

Cyber-Physical Systems Design Using Dissipativity

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Abstract: In Cyber-Physical Systems large numbers of heterogeneous cyber and physical subsystems are networked, are interacting tightly, may change dynamically and may expand or contract. Designing and preserving properties of a CPS over its lifespan is very challenging. Passivity and dissipativity are energy like concepts that offer great promise in guaranteeing properties, such as stability, in complex heterogeneous interconnected systems that are changing dynamically. Passivity indices that provide a measure of the degree of passivity are used to generalize classical results in interconnected systems, and results for continuous, discrete and switched systems in networks with delays, event triggered architectures, conic systems and systems with symmetries are shown.

1 Introduction

Recent technological developments in sensing, communications, control and computation have created an emerging class of complex systems, called Cyber-Physical Systems (CPS). Cyber-Physical Systems are characterized by large numbers of tightly integrated heterogeneous components in a network, which may expand and contract dynamically. Cyber-Physical Systems are very common and are becoming increasingly ubiquitous. Examples of CPS may be found in smart transportation systems, smart medical devices, smart buildings, smart energy systems, the smart grid. The control of such systems presents huge challenges and requires designs drawn from approaches such as those in traditional control, hybrid control systems, discrete event systems, and networked control. In addition, robustness, reliability and security issues for reconfiguring dynamical systems must also be addressed. This integration of different technologies and scientific domains presents new and challenging fundamental problems underlying the theoretical foundations for this class of systems.

There has been a series of research activities in CPS over the past 7 years and information may be found at the CPS Virtual Organization website (<http://cps-vo.org/>). It should be noted that the importance of CPS was recognized in the 2007 report of the USA President's Council of Advisors on Science and Technology (PCAST) "Leadership Under Challenge: Information Technology R&D in a Competitive World," PCAST report, August 2007 (<http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-07-nitrd-review.pdf>) and reaffirmed in the 2010 PCAST report "Designing a Digital Future: Federally Funded Research and Development in Networking and Information Technology," PCAST Report, December 2010 (<http://www.whitehouse.gov/sites/default/files/microsites/ostp/pcast-nitrd-report-2010.pdf>).

In the design of CPS one has to guarantee certain properties of the whole system even though the system consists of networked heterogeneous subsystems, the number of which may expand or contract. The energy like concepts of passivity and dissipativity appear to offer promise towards that goal and this presentation describes our recent research efforts towards establishing design methodologies for CPS. Our research work on passivity and CPS is in collaboration with

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In the following an outline of our research on the design of CPS using passivity and dissipativity is presented. In particular, Section 2 covers the problem of networking passive switched systems. This uses an accepted definition of passivity for switched systems and assumes these systems are connected over a network with possible delays, lost data, and quantization. Section 3 focuses on the application of event-triggered control to passive systems. Event-triggered control has been used to reduce communication in networks to guarantee a specified level of performance. In Section 4, passivity is applied to multi-agent systems that exhibit a form of symmetry in their interconnections. Finally, concluding remarks are made in Section 5.

2 Networking Passive Switched Systems

In CPS, physical processes are modeled using differential or difference equations with a strong dependence on time. The cyber processes evolve based on the occurrence of events, both physically and in software, and are modeled using discrete-event models such as finite automata or Petri nets. The combination of these different components results in system models that are hybrid or switched.

One challenging aspect of CPS is that these complex systems are often made up of varying components. The property of compositionality is crucial to analyzing stability of these systems. One approach to compositionality is using passivity or dissipativity theory [2]. Passivity is an energy inspired property of dynamical systems that is preserved when two systems are interconnected in parallel or in negative feedback. Under mild assumptions, passivity implies stability [3]. Using these two properties together, large-scale stable systems can be built up by sequentially connecting passive components together. Dissipativity properties can also be studied in the switched system framework. Although dissipative systems may not be stable, dissipativity is a property that is preserved in feedback and stability may be assessed from the dissipative rate of the interconnection.

The compositionality that passivity provides may be exploited in CPS by a generalized passivity property for

switched systems (see [4] and the references therein). A general model of nonlinear switched systems may be used. Switching between subsystems is assumed to be bounded on any finite time interval so to avoid the Zeno phenomenon. In general terms, a switched system is passive if the following two conditions hold.

- 1) Each subsystem is passive when it is active.
- 2) Each subsystem is dissipative (of a special form) when it is inactive.

For the second condition, the form of dissipativity is general, but it is restricted since there must exist inputs to ensure that switching only adds a finite amount of energy over the infinite time horizon. This definition generalizes the expected properties of passive systems. First, passivity is preserved when passive switched systems are interconnected in negative feedback. Second, when the definitions are made slightly more restrictive, expected stability results are shown. This includes strictly passive implying asymptotic stability and output strictly passive implying \mathcal{L}_2 stability (bounded input, bounded output stability).

Although many practical systems are passive, some applications include switched systems that aren't necessarily passive. One approach is the area of passivity indices where the feedback stability result can be extended to these non-passive systems. This framework generalizes the property by quantifying the level of passivity in a given system. In order to completely characterize the level in a system, two indices are required. The first is a measure of the level of stability of the system. The second is a measure of the extent of the minimum phase property in a system. This framework has close ties to conic systems theory [5].

The main difference in applying the indices to switched systems is that the indices become *time-varying*. Each subsystem has values for the two indices and the overall switched system takes on the values of the indices over the time intervals where that subsystem is active. With this definition, the passivity indices for switched systems are piecewise constant. With the earlier assumption that there is finite switching on any finite time interval, the switching signals are well behaved and the time varying indices are well defined. The results based on the indices generalize to the case of switched systems. Conceptually, when considering the feedback interconnection of two systems, a shortage of passivity of one system can be compensated by an excess of passivity in the other system. Specifically, a shortage of stability in one system can be compensated by an excess of the minimum phase property in the other system and the other way around [6]. Once the indices have been assessed for a given interconnection of two switched systems, the verification that the interconnection is stable is as simple as checking whether a matrix is positive definite. This means that stable feedback loops can be designed even when the systems in the loop aren't passive or even stable.

Another challenging area in CPS is that many components are interconnected over communication channels that include delayed data and lost packets. Although passivity is preserved when two switched systems are interconnected in feedback, this no longer holds when delays are introduced. Since passivity is an energy based property with a strong dependence on time, it requires instant transmission of the

energy on one side of the network to the other.

One solution to this issue is a network interface that decouples the notion of energy defined by passivity. The interface is an invertible, input-output coordinate transformation to wave variables. Instead of an inner product, energy is decoupled into a wave going out to the network and a wave coming in from the network. The wave variable transformation is well established for non-switched systems, but many areas of CPS require models that are switched. This approach can be used to handle delays that are time-varying with an upper bound and lost data due to packet drops [7].

Networking CPS has other issues such as discretization and quantization. These are common issues when continuous-time physical processes are sampled to be controlled by digital controllers or when signals are quantized to use digital networks. While discretization of passive systems has been well-studied, *quantization* has been largely ignored. The main problem is that the stability results that are provided by passivity theory no longer hold when quantization is present.

This problem is addressed in [8]. That paper introduced a control framework under which passivity for switched and non-switched systems can be maintained despite input and output quantization. The quantizers may be general with non-uniform levels as long as the gains of the quantizers are finite. This framework centers on the use of an input-output coordinate transformation to recover passivity. The transformation is not unique, but under mild assumptions, a transformation can always be found to preserve passivity despite quantization.

3 Event Triggered Control of Passive Systems

Recently, several researchers have suggested the idea of event-based control as a promising technique to reduce communication and computation load for the purpose of control in many control applications. In a typical event-based implementation, the control signals are kept constant until the violation of an "event triggering condition" on certain signals which triggers the re-computation of the control actions. Compared with time-driven control, where constant sampling period is applied to guarantee stability in the worst case scenario, the possibility of reducing the number of computations, and thus of transmissions, while guaranteeing desired levels of performance makes event-based control very appealing in networked control systems (NCSs). A comparison of time-driven and event-driven control for stochastic systems favoring the latter can be found in [9]; a deterministic event-triggered control strategy is introduced in [10]; similar results on deterministic self-triggered feedback control have been reported in [11], [12]; output-based event-triggering control with guaranteed L_∞ -gain for linear time-invariant systems has been studied in [15]; event-triggering stabilization for distributed networked control systems has been studied in [13]; in [14], a self-triggered coordination strategy for optimal deployment of mobile robotics is proposed.

Most of the results on event-triggered control are obtained under the assumption that the feedback control law provides input-to-state stability (ISS) with respect to the state mea-

surement errors. However, in many control applications the full state information is not available for measurement, so it is important to study stability and performance of event-triggered control systems with dynamic and static output feedback controllers. In [16], a static output feedback based event-triggered control scheme is introduced for stabilization of passive and output feedback passive (OFP) NCSs. A static output feedback gain and a triggering condition are derived based on the output feedback passivity indices of the plant. In [17], a dynamic output feedback based event-triggered control scheme is introduced for stabilization of Input Feed-forward Output Feedback Passive (IF-OFP) NCSs, which expands our previous work in [16] for stabilization of more general dissipative systems. The triggering condition is derived based on the passivity theorem which allows us to characterize a large class of output feedback stabilization controllers. We show that under the triggering condition derived in [17], the control system is finite gain L_2 stable in the presence of bounded external disturbances. The interactions between the triggering condition, the achievable L_2 gain of the control system and the inter-sampling time have been studied in terms of the passivity indices of the plant and the controller. Based on the results in [17], we further propose a dynamic output feedback based event-triggered control set-up for NCSs which allows us to consider network induced delays both from the sampler to the controller and from the controller to the plant [18]. We show that based on the proposed set-up, finite-gain L_2 stability can be achieved in the presence of arbitrary constant network induced delays or delays with bounded jitters.

Event-based distributed control in cooperative control of multi-agent systems is of interest because of the potential to reduce communication load and implementation cost. In [19], we propose a distributed event-driven communication strategy for stabilization of large scale networked control systems with finite-gain L_2 stability. Each subsystem broadcasts its output information to its neighbors only when the subsystem's local output error exceeds a specified threshold. The triggering condition is related to the topology of the underlying communication graph. We also provide analysis of the time intervals between two consecutive communication broadcasts (the inter-event time). Our analysis shows that the topology of the underlying communication graph plays an important role on the performance of the NCSs with event-driven communication. In [16], we study the quantized output synchronization problem of networked passive systems with event-driven communication, in which the data transmissions among networked agents are event-based and quantized measurements are exchanged among neighboring agents. We show that with the event-driven communication strategy proposed in [16], output synchronization errors of the networked passive systems are bounded by the quantization errors of the signals transmitted in the communication network.

4 Passivity and Symmetry

Symmetry, as a basic feature of shapes and graphs, appears in many real-world networks, such as the Internet and power grid, resulting from the process of tree-like or cyclic

growing. Since symmetry is related to the concept of a high degree of repetitions or regularities, the study of symmetry has been appealing in many scientific areas, such as Lie groups in quantum mechanics and crystallography in chemistry. In the classical theory of dynamical systems, symmetry has also been extensively studied. For example, to simplify the analysis and synthesis of large-scale dynamic systems, it is always of interest to reduce the dynamics of a system into smaller symmetric subsystems, which potentially simplifies control, planning or estimation tasks. When dealing with multi-agent systems with various information constraints and protocols, under certain conditions such systems can be expressed as or decomposed into interconnections of lower dimensional systems, which may lead to better understanding of system properties such as stability and controllability. Then the existence of symmetry here means that the system dynamics are invariant under transformations of coordinates.

In our work, stability conditions for large-scale systems are derived by categorizing agents into symmetry groups and applying local control laws under limited interconnections with neighbors [21]. Particularly, stability for dissipative systems is considered. Dissipativity is a generalization of passivity where the energy supplied to the system can take different forms. Several properties of dynamical systems can be captured by varying the energy supply rate. When subsystems of a symmetric system are dissipative, overall stability properties can be studied. Conditions are derived for the maximum number of subsystems that may be added while preserving stability and these results may be used in the synthesis of large-scale systems with symmetric interconnections.

Let the dynamics of interconnected nonlinear distributed systems $\Sigma_0, \Sigma_1, \dots, \Sigma_m$ be given by

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ \Sigma_i : \quad y_i &= h_i(x_i) \\ u_i &= u_{ei} - \sum_{j=0}^m H_{ij}y_j \end{aligned}$$

where $i = 0, \dots, m$, u_i is the input to subsystem i , y_i is its output, u_{ei} is an external input, and the H_{ij} are constant matrices. If we define $y = [y_1^T, \dots, y_n^T]^T$, $\tilde{H} = [H_{ij}]$, and define u, u_e similarly, then the interconnected system can be represented by

$$u = u_e - \tilde{H}y$$

Symmetries may be introduced into interconnected systems via identical dynamics of subsystems, as well as same characterizations of information structure. For instance, let the systems be interconnected with star-shaped symmetry. That is starting with the base system Σ_0 , a group of systems Σ_i are connected to it without interconnections among each other. Therefore let

$$\tilde{H} = \begin{bmatrix} H & b & \dots & b \\ c & h & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \dots & h \end{bmatrix}$$

Theorem: Consider a (Q, S, R) – *dissipative* system Σ_0 extended by m star-shaped symmetric (q, s, r) – *dissipative*

subsystems Σ_i . The whole system is asymptotically stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(c^T r c + \beta(\hat{q} - b^T R b)^{-1} \beta^T)}, \frac{\hat{q}}{b^T R b}\right)$$

where

$$\begin{aligned}\hat{Q} &= -H^T R H + S H + H^T S^T - Q > 0 \\ \hat{q} &= -h^T r h + s h + h^T s^T - q > 0 \\ \beta &= S b + c^T s^T - H^T R b - c^T r h\end{aligned}$$

The above theorem shows that there exists an upper bound on the number of subsystems that can be added so to preserve stability of dissipative systems. Besides star-shaped symmetries, there are similar results for interconnections with cyclic symmetries.

When we consider passivity as a special case of dissipativity, passivity indices can be used for interconnections of agents to assess the level of passivity. Motivated by the interest of sufficient stability conditions in [21], passivity indices for both linear and nonlinear multi-agent systems with feed-forward and feedback interconnections are derived with the distributed setup in [22]. For linear systems, the passivity indices are explicitly characterized, while the passivity indices in the nonlinear case are characterized by a set of matrix inequalities. We also focus on symmetric interconnections and specialize stability results to this case.

5 Conclusions

This paper summarizes a large body of research on the control of CPS related to the concepts of passivity, dissipativity, and symmetry. The work here focuses on key areas of CPS including networking and interconnecting systems that may change dynamically. Specific areas that are addressed include stability of interconnected passive switched systems that contain cyber and physical dynamics over delayed networks, stability of interconnected passive systems using an event-triggered scheme, and networking multi-agent agent passive systems over a structure that contains symmetries. Throughout this research, the energy concepts of passivity and dissipativity have been invaluable. These concepts will continue to be used in CPS as these areas develop.

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