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Generalized Passivity in Discrete-Time Switched Nonlinear Systems

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Abstract— **This paper analyzes the passivity and feedback passivation problems for discrete-time switched nonlinear systems with both passive and nonpassive modes that have relative degree zero. We generalize the classical passivity definition for such a switched nonlinear system provided that the increase in storage function over a finite horizon is bounded by the total supplied energy during this time period. We further extend this generalized passivity definition to switched nonlinear systems with passive, feedback passive modes and modes which can not be rendered passive using feedback (non-feedback passive modes). The switched nonlinear system is proved to be locally feedback passive if and only if its zero dynamics are locally passive. A lower bound on the ratio of total activation time between (feedback) passive and non-feedback passive modes is obtained to guarantee passive zero dynamics with Lipschitz constraints. We prove that output feedback control can be used to stabilize the equilibrium point of the switched system.**

I. INTRODUCTION

Passivity is an important property of dynamical systems because i) the free dynamics and zero dynamics of passive systems are Lyapunov stable, ii) the parallel or negative feedback interconnections of passive systems remain passive, and iii) a passive system can achieve stability using output feedback ([1], [2]). In the classical passivity theory [3]–[7], a system is said to be passive if the increase in storage function is bounded by the energy supplied to it at every time step.

However, most physical systems, especially complex systems such as switched systems, are not inherently passive. In this paper, we generalize the classical passivity definition to discrete-time switched nonlinear systems consisting of both passive and nonpassive modes that have relative degree zero. Such a system is said to be passive if the increase in storage function over a finite horizon is bounded by the total energy supplied to it during this time period. When a nonpassive mode is active, the increase in storage function is allowed to be greater than the supplied energy. We further extend this generalized definition to a switched nonlinear system consisting of passive, feedback passive modes, and modes which can not be rendered passive using feedback (non-feedback passive modes). Under the generalized passivity definition, the switched system is proved to be locally feedback passive if and only if its zero dynamics are locally passive.

We first review the literature on passivity of switched systems. There has been considerable attention on the passivity of switched systems with all passive modes. One main category uses a common storage function for all the modes [8]–[11]. To get less conservative results, the techniques of multiple storage functions for different modes have been proposed via piecewise quadratic storage functions [12], multiple storage functions and a same supply rate [13], and multiple storage functions and multiple supply rates ([14], [15]). For more general switched nonpassive systems, stability results have been presented in [16] using passivity indices and in [17] via average dwell time method. Unlike the above work, since passivity is a desirable property in addition to stability because of several additional properties that it guarantees, here we propose necessary and sufficient conditions to guarantee system passivity in the presence of both passive and nonpassive modes. We also prove that output feedback control can be used to stabilize the equilibrium point of the switched system.

Next, we summarize some related work on classical passivity and stability theory for nonlinear systems. The feedback passivity equivalence (Willems-Hill-Moylan conditions) for continuous time systems were developed in [3], [5]. The feedback equivalence for discrete-time passive systems has been proposed in [18]. In the present paper, we generalize the classical results to switched nonlinear systems with passive, feedback passive and non-feedback passive modes via the generalized passivity equivalence (i.e., necessary and sufficient conditions for passivity). This is analogous to the generalized asymptotic stability of discrete-time nonlinear time-varying systems where the Lyapunov function is non-increasing only on certain unbounded sets [19]. However, unlike [19], the passivity analysis is complicated by the fact that passivity is an input-output property and both the inputs and the outputs are time varying. Due to this difficulty, we analyze the passivity properties of a switched system based on zero dynamics ([1], [6], [20], and especially [18]) which are the internal dynamics of the system that are consistent with constraining the system output to zero.

The paper is organized as follows. Section II reviews related background on classical passivity theory. Section III introduces the problem formulation. In Section IV, we propose a generalized passivity definition and develop a necessary and sufficient condition for the passivity of the switched nonlinear systems. Section V extends the definition to switched systems with passive, feedback passive, and non-feedback passive modes. The switched system is proved to be locally feedback passive if and only if its zero dynamics are passive. In Section VI, a lower

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bound on the total activation time of the (feedback) passive versus non-feedback passive modes is derived to guarantee passive zero dynamics with Lipschitz constraints. In Section VII, we prove that the equilibrium point of the passive switched system can achieve asymptotic stability using output feedback if the system is zero state detectable. We conclude the paper in Section VIII.

II. Preliminary

Consider a system of the form

$$\begin{cases} \mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{u}(k)) \\ \mathbf{y}(k) = h(\mathbf{x}(k), \mathbf{u}(k)) \end{cases}, \quad (1)$$

where $\mathbf{x} \in \mathbf{X} \subset \mathbb{R}^n$, $\mathbf{u} \in \mathbf{U} \subset \mathbb{R}^m$ and $\mathbf{y} \in \mathbf{Y} \subset \mathbb{R}^m$ are the state, input, and output variables, respectively. $\mathbf{X}$, $\mathbf{U}$ and $\mathbf{Y}$ are the state, input, and output spaces, respectively. Let $\mathbb{Z}_+$ denote the set of nonnegative integers. The time index $k \in \mathbb{Z}_+$ and f, h are in $\mathcal{C}^\infty$. All considerations are restricted to an open set $\mathbf{X} \times \mathbf{U}$ containing an isolated equilibrium point $(\mathbf{x}^*, \mathbf{u}^*)$. Assume that $(\mathbf{x}^*, \mathbf{u}^*) = (\mathbf{0}, \mathbf{0})$ and $h(\mathbf{0}, \mathbf{0}) = \mathbf{0}$.

Definition II.1. *[21] A system of the form (1) is* locally passive *if there exists a positive definite function $V(\mathbf{x})$, called the* storage function*, such that the following inequality holds in a neighborhood of the equilibrium point $(\mathbf{0}, \mathbf{0})$,*

$$V(f(\mathbf{x}(k), \mathbf{u}(k))) - V(\mathbf{x}(k)) \le \mathbf{u}^T(k)\mathbf{y}(k), \\ \forall \mathbf{x} \in \mathbf{X}, \mathbf{u} \in \mathbf{U}, k \in \mathbb{Z}_+. \quad (2)$$

Definition II.2. *[18] A system of the form (1) is locally passive if there exists a positive definite $\mathcal{C}^2$ storage function $V(\mathbf{x})$ such that the following equation holds in a neighborhood of $(\mathbf{0}, \mathbf{0})$,*

$$V(f(\mathbf{x}(k), \mathbf{u}(k))) - V(\mathbf{x}(k)) = \mathbf{u}^T(k)\mathbf{y}(k) \\ -(l(\mathbf{x}(k)) + e(\mathbf{x})\mathbf{u}(k))^T(l(\mathbf{x}(k)) + e(\mathbf{x}(k))\mathbf{u}(k)) \\ -m(\mathbf{x}(k))^T m(\mathbf{x}(k)),\ \forall \mathbf{x} \in \mathbf{X}, \mathbf{u} \in \mathbf{U}, k \in \mathbb{Z}_+, \quad (3)$$

where $l(\mathbf{x}(k)), e(\mathbf{x}(k)), m(\mathbf{x}(k))$ are real functions that equal to zero if and only if $\mathbf{x}(k) = \mathbf{0}$.

Remark II.1. *Definitions II.1 and II.2 are consistent because the term $-(l+e\mathbf{u})^T(l+e\mathbf{u}) - m^T m$ is nonpositive. Contrarily, if a system is nonpassive, then there does not exist any $\mathcal{C}^2$ storage function such that Equation (3) holds. We may associate any $\mathcal{C}^2$ storage function with the nonpassive system and model its increase by*

$$V(f(\mathbf{x}, \mathbf{u})) - V(\mathbf{x}) = \mathbf{u}^T\mathbf{y} + (l + e\mathbf{u})^T(l + e\mathbf{u}) \\ -m^T m,\ \forall \mathbf{x} \in \mathbf{X}, \mathbf{u} \in \mathbf{U}, k \in \mathbb{Z}_+, \quad (4)$$

where the term $(l + e\mathbf{u})^T(l + e\mathbf{u}) - m^T m$ can be either positive, negative, or zero.

Let $\mathbf{u}(k) = \eta(\mathbf{x}(k), \mathbf{v}(k)) : \mathbf{X} \times \mathbf{U} \to \mathbf{U}$ denote a nonlinear feedback control law. If η is locally regular, i.e., $\frac{\partial \eta(\mathbf{x}(k), \mathbf{v}(k))}{\partial \mathbf{v}(k)} \neq \mathbf{0}$ for all $\mathbf{x} \in \mathbf{X}, \mathbf{v} \in \mathbf{U}$, the system

$$\begin{cases} \mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{u}(\mathbf{x}(k), \mathbf{v}(k))) = \bar{f}(\mathbf{x}(k), \mathbf{v}(k)) \\ \mathbf{y}(k) = h(\mathbf{x}(k), \mathbf{u}(\mathbf{x}(k), \mathbf{v}(k))) = \mathbf{v}(k) \end{cases} \quad (5)$$

is referred as the feedback transformed system. Because $h(\mathbf{x}, \mathbf{u})\Big|_{(\mathbf{x}^*, \mathbf{u}^*)} = \mathbf{0}$, $(\mathbf{x}^*, \mathbf{v}^*) = (\mathbf{0}, \mathbf{0})$ remains an isolated equilibrium point of (5).

Definition II.3. *[20] A system of the form (1) is* locally feedback passive *if exist a positive definite storage function $V(\mathbf{x})$ and a regular feedback control law $\mathbf{u}(k) = \eta(\mathbf{x}(k), \mathbf{v}(k)) : \mathbf{X} \times \mathbf{U} \to \mathbf{U}$ with $\mathbf{v}(k)$ as the new input such that the following inequality holds in a neighborhood of $(\mathbf{x}^*, \mathbf{v}^*)$, $V(\bar{f}(\mathbf{x}(k), \mathbf{v}(k))) - V(\mathbf{x}(k)) \le \mathbf{v}^T(k)\mathbf{y}(k)$, $\forall \mathbf{x} \in \mathbf{X}, \mathbf{v} \in \mathbf{U}, k \in \mathbb{Z}_+$.*

The zero dynamics ([1], [21]) of the feedback transformed system (5) are given by constricting the system output to zero using control $\mathbf{u}^*(\mathbf{x}(k), \mathbf{v}^*(k) = \mathbf{0})$, i.e.,

$$\begin{cases} \mathbf{x}(k+1) = \bar{f}(\mathbf{x}(k), \mathbf{0}) \\ \mathbf{y}(k) = \mathbf{0} \end{cases} \quad (6)$$

From Definition II.1, the system zero dynamics are passive if $V(\bar{f}(\mathbf{x}(k), \mathbf{0})) - V(\mathbf{x}(k)) \le 0$, $\forall \mathbf{x} \in \mathbf{X}, \mathbf{v} \in \mathbf{U}, k \in \mathbb{Z}_+$.

Lemma II.1. *[20] Passive zero dynamics are equivalent to a Lyapunov stable system.*

Theorem II.1. *[20] If system (1) is locally passive, then its zero dynamics (6) are also locally passive.*

We now consider a discrete-time nonlinear system with affine input and local relative degree zero [20]

$$\begin{cases} \mathbf{x}(k+1) = f(\mathbf{x}(k)) + g(\mathbf{x}(k))\mathbf{u}(k) \\ \mathbf{y}(k) = h(\mathbf{x}(k)) + J(\mathbf{x}(k))\mathbf{u}(k) \end{cases}. \quad (7)$$

Theorem II.2. *[20] Suppose there exists a positive definite $\mathcal{C}^2$ storage function where $V(f(\mathbf{x}(k)) + g(\mathbf{x}(k))\mathbf{u}(k))$ is quadratic in $\mathbf{u}(k)$, $\forall f, g$. System (7) is locally feedback passive if and only if its zero dynamics are locally passive in a neighborhood of $\mathbf{x}^* = \mathbf{0}$.*

III. Problem Formulation

Consider a discrete-time switched nonlinear system which is affine in control input

$$\begin{cases} \mathbf{x}(k+1) = f_{\sigma(k)}(\mathbf{x}(k)) + g_{\sigma(k)}(\mathbf{x}(k))\mathbf{u}(k) \\ \mathbf{y}(k) = h_{\sigma(k)}(\mathbf{x}(k)) + J_{\sigma(k)}(\mathbf{x}(k))\mathbf{u}(k) \end{cases}, \quad (8)$$

where $\sigma(k) = \{1, 2, \cdots, N\}$ is the switching signal. $f_{\sigma(k)}$, $g_{\sigma(k)}$, $h_{\sigma(k)}$ and $J_{\sigma(k)}$ are in $\mathcal{C}^\infty$. Assume that $(\mathbf{x}^*, \mathbf{u}^*) = (\mathbf{0}, \mathbf{0})$ is a common isolated equilibrium for all the modes. The system is assumed to have local relative degree zero and $J_{\sigma(k)}$ is invertible in a neighborhood of the equilibrium. Let $\mathcal{S}_1$, $\mathcal{S}_2$ denote the set of switching signals of nonpassive and passive modes, respectively. Furthermore, let $\mathcal{S}_1^*$ denote the set of switching signals of feedback passive modes. Therefore, the set of switching signals of non-feedback passive modes is denoted by

$\mathcal{S}_1 \backslash \mathcal{S}_1^*$. Assume that the system starts in one of the passive or feedback passive modes. According to the classical definition of passivity, this system is nonpassive because the increase in storage function is not necessarily bounded by the energy supplied to it when a nonpassive mode is active.

Remark III.1. *The assumption of local relative degree zero and locally invertible $J_{\sigma(k)}$ is reasonable because it is shown in [20] that a discrete-time nonlinear system can be rendered passive if and only if it has relative degree zero and passive zero dynamics.*

Choose the feedback control law $\mathbf{u}(k) = -J_{\sigma(k)}^{-1} h_{\sigma(k)} + J_{\sigma(k)}^{-1} \mathbf{v}(k)$. The transformed system dynamics is

$$\begin{cases} \mathbf{x}(k+1) = f_{\sigma(k)}^*(\mathbf{x}(k)) + g_{\sigma(k)}^*(\mathbf{x}(k))\mathbf{v}(k) \\ \mathbf{y}(k) = \mathbf{v}(k) \end{cases}, \tag{9}$$

where we have $f_{\sigma(k)}^* = f_{\sigma(k)} - g_{\sigma(k)} J_{\sigma(k)}^{-1} h_{\sigma(k)}$ and $g_{\sigma(k)}^* = g_{\sigma(k)} J_{\sigma(k)}^{-1}$. The zero dynamics are

$$\begin{cases} \mathbf{x}(k+1) = f_{\sigma(k)}^*(\mathbf{x}(k)) \\ \mathbf{y}(k) = \mathbf{0} \end{cases}. \tag{10}$$

We assume $f_\sigma^*(\mathbf{0}) = \mathbf{0}$.

IV. Generalized Passivity

In this section, we define the generalized passivity for system (8) which has both passive and nonpassive modes. We also derive a necessary and sufficient condition for system (8) to be passive under the generalized definition.

Definition IV.1. *A switched system of the form (8) is* locally passive *if there exists a positive definite storage function V such that the following passivity inequality holds*

$$V(\mathbf{x}(T+1)) - V(\mathbf{x}(0)) \leq \sum_{k=0}^{T} \mathbf{u}^T(k)\mathbf{y}(k), \quad \forall \mathbf{x} \in \mathbf{X}, \mathbf{u} \in \mathbf{U}, T \in \mathbb{Z}_+. \tag{11}$$

Remark IV.1. *This definition allows system (8) to have nonpassive modes as long as the increase in storage function over a finite horizon T is bounded by the total energy supplied to the system in the period $[0, T]$.*

Theorem IV.1. *Suppose there exists a $\mathcal{C}^2$ storage function V, which is positive definite and $V(f_{\sigma(k)} + g_{\sigma(k)}\mathbf{u}(k))$ is quadratic in $\mathbf{u}$. Then the switched system (8) is passive with the storage function V if and only if there exist real functions $l_{\sigma(k)}(\mathbf{x}(k))$, $m_{\sigma(k)}(\mathbf{x}(k))$, and $e_{\sigma(k)}(\mathbf{x}(k))$ such that $\forall T \in \{0\} \bigcup \mathbb{Z}_+,\ k = 0, 1, \cdots, T$,*

If $\sigma(k) \in \mathcal{S}_1$,

$$V(f_{\sigma(k)}) - V(\mathbf{x}(k)) = l_{\sigma(k)}^T l_{\sigma(k)} - m_{\sigma(k)}^T m_{\sigma(k)} \tag{12}$$

$$\frac{\partial V(z)}{\partial z}\Big|_{z=f_{\sigma(k)}} g_{\sigma(k)} = h_{\sigma(k)}^T + 2 l_{\sigma(k)}^T e_{\sigma(k)} \tag{13}$$

$$g_{\sigma(k)}^T \frac{\partial^2 V(z)}{\partial z^2}\Big|_{z=f_{\sigma(k)}} g_{\sigma(k)} = J_{\sigma(k)}^T + J_{\sigma(k)} + 2 e_{\sigma(k)}^T e_{\sigma(k)} \tag{14}$$

If $\sigma(k) \in \mathcal{S}_2$,

$$V(f_{\sigma(k)}) - V(\mathbf{x}(k)) = -l_{\sigma(k)}^T l_{\sigma(k)} - m_{\sigma(k)}^T m_{\sigma(k)} \tag{15}$$

$$\frac{\partial V(z)}{\partial z}\Big|_{z=f_{\sigma(k)}} g_{\sigma(k)} = h_{\sigma(k)}^T - 2 l_{\sigma(k)}^T e_{\sigma(k)} \tag{16}$$

$$g_{\sigma(k)}^T \frac{\partial^2 V(z)}{\partial z^2}\Big|_{z=f_{\sigma(k)}} g_{\sigma(k)} = J_{\sigma(k)}^T + J_{\sigma(k)} - 2 e_{\sigma(k)}^T e_{\sigma(k)} \tag{17}$$

$$\begin{aligned} &\sum_{\substack{k:\sigma(k)\in\mathcal{S}_1 \\ k \leq T}} (l_{\sigma(k)} + e_{\sigma(k)}\mathbf{u}(k))^T (l_{\sigma(k)} + e_{\sigma(k)}\mathbf{u}(k)) \\ &- \sum_{\substack{k:\sigma(k)\in\mathcal{S}_2 \\ k \leq T}} (l_{\sigma(k)} + e_{\sigma(k)}\mathbf{u}(k))^T (l_{\sigma(k)} + e_{\sigma(k)}\mathbf{u}(k)) \\ &\leq \sum_{k=0}^{T} m_{\sigma(k)}^T m_{\sigma(k)} \end{aligned} \tag{18}$$

Proof: The switched system (8) is composed of both passive and nonpassive modes. If the system is in a passive mode, i.e., $\sigma(k) \in \mathcal{S}_2$, Equation (3) holds. Similarly, if the system is in a nonpassive mode ($\sigma(k) \in \mathcal{S}_1$), Equation (4) holds for any choice of $\mathcal{C}^2$ storage function V.

(Necessity) We will derive i) Equations (12)-(14) from Equation (4), ii) Equations (15)-(17) from Equation (3), and iii) the inequality (18) based on Equations (3) and (4).

i) Equation (12) is obtained by setting $\mathbf{u}(k) = \mathbf{0}$ in Equation (4). Next, we take the first-order derivative of Equation (4) with respect to $\mathbf{u}(k)$ and then set $\mathbf{u}(k) = \mathbf{0}$ to derive Equation (13). We further take the second-order derivative of Equation (4) with respect to $\mathbf{u}(k)$ and set $\mathbf{u}(k) = \mathbf{0}$ and obtain Equation (14).

ii) Similarly, by setting $\mathbf{u}(k) = \mathbf{0}$ in Equation (3), we obtain Equation (15). Take the first and second-order derivatives of Equation (3) with respect to $\mathbf{u}(k)$ and set $\mathbf{u}(k) = \mathbf{0}$, we derive Equations (16) and (17).

iii) Sum Equations (3) and (4) from $k = 0$ to T. To have the generalized passivity inequality (11) hold, the inequality (18) must be satisfied.

(Sufficiency) We derive i) Equation (4) from Equations (12)-(14), and ii) Equation (3) from Equations (15)-(17).

i) We right multiply Equation (13) by $\mathbf{u}(k)$ and add Equation (12) to obtain

$$\begin{aligned} &V(f_{\sigma(k)}) - V(\mathbf{x}(k)) + \frac{\partial V(z)}{\partial z}\Big|_{z=f_{\sigma(k)}} g_{\sigma(k)}\mathbf{u}(k) \\ &= h_{\sigma(k)}^{\mathrm{T}}\mathbf{u}(k) + (l_{\sigma(k)} + e_{\sigma(k)}\mathbf{u}(k))^{\mathrm{T}}(l_{\sigma(k)} + e_{\sigma(k)}\mathbf{u}(k)) \\ &\quad - m_{\sigma(k)}^{\mathrm{T}} m_{\sigma(k)} - \mathbf{u}^{\mathrm{T}}(k) e_{\sigma(k)}^{\mathrm{T}} e_{\sigma(k)} \mathbf{u}(k). \end{aligned} \tag{19}$$

Similarly, we left multiply Equations (14) by $\mathbf{u}^{\mathrm{T}}(k)$ and right multiply by $\mathbf{u}(k)$. We then multiply the resulting equation by $\frac{1}{2}$ and add it to Equation (19), it follows

$$\begin{aligned}
&V(f_{\sigma(k)}) - V(\mathbf{x}(k)) + \frac{\partial V(z)}{\partial z}\Big|_{z=f_{\sigma(k)}} g_{\sigma(k)}\mathbf{u}(k) \\
&+\frac{1}{2}\mathbf{u}^{\mathrm{T}}(k) g^{\mathrm{T}}_{\sigma(k)} \frac{\partial^2 V(z)}{\partial z^2}\Big|_{z=\mathbf{f}_{\sigma(k)}} g_{\sigma(k)}\mathbf{u}(k) \\
=\ & h^{\mathrm{T}}_{\sigma(k)}\mathbf{u}(k) + \frac{1}{2}\mathbf{u}^{\mathrm{T}}(k)\left(J^{\mathrm{T}}_{\sigma(k)} + J_{\sigma(k)}\right)\mathbf{u}(k) \\
&+(l_{\sigma(k)} + e_{\sigma(k)}\mathbf{u}(k))^{\mathrm{T}}(l_{\sigma(k)} + e_{\sigma(k)}\mathbf{u}(k)) \\
&-m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)}. \qquad (20)
\end{aligned}$$

Because the storage function $V(f_{\sigma(k)} + g_{\sigma(k)}\mathbf{u}(k))$ is quadratic in $\mathbf{u}(k)$, Equation (20) is the Taylor series expansion of Equation (4) at $\mathbf{u}(k) = \mathbf{0}$.

ii) Following similar derivation, we can obtain Equation (3) from Equations (15)-(17).

iii) Sum up Equation (4) for all the time steps when $\sigma(k) \in \mathcal{S}_1$ and Equation (3) for all the time steps when $\sigma(k) \in \mathcal{S}_2$ up to T. According to inequality (18), we have $V(\mathbf{x}(T+1)) - V(\mathbf{x}(0)) \le \sum_{k=0}^{T}\mathbf{u}(k)\mathbf{y}(k)$, i.e., the switched system (8) is passive under the generalized definition IV.1.

V. Generalized Feedback Passivity

In this section, we extend the generalized passivity results to generalized feedback passivity by passivating some of the nonpassive modes using feedback. We relate the feedback passivity of system (8) with its zero dynamics (10). From Definition IV.1, the system zero dynamics (10) are *locally passive* if

$$V(\mathbf{x}(T+1)) - V(\mathbf{x}(0)) \le 0,\ \forall \mathbf{x} \in \mathbf{X}, T \in \mathbb{Z}_+. \quad (21)$$

Consider a new control input $\mathbf{w} \in \mathbf{U}$ for the transformed dynamics (9), $\mathbf{y}(k) = \mathbf{v}(k) = \bar{h}_{\sigma(k)} + \bar{J}_{\sigma(k)}\mathbf{w}(k)$, where $\bar{J}_{\sigma(k)}$ is assumed to be symmetric and

$$\begin{aligned}
\bar{J}_{\sigma(k)} &= \left(\frac{1}{2}{g^*_{\sigma(k)}}^{\mathrm{T}}\frac{\partial^2 V}{\partial z^2}\Big|_{z=f^*_{\sigma(k)}} g^*_{\sigma(k)}\right)^{-1} \\
\bar{h}_{\sigma(k)} &= -\bar{J}_{\sigma(k)}\left(\frac{\partial V}{\partial z}\Big|_{z=f^*_{\sigma(k)}} g^*_{\sigma(k)}\right)^{\mathrm{T}} \qquad (22)
\end{aligned}$$

The new system dynamics is given by

$$\left\{\begin{array}{l} \mathbf{x}(k+1) = f^*_{\sigma(k)} + g^*_{\sigma(k)}\bar{h}_{\sigma(k)} + g^*_{\sigma(k)}\bar{J}_{\sigma(k)}\mathbf{w}(k) \\ \mathbf{y}(k) = \bar{h}_{\sigma(k)} + \bar{J}_{\sigma(k)}\mathbf{w}(k) \end{array}\right. \quad (23)$$

Definition V.1. *The switched system (8) is* locally feedback passive *if there exist a positive definite storage function* $V(\mathbf{x})$ *and a control law* $\mathbf{w}$ *such that the following inequality holds*

$$V(\mathbf{x}(T+1)) - V(\mathbf{x}(0)) \le \sum_{k=0}^{T}\mathbf{w}^T(k)\mathbf{y}(k), \quad \forall \mathbf{x} \in \mathbf{X}, \mathbf{w} \in \mathbf{U}, T \in \mathbb{Z}_+. \quad (24)$$

Lemma V.1. *If the switched system (8) is locally feedback passive, then its zero dynamics (10) are also locally passive.*

Proof: Because system (8) is locally feedback passive, the inequality (24) holds. The zero dynamics enforces $\mathbf{y}(k) = \mathbf{0}$. Hence, the inequality (24) is converted to the inequality (21). That is, the zero dynamics (10) are locally passive.

Lemma V.2. *The passive and feedback passive modes of the switched system (8) correspond to the passive modes of the zero dynamics (10). The non-feedback passive modes of (8) correspond to the nonpassive modes of (10).*

Proof: According to Theorem II.1, if a mode is locally passive, then the corresponding zero dynamics are also locally passive. According to Theorem II.2, a locally feedback (non)passive mode has locally (non)passive zero dynamics.

Theorem V.1. *Suppose there exists a $\mathcal{C}^2$ positive definite storage function $V(\mathbf{x})$ where $V(f_{\sigma(k)} + g_{\sigma(k)}\mathbf{u}(k))$ is quadratic in $\mathbf{u}$. Then the switched nonlinear system (8) is locally feedback passive if and only if its zero dynamics (10) are passive.*

Proof: The necessity is given by Lemma V.1. We next prove the sufficiency based on Theorem IV.1. Because $V(f_{\sigma(k)} + g_{\sigma(k)}\mathbf{u}(k))$ is quadratic in $\mathbf{u}$, the Taylor series expansion for $V(f^*_{\sigma(k)} + g^*_{\sigma(t)}\bar{h}_{\sigma(k)})$ can be expressed as

$$\begin{aligned}
&V(f^*_{\sigma(k)} + g^*_{\sigma(k)}\bar{h}_{\sigma(k)}) \\
&= V(f^*_{\sigma(k)}) + \frac{\partial V}{\partial z}\Big|_{z=f^*_{\sigma(k)}} g^*_{\sigma(k)}\bar{h}_{\sigma(t)} \\
&+\frac{1}{2}\bar{h}^{\mathrm{T}}_{\sigma(k)}(g^*_{\sigma(k)})^{\mathrm{T}}\frac{\partial^2 V}{\partial z^2}\Big|_{z=f^*_{\sigma(k)}} g^*_{\sigma(k)}\bar{h}_{\sigma(k)}. \qquad (25)
\end{aligned}$$

Use Equation (22) to Equation (25), we have

$$V(f^*_{\sigma(k)} + g^*_{\sigma(k)}\bar{h}_{\sigma(k)}) = V(f^*_{\sigma(k)}). \quad (26)$$

According to Lemma V.2, when $\sigma(k) \in \mathcal{S}^*_1 \bigcup \mathcal{S}_2$, the corresponding zero dynamics are passive. Based on Equations (4) and Equation (26), it follows that

$$\begin{aligned}
&V(f^*_{\sigma(k)}) - V(\mathbf{x}(k)) = V(f^*_{\sigma(k)} + g^*_{\sigma(k)}\bar{h}_{\sigma(k)}) - V(\mathbf{x}(k)) \\
&= -\left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right)^{\mathrm{T}}\left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right) \\
&\quad -m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)}. \qquad (27)
\end{aligned}$$

Similarly, if $\sigma(k) \in \mathcal{S}_1 \setminus \mathcal{S}^*_1$, we have

$$\begin{aligned}
&V(f^*_{\sigma(k)}) - V(\mathbf{x}(k)) = \left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right)^{\mathrm{T}} \\
&\cdot\left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right) - m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)}. \qquad (28)
\end{aligned}$$

i) We first consider the case when $\sigma(k) \in \mathcal{S}^*_1 \bigcup \mathcal{S}_2$. Equation (27) gives the passivity condition (15) with $l_{\sigma(k)} = l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}$ and $m_{\sigma(k)} = m_{\sigma(k)}$.

Next, subtract V from Equation (25) and substitute Equation (27) into the resulting equation, we have

$$V(f^*_{\sigma(k)} + g^*_{\sigma(k)}\bar{h}_{\sigma(k)}) - V(\mathbf{x}(k))$$

$$
\begin{aligned}
&= -\left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right)^{\mathrm{T}}\left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right) \\
&\quad -m_{\sigma(k)}^{\mathrm{T}} m_{\sigma(k)} + \frac{\partial V}{\partial z}\Big|_{z=f^*_{\sigma(k)}} g^*_{\sigma(k)}\bar{h}_{\sigma(k)} \\
&\quad +\frac{1}{2}\bar{h}^{\mathrm{T}}_{\sigma(k)}(g^*_{\sigma(k)})^{\mathrm{T}}\frac{\partial^2 V}{\partial z^2}\Big|_{z=f^*_{\sigma(k)}} g^*_{\sigma(k)}\bar{h}_{\sigma(k)}. \qquad (29)
\end{aligned}
$$

Differentiate both sides of Equation (29) with respect to $\bar{h}_{\sigma(k)}$, right multiply the result by $\bar{J}_{\sigma(k)}$, and substitute Equation (22), we have

$$
\begin{aligned}
&\frac{\partial V}{\partial z}\Big|_{z=f^*_{\sigma(k)}+g^*_{\sigma(k)}\bar{h}_{\sigma(k)}} g^*_{\sigma(k)}\bar{J}_{\sigma(k)} \\
&= \bar{h}^{\mathrm{T}}_{\sigma(k)} - 2\left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right)^{\mathrm{T}} e_{\sigma(k)}\bar{J}_{\sigma(k)}. \qquad (30)
\end{aligned}
$$

Therefore, Equation (30) gives the passivity condition (16) with $l_{\sigma(k)} = l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}$ and $e_{\sigma(k)} = e_{\sigma(k)}\bar{J}_{\sigma(k)}$.

Take the second-order derivatives of Equation (29) with respect to $\bar{h}_{\sigma(k)}$. Left multiply the results by $\bar{J}^{\mathrm{T}}_{\sigma(k)}$ and right multiply by $\bar{J}_{\sigma(k)}$, use Equation (22), we yield the following equation

$$
\begin{aligned}
&\left[g^*_{\sigma(k)}\bar{J}_{\sigma(k)}\right]^{\mathrm{T}}\frac{\partial V}{\partial z}\Big|_{z=f^*_{\sigma(k)}+g^*_{\sigma(k)}\bar{h}_{\sigma(k)}} g^*_{\sigma(k)}\bar{J}_{\sigma(k)} \\
&= \bar{J}^{\mathrm{T}}_{\sigma(k)} + \bar{J}_{\sigma(k)} + 2\bar{J}^{\mathrm{T}}_{\sigma(k)} e^{\mathrm{T}}_{\sigma(k)} e_{\sigma(k)}\bar{J}_{\sigma(k)} \qquad (31)
\end{aligned}
$$

Therefore, Equation (31) gives the passivity condition (17) with $e_{\sigma(k)} = e_{\sigma(k)}\bar{J}_{\sigma(k)}$.

ii) Similarly, when $\sigma(k) \in \mathcal{S}_1 \setminus \mathcal{S}_1^*$, we can derive the passivity conditions (12)-(14) following the above procedure.

iii) We now prove that there exists a control $\mathbf{w}(k)$ such that the passivity condition (18)

$$
\begin{aligned}
&\sum_{\substack{k:\sigma(k)\in\mathcal{S}_1\setminus\mathcal{S}_1^* \\ k\le T}} (l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}(k) + e_{\sigma(k)}\bar{J}_{\sigma(k)}(k)\mathbf{w}(k))^{\mathrm{T}} \\
&\quad \cdot(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}(k) + e_{\sigma(k)}\bar{J}_{\sigma(k)}(k)\mathbf{w}(k)) - \\
&\sum_{\substack{k:\sigma(k)\in\mathcal{S}_2\bigcup\mathcal{S}_1^* \\ k\le T}} (l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}(k) + e_{\sigma(k)}\bar{J}_{\sigma(k)}(k)\mathbf{w}(k))^{\mathrm{T}} \\
&\quad \cdot(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}(k) + e_{\sigma(k)}\bar{J}_{\sigma(k)}(k)\mathbf{w}(k)) \\
&\le \sum_{k=0}^{T} m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)} \qquad (32)
\end{aligned}
$$

holds with $l_{\sigma(k)} = l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}$, $e_{\sigma(k)} = e_{\sigma(k)}\bar{J}_{\sigma(k)}$ and $m_{\sigma(k)} = m_{\sigma(k)}$. The equality holds if and only if $l_{\sigma(k)} = e_{\sigma(k)} = m_{\sigma(k)} = \mathbf{0}$.

Because the zero dynamics are passive, we have

$$
\begin{aligned}
&\sum_{\substack{k:\sigma(k)\in\mathcal{S}_1\setminus\mathcal{S}_1^* \\ k\le T}} (l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}(k))^{\mathrm{T}}(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}(k)) \\
-&\sum_{\substack{k:\sigma(k)\in\mathcal{S}_2\bigcup\mathcal{S}_1^* \\ k\le T}} (l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}(k))^{\mathrm{T}}(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}(k)) \\
&\le \sum_{k=0}^{T} m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)}. \qquad (33)
\end{aligned}
$$

Note that when $\sigma(k) \in \mathcal{S}_1^* \bigcup \mathcal{S}_2$, the zero dynamics are passive and the following equation holds $V(f^*_{\sigma(k)}) - V(\mathbf{x}(k)) = -l^{\mathrm{T}}_{\sigma(k)} l_{\sigma(k)} - m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)}$. According to Equation (27), we have $-l^{\mathrm{T}}_{\sigma(k)} l_{\sigma(k)} - m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)} = -\left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right)^{\mathrm{T}}\left(l_{\sigma(k)} + e_{\sigma(k)}\bar{h}_{\sigma(k)}\right) - m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)}$. For this to hold for any $\bar{h}_{\sigma(k)}$, it follows that $e_{\sigma(k)} = \mathbf{0}$. Similar derivation holds when $\sigma(k) \in \mathcal{S}_1 \setminus \mathcal{S}_1^*$.

Thus, with $e_{\sigma(k)} = \mathbf{0}$, the inequality (32) reduces to $\sum_{\substack{k:\sigma(k)\in\mathcal{S}_1\setminus\mathcal{S}_1^* \\ k\le T}} l^{\mathrm{T}}_{\sigma(k)} l_{\sigma(k)} - \sum_{\substack{k:\sigma(k)\in\mathcal{S}_2\bigcup\mathcal{S}_1^* \\ k\le T}} l^{\mathrm{T}}_{\sigma(k)} l_{\sigma(k)} \le \sum_{k=0}^{T} m^{\mathrm{T}}_{\sigma(k)} m_{\sigma(k)}$. Because the zero dynamics of the switched system are passive under the generalized passivity definition, the above inequality naturally holds. Hence, all the conditions in Theorem IV.1 are satisfied and the switched system (8) is locally feedback passive.

VI. Switching Signal for Passive Zero Dynamics

In this section, we derive a lower bound on the ratio of the total activation time of (feedback) passive versus nonpassive modes such that the system zero dynamics (10) are passive.

Assume that the zero dynamics of each subsystem are Lipschitz, i.e.,

$$
\begin{aligned}
&|f^*_\sigma(\mathbf{x}) - f^*_\sigma(\mathbf{x}')| \le L_\sigma|\mathbf{x}-\mathbf{x}'|,\ \forall \mathbf{x},\mathbf{x}', \\
&\begin{cases} 0 \le L_\sigma < 1 & \text{if } \sigma\in\mathcal{S}_2\bigcup\mathcal{S}_1^* \\ L_\sigma \ge 1 & \text{if } \sigma\in\mathcal{S}_1\setminus\mathcal{S}_1^* \end{cases}. \qquad (34)
\end{aligned}
$$

Let

$$
\begin{aligned}
L_1 &= \max_\sigma\{L_\sigma|\sigma\in\mathcal{S}_2\bigcup\mathcal{S}_1^*\},\ L_1\in[0,1), \\
L_2 &= \max_\sigma\{L_\sigma|\sigma\in\mathcal{S}_1\setminus\mathcal{S}_1^*\}, L_2\in[1,+\infty)
\end{aligned}
$$

with $\sigma = 1, 2, \cdots, N$.

Theorem VI.1. *Design the switching signal such that*

$$
\frac{K^-(0,T)}{K^+(0,T)} \ge \frac{\ln L_2 - \ln L_0}{\ln L_0 - \ln L_1}, \qquad (35)
$$

where $L_0 \in (L_1, 1)$, $K^-(0,T)$ is the total activation time of the (feedback) passive modes, and $K^+(0,T)$ is the total activation time of the non-feedback passive modes during time interval $[0,T)$, $\forall T\in\{0\}\bigcup\mathbb{Z}_+$. The zero dynamics (10) are passive under the switching signal (35).

Proof: Let $0 = k_0 < k_1 < k_2 \cdots$ denote the switching points and $\sigma(k_{i-1}) = p_i$. Assume k_i is the i_{th} switch, at time $T \in [k_i, k_{i+1})$, the state of the zero dynamics evolve as

$$
\mathbf{x}(T) = (\underbrace{f^*_{p_{i+1}}\circ f^*_{p_{i+1}}\dots f^*_{p_{i+1}}\circ f^*_{p_{i+1}}}_{T-k_i}\underbrace{f^*_{p_i}\circ f^*_{p_i}\dots f^*_{p_i}\circ f^*_{p_i}}_{k_i-k_{i-1}} \cdots \underbrace{f^*_{p_1}\circ f^*_{p_1}\cdots f^*_{p_1}\circ f^*_{p_1}}_{k_1-k_0})\mathbf{x}(0).
$$

Since we assume that $f^*_\sigma(\mathbf{0}) = \mathbf{0}$, based on (34), we have $|\mathbf{x}(T)| \le L_1^{K^-(0,T)} L_2^{K^+(0,T)}|\mathbf{x}(0)|$. From the switching signal (35), we have

$$|\mathbf{x}(T)| \leq L_1^{K^-(0,T)} L_2^{K^+(0,T)} |\mathbf{x}(0)| \leq L_0^T |\mathbf{x}(0)|. \quad (36)$$

Hence, the zero dynamics (10) are stable with $L_0 \in (L_1, 1)$. According to Lemma II.1, the zero dynamics are passive.

VII. Stability of the Switched Systems

Theorem VII.1. *Under the switching signal (35), system (8) with zero dynamics satisfying the Lipschitz condition (34) is locally feedback passive with a $\mathcal{C}^2$ storage function. Let $\phi : \mathbb{R}^m \to \mathbb{R}^m$ be any smooth function such that $\phi(\mathbf{0}) = \mathbf{0}$ and $\mathbf{y}^T\phi(\mathbf{y}) > 0$ for all $\mathbf{y} \neq \mathbf{0}$. Then the smooth feedback $\mathbf{w}(k) = -\phi(\mathbf{y}(k))$ locally asymptotically stabilizes the equilibrium $\mathbf{x}^* = \mathbf{0}$, provided that (8) is locally locally zero state detectable (ZSD) ([7]).*

Proof: According to Theorem VI.1, under the switching signal (35), system zero dynamics (10) satisfying the Lipschitz condition (34) are locally passive. From Theorem V.1, the switched system (8) is locally feedback passive given the passive zero dynamics (10). Under the feedback law $\mathbf{w}(k) = -\phi(\mathbf{y}(k))$, we have $V(\mathbf{x}(T+1)) - V(\mathbf{x}(0)) \leq \sum_{k=0}^{T} \mathbf{y}^{\mathrm{T}}(k)\mathbf{w}(k) = -\sum_{k=0}^{T} \mathbf{y}^{\mathrm{T}}(k)\phi(\mathbf{y}(k)) \leq 0,\ \forall \mathbf{x} \in \mathbf{X}, \mathbf{w} \in \mathbf{U}$.

Define the sequence of time steps $\{t_i\}$ such that $t_0 = 1$ and $t_i =$ the least time $> t_{i-1}$ such that $\sigma(t_i - 1) \in \mathcal{S}_1 \setminus \mathcal{S}_1^*$ and $\sigma(t_i) \in \mathcal{S}_1^* \bigcup \mathcal{S}_2$. Choosing $T + 1 = t_1$ yields in particular $V(\mathbf{x}(t_1)) - V(\mathbf{x}(0)) \leq \mathbf{0}, \forall \mathbf{x}(\cdot) \in \mathbf{X}$. We can repeat the same argument starting from time t_i with $\mathbf{x}(t_i)$ as the initial condition. Thus we obtain the series of inequalities $V(\mathbf{x}(t_{i+1})) - V(\mathbf{x}(t_i)) \leq 0,\ \forall i = 0, 1, \cdots, \forall \mathbf{x} \in \mathbf{X}$. Since $\{k_i\}$ is an infinite sequence, $V(\mathbf{x}(\cdot))$ is a Lyapunov function for the switched system.

The asymptotic stability follows from ZSD. Observe that all the trajectories of the (feedback) passive system eventually approach the invariant set $I = \{\mathbf{x} \in \mathbb{R}^n : V(\mathbf{x}(k+1)) = V(\mathbf{x}(k))\}$, which implies $\mathbf{y}^{\mathrm{T}}(k)\phi(\mathbf{y}(k)) = 0,\ \forall k \in \{0\} \bigcup \mathbb{Z}_+$. Hence $\mathbf{y}(k) = 0$ and $\mathbf{w}(k) = -\phi(\mathbf{y}(k)) = \mathbf{0}\ \forall k \in \{0\} \bigcup \mathbb{Z}_+$. Thus by ZSD $\lim_{k\to\infty} \mathbf{x}(k) = \mathbf{0}$.

VIII. Conclusion

In this paper, we generalize the classical passivity definition for a class of discrete-time switched nonlinear systems with both passive and nonpassive modes. We propose necessary and sufficient conditions for such systems to be locally passive. Using the passivation methods, some of the nonpassive modes are rendered passive using feedback. The switched nonlinear system is proved to be locally feedback passive if and only if its zero dynamics are locally passive. A lower bound on the total activation time of non-feedback passive modes versus (feedback) passive modes is derived to guarantee passive zero dynamics if it satisfies the Lipschitz condition. We prove that the system equilibrium point can achieve asymptotic stability using output feedback.

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References

[1] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, 2002.

[2] J. Bao and P. L. Lee, *Process Control: the Passive Systems Approach*. Springer, 2007.

[3] J. C. Willems, "Dissipative Dynamical Systems Part I: General Theory," *Archive for Rational Mechanics and Analysis*, vol. 45, no. 5, pp. 321–351, 1972.

[4] J. C. Willems, "Dissipative Dynamical Systems Part II: Linear Systems with Quadratic Supply Rates," *Archive for Rational Mechanics and Analysis*, vol. 45, no. 5, pp. 352–393, 1972.

[5] D. J. Hill and P. J. Moylan, "The Stability of Nonlinear Dissipative Systems," *IEEE Transactions on Automatic Control*, vol. 21, no. 5, pp. 708–711, 1976.

[6] C. I. Byrnes, A. Isidiri, and J. C. Willems, "Passivity, Feedback Equivalence, and the Global Stabilization of Minimum Phase Nonlinear Systems," *IEEE Transactions on Automatic Control*, vol. 36, pp. 1228–1240, November 1991.

[7] W. Lin and C. I. Byrness, "Passivity and Absolute Stabilization of a Class of Discrete-time Nonlinear Systems," *Automatica*, vol. 31, no. 2, pp. 263–267, 1995.

[8] A. Y. Pogromsky, M. Jirstrand, and P. Spangeus, "On Stability and Passivity of a Class of Hybrid Systems," *37th IEEE Conference on Decision and Control*, pp. 3705–3710, 1998.

[9] W. Chen and M. Saif, "Passivity and Passivity Based Controller Design of a class of Switched Control Systems," *Proceedings of 16th IFAC World Congress*, pp. 143–147, 2005.

[10] S. Mastellone, D. Lee, and M. W. Spong, "Master-Slave Synchronization with Switching Communication through Passive Model-Based Control Design," *American Control Conference*, pp. 3203–3208, 2006.

[11] J. Zhao and D. Hill, "Dissipativity Based Stability of Switched Systems with Statedependent Switchings," *46th IEEE Conference on Decision and Control*, pp. 4027–4032, 2007.

[12] A. Bemporad, G. Bianchini, and F. Brogi, "Passivity Analysis andd Passification of Discrete-time Hybrid Systems," *IEEE Transactions on Automatic Control*, vol. 53, pp. 1004–1009, May 2008.

[13] M. Žefran, F. Bullo, and M. Stein, "A Notion of Passivity for Hybrid Systems," *40th IEEE Conference on Decision and Control*, 2001.

[14] J. Zhao and D. J. Hill, "Dissipativity Theory for Switched Systems," *IEEE Transactions on Automatic Control*, vol. 53, pp. 941–953, May 2008.

[15] Y. Liu, G. S. Stojanovski, M. J. S. abd G. M. Dimirovski, and J. Zhao, "Passivity and Feedback Equivalence of Switched Nonlinear Systems with Storage-Like Functions," *IEEE International Conference on Systems Man and Cybernetics (SMC)*, pp. 4137 – 4141, 2010.

[16] M. J. McCourt and P. J. Antsaklis, "Control Design for Switched Systems using Passivity Indices," *American Control Conference*, pp. 2499–2504, June/July 2010.

[17] Y. Liu and J. Zhao, "Stabilization of Switched Nonlinear Systems with Passive and Non-Passive Subsystems," *Nonlinear Dynamics*, vol. 67, no. 3, pp. 1709–1716, 2012.

[18] E. M. Navarro-López and E. Fossas-Colet, "Dissipativity, Passivity and Feedback Passivity in the Nonlinear Discrete-Time Setting," *15th IFAC Triennial World Congress*, July 2002.

[19] D. Aeyels and J. Peuteman, "A New Asymptotic Stability Criterion for Nonlinear Time-Variant Differential Equations," *IEEE Transactions on Automatic Control*, vol. 43, no. 7, pp. 968–971, 1998.

[20] E. M. Navarro-López, *Dissipativity and Passivity-Related Properties in Nonlinear Discrete-Time Systems*. PhD thesis, Universitat Politécnica de Catalunya, June 2002.

[21] C. I. Bymes and W. Lin, "Losslessness, Feedback Equivalence, and the Global Stabilization of Discrete-Time Nonlinear Systems," *IEEE Transactions on Automatic Control*, vol. 39, pp. 83–98, January 1994.