Distributed Formation Control of Networked Passive Systems with Event-driven Communication

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Abstract— **This paper is focused on the formation control problem of networked passive systems with event-driven communication. The data transmissions between agents are event-based and distributed control laws to achieve formation under the event-driven communication strategy are obtained. We first derive a triggering condition to achieve distance-based formation among the agents with an ideal network model being assumed; we then consider the case when there are constant network induced delays between coupled agents. Simulations are provided to validate our results.**

I. Introduction

A multi-agent system, in general, can be defined as a network of a number of loosely coupled dynamic units that are called agents. In real-life, each agent can be a robot, a vehicle, or a dynamic sensor, etc. The main purpose of using multi-agent systems is to collectively reach goals that are difficult to achieve by an individual agent or a monolithic system. When the main problem of interest in control of multi-agent systems is to establish a well-structured motion, the term *swarm* or sometimes *formation* is used. There exists a number of different formation coordination and control approaches investigated in the system and control literature, see [3], [4] and [9]-[14]. Most of these work assumed a synchronous implementation strategy regarding the control action updates and the scheduling of data transmissions among the coupled agents. Note that multi-agent dynamic systems are distributed systems which usually act in an asynchronous manner and in general, it is difficult to implement synchronous motions on them. However, analyzing the dynamics of asynchronous systems is more difficult compared to their synchronous counterparts.

A deterministic event-triggered control strategy is introduced by Tabuada in [6] for a single loop sensor-actuator networked control system. In [6], the control actuation is triggered whenever a certain error becomes large enough concerning the norm of the state. It is assumed that the nominal system is Input-to-State Stable with respect to measurement errors. Extensions to output feedback based event-triggering control has been studied in [17] and [18]-[19]. Event-triggering stabilization for distributed networked control systems has been studied in [16]. Event-triggered consensus problem is reported in [5]. However, there has not been much published work on formation control of multi-agent systems with distributed event-driven control.

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In the present paper, we propose a distributed event-driven control strategy for formation control of networked passive systems. The distributed triggering condition is derived based on the observation that the entire networked control system is Output Strictly Passive (OSP) with some error signal as input and some disagreement signal as output when an ideal network model is assumed. We further propose a set-up to render the entire networked control system OSP in the presence of constant network induced delays and derive distributed triggering conditions to achieve distance-based formation when constant network induced delays are considered.

The rest of this paper is organized as follows. After some mathematical preliminaries on passive systems and graph theory, the models of the agents (passive systems) and the model of the communication network (graph Laplacian) are given in Section II. The main assumptions and the problem statement are provided in Section III. In Section IV, we derive a triggering condition to achieve distance-based formation among the agents when an ideal network model is assumed; in Section V, we consider the case when there are constant network induced delays between coupled agents and simulations are provided to validate our results; finally, concluding remarks are made in Section VI.

II. Background Material

A. Passivity

Consider the following dynamical system which can be used to describe both linear and nonlinear control systems:

$$H : \begin{cases} \dot{x} = f(x,u) \\ y = h(x,u) \end{cases} \tag{1}$$

where $x \in \mathbb{X} \subset \mathbb{R}^n$, $u \in \mathbb{U} \subset \mathbb{R}^m$ and $y \in \mathbb{Y} \subset \mathbb{R}^m$ are the state, input and output variables, respectively, and $\mathbb{X}$, $\mathbb{U}$ and $\mathbb{Y}$ are the state, input and output spaces, respectively. The representation $\phi(t, t_0, x_0, u)$ is used to denote the state at time t reached from the initial state x_0 at t_0 under the control u.

***Definition** 1:* [1] The supply rate $\omega(t) = \omega(u(t), y(t))$ is a real valued function defined on $\mathbb{U}\times\mathbb{Y}$, such that for any $u(t) \in \mathbb{U}$ and $x_0 \in \mathbb{X}$ and $y(t) = h(\phi(t, t_0, x_0, u), u)$, $\omega(t)$ satisfies

$$\int_{t_0}^{t_1} |\omega(\tau)| d\tau < \infty. \tag{2}$$

***Definition** 2:* [1] System H with supply rate $\omega(t)$ is said to be dissipative if there exists a nonnegative real function $V : \mathbb{X} \to \mathbb{R}^+$ ($\mathbb{R}^+$ is the set of nonnegative real numbers), called the storage function, such that, for all $t_1 \geq t_0 \geq 0$, $x_0 \in \mathbb{X}$

and $u \in \mathbb{U}$,

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} \omega(\tau) d\tau \tag{3}$$

where $x_1 = \phi(t_1, t_0, x_0, u)$. If V is C^1, then $\dot{V}(x) \leq \omega(t), \ \ \forall t \geq 0$. Passive systems are special cases of dissipative systems defined as follows.

***Definition** 3:* [1] System H is said to be **passive** if there exists a storage function V such that

$$V(x_1) - V(x_0) \leq \int_{t_0}^{t_1} u(\tau)^T y(\tau) d\tau. \tag{4}$$

If V is C^1, then

$$\dot{V} \leq u(t)^T y(t), \ \forall t \geq 0. \tag{5}$$

B. Graph Theory

The information exchange topology between agents can be modeled as a graph. In the following, we give some basic terminologies and definitions from graph theory [7].

A *directed graph* is a graph whose edges have direction and are called arcs. Consider a finite weighted directed graph $G := (V, E)$ with no self-loops and *adjacency matrix* A, where V denotes the set of all vertices, E denotes the set of all edges, and $A := [a_{ij}]$ with $a_{ij} > 0$ if there is a directed edge from vertex i into vertex j, and $a_{ij} = 0$ otherwise. The *in-degree* and *out-degree* of vertex k are given by $d_i(k) = \sum_j a_{jk}$ and $d_o(k) = \sum_j a_{kj}$ respectively.

The *Laplacian* matrix of a directed graph is defined as $L = D - A$, where D is the diagonal matrix of vertex out-degrees.

***Definition** 4:* A directed graph is *strongly connected* if for any pair of distinct vertices v_i and v_j, there is a directed path from v_i to v_j.

***Definition** 5:* A vertex is *balanced* if its in-degree is equal to its out-degree. A directed graph is balanced if every vertex is balanced.

***Definition** 6:* A *path* of length r in a directed graph is a sequence $v_0, \ldots, v_r$ of $r+1$ distinct vertices such that for every $i \in \{0, \ldots, r-1\}$, (v_i, v_{i+1}) is an edge. A *weak path* is a sequence $v_0, \ldots, v_r$ of $r+1$ distinct vertices such that for each $i \in \{0, \ldots, r-1\}$, either (v_i, v_{i+1}) or (v_{i+1}, v_i) is an edge. A directed graph is *weakly connected* if any two vertices can be joined by a weak path.

***Lemma** 1:* Let G be a directed graph and assume it is balanced. Then G is strongly connected if and only if it is weakly connected.

III. Assumptions and Problem Statement

The evolution of multi-agent systems depends fundamentally on their information exchange topology. We list below two assumptions regarding the information exchange topology that we will make in the sequel. The specific assumption(s) used will be made clear in the statement of each result.

A1. The underlying communication graph is weakly connected in time and form a directed balanced graph with respect to information exchange.

A2. The underlying communication graph is weakly connected in time, bidirectional and balanced.

***Definition** 7:* For a group of N agents, the agents are said to establish a *distance-based formation* if

$$\lim_{t \to \infty} \|p_j(t) - p_i(t)\|_2 = d_{ij}, \ \forall j \in \mathcal{N}_i,$$

for $i = 1, \ldots, N$, where $\mathcal{N}_i$ denotes the set of agents sending information to agent i; $p_i(t)$ denotes the spatial coordinates of agent i; $d_{ij} \in \mathbb{R}^+$ denotes the desired distance between agent i and agent j; $d_{ij} = d_{ji}$ if both $i \in \mathcal{N}_j$ and $j \in \mathcal{N}_i$.

Assume that the i-th agent's state includes the spatial variable p_i, and the agent's dynamics is passive with input u_i, output $q_i = \dot{p}_i$ and the storage function is V_i, $i = 1, \ldots, N$. The agents are able to communicate with each other through a network. The topology of the underlying information exchange graph is modeled by a graph Laplacian. The problem investigated in the present paper is to achieve distance-based formation among the networked agents via event-based communication.

IV. Main Result I: Ideal Network Model

In this section, with an ideal network model being assumed (no delay, no data packet drop-out), we propose the following control law for each agent:

$$\begin{aligned} u_i(t) = &\sum_{j \in \mathcal{N}_i} K_p \frac{\|\widehat{p}_j - p_i(t)\|_2 - d_{ij}}{\|\widehat{p}_j - p_i(t)\|_2} [\widehat{p}_j - p_i(t)] \\ &+ \sum_{j \in \mathcal{N}_i} K_d (\widehat{q}_j - \widehat{q}_i), \end{aligned} \tag{6}$$

where $\widehat{q}_i = q_i(t_k^i)$, for $t \in [t_k^i, t_{k+1}^i]$, t_k^i denotes the last event time of agent i by the time t; $\widehat{p}_j = p_j(t_{k'}^j)$ and $\widehat{q}_j = q_j(t_{k'}^j)$, for $t \in [t_{k'}^j, t_{k'+1}^j]$, where $t_{k'}^j$ denotes the last event time of agent j by the time t ($j \in \mathcal{N}_i$); $K_p \in \mathbb{R}^+ \setminus \{0\}$ and $K_d \in \mathbb{R}^+ \setminus \{0\}$ are the control gains. Under the proposed control law (6), a distributed triggering condition to achieve distance-based formation is provided in the following theorem.

***Theorem** 1:* Consider a group of N passive agents with control law (6), where each agent i is passive with input $u_i \in \mathbb{R}^m$, output $q_i = \dot{p}_i \in \mathbb{R}^m$ and a C^1 storage function V_i, for $i = 1, \ldots, N$. Under assumption **A1.** and with an ideal network model being assumed, if each agent i transmits its current information p_i and q_i to its neighbors $\mathcal{Z}_i$ (where $\mathcal{Z}_i$ denotes the set of agents receiving information from agent i) whenever the following triggering condition is satisfied

$$\|e_i(t)\|_2 > \frac{\gamma_1 \sum_{j \in \mathcal{N}_i} \left\|\widehat{q}_j - \widehat{q}_i\right\|_2^2}{\left\|\sum_{j \in \mathcal{N}_i} (\widehat{q}_j - \widehat{q}_i)\right\|_2}, \ \forall i, \tag{7}$$

where $\gamma_1 \in (0, 0.5)$, $e_i(t) = q_i(t) - \widehat{q}_i$, then the networked agents will achieve distance-based formation asymptotically.

Proof: Due to the length constraints, proof of Theorem 1 is eliminated from the final submitted version. The main idea is that when ideal communication network is assumed, we can find a storage function to show that the entire networked control system is Output Strictly Passive(OSP)[15] with input being the error signal $e_i(t)$ and output being the disagreement signal $(\widehat{q}_j - \widehat{q}_i)$, $\forall j \in \mathcal{N}_i$, $i = 1, 2, \ldots, N$. So if we derive a triggering condition which renders the size of $\|e_i(t)\|_2$ properly bounded, then the storage function of the networked control system will be decrescent and this further yields the distance-based formation control result. Interested readers should refer to [20] for detailed proof. ■

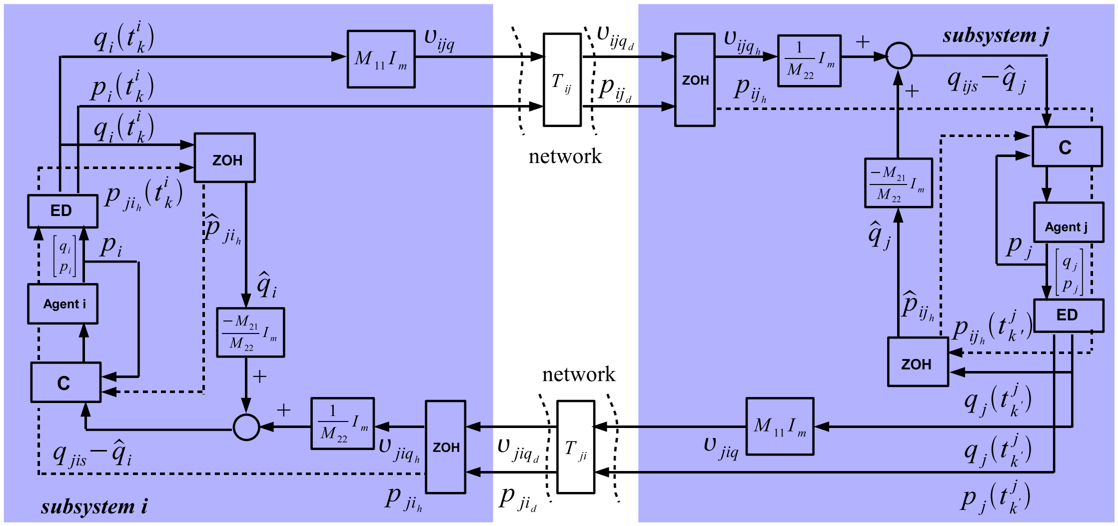

Fig. 1: Proposed Set-up to Deal With Constant Network Induced Delays

V. Main Results II: Network Induced Delays

In this section, we propose a set-up to achieve formation control of multi-agent systems with event-driven communication in the presence of constant network induced delays. The set-up for a pair of interconnected agents (Agent i and Agent j, where each agent is passive with m-inputs and m-outputs) is illustrated schematically in Fig.1: the "ED" block represents the "event-detector" (which could be implemented by embedded hardware in the microprocessor and it is able to monitor the output of the agent with sufficiently fast sampling rate); whenever the event-detector detects that the agent satisfies its specific triggering condition, state information of the agent at that "event time" will be obtained (t^i_k is used to denote the event-time of agent i, while the event-time of agent j is denoted by $t^j_{k'}$); the "ZOH" block represents the zero-order hold; the "C" block represents the distributed controller implemented in the agent; T_{ji} represents the network induced delays from agent j to agent i while T_{ij} represents the network induced delays from agent i to agent j (T_{ij} and T_{ji} are assumed to be constant but not necessarily equal to each other). As the information is transmitted through the network, we have

$$\begin{aligned} &\upsilon_{jiq_d}(t)=\upsilon_{jiq}(t-T_{ji}) \qquad \upsilon_{ijq_d}(t)=\upsilon_{ijq}(t-T_{ij}) \quad \text{and} \\ &p_{ji_d}(t)=p_j(t-t^j_{k'}-T_{ji}) \quad p_{ij_d}(t)=p_i(t-t^i_k-T_{ij}) \end{aligned} \tag{8}$$

$\forall (i,j)\in E(G)$. The control laws for a pair of coupled agent i and agent j are given by

$$\begin{aligned} u_i(t) &= \sum_{j\in\mathcal{N}_i}\left[K_p\frac{\left\|\widehat{p}_{jih}-p_i(t)\right\|_2-d_{ij}}{\left\|\widehat{p}_{jih}-p_i(t)\right\|_2}\Big(\widehat{p}_{jih}-p_i(t)\Big)+K_d\Big(q_{jis}(t)-\widehat{q}_i\Big)\right] \\ &= \sum_{j\in\mathcal{N}_i}K_p\phi(\widehat{p}_{jih}-p_i(t))+\sum_{j\in\mathcal{N}_i}K_d[q_{jis}(t)-\widehat{q}_i] \end{aligned} \tag{9}$$

where $\widehat{p}_{jih}=p_{jih}(t^i_k)$, for $t\in[t^i_k,t^i_{k+1}]$, $\phi(\widehat{p}_{jih}-p_i(t))=\frac{\left\|\widehat{p}_{jih}-p_i(t)\right\|_2-d_{ij}}{\left\|\widehat{p}_{jih}-p_i(t)\right\|_2}[\widehat{p}_{jih}-p_i(t)]$, and

$$\begin{aligned} u_j(t) &= \sum_{i\in\mathcal{N}_j}\left[K_p\frac{\left\|\widehat{p}_{ijh}-p_j(t)\right\|_2-d_{ij}}{\left\|\widehat{p}_{ijh}-p_j(t)\right\|_2}\Big(\widehat{p}_{ijh}-p_j(t)\Big)+K_d\Big(q_{ijs}(t)-\widehat{q}_j\Big)\right] \\ &= \sum_{i\in\mathcal{N}_j}K_p\phi(\widehat{p}_{ijh}-p_j(t))+\sum_{i\in\mathcal{N}_j}K_d[q_{ijs}(t)-\widehat{q}_j], \end{aligned} \tag{10}$$

where $\widehat{p}_{ijh}=p_{ijh}(t^j_{k'})$, for $t\in[t^j_{k'},t^j_{k'+1}]$, $\phi(\widehat{p}_{ijh}-p_j(t))=\frac{\left\|\widehat{p}_{ijh}-p_j(t)\right\|_2-d_{ij}}{\left\|\widehat{p}_{ijh}-p_j(t)\right\|_2}[\widehat{p}_{ijh}-p_j(t)]$, $\forall (i,j)\in E(G)$. Whenever the "ED" detects that the triggering condition of the agent is satisfied, state information of the agent at that event time(i.e., $\upsilon_{ijq}=M_{11}q_i(t^i_k)$ and $p_i(t^i_k)$) will be transmitted through the network, and the neighboring agents will use their received information to update their own control action. The transmissions of exchanged information and the updates of the control actions are generated through the following transformation:

$$\text{in agent } i: \begin{cases} \dfrac{1}{M_{22}}\upsilon_{jiq_h}(t)-\dfrac{M_{21}}{M_{22}}\widehat{q}_i=q_{jis}(t)-\widehat{q}_i \\ \upsilon_{ijq}(t)=M_{11}q_i(t^i_k), \quad \text{at } t=t^i_k, \end{cases} \tag{11}$$

$$\text{in agent } j: \begin{cases} \dfrac{1}{M_{22}}\upsilon_{ijq_h}(t)-\dfrac{M_{21}}{M_{22}}\widehat{q}_j=q_{ijs}(t)-\widehat{q}_j \\ \upsilon_{jiq}(t)=M_{11}q_j(t^j_{k'}), \quad \text{at } t=t^j_{k'}, \end{cases} \tag{12}$$

υ_{ijq_h} and υ_{jiq_h} hold the last sample of υ_{ijq_d} and υ_{jiq_d} respectively. Thus, through (11)-(12), agent i and agent j can extract variables $q_{ijs}(t)$ and $q_{jis}(t)$ from their received variables $\upsilon_{ijq_h}(t)$ and $\upsilon_{jiq_h}(t)$, and update their control action accordingly. One should notice that agent i is participating in $|\mathcal{N}_i|$ closed-loops as the one illustrated in Fig.1, where $|\mathcal{N}_i|$ is the number of neighboring agents communicating with agent i. A distributed triggering condition to achieve formation control among agents in the presence of constant network induced delays is presented in the following theorem.

***Theorem** 2:* Consider the set-up of event-driven communication between any pair of coupled agent i and agent j with m inputs and m outputs as shown in Fig.1, $\forall (i,j)\in E(G)$ (each agent i is passive with input $u_i\in\mathbb{R}^m$, output $q_i=\dot{p}_i\in\mathbb{R}^m$ and a C^1 storage function V_i, for $i=1,\ldots,N$). Assume that the network induced delays between any coupled agents are constant and finite. $M_{11}=\frac{\sqrt{K_d}}{2}$, $M_{22}=\sqrt{K_d}$, $M_{21}=\frac{\sqrt{K_d}}{2}$. If agent i transmits its current state information $p_i(t)$ and $q_i(t)$ to its neighbors whenever the following triggering condition is satisfied

$$\|e_i(t)\|_2>\frac{\gamma_2\sum_{j\in\mathcal{N}_i}\left\|q_{jis}(t)-\widehat{q}_i\right\|_2^2}{\left\|\sum_{j\in\mathcal{N}_i}(q_{jis}(t)-\widehat{q}_i)\right\|_2}, \tag{13}$$

where $e_i(t)=q_i(t)-\widehat{q}_i$, $i=1,2,\ldots,N$, and $\gamma_2\in(0,1)$, then under assumption **A2.**, the networked agents will achieve distance-based formation asymptotically.

Proof: Since agent j transmits its current state information to agent i at its event time $t^j_{k'}$, see Fig.1, we have

$$\int_0^t \left\|\upsilon_{jiq}(\tau)\right\|_2^2 d\tau = \sum_{k'=0}^{n_{ji}} \delta(t-t^j_{k'}) M_{11}^2 \left\|q_j(t^j_{k'})\right\|_2^2, \tag{14}$$

where $\delta(\cdot)$ is the Dirac delta function, n_{ji} is the number of data packets sent from agent j to agent i during the time interval $[0,t]$. Thus, we have

$$\begin{aligned}\int_0^t \left\|\upsilon_{jiq}(\tau)\right\|_2^2 d\tau &\le \sum_{k'=0}^{n_{ji}} \int_{t^j_{k'}}^{t^j_{k'+1}} M_{11}^2 \left\|q_j(t^j_{k'})\right\|_2^2 d\tau \\ &= \sum_{k'=0}^{n_{ji}} \int_{t^j_{k'}}^{t^j_{k'+1}} M_{11}^2 \left\|\widehat{q}_j\right\|_2^2 d\tau.\end{aligned} \tag{15}$$

Similarly, one can obtain

$$\begin{aligned}\int_0^t \left\|\upsilon_{ijq}(\tau)\right\|_2^2 d\tau &\le \sum_{k=0}^{n_{ij}} \int_{t^i_{k}}^{t^i_{k+1}} M_{11}^2 \left\|q_i(t^i_{k})\right\|_2^2 d\tau \\ &= \sum_{k=0}^{n_{ij}} \int_{t^i_{k}}^{t^i_{k+1}} M_{11}^2 \left\|\widehat{q}_i\right\|_2^2 d\tau,\end{aligned} \tag{16}$$

where n_{ij} is the number of data packets sent from agent i to agent j during the time interval $[0,t]$. Denote

$$\int_0^t \left\|\widetilde{\upsilon}_{jiq}(\tau)\right\|_2^2 d\tau = \sum_{k'=0}^{n_{ji}} \int_{t^j_{k'}}^{t^j_{k'+1}} M_{11}^2 \left\|\widehat{q}_j\right\|_2^2 d\tau \tag{17}$$

$$\int_0^t \left\|\widetilde{\upsilon}_{ijq}(\tau)\right\|_2^2 d\tau = \sum_{k=0}^{n_{ij}} \int_{t^i_{k}}^{t^i_{k+1}} M_{11}^2 \left\|\widehat{q}_i\right\|_2^2 d\tau. \tag{18}$$

Let $\widehat{n}_{ij}$ denote the number of data packets received by agent j from agent i during the time interval $[0,t]$. Since υ_{ijq_h} holds the last sample of υ_{ijq_d}, and $\upsilon_{ijq_d}(t) = \upsilon_{ijq}(t-T_{ij})$ (similar relations hold among υ_{jiq_h}, υ_{jiq_d} and υ_{jiq}), we can get

$$\begin{aligned}\int_0^t \left\|\upsilon_{ijq_h}(\tau)\right\|_2^2 d\tau &= \sum_{k=0}^{\widehat{n}_{ij}} \int_{t^i_{k}+T_{ij}}^{t^i_{k+1}+T_{ij}} M_{11}^2 \left\|q_i(t^i_{k})\right\|_2^2 d\tau \\ &= \sum_{k=0}^{\widehat{n}_{ij}} \int_{t^i_{k}}^{t^i_{k+1}} M_{11}^2 \left\|\widehat{q}_i\right\|_2^2 d\tau,\end{aligned} \tag{19}$$

$$\begin{aligned}\int_0^t \left\|\upsilon_{jiq_h}(\tau)\right\|_2^2 d\tau &= \sum_{k'=0}^{\widehat{n}_{ji}} \int_{t^j_{k'}+T_{ji}}^{t^j_{k'+1}+T_{ji}} M_{11}^2 \left\|q_j(t^j_{k'})\right\|_2^2 d\tau \\ &= \sum_{k'=0}^{\widehat{n}_{ji}} \int_{t^j_{k'}}^{t^j_{k'+1}} M_{11}^2 \left\|\widehat{q}_j\right\|_2^2 d\tau,\end{aligned} \tag{20}$$

where $\widehat{n}_{ji}$ denotes the number of data packets received by agent i from agent j during the time interval $[0,t]$. Due to the delay in the network, we have $n_{ij} \ge \widehat{n}_{ij}$ and $n_{ji} \ge \widehat{n}_{ji}$, thus we can define V^{ijq} such that

$$\begin{aligned}V^{ijq} &= \int_0^t \left\|\widetilde{\upsilon}_{ijq}(\tau)\right\|_2^2 d\tau - \int_0^t \left\|\upsilon_{ijq_h}(\tau)\right\|_2^2 d\tau \\ &+ \int_0^t \left\|\widetilde{\upsilon}_{jiq}(\tau)\right\|_2^2 d\tau - \int_0^t \left\|\upsilon_{jiq_h}(\tau)\right\|_2^2 d\tau,\end{aligned} \tag{21}$$

and $V^{ijq} \ge 0$, $\forall (i,j) \in E(G)$.

In view of (11), we have

$$\upsilon_{jiq_h}(t) = M_{21}\widehat{q}_i + M_{22}[q_{jis}(t) - \widehat{q}_i] \tag{22}$$

thus

$$\begin{aligned}\int_0^t \left\|\upsilon_{jiq_h}(\tau)\right\|_2^2 d\tau &= \int_0^t \left\|M_{21}\widehat{q}_i + M_{22}[q_{jis}(\tau) - \widehat{q}_i]\right\|_2^2 d\tau \\ &= \sum_{k=0}^{n_{ij}} \int_{t^i_k}^{t^i_{k+1}} \Big[(M_{21}^2 - 2M_{21}M_{22} + M_{22}^2)\|\widehat{q}_i\|_2^2 \\ &+ (2M_{21}M_{22} - 2M_{22}^2)\widehat{q}_i^T q_{jis}(\tau) \\ &+ M_{22}^2 \|q_{jis}(\tau)\|_2^2\Big] d\tau,\end{aligned} \tag{23}$$

similarly, we can get

$$\begin{aligned}\int_0^t \left\|\upsilon_{ijq_h}(\tau)\right\|_2^2 d\tau &= \sum_{k'=0}^{n_{ji}} \int_{t^j_{k'}}^{t^j_{k'+1}} \Big[(M_{21}^2 - 2M_{21}M_{22} + M_{22}^2)\|\widehat{q}_j\|_2^2 \\ &+ (2M_{21}M_{22} - 2M_{22}^2)\widehat{q}_j^T q_{ijs}(\tau) \\ &+ M_{22}^2 \|q_{ijs}(\tau)\|_2^2\Big] d\tau.\end{aligned} \tag{24}$$

Replace (17)-(18) and (23)-(24) into (21), with $M_{11}^2 = M_{21}^2 - 2M_{21}M_{22} + M_{22}^2$, $M_{22}^2 = 2M_{21}M_{22}$, we can obtain

$$\begin{aligned}V^{ijq} &= \sum_{k=0}^{n_{ij}} \int_{t^i_k}^{t^i_{k+1}} \Big[M_{22}^2 \widehat{q}_i^T q_{jis}(\tau) - M_{22}^2 \|q_{jis}(\tau)\|_2^2\Big] d\tau \\ &+ \sum_{k'=0}^{n_{ji}} \int_{t^j_{k'}}^{t^j_{k'+1}} \Big[M_{22}^2 \widehat{q}_j^T q_{ijs}(\tau) - M_{22}^2 \|q_{ijs}(\tau)\|_2^2\Big] d\tau.\end{aligned}$$

Consider a storage function for the entire networked system given by

$$V = \sum_{i=1}^N V_i + \widehat{V} + \frac{1}{2} \sum_{(i,j)\in E(G)} V^{ijq}$$

where

$$\widehat{V} = \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{K_p}{2} \Big(\left\|\widehat{p}_{jih} - p_i(t)\right\|_2 - d_{ij}\Big)^2 \tag{25}$$

then we can get

$$\begin{aligned}\sum_{i=1}^N \dot{V}_i &\le \sum_{i=1}^N u_i^T(t) q_i(t) \\ &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \Big[K_p \phi(\widehat{p}_{jih} - p_i(t)) + K_d(q_{jis}(t) - \widehat{q}_i)\Big]^T q_i(t) \\ &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \Big[K_p \phi(\widehat{p}_{jih} - p_i(t))^T q_i(t) + K_d(q_{jis}(t) - \widehat{q}_i)^T \widehat{q}_i \\ &+ K_d(q_{jis}(t) - \widehat{q}_i)^T e_i(t)\Big],\end{aligned} \tag{26}$$

$$\begin{aligned}\dot{\widehat{V}} &= \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} K_p \Big(\left\|\widehat{p}_{jih} - p_i(t)\right\|_2 - d_{ij}\Big) \frac{d}{dt} \left\|\widehat{p}_{jih} - p_i(t)\right\|_2 \\ &= -\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} K_p \phi(\widehat{p}_{jih} - p_i(t))^T q_i(t).\end{aligned} \tag{27}$$

With $M_{22} = \sqrt{K_d}$, then we have

$$\frac{1}{2} \sum_{(i,j)\in E(G)} \dot{V}^{ijq} = \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \Big[K_d \widehat{q}_i^T q_{jis}(t) - K_d \|q_{jis}(t)\|_2^2\Big], \tag{28}$$

in view of (26)-(28), we can further get

$$\dot{V} \le \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} \Big[K_d (q_{jis}(t) - \widehat{q}_i)^T \widehat{q}_i + K_d (q_{jis}(t) - \widehat{q}_i)^T e_i(t) + K_d \widehat{q}_i^T q_{jis}(t) - K_d \big\| q_{jis}(t) \big\|_2^2 \Big],$$

thus

$$\begin{aligned} \dot{V} &\le \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} \Big[K_d (q_{jis}(t) - \widehat{q}_i)^T e_i(t) - K_d \big\| q_{jis}(t) - \widehat{q}_i \big\|_2^2 \Big] \\ &\le \sum_{i=1}^{N} \Big[\big\| e_i(t) \big\|_2 \Big\| \sum_{j\in\mathcal{N}_i} K_d (q_{jis}(t) - \widehat{q}_i) \Big\|_2 \\ &\quad - \sum_{j\in\mathcal{N}_i} K_d \big\| q_{jis}(t) - \widehat{q}_i \big\|_2^2 \Big], \end{aligned} \tag{29}$$

so if

$$\big\| e_i(t) \big\|_2 \le \frac{\sum_{j\in\mathcal{N}_i} \big\| q_{jis}(t) - \widehat{q}_i \big\|_2^2}{\big\| \sum_{j\in\mathcal{N}_i} (q_{jis}(t) - \widehat{q}_i) \big\|_2} \tag{30}$$

$\forall i$, then $\dot{V} \le 0$ and we can further conclude that $\lim_{t\to\infty} V$ exists and is finite because $V \ge 0$. Note that the triggering condition (13) will assure that (30) is satisfied. Moreover, with $\lim_{t\to\infty} V$ exists, $V \ge 0$ and $\dot{V} \le 0$, we can conclude that $\lim_{t\to\infty} \dot{V} = 0$. Thus, under the triggering condition we can further get

$$0 = \lim_{t\to\infty} \dot{V} \le -(1-\gamma_2) \lim_{t\to\infty} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} K_d \big\| q_{jis}(t) - \widehat{q}_i \big\|_2^2 \le 0, \tag{31}$$

then under assumption **A2.**, we have

$$\lim_{t\to\infty} [q_{jis}(t) - \widehat{q}_i] = 0, \quad \forall (i,j) \in E(G). \tag{32}$$

In view of (30), this further implies that

$$\lim_{t\to\infty} e_i(t) = \lim_{t\to\infty} \Big[q_i(t) - \widehat{q}_i \Big] = 0, \ \forall i. \tag{33}$$

Since $\lim_{t\to\infty} \upsilon_{ijq_h}(t) = \lim_{t\to\infty} M_{11}\widehat{q}_i$, in view of (12), we have

$$\begin{aligned} \lim_{t\to\infty} \upsilon_{ijq_h}(t) &= \lim_{t\to\infty} \Big[(M_{21} - M_{22})\widehat{q}_j + M_{22} q_{ijs}(t) \Big] \\ &= \lim_{t\to\infty} M_{11}\widehat{q}_i, \end{aligned} \tag{34}$$

thus

$$\lim_{t\to\infty} q_{ijs}(t) = \lim_{t\to\infty} \Big[\frac{M_{11}}{M_{22}} \widehat{q}_i - \frac{M_{21} - M_{22}}{M_{22}} \widehat{q}_j \Big], \tag{35}$$

with $M_{11} = \frac{\sqrt{K_d}}{2}$, $M_{22} = \sqrt{K_d}$, $M_{21} = \frac{\sqrt{K_d}}{2}$, we can get

$$\lim_{t\to\infty} q_{ijs}(t) = \lim_{t\to\infty} \frac{1}{2} (\widehat{q}_i + \widehat{q}_j), \ \forall i \in \mathcal{N}_j. \tag{36}$$

In view of (32), we have

$$\lim_{t\to\infty} [q_{jis}(t) - \widehat{q}_i] = \lim_{t\to\infty} \frac{1}{2} (\widehat{q}_j - \widehat{q}_i) = 0, \tag{37}$$

and based on (33), we can get

$$\lim_{t\to\infty} [q_i(t) - q_j(t)] = 0, \quad \forall (i,j) \in E(G). \tag{38}$$

Furthermore, with $\lim_{t\to\infty} V$ exists, $V, \widehat{V}, V_i, V^{ijq} \ge 0$, we can conclude that $\lim_{t\to\infty} \widehat{V}$, $\lim_{t\to\infty} \sum_{(i,j)\in E(G)} V^{ijq}$ and $\lim_{t\to\infty} \sum_{i=1}^{N} V_i$ exist; with $\lim_{t\to\infty} \dot{V} = 0$, we can conclude that $\lim_{t\to\infty} \dot{\widehat{V}} = 0$, $\lim_{t\to\infty} \frac{1}{2} \sum_{(i,j)\in E(G)} \dot{V}^{ijq} = 0$ and $\lim_{t\to\infty} \sum_{i=1}^{N} \dot{V}_i = 0$. Under the triggering condition (13), this further yields: $0 = \lim_{t\to\infty} \dot{V} \le \lim_{t\to\infty} \sum_{i=1}^{N} u_i^T(t) q_i(t) \le -(1-\gamma_2) \lim_{t\to\infty} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} K_d \big\| q_{jis}(t) - \widehat{q}_i \big\|_2^2 \le 0$, and we can obtain $\lim_{t\to\infty} \sum_{i=1}^{N} u_i^T(t) q_i(t) = 0$. Thus, the solutions of the networked system should converge to the set

$$S = \{ p_i, q_i \in \mathbb{R}^m | \ q_i = 0 \cup \|\widehat{p}_{jih} - p_i(t)\|_2 - d_{ij} = 0, \forall (i,j) \in E(G) \},$$

which further implies that

$$\lim_{t\to\infty} q_j(t) = \lim_{t\to\infty} q_i(t) = 0, \ \text{and} \ \lim_{t\to\infty} \|\widehat{p}_{jih} - p_i(t)\|_2 - d_{ij} = 0. \tag{39}$$

Assume that at time t_f^j, where $t_f^j \to \infty$, we have $\big\| \widehat{p}_{jih} - p_i(t_f^j) \big\|_2 - d_{ij} = 0$, $\lim_{t\to\infty} \widehat{p}_{jih} = \lim_{t\to\infty} p_j(t_f^j)$, and $q_i(t) = q_j(t) = 0$, $\forall t \ge t_f^j$, for some $(i,j) \in E(G)$. Since

$$p_i(t) = p_i(t_f^j) + \int_{t_f^j}^{t} q_i(\tau) d\tau, \quad p_j(t) = p_j(t_f^j) + \int_{t_f^j}^{t} q_j(\tau) d\tau$$

thus $\lim_{t\to\infty} \big\| p_i(t) - p_j(t) \big\|_2 = \big\| p_i(t_f^j) - p_j(t_f^j) \big\|_2 = \lim_{t\to\infty} \big\| \widehat{p}_{jih} - p_i(t_f^j) \big\|_2 = d_{ij}$, and under assumption **A2.**, one can further conclude that

$$\lim_{t\to\infty} \big\| p_i(t) - p_j(t) \big\|_2 = d_{ij}, \quad \forall (i,j) \in E(G),$$

which completes the proof. ■

Remark 1: When network induced delays are considered, in view of (29), one can find that the proposed set-up actually renders the entire networked system OSP with the error signal $e_i(t)$ being the input and the disagreement signal $(q_{jis}(t) - \widehat{q}_i)$ being the output, $\forall j \in \mathcal{N}_i$, $i = 1, 2, \ldots, N$. So again, we can derive distributed triggering conditions to make the storage function V of the networked system be decrescent by controlling the size of $\|e_i(t)\|_2$, $\forall i$, as seen in (30).

Remark 2: In view of (36), one can conclude that when there is no network induced delays in the network, we will have

$$q_{ijs}(t) = \frac{1}{2} (\widehat{q}_i + \widehat{q}_j) \ \text{ and } \ q_{jis}(t) = \frac{1}{2} (\widehat{q}_i + \widehat{q}_j) \tag{40}$$

$\forall (i,j) \in E(G)$, and the triggering condition (13) will be the same as the triggering condition (7) shown in Theorem 1. However, the results in Theorem 2 assumes that the underlying information exchange graph being bidirectional, which is not required in Theorem 1.

Example : Consider a group of 3 agents trying to establish a 2D equilateral triangle formation with side's length equal to 40. The dynamics of agent i is given by

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = u_i(t), \quad u_i(t), \ q_i(t), \ p_i(t) \in \mathbb{R}^2, \end{cases} \tag{41}$$

$i = 1, 2, 3$. If we choose the output as $q_i(t)$, then the agent with input $u_i(t)$ and output $q_i(t)$ is passive with storage function $V_i = \frac{1}{2} q_i^T q_i$. The initial conditions of agents are given by

$$\begin{aligned} p_1(0) &= [-2, \ 1]^T, \quad q_1(0) = [0.1, \ 0.2]^T, \\ p_2(0) &= [1, \ 1]^T, \quad q_2(0) = [0.3, \ 1]^T, \\ p_3(0) &= [0.3, \ 3]^T, \quad q_3(0) = [0, \ -0.6]^T. \end{aligned} \tag{42}$$

The Laplacian matrix of the underlying information exchange graph is given by

$$L = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \tag{43}$$

which satisfies assumption **A2**. Let $\gamma_2 = 0.95$, $K_p = 15$ and $K_d = 10$. The network induced delays between coupled agents are given by: $T_{12} = 0.5s$, $T_{21} = 0.4s$, $T_{13} = 0.3s$, $T_{31} = 0.6s$, $T_{23} = 0.8s$, $T_{32} = 0.6s$. Applying the results in Theorem 2, we get the simulation results shown in Fig.2-Fig.4. In Fig.2, the x-axis shows the event-time t_k^i of each agent and the y-axis shows the evolutions of inter-event time $[t_{k+1}^i - t_k^i]$; Fig.3 shows the evolution of the distances between agent 1 and agent 2 (d_{12}), agent 2 and agent 3 (d_{23}), and agent 1 and agent 3 (d_{13}); in Fig.4, the "squares" represent the initial positions and "circles" represent the final positions of the agents.

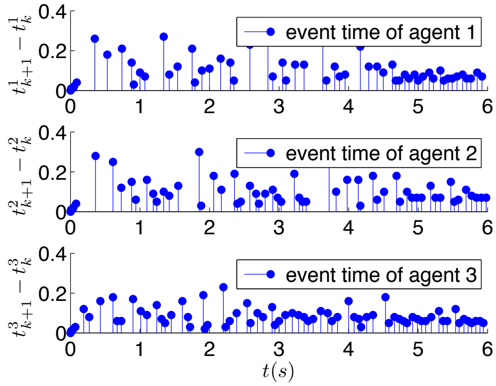

Fig. 2: event-time of each agent

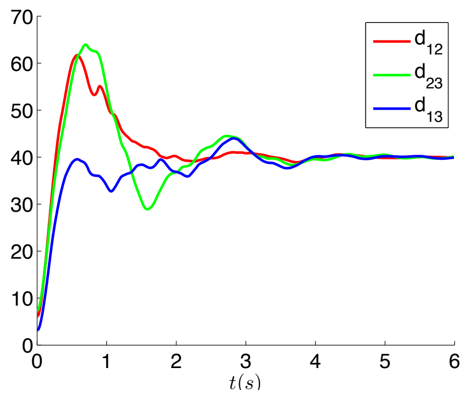

Fig. 3: distance evolution

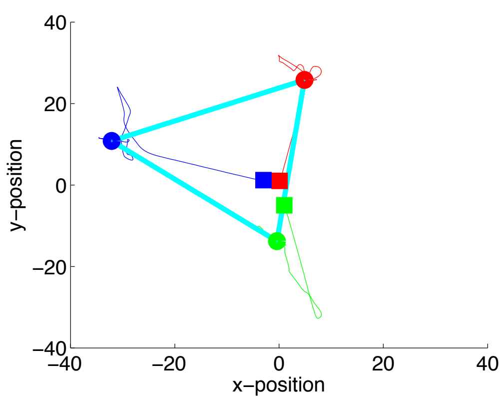

Fig. 4: formation evolution

VI. Conclusion

In this paper, we studied the formation control problem of networked passive systems with event-driven communication. We first derived a triggering condition to achieve distance-based formation among the agents assuming an ideal network model; we then considered the case when there are constant network induced delays between coupled agents.

VII. Acknowledgement

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