Decentralized Model-Based Event-Triggered Control of Networked Systems

Technical Report of the ISIS Group

at the University of Notre Dame

ISIS-2012-002

February, 2012

Eloy Garcia and Panos J. Antsaklis

Department of Electrical Engineering

University of Notre Dame

Notre Dame, IN 46556

Email: {egarcia7, antsaklis.1}@nd.edu

Interdisciplinary Studies in Intelligent Systems-Technical Report

Decentralized Model-Based Event-Triggered Control of Networked Systems

Eloy Garcia and Panos J. Antsaklis, *Fellow, IEEE*

Abstract

This paper presents a model-based event-triggered (MB-ET) control framework for stabilization of networked systems. The controller and the events are designed in a decentralized manner, based only on local information. The knowledge of a priori model of the interconnected subsystems or agents is used at every controller node to generate estimates of the state of distant subsystems in order to reduce the frequency at which measurements need to be broadcasted. This framework allows for considerable reduction of bandwidth since every agent broadcasts its local information to other agents only when it is necessary, based on the difference between real and estimated variables. The use of models of the systems in the controller nodes not only generalizes the Zero-Order-Hold (ZOH) implementation in traditional event-triggered control schemes but it also provides stability thresholds that are robust to model uncertainties.

I. INTRODUCTION

In recent years there has been a strong interest in the analysis, development, and controller synthesis for networks of interconnected systems. The importance and challenges of networks comprised from several to many subsystems or agents was recognized early by the research community [1]. Examples of such systems can be found in a wide variety of applications such as: power networks, multi-agent robotic systems and coordination of autonomous vehicles, large chemical processes comprised of several subsystems interacting one with each other, and also in areas that consider economic and/or social systems. In addition, the availability of cheap, fast, embedded sensor and controller subsystems that are capable to communicate via a shared digital network allow for the different subsystems to share their local information with other (possibly the rest of) subsystems so it can be used to achieve a common objective in a more efficient way [2]. However, digital communication networks have limited bandwidth and not all agents can communicate at a given time instant. It becomes necessary to be able to schedule the broadcast of information by the different nodes in such a way that bandwidth constraints are not violated.

Previous research in Networked Control Systems (NCS) has addressed the problem of minimum bit rate stabilization [3], [4] using periodic communication. Important extensions have considered the stochastic properties of some networks to include unknown but bounded time-varying sampling intervals [5] or purposely giving random access to several nodes as long as access to the network is obtained before a Maximum Allowable Transfer Time (MATI) [6]. Many authors have demonstrated that important reduction of bandwidth utilization without significant loss of performance can be achieved using one of the following approaches: Model-Based Networked Control Systems (MB-NCS) and event-triggered control. The MB-NCS approach that was introduced by Montestruque and Antsaklis [7], [8] has been extended to consider networks of coupled systems [9] using periodic communication. Event-triggered control has been used for stabilization of dynamical systems while reducing the number of measurements that the sensor needs to send to the controller. The events that are designed based on state errors have been used extensively [10]-[13]. The same approach has been extended to consider networked interconnected systems [14]-[17]. A common characteristic in the previous work on event-triggered control is the use of a Zero-Order-Hold (ZOH) model in the controller node and the assumption that the model being used are the same as the plants they represent, i.e. no model uncertainties are considered.

In [18] we proposed a combined model-based event-triggered (MB-ET) control framework that considered model uncertainties and events based on state errors. The work in [19] presents a similar idea but model uncertainties are not considered, instead, disturbance rejection is the focus assuming the model matches the system parameters. Here, we extend the approach that we presented in [18] to consider multiple interconnected subsystems. Similar work was presented in [20]-[21] where the same framework is used but the design of events is quite different from the ones derived in the present paper. Our approach offers a considerable reduction of network communication compared to [20]-[21] especially when the number of agents grows. We achieve this reduction by only broadcasting an agent's state to update distant nodes instead of making an agent request updates and therefore making all other agents to send information, all at the same time as it is shown in [20]-[21]. Moreover, in those papers the design of the control law is not practical and sufficiently robust, and may lead to undesirable behavior. Although the controller uses feedback from the model states, the controller parameters are found using the real system parameters which are unknown. In this paper we consider a robust by construction controller, by designing the controller parameters based on the model (available information) and analyzing the worst case behavior in the presence of uncertainties.

The paper is organized as follows: in section II we formulate the problem and provide more details of our decentralized MB-ET approach. Section III provides a solution in which every node is able to trigger a message broadcasting based on local measurements but the stability conditions are based on the dynamics of the overall system-model. In section IV we present a fully decentralized solution in which the controller and the event thresholds are designed based only on local models and uncertainty bounds. Illustrative examples are presented in section V and conclusions are offered in section VI.

II. Problem Description

The MB-NCS configuration [7]-[8] makes use of an explicit model of the plant which is added to the controller node to compute the control input based on the state of the model rather than on the plant state. The plant and model can be described respectively by:

$$\dot{x} = Ax + Bu \tag{1}$$

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u \tag{2}$$

where $x, \hat{x} \in \mathbb{R}^n$, $u = K\hat{x}$, and the matrices $\hat{A}, \hat{B}$ represent the available model of the system matrices A,B. The plant may be unstable i.e. not all eigenvalues of A need to have negative real parts. Here, we extend a similar approach that we presented in [18] in order to consider multiple interconnected subsystems as shown in Fig. 1 where the measurements can be transmitted over a network and the dark lines represent the physical interconnections or coupling of the subsystems. In the present paper we address the problem of robust stabilization of networks of coupled unstable and uncertain systems by also considering a significant reduction of network communication among subsystems. In addition we use an event-triggered strategy in order to determine the time instants at which each subsystem needs to send its state measurement to the other agents in the network. We use the estimate of the state given by the model of the plant to compare it with the actual state, and then the sensor transmits the state of the plant if the error is above some predefined tolerance.

The MB-ET framework allows a system to work in open-loop mode for extended periods of time by computing the control input based on the state estimate provided by the local model and updating the model only when it is necessary according to the local state error. By increasing the update interval (reducing communication rate) we release the network for other uses. In case we have several control systems implemented over the network, by reducing network traffic, we are also reducing the size of time delays and reducing the probability of packets being lost. In addition, the conditions to select a stabilizing threshold are given in terms of the nominal model parameters and bounds on the model uncertainties, assuming the dimension of the system is known. In this paper we consider a network of N interconnected agents or subsystems. Each subsystem can be described by a state-space representation as follows:

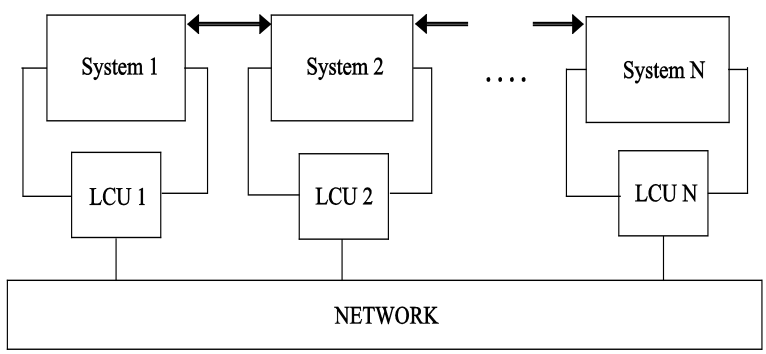

Fig. 1. Model-based network interconnected systems.

$$\dot{x}_i = A_i x_i + B_i u_i + \sum_{j=1, j\neq i}^{N} A_{ij} x_j \tag{3}$$

for each $i \in \mathrm{N}$, N denotes the set $\{1, 2, \ldots N\}$ of N integers where $x_i \in \mathbb{R}^{ni}$ represents the state of the *i-th* subsystem, $u_i \in \mathbb{R}^{mi}$ represents the local input for subsystem *i*.

$A_i \in \mathbb{R}^{ni \times ni}, A_{ij} \in \mathbb{R}^{ni \times nj}, B_i \in \mathbb{R}^{ni \times mi}$ represent respectively the state, coupling, and input matrices for the *i-th* subsystem.

In this framework each Local Control Unit (LCU) contains copies of the models of all subsystems including the model corresponding to its own local dynamics in order to generate estimates of the states of all subsystems in the network. The results in this paper can be easily applied to the case in which the dynamics of a given subsystem are affected only by a small neighborhood of subsystems. In this case each LCU does not need to implement models of all subsystems. Each LCU only needs to implement models (and receive updates from the corresponding systems) that are needed to estimate the variables that affect its own dynamics. Similarly, agent *i* needs to send updates only to the agents that need to implement a model of agent *i*.

The model of each subsystem is represented by:

$$\dot{\hat{x}}_i = \hat{A}_i \hat{x}_i + \hat{B}_i \hat{u}_i + \sum_{j=1, j \neq i}^{N} \hat{A}_{ij} \hat{x}_j \tag{4}$$

where $\hat{x}_i \in \mathbb{R}^{ni}$ represents the state of the *i-th* model, $\hat{u}_i \in \mathbb{R}^{mi}$ represents the local input for model *i*. The matrices $\hat{A}_i \in \mathbb{R}^{ni \times ni}, \hat{A}_{ij} \in \mathbb{R}^{ni \times nj}, \hat{B}_i \in \mathbb{R}^{ni \times mi}$ represent the nominal parameters of the dynamics of the *i-th* subsystem. Note that the subsystems could have different dynamics and different dimensions, the dimensions *mi* and *ni* could be all different in general. Note also that each LCU*i* has access to its local state x_i at all times which is used to compute the local subsystem control input:

$$u_i = K_i x_i + \sum_{j=1, j \neq i}^{N} K_{ij} \hat{x}_j \tag{5}$$

and the local model-state error which is defined as:

$$e_i = \hat{x}_i - x_i \tag{6}$$

where K_i and K_{ij} are the stabilizing control gains to be designed.

By measuring its local error, each LCU*i* is able to decide the appropriate times at which it should broadcast the current measured state x_i to all other units so all LCU's can update the state of their local models $\hat{x}_i$ corresponding to x_i. At the same time the LCU that broadcasted its state needs to update its own local model corresponding to x_i, and the error e_i is set to zero. We assume that the communication delay is negligible and the initial conditions of the plant are nonzero but finite.

This strategy represents a considerable saving on bandwidth compared to similar work in [20]-[21]; there the same problem is considered but the proposed solution requires the opposite updating strategy as the one we present in this paper; all agents need to communicate or send their information to agent *i* when the error e_i grows large, that is, agent *i* needs to send a request for updates to all other agents, then all agents need to respond and send their current measurements to agent *i* all of them at the same time instant, which may produce packet collisions and loss of information. In addition, since the update request is based on the local error e_i, we could be requesting all other agents to send their information to agent *i* when it is not necessary, i.e. their local errors are small and the growth of e_i may be due to large local parameter uncertainty or due to only one or very few errors from other agents.

The strategy proposed here avoids the unnecessary increase in communication by simply making agent *i* to broadcast its state according to its local error. If all agents including agent *i* have the same estimate $\hat{x}_i$ of x_i then when the error e_i is large by an appropriate measure we know it is necessary for all agents to receive the real state and update the state of the model $\hat{x}_i$. Note that when the LCUs update the state of the model corresponding to x_i then the error e_i is set to zero and therefore it is less than the positive threshold that is used to determine the update instants.

Using this framework we can see from (5) that the input u_i for the agent *i* is not an appropriate input for the corresponding model. The input for the local subsystem is a function of the real state which is not always available to the other agents. In order to make sure that every agent in the network computes the same estimate of the states of all agents we need to use the same parameters for the model equations (4) and we also need to implement control inputs for the models that can be executed at every LCU. We define the model inputs

$$\hat{u}_i = K_i\hat{x}_i + \sum_{j=1, j\neq i}^{N} K_{ij}\hat{x}_j. \tag{7}$$

These control inputs are applied to all models in all LCUs whereas (5) is applied to each local subsystem. It is clear now that although LCU*i* computes an estimate $\hat{x}_i$ of x_i, this estimated state is not used to control subsystem i since we have the real state available. At LCUi we use $\hat{x}_i$ as input for the models ensuring that the same model equations with the same model control inputs are implemented at all LCUs.

III. CENTRALIZED STABILIZING MODEL-BASED EVENT-TRIGGERED CONTROLLER

The first approach to compute the stabilizing controllers and thresholds is presented in this section and it is based on the dynamics of the overall system and model. The time instants at which each agent needs to send its information to the network can be computed locally by each LCU.

Let us introduce the augmented vectors:

$$\begin{aligned} x &= [x_1^T \ x_2^T \ \ldots x_n^T]^T \\ \hat{x} &= [\hat{x}_1^T \ \hat{x}_2^T \ \ldots \hat{x}_n^T]^T \\ e &= [e_1^T \ e_2^T \ \ldots e_n^T]^T. \end{aligned} \tag{8}$$

The dynamics of the overall system and model can be represented by:

$$\dot{x} = Ax + Bu \tag{9}$$

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}\hat{u}. \tag{10}$$

Notice that contrary to (1)-(2) the model control input is not the same as the real system control input. The form of the matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$, where $n = \sum_{i=1}^{N} ni$ and $m = \sum_{i=1}^{N} mi$, are as follows:

$$A = \begin{bmatrix} A_1 & A_{12} & \ldots & A_{1n} \\ A_{21} & A_2 & \ldots & A_{2n} \\ \vdots & & \ddots & \vdots \\ A_{n1} & A_{n2} & \ldots & A_n \end{bmatrix} \quad B = \begin{bmatrix} B_1 & 0 & \cdots & 0 \\ 0 & B_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \ldots & B_n \end{bmatrix} \tag{11}$$

and similarly for $\hat{A}$ and $\hat{B}$. We describe the dynamics of the overall system in the next proposition.

Proposition 1. Assume $(\hat{A}, \hat{B})$ is stabilizable. The dynamics of the overall system can be represented by:

$$\dot{x} = (A + BK)x + BK_{off}e \tag{12}$$

where $K_{off} = K - K_{diag}$. $K_{diag} = diag(K_i)$ is a matrix containing the controller gains K_i as main diagonal sub matrices. The controller K is a stabilizing controller for the overall model dynamics, i.e. $\hat{A} + \hat{B}K$ is Hurwitz.

Proof. We rewrite (9) in the next form

$$\dot{x} = Ax + Bu = Ax + B(K_{diag}x + (K - K_{diag})\hat{x})$$

where u is the augmented vector containing each agent local subsystem control inputs

$$u = [u_1^T \; u_2^T \; \ldots u_n^T]^T. \tag{13}$$

From (8) we have that $e = \hat{x} - x$ and we can write

$$\dot{x} = Ax + BK(x + e) - BK_{diag}e = (A + BK)x + BK_{off}e \blacksquare$$

In order to asymptotically stabilize the states of all agents from their finite initial conditions we implement a similar strategy as in [18]. The main idea is to reduce the threshold value as we approach the equilibrium point of the system. This can be achieved by comparing the norm of the error to a function of the norm of the state. Previous work on event-triggered control dealt with systems controlled by static gains that generate piecewise constant inputs due to the fact that the update is held constant in the controller. The main difference in this section is that we use Model-Based controllers i.e. models of the subsystems and static gains; the models provide estimates of the states between updates and the model/gain controllers provide an input for the plant that does not remain constant between measurement updates. The norm used in the next results is the Euclidean norm.

Consider again the plant and model described by (9) and (10) and by using the control input $u = K_{diag}x + K_{off}\hat{x}$ we obtain expression (12) for the plant with K rendering $\hat{A} + \hat{B}K$ Hurwitz, i.e. the model is globally asymptotically stable. Then we can always find a P which is symmetric positive definite and is the solution of the closed loop model Lyapunov function:

$$(\hat{A} + \hat{B}K)^T P + P(\hat{A} + \hat{B}K) = -Q \tag{14}$$

where Q is a symmetric positive definite matrix.

Let us first analyze the case when $\hat{B} = B$ for simplicity, and define the uncertainty $\tilde{A} = A - \hat{A}$. Also assume that the next bound on the uncertainty $\left|\tilde{A}^T P + P\tilde{A}\right| \le \Delta < q$ holds where $q = \underline{\sigma}(Q)$, the smallest singular value of Q in the Lyapunov equation (14) and Δ is a bound on the norm of the uncertainty. This bound can be seen as a measure of how close A and $\hat{A}$ should be.

The next theorem provides conditions on the error and its threshold value so the networked system is asymptotic stable. The error threshold is defined as a function of the norm of the state and Δ. Additionally, the error events can be computed locally, that is, once the thresholds have been designed, each agent can decide when to broadcast its current measurement to the rest of the agents based only on the measurements of its own state and its own error.

Theorem 2. System (9) with $u = K_{diag}x + K_{off}\hat{x}$ and feedback based on error events generated when the relation:

$$\left|e_i\right| > \alpha \left|x_i\right| \tag{15}$$

is first satisfied, is globally asymptotically stable, where $\alpha = \sigma(q - \Delta)/b$, $b = 2\left|P\hat{B}K_{off}\right|$ and $0 < \sigma < 1$.

Proof. In order to prove this theorem we will set a bound on the derivative of $V = x^T Px$ along the trajectories of the system (12) which is equal to (9) when the input $u = K_{diag}x + K_{off}\hat{x}$ has already been substituted and expressed in terms of the state error, then we can easily show that this bound can be appropriately tuned by the choice of the threshold on the error.

$$\begin{aligned}\dot{V} &= x^T[(A + BK)^T P + P(A + BK)]x + e^T K_{off}{}^T B^T Px + x^T PBK_{off}e \\ &= x^T[(\hat{A} + \tilde{A} + \hat{B}K)^T P + P(\hat{A} + \tilde{A} + \hat{B}K)]x + 2x^T P\hat{B}K_{off}e \\ &= -x^T Qx + x^T(\tilde{A}^T P + P\tilde{A})x + 2x^T P\hat{B}K_{off}e.\end{aligned}$$

We have just expressed $\dot{V}$ in terms of the model parameters and the uncertainty of the state matrix A. We now proceed to evaluate the contributions of each, the model, the uncertainty, and the error.

$$\begin{aligned}\dot{V} &\le -q\left|x\right|^2 + \left|\tilde{A}^T P + P\tilde{A}\right|\left|x\right|^2 + 2\left|P\hat{B}K_{off}\right|\left|e\right|\left|x\right| \\ &\le (-q + \Delta)\left|x\right|^2 + b\left|e\right|\left|x\right|.\end{aligned} \tag{16}$$

Now, by sending updates according to (15), which sets the error equal to zero at every update time, we can see first that

$$|e_i|^2 \le \alpha^2 |x_i|^2$$

and

$$|e|^2 = \sum_{i=1}^{N} |e_i|^2 \le \sum_{i=1}^{N} \alpha^2 |x_i|^2 = \alpha^2 |x|^2 .$$

Since $\alpha > 0$ and the norms are always nonnegative then we have that

$$|e| \le \alpha |x|. \tag{17}$$

We can use (17) in (16) to obtain

$$\dot{V} \le (\sigma - 1)(q - \Delta)|x|^2 . \tag{18}$$

Then V is guaranteed to decrease for any σ such $0 < \sigma < 1$ and updating the elements of the state of the models in all LCUs according to the events in (15). ■

The extension to consider the case of $\hat{A} \ne A$ and $\hat{B} \ne B$ is straightforward by assuming that the next bounds on the uncertainty matrices hold:

$$\left|(\tilde{A} + \tilde{B}K)^T P + P(\tilde{A} + \tilde{B}K)\right| \le \Delta < q \tag{19}$$

$$\left|\tilde{B}\right| \le \beta \tag{20}$$

where $\tilde{B} = B - \hat{B}$. In order to obtain (18) the local errors are set to satisfy (triggering an update otherwise):

$$|e_i| \le \frac{\sigma(q - \Delta)}{\bar{b}} |x_i| \tag{21}$$

where $\bar{b} = b + 2\beta |K||P|$.

By following the approach described above each LCU is capable of determining the time instants at which it should send its current measurement to the network. An important disadvantage in Theorem 2 is that the controller is designed based on the model dynamics of the overall system and the threshold is calculated as a function of the bounds on the uncertainty of the augmented system as well. In the next section we offer a complete decentralized solution, that is, not only the LCUs update their state based on local information but the local controllers and the local thresholds, which are not necessarily the same for every agent as in Theorem 2, can also be designed based on local model dynamics and uncertainty bounds.

IV. Decentralized Stabilizing Model-Based Event-Triggered Controller

The decentralized method described in this section extends the results provided by [22]. In [22] only a ZOH model is used, that is, the control input for each agent remains constant between updates. With respect to previous work in event-triggered control, the implementation of this strategy using MB-NCS accounts for the unavoidable existence of model uncertainties in the stability analysis.

Consider the network of coupled subsystems represented by (3) with models (4). We assume that every pair $(\hat{A}_i, \hat{B}_i)$ is stabilizable. We design controllers K_i that render the matrices $\hat{A}_i + \hat{B}_i K_i$ Hurwitz. Then for every agent *i*, $i \in \mathrm{N}$, there exists a symmetric and positive definite P_i which is the solution of the closed loop local decoupled model

$$(\hat{A}_i + \hat{B}_i K_i)^T P_i + P_i(\hat{A}_i + \hat{B}_i K_i) = -Q_i \tag{22}$$

where Q_i is a symmetric and positive definite matrix.

Theorem 3. Let (5) be the control input for each agent in the networked system (3). Assume that the following bounds are satisfied: $\left|(\tilde{A}_i + \tilde{B}_i K_i)^T P_i + P_i(\tilde{A}_i + \tilde{B}_i K_i)\right| \le \Delta_i < q_i$ and $\sum_{j=1, j\neq i}^{N} \left|P_j \tilde{A}_{ji}\right|^2 \le Wi \le \frac{f_i^2}{8(N-1)}$, where $q_i = \underline{\sigma}(Q_i)$, $f_i = q_i - \Delta_i$ and $\tilde{A}_{ji} = A_{ji} - B_j K_{ji}$. Then the networked system (3) is globally asymptotically stable when the local events are triggered by

$$|e_i|^2 > \frac{\chi_i}{\beta_i}|x_i|^2 \tag{23}$$

where $\chi_i = f_i - 2(N-1)\delta_i - \frac{W_i}{\delta_i}$, $\beta_i = \sum_{j=1, j\neq i}^{N} \frac{\left|P_j B_j K_{ji}\right|^2}{\delta_i}$, and δ_i is such that

$$\begin{cases} \delta_{i1} < \delta_i < \delta_{i2} & if \quad \delta_{i1} > 0 \\ 0 < \delta_i < \delta_{i2} & if \quad \delta_{i1} \le 0 \end{cases} \tag{24}$$

with

$$\delta_{i1} = \frac{f_i}{N-1}\left(\frac{1}{4} - \sqrt{\frac{1}{16} - \frac{(N-1)W_i}{2f_i^2}}\right) \tag{25}$$

$$\delta_{i2} = \frac{f_i}{N-1}\left(\frac{1}{4} + \sqrt{\frac{1}{16} - \frac{(N-1)W_i}{2f_i^2}}\right). \tag{26}$$

Proof. We consider the candidate Lyapunov function

$$V(x) = \sum_{i=1}^{N} V_i(x_i) \tag{27}$$

and use the next proposition (see appendix for proof).

Proposition 4. The derivative of $V_i = x_i^T P_i x_i$ along the trajectories of subsystem i in (3) with control input (5) satisfies the next inequality.

$$\begin{aligned} \dot{V}_i \le & -(f_i - 2(N-1)\delta_i)\left|x_i\right|^2 \\ & + \sum_{j=1, j\neq i}^{N} \frac{\left|P_i\tilde{A}_{ij}\right|^2}{\delta_i}\left|x_j\right|^2 + \sum_{j=1, j\neq i}^{N} \frac{\left|P_iB_iK_{ij}\right|^2}{\delta_i}\left|e_j\right|^2 . \bullet \end{aligned} \tag{28}$$

Now, taking the derivative of the Lyapunov function V along the trajectories of the state $x = [x_1^T \; x_2^T \; \ldots \; x_n^T]^T$ results in the next expression

$$\begin{aligned} \dot{V}(x) &= \sum_{i=1}^{N} \dot{V}_i(x_i) \\ &\le \sum_{i=1}^{N} -(f_i - 2(N-1)\delta_i)\left|x_i\right|^2 \\ &+ \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \frac{\left|P_i\tilde{A}_{ij}\right|^2}{\delta_i}\left|x_j\right|^2 + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} \frac{\left|P_iB_iK_{ij}\right|^2}{\delta_i}\left|e_j\right|^2 . \end{aligned}$$

We consider the case where all subsystems can receive measurement updates from the rest of the agents in the network, although this is not a necessary condition for the validity of the results in this theorem. It suffices for each agent i, $i \in \mathrm{N}$ to establish a bidirectional communication to those agents for which exchange of information is needed, that is, those agents that need to estimate the state x_i in any of their models and the agents for which agent i needs to estimate their state to use in any of their models. Then we can use the symmetry property of this type of interconnection to obtain

$$
\begin{aligned}
\dot{V}(x) &\le \sum_{i=1}^{N} -(f_i - 2(N-1)\delta_i)\left|x_i\right|^2 \\
&+ \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \frac{\left|P_j\tilde{A}_{ji}\right|^2}{\delta_i}\left|x_i\right|^2 + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \frac{\left|P_jB_jK_{ji}\right|^2}{\delta_i}\left|e_i\right|^2 \\
&\le -\left(\sum_{i=1}^{N} \chi_i\left|x_i\right|^2 - \sum_{i=1}^{N} \beta_i\left|e_i\right|^2\right).
\end{aligned} \tag{29}
$$

It is clear that coefficients β_i are positive and, in order for condition (23) to be a valid threshold we need the χ_i coefficients to be positive as well, which requires to solve the following inequalities for the real parameter δ_i

$$
\begin{gathered}
\chi_i = f_i - 2(N-1)\delta_i - \frac{W_i}{\delta_i} > 0 \\
\delta_i > 0
\end{gathered} \tag{30}
$$

It can be verified that the solution for the above inequalities is given by (24) with $\delta_{i1,i2}$ as in (25)-(26), moreover the solution is a real number by the assumption on the bounds W_i. Since we showed that $\chi_i, \beta_i > 0$ then the Lyapunov function is guaranteed to decrease by updating the models in all LCUs corresponding to the state x_i according to the threshold (23). ■

The parameters Δ_i represent given bounds on the norm of the uncertainty for every agent and they can be seen as a measure of how close the model and system dynamics are. The bound W_i represents a measure of how close we are able to cancel the effects of other subsystems on system *i* using the control gains that are designed based on the nominal models.

V. EXAMPLES

Example 1. We consider a network of *N*=10 unstable subsystems represented as in equation (3) all with different dynamics. The dimensions of the systems vary from 1 to 3 as well. The models for all different parameters represent an uncertainty as follows: 12% alteration in the A_i matrices, 10% in A_{ij}, and 6% in B_i.

The unknown dynamics of the subsystems are given by:

$$A_1 = 0.4, \quad B_1 = 1$$
$$A_2 = 0.5, \quad B_2 = 1$$
$$A_3 = 0.2, \quad B_3 = 1$$
$$A_4 = 1, \quad B_4 = 1$$
$$A_5 = \begin{bmatrix} 0.21 & -0.4 \\ 0.3 & -0.7 \end{bmatrix}, \quad B_5 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$A_6 = \begin{bmatrix} 0.3 & 2 \\ 0.6 & -1.8 \end{bmatrix}, \quad B_6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$A_7 = \begin{bmatrix} 0.05 & 0.003 \\ 0.0023 & -0.7 \end{bmatrix}, \quad B_7 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$A_8 = \begin{bmatrix} 0.11 & -0.41 & 0 \\ 0 & 0.3 & -0.7 \\ 0 & -0.3 & -0.5 \end{bmatrix}, \quad B_8 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$A_9 = \begin{bmatrix} 0.3 & 2 & -0.07 \\ 0 & 0.09 & -1 \\ 0.1 & 0 & 0.2 \end{bmatrix}, \quad B_9 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$A_{10} = \begin{bmatrix} 0.05 & 0.003 & 0.5 \\ 0 & 0.0023 & -0.7 \\ -0.3 & 0 & -0.5 \end{bmatrix}, \quad B_{10} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The nominal model parameters are given by:

$$\hat{A}_1 = 0.352, \quad \hat{B}_1 = 0.95$$
$$\hat{A}_2 = 0.44, \quad \hat{B}_2 = 0.95$$
$$\hat{A}_3 = 0.176, \quad \hat{B}_3 = 0.95$$
$$\hat{A}_4 = 0.88, \quad \hat{B}_4 = 0.95$$
$$\hat{A}_5 = \begin{bmatrix} 0.1848 & -0.352 \\ 0.264 & -0.616 \end{bmatrix}, \quad \hat{B}_5 = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix}$$
$$\hat{A}_6 = \begin{bmatrix} 0.264 & 1.76 \\ 0.528 & -1.584 \end{bmatrix}, \quad \hat{B}_6 = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix}$$
$$\hat{A}_7 = \begin{bmatrix} 0.044 & 0.0026 \\ 0.002 & -0.616 \end{bmatrix}, \quad \hat{B}_7 = \begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix}$$

$$\hat{A}_8 = \begin{bmatrix} 0.0968 & -0.3608 & 0 \\ 0 & 0.264 & -0.616 \\ 0 & -0.264 & -0.44 \end{bmatrix}, \quad \hat{B}_8 = \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 0.95 \end{bmatrix}$$

$$\hat{A}_9 = \begin{bmatrix} 0.264 & 1.76 & -0.0616 \\ 0 & 0.0792 & -0.88 \\ 0.88 & 0 & 0.176 \end{bmatrix}, \quad \hat{B}_9 = \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 0.95 \end{bmatrix}$$

$$\hat{A}_{10} = \begin{bmatrix} 0.044 & 0.0026 & 0.44 \\ 0 & 0.002 & -0.616 \\ -0.264 & 0 & -0.44 \end{bmatrix}, \quad \hat{B}_{10} = \begin{bmatrix} 0.95 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 0.95 \end{bmatrix}$$

Every agent is coupled to all other agents including those with different dimensions by corresponding coupling matrices A_{ij}. The unknown coupling matrices are given by:

$$A_{ij} = c_1 \quad for\ i = 1,2,3,4;\ j = 1,2,3,4;\ i \neq j.$$

$$A_{ij} = \begin{bmatrix} c_1 & c_1 \end{bmatrix} \quad for\ i = 1,2,3,4;\ j = 5,6,7.$$

$$A_{ij} = \begin{bmatrix} c_2 & c_2 & c_2 \end{bmatrix} \quad for\ i = 1,2,3,4;\ j = 8,9,10.$$

$$A_{ij} = \begin{bmatrix} c_1 & c_1 \end{bmatrix}^T \quad for\ i = 5,6,7;\ j = 1,2,3,4.$$

$$A_{ij} = \begin{bmatrix} c_2 & 0 \\ 0 & c_2 \end{bmatrix} \quad for\ i = 5,6,7;\ j = 5,6,7;\ i \neq j.$$

$$A_{ij} = \begin{bmatrix} c_3 & 0 & c_3 \\ 0 & c_3 & 0 \end{bmatrix} \quad for\ i = 5,6,7;\ j = 8,9,10.$$

$$A_{ij} = \begin{bmatrix} c_2 & c_2 & c_2 \end{bmatrix}^T \quad for\ i = 8,9,10;\ j = 1,2,3,4.$$

$$A_{ij} = \begin{bmatrix} c_3 & 0 & c_3 \\ 0 & c_3 & 0 \end{bmatrix}^T \quad for\ i = 8,9,10;\ j = 5,6,7.$$

$$A_{ij} = \begin{bmatrix} c_3 & 0 & 0 \\ 0 & c_3 & 0 \\ 0 & 0 & c_3 \end{bmatrix} \quad for\ i = 8,9,10;\ j = 8,9,10;\ i \neq j.$$

for $c_1 = 0.5, c_2 = 0.4, c_3 = 0.1$. The nominal coupling matrices $\hat{A}_{ij}$ are of the same form but with $\hat{c}_1 = 0.45, \hat{c}_2 = 0.36, \hat{c}_3 = 0.09$.

The local controllers and thresholds are designed following the decentralized approach in section IV, where only the model parameters and the bounds on the uncertainties are used. The results of simulations are shown in Fig. 2 and Fig. 3. In the top portion of Fig. 2 it can be seen the norm of the augmented state, that is, the response of all states of all subsystems. Fig. 3 and the bottom portion of Fig. 2 show the broadcasting periods for every agent in the networked system. Fig. 2 (bottom) represent the broad-casting periods for the 4 first order systems, Fig. 3 (top) for the 3 second order systems, and Fig. 3 (bottom) for the 3 third order systems.

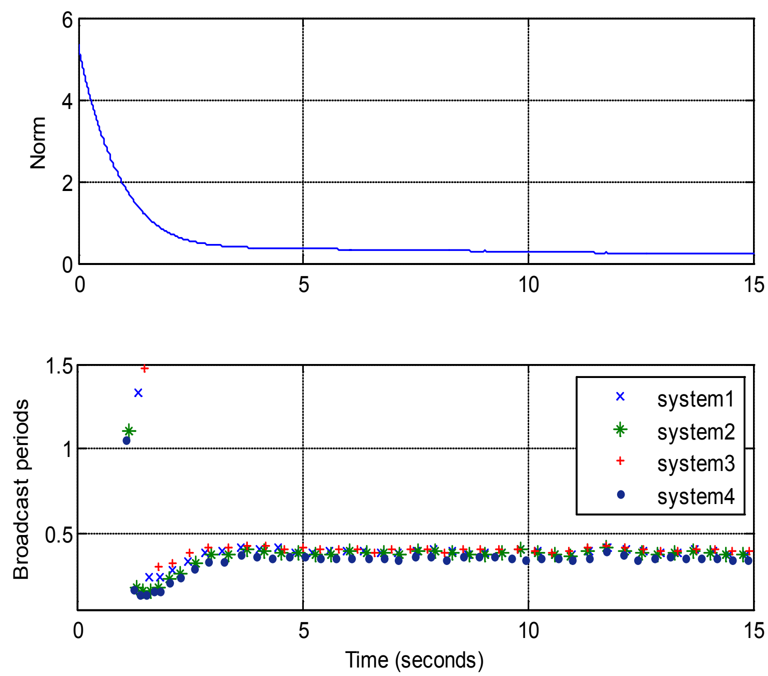

Fig. 2. The norm of the state of the overall system is shown in the top. The broadcasting period (in seconds) for subsystems 1-4 is shown at the bottom.

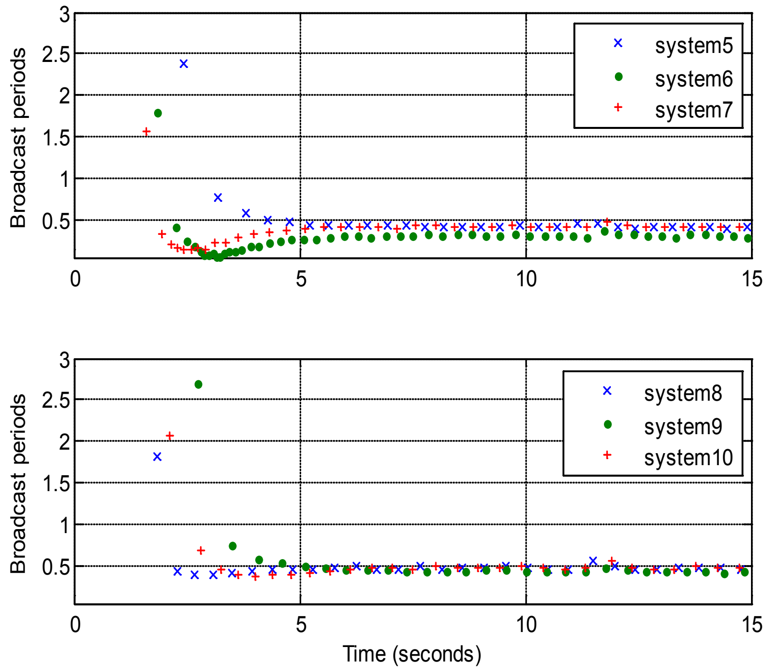

Fig. 3. Broadcasting period (in seconds) for subsystems 5-7 (top) and 8-10 (bottom).

Example 2. Consider a similar set of subsystems as in example 1, with the same system and model parameters $A_i, B_i, \hat{A}_i, \hat{B}_i$, but in this case we assume that the dynamics of every subsystem is affected by only a subset of the N=10 systems. In particular, each system is coupled with systems that have their same dimension by means of corresponding matrices A_{ij} as in example 1 (using the corresponding model $\hat{A}_{ij}$ to implement the models) e.g. system 1 is coupled only to systems 2,3, and 4. In addition system 4 affects system 5 and system 7 affects system 8.

In this case each LCU does not need to implement models of all subsystems. Each LCU only needs to implement those models that are needed to estimate the variables that affect its own dynamics. Results of simulations are shown in Fig. 4 and Fig. 5. In general, the subsystems need to broadcast their states less often compared to example 1 since the systems are coupled with a fewer number of systems and their errors grow slower than in the previous example.

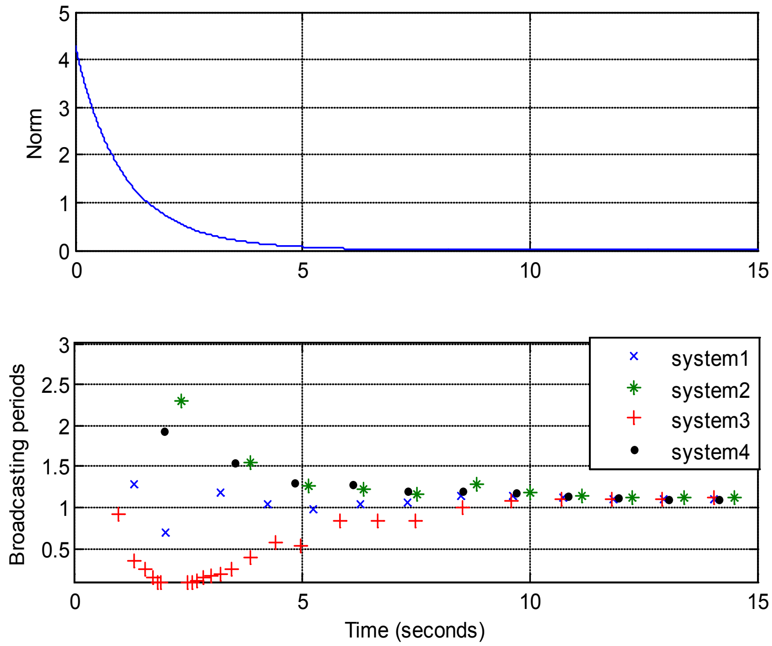

Fig. 4. The norm of the state of the overall system for example 2 is shown in the top. The broadcasting period (in seconds) for subsystems 1-4 is shown at the bottom.

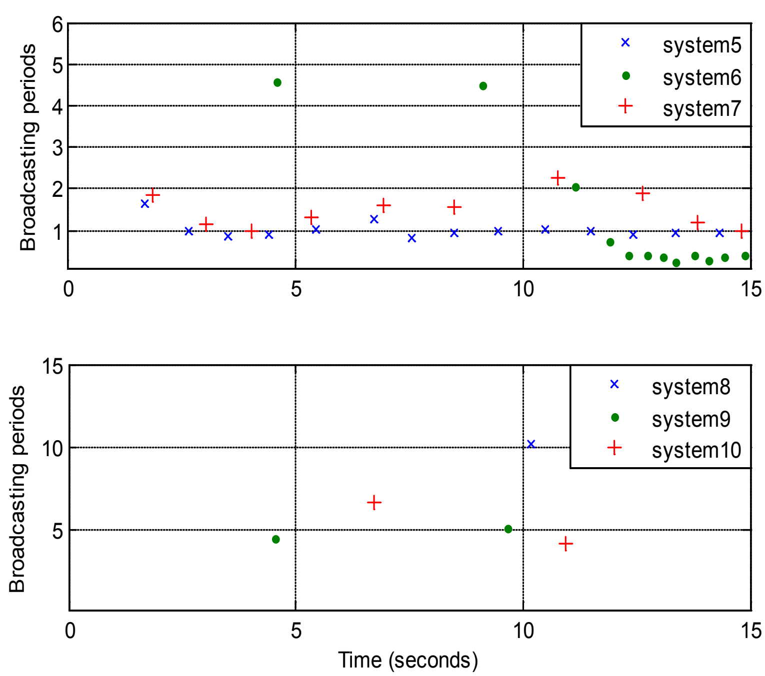

Fig. 5. Broadcasting period (in seconds) for subsystems 5-7 (top) and 8-10 (bottom) in example 2.

Example 3. In this example we consider a collection of three coupled carts. The physical coupling corresponds to the springs used to connect the carts to each other, and assume that at the equilibrium of the system, all springs are not stretched. The dynamics of each cart and its corresponding model can be described by (3) and (4), respectively with

$$A_i = \begin{bmatrix} 0 & 1 \\ -c_i k & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A_{ij} = \begin{bmatrix} 0 & 0 \\ d_{ij} & 0 \end{bmatrix},$$

$$\hat{A}_i = \begin{bmatrix} 0 & 1 \\ -c_i \hat{k} & 0 \end{bmatrix}, \quad \hat{B}_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \hat{A}_{ij} = \begin{bmatrix} 0 & 0 \\ \hat{d}_{ij} & 0 \end{bmatrix},$$

where $c_1 = c_3 = 1, c_2 = 2$, $k = 5$, $d_{12} = d_{32} = d_{21} = d_{23} = 1$, and $d_{13} = d_{31} = 0$. The model parameters are $\hat{k} = 4.95$, $\hat{d}_{13} = \hat{d}_{31} = 0$, and the remaining $\hat{d}_{ij} = 1.01$. Results of simulations of this example are shown in Fig. 6.

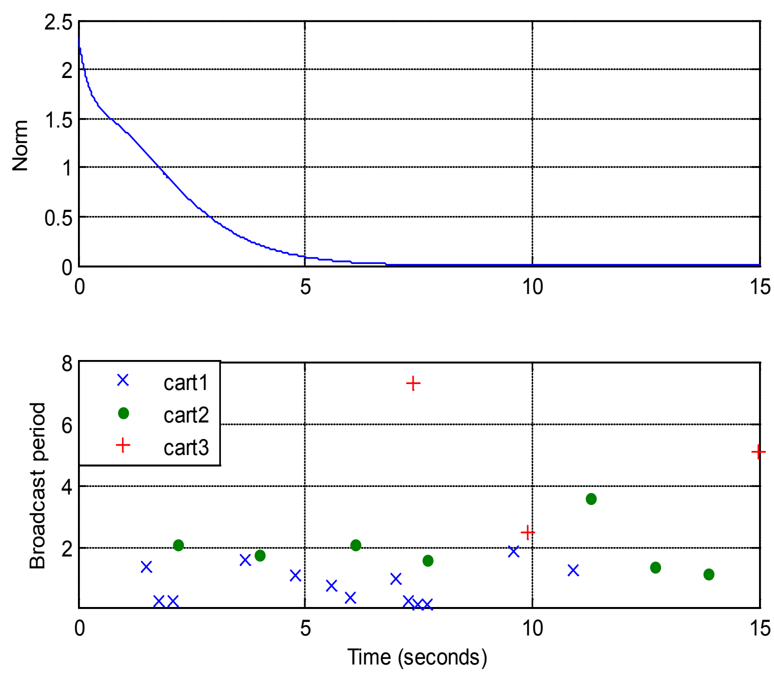

Fig. 6. The norm of the state of the overall system in example 3 is shown in the top. The broadcasting period (in seconds) for each cart is shown at the bottom.

VI. CONCLUSION

The model-based event-triggered control framework introduced in [18] was extended in this paper to deal with networks of coupled subsystems. Our framework is capable of significantly reducing network communication by using available models of other subsystems at each controller node and by dynamically scheduling measurement broadcasting based only on local information. The use of errors in the state in order to generate communication events relaxes the requirement for a fixed and, many times, conservative schedule of network message broadcasting. Future work will address important aspects of network communication such as time delays, data quantization, and packet dropouts.

APPENDIX

Proof of proposition 4. Consider the local Lyapunov function $V_i = x_i^T P_i x_i$ and compute its derivative along the trajectories of subsystem i in (3) using the local control input (5) to obtain

$$\begin{aligned}\dot{V}_i &= x_i^T((\hat{A}_i + \hat{B}_i K_i)^T P_i + P_i(\hat{A}_i + \hat{B}_i K_i))x_i \\ &+ x_i^T((\tilde{A}_i + \tilde{B}_i K_i)^T P_i + P_i(\tilde{A}_i + \tilde{B}_i K_i))x_i \\ &+ x_i^T P_i \left(\sum_{j=1\, j\neq i}^{N} \tilde{A}_{ij} x_j + \sum_{j=1\, j\neq i}^{N} B_i K_{ij} x_j \right) \\ &+ \left(\sum_{j=1\, j\neq i}^{N} \tilde{A}_{ij} x_j + \sum_{j=1\, j\neq i}^{N} B_i K_{ij} x_j \right)^T P_i x_i\end{aligned} \tag{A.1}$$

and consider the next inequality involving the vectors $\mu \in \mathbb{R}^n, \nu \in \mathbb{R}^m$

$$\left| \delta\mu - \Pi\nu \right|^2 \geq 0 \tag{A.2}$$

where $\Pi \in \mathbb{R}^{n\times m}$ and δ is any positive real constant. (A.2) can be expanded to yield

$$\mu^T \Pi \nu \leq \frac{|\Pi\nu|^2}{2\delta} + \frac{\delta |\mu|^2}{2} \tag{A.3}$$

Applying (A.3) to (A.1) we obtain

$$\dot{V}_i \leq -q_i |x_i|^2 + \Delta_i |x_i|^2 + \sum_{j=1, j\neq i}^{N} \left(\frac{|P_i \tilde{A}_{ij}|^2}{\delta_i} |x_j|^2 + \delta_i |x_i|^2 \right)$$

$$+ \sum_{j=1, j\neq i}^{N} \left(\frac{|P_i B_i K_{ij}|^2}{\delta_i} |e_j|^2 + \delta_i |x_i|^2 \right)$$

Finally, we write the terms involving $|x_i|^2$ together and we obtain (28).■

REFERENCES

[1] N. R. Sandell, P. Varaiya, M. Athans, and M. G. Safonov, "Survey of decentralized control methods for large scale systems," *IEEE Trans. Automatic Control,* vol. AC-23, no. 2, pp. 108-128, April 1978.

[2] B. Sinopoli, C. Sharp, L. Schenato, S. Schaffert, and S. Sastry, "Distributed control applications within sensor networks," *Proc. IEEE,* vol. 91, no. 8, pp. 1235-1246, August 2003.

[3] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in Networked Control Systems," *Proc. IEEE*, vol. 95, no.1, pp. 138-162, January 2007.

[4] N. Elia and S. K. Mitter, "Stabilization of linear systems with limited information" *IEEE Trans. Automatic Control,* vol. 46, pp. 1384-1400, Sept. 2001.

[5] M. B. G. Cloosterman, L. Hetel, N. van de Wouw, W. P. M. H.Heemels, J. Daafouz, and H. Nijmeijer, "Controller synthesis for networked systems," *Automatica*, vol. 46, pp. 1584-1594, 2010.

[6] G. C. Walsh, H. Ye, and L. Bushnell, "Stability analysis of Networked Control Systems". *IEEE Trans. Control Systems Technology*, vol. 10, no. 3, pp. 438-446, May 2002.

[7] L. A. Montestruque and P. J. Antsaklis. "State and output feedback control in model-based networked control systems," *Proc. of the 41st IEEE Conference on Decision and Control*, 2002.

[8] L. A. Montestruque, P. J. Antsaklis "On the model-based control of networked systems," *Automatica*, vol. 39, no. 10, pp. 1837 - 1843, 2003.

[9] Y. Sun and N. H. El-Farra, "Quasi-decentralized model-based networked control of process systems," *Computers and Chemical Engineering,* vol. 32, pp. 2016-2029, 2008.

[10] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Trans. Automatic Control,* vol. 52, no. 9, pp. 1680-1685, Sept. 2007.

[11] A. Anta and P. Tabuada, "To sample or not to sample: Self-triggered control for nonlinear systems," *IEEE Trans. Automatic Control*, vol. 55, no. 9, pp. 2030-2042, Sept. 2010.

[12] X. Wang and M. D. Lemmon, "Event Design in Event-Triggered Feedback Control Systems," in *Proc. of the 47th IEEE Conference on Decision and Control*, 2008.

[13] A. Cervin and T. Henningson, "Scheduling of event-triggered controllers on a shared network," in *Proc. 47th IEEE Conference on Decision and Control*, 2008, pp. 3601-3606.

[14] X. Wang and M. D. Lemmon, "Event-triggering in distributed networked control systems," *IEEE Trans. Automatic Control*, vol. 56, pp. 586-601, March 2011.

[15] X. Wang and M. Lemmon, "Event-triggering in distributed networked systems with data dropouts and delays," in *Hybrid Systems: Computation and Control,* 2009.

[16] M. Mazo and P. Tabuada, "Decentralized event-triggered control over wireless sensor/actuator networks," *IEEE Trans. Automatic Control,* vol. 56, no. 10, pp. 2456-2461, 2011.

[17] D. V. Dimarogonas and K. H. Johansson, "Event-triggered control of multi-agent systems," *Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 2009.

[18] E. Garcia and P. J. Antsaklis, "Model-based event-triggered control with time-varying network delays," in *Proc. of the IEEE 50th IEEE Conference on Decision and Control-European Control Conference*, 2011.

[19] J. Lunze and D. Lehmann, "A state-feedback approach to event-based control," *Automatica*, vol. 46, no.1, pp.211-215, 2010.

[20] Y. Sun and N. H. El-Farra, "Quasi-decentralized networked process control using an adaptive communication policy," in *Proc. American Control Conference,* 2010.

[21] Y. Sun and N. H. El-Farra, "Dynamic quasi-decentralized control of networked process systems with limited measurements," in *Proc. American Control Conference,* 2011.

[22] X. Wang and M. D. Lemmon, "Event triggered Broadcasting across distributed networked control systems," in *Proc. of the American Control Conference,* 2008.