{R}^+$, called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_0 \in \mathbb{X}$ and $u \in \mathbb{U}$,

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} \omega(\tau) d\tau \tag{3}$$

where $x_1 = \phi(t_1, t_0, x_0, u)$.
Passive systems are special cases of dissipative systems which are defined as follows.

Definition 6 (Passive System[1]): System H is *passive* if there exists a storage function V such that for all $t_1 \geq t_0 \geq 0$, $x_0 \in \mathbb{X}$ and $u \in \mathbb{U}$

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} u(\tau)^T y(\tau) d\tau, \tag{4}$$

if V is $\mathcal{C}^1$, then we have

$$\dot{V} \leq u(t)^T y(t), \ \forall t \geq 0. \tag{5}$$

If $V(x_1) - V(x_0) = \int_{t_0}^{t_1} u(\tau)^T y(\tau) d\tau$ (or with V being $\mathcal{C}^1$, $\dot{V} = u^T(t) y(t)$), then the system is *lossless*.

Definition 7 (Passive Memoryless Functions[15]): The memoryless function $y = h(t, u)$ is passive if $u^T y \geq 0$.

Definition 8 (Output Synchronization[3]): For a network of N agents, the agents achieve output synchronization if

$$y_j(t) - y_i(t) \to 0 \text{ as } t \to \infty, \ \forall i, j = 1, \dots, N.$$

III. Problem Statement and Assumptions

The evolution of multi-agent networked control systems depends fundamentally on the topology of their information exchange graph. The following assumption will be used through the rest of this paper.

A1. The topology of the underlying communication graph is weakly connected point-wise in time and form a balanced graph with respect to information exchange.

It has been shown in [3] that for a group of N networked passive systems, if the agents are coupled together using the control law

$$u_i(t) = \sum_{j \in \mathcal{N}_i} K\big[y_j(t) - y_i(t)\big], \quad i = 1, 2, \dots, N, \tag{6}$$

where K is a positive constant and $\mathcal{N}_i$ is the set of agents transmitting their outputs to agent H_i, then under assumption **A1.**, the networked passive systems are globally stable and the agents achieve output synchronization asymptotically.

The output synchronization results in [3] require that each agent communicates with its neighboring agents continuously. In this paper, we redefine the above control problem and propose an event-driven communication strategy. Consider a networked control system which consists of N agents each denoted by H_i, for $i = 1, 2, \dots, N$. Agent H_i transmits its current output information to its neighbors $\mathcal{Z}_i$($\mathcal{Z}_i$ is the set of agents receiving output information from H_i) whenever its triggering condition is satisfied. The sequence of data transmission time (event time) for H_i is denoted by $\{t_{k_i}\}$, for $k_i = 0, 1, 2, \dots$. We summarize the problems we try to solve in this paper as follows:

1) Assuming the dynamics of agents are passive, what are the triggering condition and the coordinate control law to achieve output synchronization with event-driven communication?
2) When quantized output measurements are exchanged between networked agents, what should be the triggering condition for output synchronization?

IV. Triggering Condition for Output Synchronization of Passive Systems with Event-driven Communication

In this section, we first assume that exact output measurements are transmitted at each event time and the coordinate control law of agent H_i is given by

$$u_i(t) = \sum_{j \in \mathcal{N}_i} a\big[\widehat{y}_j(t) - \widehat{y}_i(t)\big], \tag{7}$$

where a is a positive constant representing the coupling between agent H_j and agent H_i as defined in the adjacency matrix of the underlying communication graph; $\widehat{y}_j(t) = y_j(t_{k_j})$, for $t \in [t_{k_j}, t_{k_j+1}]$ with $j \in \mathcal{N}_i$, where $y_j(t_{k_j})$ denotes the last broadcasted output information of agent j at its event time t_{k_j} ; $\widehat{y}_i = y_i(t_{k_i})$, for $t \in [t_{k_i}, t_{k_i+1}]$, where

$y_i(t_{k_i})$ denotes the last broadcasted output information of agent i at its event time t_{k_i}.

We also assume that there is no transmission delay in the network and the topology of the underlying communication graph is fixed. A triggering condition to achieve output synchronization is stated in the theorem below.

***Theorem** 1:* Consider a multi-agent system composed of N mobile agents, each agent is passive with a $\mathcal{C}^1$ storage function, and the control law is given in (7). Under assumption **A1.**, if agent H_i transmits its current output information y_i to its neighbors ($\mathcal{Z}_i$) whenever the following triggering condition is satisfied

$$\left\|e_i(t)\right\|_2 > \frac{\delta_1 \sum_{j\in\mathcal{N}_i} \left\|\widehat{y}_j - \widehat{y}_i\right\|_2^2}{\left\| \sum_{j\in\mathcal{N}_i} (\widehat{y}_j - \widehat{y}_i)\right\|_2}, \ \forall t \geq 0, \tag{8}$$

where $\delta_1 \in (0, 0.5)$, $e_i(t) = y_i(t) - \widehat{y}_i$, then the multi-agent system will achieve output synchronization asymptotically.

Proof: Since each agent is passive with a $\mathcal{C}^1$ storage function, we have $\dot{V}_i \leq u_i^T(t)y_i(t)$, where $V_i(t)$ is the storage function of agent H_i, $i = 1, 2, \dots, N$. Consider a storage function for the entire networked system given by $V = \sum_{i=1}^N V_i$, then we can get

$$\begin{aligned}\dot{V} &= \sum_{i=1}^N \dot{V}_i \leq \sum_{i=1}^N u_i^T y_i = \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a(\widehat{y}_j - \widehat{y}_i)^T y_i \\ &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a(\widehat{y}_j - \widehat{y}_i)^T (e_i + \widehat{y}_i),\end{aligned} \tag{9}$$

and we can further get

$$\begin{aligned}\dot{V} &\leq \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a(\widehat{y}_j - \widehat{y}_i)^T e_i + \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a(\widehat{y}_j - \widehat{y}_i)^T \widehat{y}_i \\ &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a(\widehat{y}_j - \widehat{y}_i)^T e_i + \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\widehat{y}_j^T \widehat{y}_i \\ &\quad - \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\widehat{y}_i^T \widehat{y}_i.\end{aligned} \tag{10}$$

As the information exchange graph is balanced, we have

$$\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \widehat{y}_i^T \widehat{y}_i = \frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \widehat{y}_i^T \widehat{y}_i + \frac{1}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \widehat{y}_j^T \widehat{y}_j, \tag{11}$$

which further yields

$$\begin{aligned}\dot{V} &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a(\widehat{y}_j - \widehat{y}_i)^T e_i - \frac{a}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} (\widehat{y}_i - \widehat{y}_j)^T (\widehat{y}_i - \widehat{y}_j) \\ &\leq a\sum_{i=1}^N \left\|e_i\right\|_2 \left\| \sum_{j\in\mathcal{N}_i} (\widehat{y}_j - \widehat{y}_i)\right\|_2 - \frac{a}{2}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|\widehat{y}_i - \widehat{y}_j\right\|_2^2,\end{aligned} \tag{12}$$

so if

$$\left\|e_i\right\|_2 \leq \frac{\sum_{j\in\mathcal{N}_i} \left\|\widehat{y}_i - \widehat{y}_j\right\|_2^2}{2\left\| \sum_{j\in\mathcal{N}_i} (\widehat{y}_j - \widehat{y}_i)\right\|_2}, \quad \forall i \tag{13}$$

then $\dot{V} \leq 0$, $\forall t \geq 0$. Note that the triggering condition (8) actually guarantees that (13) is satisfied. Furthermore, $\dot{V} \leq 0$ implies that $\lim_{t\to\infty} V$ exists and is finite, and since $V \geq 0$, one can further conclude that $\lim_{t\to\infty} \dot{V} = 0$. Under the triggering condition (8), we can get

$$0 = \lim_{t\to\infty} \dot{V} \leq -(0.5 - \delta_1)a \lim_{t\to\infty} \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \left\|\widehat{y}_i - \widehat{y}_j\right\|_2^2 \leq 0, \tag{14}$$

thus, under assumption **A1.**, we can conclude that

$$\lim_{t\to\infty} \widehat{y}_i = \lim_{t\to\infty} \widehat{y}_j, \quad \forall i, j. \tag{15}$$

Based on (13) and (15), we can further get

$$0 = \lim_{t\to\infty} e_i = \lim_{t\to\infty} \big(y_i - \widehat{y}_i\big), \quad \forall i. \tag{16}$$

In view of (15), (16) further yields

$$\lim_{t\to\infty} y_i = \lim_{t\to\infty} y_j, \quad \forall i, j \tag{17}$$

which completes the proof. ∎

V. Case Study: Event-driven Consensus Problem with a Passive Quantizer

In this section, we use the event-driven consensus problem with passive quantizers as a case study to illustrate the results obtained in the previous section. Since the data transmissions among the networked agents are event-based rather than synchronized, one could consider the control problem studied in this section as "quantized asynchronous consensus" problem.

The system considered consists of N agents, with $x_i \in \mathbb{R}$ denoting the state of agent H_i. Note that the results derived in this section are extendable to arbitrary dimensions by using Kronecker algebra. The agent's motion obeys a single integrator model

$$\dot{x}_i(t) = u_i(t), \quad y_i(t) = x_i(t) \tag{18}$$

with coordinate control law

$$u_i(t) = \sum_{j\in\mathcal{N}_i} a\big[q(\widehat{y}_j) - q(\widehat{y}_i)\big], \tag{19}$$

where $q(y_i(t_{k_i}))$ is the quantized value of agent $H_i{}'s$ last transmitted output measurement, $q(\widehat{y}_i) = q(y_i(t_{k_i}))$ for $t \in [t_{k_i}, t_{k_i+1}]$; $q(y_j(t_{k_j}))$ is the quantized value of agent $H_j{}'s$ last transmitted output measurement, and $q(\widehat{y}_j) = q(y_j(t_{k_j}))$ for $t \in [t_{k_j}, t_{k_j+1}]$. Before we present the main result of this section, we have the following lemma.

***Lemma** 2:* The cascade connection of an integrator and a passive memoryless function h as shown in Fig.1, is still lossless from u to $h(x)$.

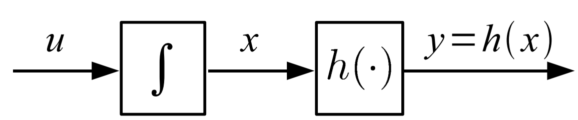

Fig. 1: cascade interconnection of an integrator with a passive memoryless function

Proof: Passivity of h guarantees that $\int_0^x h(\sigma)d\sigma \geq 0$ for all x. With $V(x) = \int_0^x h(\sigma)d\sigma$ as the storage function, we have $\dot{V} = h(x)\dot{x} = yu$. Hence the entire interconnection is lossless. Similar analysis can be found in [15]. ■

***Remark** 1:* Lemma 2 indicates that the cascade interconnection of an integrator with a passive memoryless quantizer can be studied as a lossless(passive) system with the quantized output as the new output of the cascade system. This result enables us to derive the triggering condition for the event-driven consensus problem with quantization.

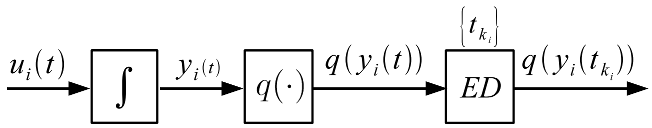

Fig. 2: cascade interconnection of an integrator with a passive memoryless quantizer

Assume that each agent is equipped with a passive memoryless quantizer $q(\cdot)$ and an event detector which is denoted by "ED" as shown in Fig.2. The event detector continuously (or with adequately fast sampling rate) monitors the output of the quantizer associated with the agent. Whenever it detects that the triggering condition of the agent is satisfied, a quantized output information $q(y_i(t_{k_i}))$ at that event time (t_{k_i}) will be transmitted to the agent's neighbors $\mathcal{Z}_i$. The theorem below provides a triggering condition to achieve average consensus among the networked agents.

***Theorem** 2:* Consider a network of N agents with each agent's dynamics described by (18)-(19), and each agent is equipped with a passive memoryless quantizer $q(\cdot)$ and an event detector as shown in Fig.2. Assume there is no data transmission delay in the network. Under assumption **A1.**, if each agent H_i transmits its current output information to its neighboring agents $\mathcal{Z}_i$ whenever the following triggering condition is satisfied

$$\|\varepsilon_i(t)\|_2 > \frac{\delta_2 \sum_{j\in\mathcal{N}_i} \big\|q(\widehat{y}_j) - q(\widehat{y}_i)\big\|_2^2}{\big\| \sum_{j\in\mathcal{N}_i} \big[q(\widehat{y}_j) - q(\widehat{y}_i)\big]\big\|_2}, \quad \forall t \geq 0, \qquad (20)$$

for some $\delta_2 \in (0, 0.5)$, where $\varepsilon_i(t) = q(y_i(t)) - q(\widehat{y}_i)$, then the quantized outputs of those networked agents achieve output synchronization asymptotically, i.e.,

$$\lim_{t\to\infty} q(y_i(t)) = \lim_{t\to\infty} q(y_j(t)), \quad \forall i, j. \qquad (21)$$

Proof: Based on Lemma 2, choose $V_i(x_i) = \int_0^{x_i} q(\sigma)d\sigma$ as the storage function for each agent, then we have $\dot{V}_i = u_i(t)q(y_i(t))$, $\forall t \geq 0$. Consider a storage function for the multi-agent system given by $V = \sum_{i=1}^N V_i$, then we have $\dot{V} = \sum_{i=1}^N u_i(t)q(y_i(t))$, and thus

$$\begin{aligned}\dot{V} &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\big[q(\widehat{y}_j) - q(\widehat{y}_i)\big]\big[\varepsilon_i(t) + q(\widehat{y}_i)\big] \\ &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\big[q(\widehat{y}_j) - q(\widehat{y}_i)\big]\varepsilon_i(t) + \\ &+ \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} aq(\widehat{y}_i)q(\widehat{y}_j) - \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} aq(\widehat{y}_i)^2,\end{aligned} \qquad (22)$$

as the underlying information exchange graph is balanced, we have

$$\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} aq(\widehat{y}_i)^2 = 0.5\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \big[aq(\widehat{y}_i)^2 + aq(\widehat{y}_j)^2\big],$$

and therefore it follows that

$$\begin{aligned}\dot{V} &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\big[q(\widehat{y}_j) - q(\widehat{y}_i)\big]\varepsilon_i(t) \\ &- \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} 0.5a\big\|q(\widehat{y}_j) - q(\widehat{y}_i)\big\|_2^2 \\ &\leq \sum_{i=1}^N \big\|\varepsilon_i(t)\big\|_2 \big\| \sum_{j\in\mathcal{N}_i} a\big[q(\widehat{y}_j) - q(\widehat{y}_i)\big]\big\|_2 \\ &- \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} 0.5a\big\|q(\widehat{y}_j) - q(\widehat{y}_i)\big\|_2^2,\end{aligned} \qquad (23)$$

so if we can guarantee that

$$\|\varepsilon_i(t)\|_2 \leq \frac{\sum_{j\in\mathcal{N}_i} 0.5\big\|q(\widehat{y}_j) - q(\widehat{y}_i)\big\|_2^2}{\big\| \sum_{j\in\mathcal{N}_i} \big[q(\widehat{y}_j) - q(\widehat{y}_i)\big]\big\|_2}, \quad \forall i \qquad (24)$$

then we will have $\dot{V} \leq 0$, which means $\lim_{t\to\infty} V$ exists and is finite. With $V \geq 0$, this further implies that $\lim_{t\to\infty} \dot{V} = 0$. Note that the triggering condition (20) will guarantee that (24) is satisfied. Under the triggering condition, we can get

$$0 = \lim_{t\to\infty} \dot{V} \leq -(0.5 - \delta_2)a \lim_{t\to\infty} \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \big\|q(\widehat{y}_i) - q(\widehat{y}_j)\big\|_2^2 \leq 0,$$

thus, under assumption **A1.**, we can conclude that

$$\lim_{t\to\infty} q(\widehat{y}_i) = \lim_{t\to\infty} q(\widehat{y}_j), \quad \forall i, j. \qquad (25)$$

Based on (24) and (25), we can further get

$$0 = \lim_{t\to\infty} \varepsilon_i = \lim_{t\to\infty} \big(q(y_i) - q(\widehat{y}_i)\big), \quad \forall i. \qquad (26)$$

In view of (25), (26) further yields

$$\lim_{t\to\infty} q(y_i) = \lim_{t\to\infty} q(y_j), \quad \forall j \in \mathcal{N}_i, \qquad (27)$$

which completes the proof. ■

***Remark** 2:* Since

$$\begin{aligned}\frac{1}{N}\sum_{i=1}^N \dot{x}_i(t) = \frac{1}{N}\sum_{i=1}^N u_i(t) &= \frac{1}{N}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\big[q(\widehat{y}_j) - q(\widehat{y}_i)\big] \\ &= \frac{1}{N}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\big[q(x_j(t)) - q(x_i(t))\big] \\ &- \frac{1}{N}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\big[\varepsilon_j(t) - \varepsilon_i(t)\big],\end{aligned} \qquad (28)$$

with the underlying communication graph being balanced, one can conclude that $\frac{1}{N}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\big[q(x_j(t)) - q(x_i(t))\big] = 0$ and $\frac{1}{N}\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} a\big[\varepsilon_j(t) - \varepsilon_i(t)\big] = 0$, thus $\frac{1}{N}\sum_{i=1}^N \dot{x}_i(t) = 0$, which further yields $\frac{1}{N}\sum_{i=1}^N x_i(t) = \frac{1}{N}\sum_{i=1}^N x_i(0)$. In view of (21), we can further conclude that as long as the quantizer is designed with bounded quantization error, the state of each agent will converge to a bounded region around their initial average asymptotically.

Example 1. We consider the event-driven consensus problem with signal quantization as discussed above, the underlying information exchange graph is given by

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \tag{29}$$

which satisfies assumption **A1**. Assume that each agent is equipped with a uniform mid-tread quantizer with quantization level 0.5 (one can verify that a uniform mid-tread quantizer is passive since $y_i(t)q(y_i(t)) \geq 0$). Under the triggering condition (20), the simulation results are shown in Fig.3-Fig.5. In Fig.3, the x-axis shows the time instants of events while the y-axis shows the length of inter-event time of each agent. Fig.4 shows the evolution of quantized output of each agent, Fig.5 shows the evolution of the state of each agent.

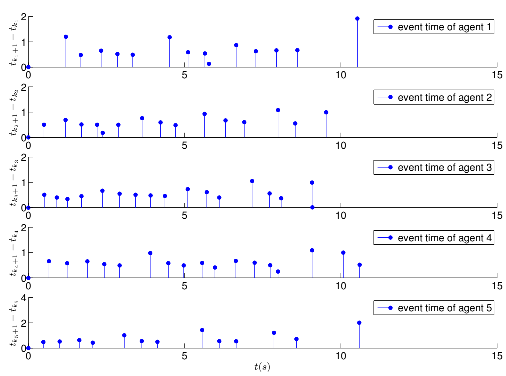

Fig. 3: Example 1-event time

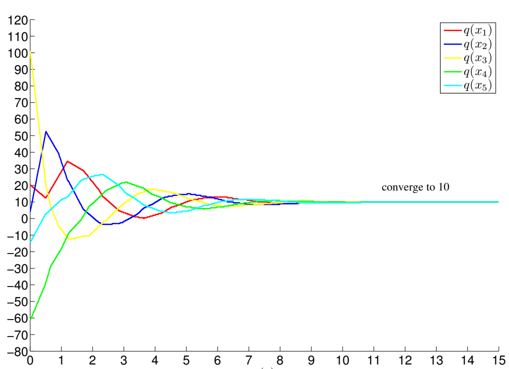

Fig. 4: Example 1-evolution of quantized output of each agent

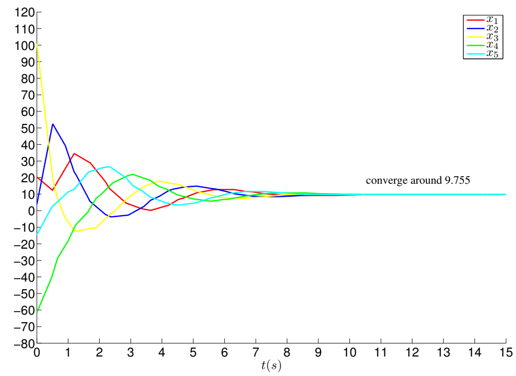

Fig. 5: Example 1-evolution of the state of each agent

With initial state $x_1(0) = 20, x_2(0) = 4, x_3(0) = 100, x_4(0) = -60, x_5(0) = -15$, we have $\frac{1}{N}\sum_{i=1}^{N} x_i(0) = 9.8$. And one can see from Fig.4-Fig.5 that while the quantized output of each agent converges to 10, the state of each agent finally converges to a value around 9.8.

VI. Triggering Condition for Quantized Output Synchronization

In the previous sections, we derived a triggering condition for output synchronization of passive systems. The event-driven consensus problems with passive quantizers have been singled out as a case study since the cascade interconnection of an integrator with a passive memoryless quantizer can still be analyzed as a passive system. Unfortunately, the cascade interconnection of two passive systems may not be passive in general[16]. Thus, the triggering condition derived in section IV may only apply to a restricted class of systems. In this section, a more general case is considered. We derive a triggering condition to achieve practical output synchronization when quantized output measurements are exchanged among networked agents.

Assume that the coordinate control input to agent H_i is given by

$$u_i(t) = \sum_{j\in\mathcal{N}_i} a\big[q(\widehat{y}_j) - q(\widehat{y}_i)\big], \tag{30}$$

where $q(y_i(t_{k_i}))$ is the quantized value of agent $H_i{}^{'}s$ last transmitted output measurement, and $q(\widehat{y}_i) = q(y_i(t_{k_i}))$ for $t \in [t_{k_i}, t_{k_i+1}]$, for $i = 1, 2, \ldots, N$. The quantization function $q(\cdot)$ is designed with bounded quantization error and it acts component-wise on the input vector (note that $q(\cdot)$ does not have to be a passive memoryless function). Let $e_i(t) = y_i(t) - \widehat{y}_i$ denote the output novelty error with respect to the last transmitted output information, let $\varepsilon_i = \widehat{y}_i - q(\widehat{y}_i)$ denote the quantization error with respect to the last transmitted output measurement, let $\widetilde{e}_i(t) = y_i(t) - q(\widehat{y}_i)$ denote the output novelty error between the current output measurement and the latest transmitted quantized output measurement. One can verify that $\widetilde{e}_i(t) = y_i(t) - \widehat{y}_i + \varepsilon_i$. With event-driven communication and quantized output measurement exchanged among networked agents, we have the following theorem.

***Theorem** 3:* Consider a network of N passive agents with coordinate control law (30). Under assumption **A1.**, if each agent H_i transmits its current output information to its neighbors $\mathcal{Z}_i$ whenever the following triggering condition is satisfied

$$\|e_i(t)\|_2 > \delta_3\big(\frac{1-\kappa}{2} - \frac{1}{2\beta}\big)\frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}\big\|q(\widehat{y}_j) - q(\widehat{y}_i)\big\|_2, \tag{31}$$

$\forall t \geq 0$, where $\delta_3 \in (0,1)$, $0 < \kappa < 1$ and $1 < \frac{1}{1-\kappa} < \beta$, then the output synchronization error of the studied multi-agent system is bounded by the quantization errors of the agents' last transmitted output measurements in the network.

Proof: Due to the limit of the length, the proof is eliminated from the current paper. Interested readers are referred to the complete online version [20] for more details. ∎

VII. Conclusion

In this paper, we studied the quantized output synchronization problem of networked passive systems with event-driven communication. We first derived a triggering condition for output synchronization of networked passive systems. The event-driven consensus problem with passive quantizers was examined as a case study, since the cascade interconnection of an integrator with a passive memoryless quantizer is still passive. We then considered the more general case when quantized output measurements are exchanged in the network, and we derived a triggering condition for practical output synchronization. We showed that under the derived triggering condition, the output synchronization error of the networked passive systems is bounded by the quantization errors of the signals transmitted in the network.

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