CDC '12

Formation Control of Multi-agent Systems with Connectivity Preservation by Using both Event-driven and Time-driven Communication

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Abstract— **In this paper, distributed control algorithms and event-based communication strategies are developed to achieve formation control with connectivity preservation among a group of networked mobile agents. Each agent transmits its current state information to its neighbors when its own triggering condition is satisfied or when the time elapsed from its last event time is going to exceed the agent's maximal admissible inter-event time. We have focused our studies on two types of systems' dynamics: agents that can be modeled as first order integrators and agents that can be modeled as double integrators. Simulation results are provided to validate our results.**

I. Introduction

Existing results on distributed coordination control of multi-agent systems critically rely on maintaining a connected communication network among the agents, either for all time (i.e., [6],[5],[8]) or over sequence of bounded time intervals (i.e., [2],[9]). However, for a given set of initial conditions, those assumptions on connectivity of the networks are difficult to verify. In particular, connectivity of the initial deployment of the multi-agent systems cannot guarantee connectivity of the systems in future times.

Motivated by the importance of network connectivity in the control of multi-agent systems, many researchers have emphasized connectivity preservation in networked dynamical systems. In [11], network connectivity is maintained by means of potential fields that guarantee that the second smallest eigenvalue of the graph Laplacian matrix is positive definite; in [7], a measure of local connectivity of a network is introduced and under certain conditions it is also sufficient for global connectivity; distributed maintenance of nearest neighbor links by means of unbounded "edge tension" functions is addressed in [3], where a control hysteresis is introduced to avoid infinite control inputs when new links are about to be inserted to the network; similarly, in [1], a system of interconnected unicycles is steered to a common configuration by means of non-smooth, potential-based control inputs that turn unbounded when the distance between adjacent agents approaches a certain threshold; in [4], a distributed coordination algorithm that allows the robots to decide whether a desired collective motion breaks connectivity is proposed, and this procedure is used to design a second coordination algorithm that allows the robots to modify a desired collective motion to guarantee that connectivity is preserved. Other related recent work have been reported in [12]-[16].

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While connectivity preservation for coordinated control of mobile agents has been extensively studied in the literature, one should notice that continuous or frequent communications between coupled agents are still required in most of these works; moreover, the control action updates and the data transmissions between agents are usually assumed to be implemented in a synchronous fashion. Note that multi-agent dynamic systems are distributed systems which usually act in an asynchronous manner and in general, it is difficult to implement synchronous motions in them. However, analyzing the dynamics of asynchronous systems is more difficult compared to their synchronous counterparts.

This paper studies formation control of multi-agent systems with connectivity preservation by using both event-driven and time-driven communication. We have derived distributed triggering conditions and whenever an agent satisfies its triggering condition, it will send its current state information to its neighbors at that time. Moreover, there exists an upper bound on the inter-event time of each agent. Hence, an agent will transmit its current state information to its neighbors whenever it satisfies its own triggering condition or if the time elapsed from its last event time is going to exceed the agent's maximal admissible inter-event time. We have derived distributed control actions to achieve both formation control and connectivity preservation under the proposed data transmission strategy provided that the initial deployment of the agents are within the communication radius of their neighbors. Note that the event-driven control approach has been extensively studied in the area of networked control systems, see [18]-[27]. However, to the best of our knowledge, not much work have been reported on the formation control problem studied in the present paper.

The rest of this paper is organized as follows. Section II provides some background material. Section III describes the problems studied in this paper. The main results are stated in Section IV and Section V. Simulation studies are included in Section VI. Finally, concluding remakes are provided in Section VII.

II. Background Material

The information exchange topology between agents can be modeled by a graph. In the following, we give some basic terminologies and definitions from graph theory [17].

A *directed graph* is a graph whose edges have direction and are called arcs. A *bi-directed graph* is a graph in which each edge is given an independent orientation at each end. Consider a finite weighted directed graph $G := (V, E)$ with no self-loops and *adjacency matrix* A, where V denotes the set of all vertices, E denotes the set of all edges, and $A := [a_{ij}]$ with $a_{ij} > 0$ if there is a directed edge from vertex i into vertex j, and $a_{ij} = 0$ otherwise. The *in-degree* and *out-degree* of vertex k are given by $d_i(k) = \sum_j a_{jk}$ and $d_o(k) = \sum_j a_{kj}$ respectively.

The *Laplacian* matrix of a directed graph is defined as $L = D - A$, where D is the diagonal matrix of vertex out-degrees.

***Definition** 1:* A directed graph is *strongly connected* if for any pair of distinct vertices v_i and v_j, there is a directed path from v_i to v_j.

***Definition** 2:* A vertex is *balanced* if its in-degree is equal to its out-degree. A directed graph is balanced if every vertex is balanced.

***Definition** 3:* A *path* of length r in a directed graph is a sequence $v_0, \ldots, v_r$ of $r+1$ distinct vertices such that for every $i \in \{0, \ldots, r-1\}$, (v_i, v_{i+1}) is an edge. A *weak path* is a sequence $v_0, \ldots, v_r$ of $r+1$ distinct vertices such that for each $i \in \{0, \ldots, r-1\}$, either (v_i, v_{i+1}) or (v_{i+1}, v_i) is an edge. A directed graph is *weakly connected* if any two vertices can be joined by a weak path.

***Lemma** 1:* Let G be a directed graph and assume it is balanced. Then G is strongly connected if and only if it is weakly connected.

III. Problem Statement

The evolution of multi-agent systems depends fundamentally on their information exchange topology. In this paper, we have the following assumption with respect to the underlying information exchange graph:

Assumption A. The underlying communication graph is bidirectional and balanced, and weakly connected in time.

***Definition** 4:* Let $p_i(t)$ denotes the position of agent i at time t; $\mathcal{N}_i$ denotes the set of agents sending information to agent i; $d_{ij} \in \mathbb{R}^+$ denotes the desired distance between agent i and agent j; $d_{ij} = d_{ji}$ if both $i \in \mathcal{N}_j$ and $j \in \mathcal{N}_i$. For a group of N agents, the agents are said to establish a *distance-based formation* if

$$\lim_{t\to\infty} \|p_j(t) - p_i(t)\|_2 = d_{ij}, \ \forall j \in \mathcal{N}_i,$$

for $i = 1, \ldots, N$.

Consider a group of mobile multi-agents, where the agents may have different communication capabilities (i.e., communication radius) and different limitations on mobility (i.e., maximal allowable speed). The underlying communication network is modeled by a graph Laplacian. Assume that each agent has access to its current state information (i.e., current position or speed), and it can also exchange information with its neighbors (agents that are within its communication radius are defined as neighbors in the communication graph). The problem investigated in the present paper is to achieve distance-based formation among the networked agents with event-driven and/or time-driven communication while preserving connectivity of the underlying information exchange graph.

The fundamental challenges regarding the problem studied in the current paper are the design of the distributed control laws and the distributed data transmission strategy to achieve both formation and connectivity preservation based on the local information available to each agent. The distributed data transmission strategy will determine the *event-time* at which an agent transmits its current state information to its neighbors. Since connectivity preservation is required, intuitively, one may expect that each agent should have some sort of mechanism to estimate the current maximal distance from its neighbors based on the last state information it has received from its neighbors. Moreover, one may also expect that each agent should be able to update its control actions and schedule its data transmissions based on this estimate in order to preserve connectivity with its neighbors(i.e., keep the distance from its neighbors within its communication radius).

The connectivity preservation control algorithms reported in the literature have been mostly devoted to two types of systems' dynamics: agents that can be modeled as first order integrators and agents that can be modeled as double integrators. In the following sections, we will also focus our studies on these two types of multi-agent systems.

IV. Formation Control with Connectivity Preservation: First Order Integrator

The formation control problem studied in the present paper is focused in the *2D* space. We first consider the case when the dynamics of the agents can be modeled as first order integrators given by

$$\dot{p}_i(t) = u_i(t), \quad p_i(t),\ u_i(t) \in \mathbb{R}^2,\ i = 1, 2, \ldots, N. \tag{1}$$

Define an edge-tension function between agent i and agent j as

$$V_{ij}(\delta_i, p_i) = \frac{(\|p_i(t) - \widehat{p}_j\|_2 - d_{ij})^2}{\delta_i - \|p_i(t) - \widehat{p}_j\|_2 - \upsilon_m^j \tau_m^j}, \quad \forall (i,j) \in E(G), \tag{2}$$

where $\widehat{p}_j = p_j(t^j_{k'})$, for $t \in [t^j_{k'}, t^j_{k'+1}]$, $\{t^j_{k'}\}_{k'=0,1,2,\ldots}$ is the event-time of agent j; $\delta_i \in \mathbb{R}^+/\{0\}$ is the communication radius of agent i; $\upsilon_m^j \in \mathbb{R}^+/\{0\}$ is the maximal allowable magnitude of the velocity of agent j; $\tau_m^j \in \mathbb{R}^+/\{0\}$ is an upper bound on the admissible inter-event time of agent j; $d_{ij} \in \mathbb{R}^+/\{0\}$ is the desired distance between agent i and agent j, $d_{ij} + \upsilon_m^j \tau_m^j < \delta_i$, $\forall j \in \mathcal{N}_i$; if $(i,j) \in E(G)$, then $d_{ij} = d_{ji}$.

Let $l_{ij} = \|p_i(t) - \widehat{p}_j\|_2$, then one can verify that

$$\frac{\partial V_{ij}}{\partial p_i} = \frac{(l_{ij} - d_{ij})(2\delta_i - l_{ij} - 2\upsilon_m^j \tau_m^j - d_{ij})}{(\delta_i - l_{ij} - \upsilon_m^j \tau_m^j)^2 l_{ij}} (p_i - \widehat{p}_j). \tag{3}$$

Let

$$\varphi_{ij} = \frac{(l_{ij} - d_{ij})(2\delta_i - l_{ij} - 2\upsilon_m^j \tau_m^j - d_{ij})}{(\delta_i - l_{ij} - \upsilon_m^j \tau_m^j)^2 l_{ij}}, \tag{4}$$

our designed control input for agent i is given by

$$u_i(t) = \begin{cases} \displaystyle\sum_{j\in\mathcal{N}_i} -\varphi_{ij}[\widehat{p}_i - \widehat{p}_j] & \text{if } \Big\| \displaystyle\sum_{j\in\mathcal{N}_i} -\varphi_{ij}[\widehat{p}_i - \widehat{p}_j] \Big\|_2 \le \upsilon_m^i, \\ -\dfrac{\upsilon_m^i}{\sqrt{2}} sgn\Big(\displaystyle\sum_{j\in\mathcal{N}_i} \dfrac{\partial V_{ij}}{\partial p_i} \Big) & \text{if } \Big\| \displaystyle\sum_{j\in\mathcal{N}_i} -\varphi_{ij}[\widehat{p}_i - \widehat{p}_j] \Big\|_2 > \upsilon_m^i, \end{cases} \tag{5}$$

where $\widehat{p_i} = p_i(t_k^i)$, for $t \in [t_k^i, t_k^{i+1}]$, $\{t_k^i\}_{k=0,1,2,...,}$ is the event-time of agent i. Define $h_i = \left\| \sum_{j\in\mathcal{N}_i} -\varphi_{ij}[\widehat{p_i} - \widehat{p_j}] \right\|_2 - \upsilon_m^i$, we can rewrite (5) as

$$\begin{aligned} u_i(t) = &-\frac{1-\widetilde{sgn}(h_i)}{2}\sum_{j\in\mathcal{N}_i}\varphi_{ij}(\widehat{p_i}-\widehat{p_j}) \\ &-\frac{1+\widetilde{sgn}(h_i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}sgn\Big(\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big), \end{aligned} \tag{6}$$

where

$$\widetilde{sgn}(h_i) = \begin{cases} 1, & \text{if} \quad h_i > 0; \\ -1, & \text{if} \quad h_i \le 0. \end{cases}$$

***Remark** 1:* One can see that the control law (6) requires that each agent knows its own communication radius (δ_i), its current position ($p_i(t)$), its last transmitted state information ($\widehat{p_i}$), the latest received information from its neighbors ($\widehat{p}_j$, $j \in \mathcal{N}_i$), the maximal magnitude of the velocity of its neighbors (υ_m^j, $j \in \mathcal{N}_i$) and the maximal admissible inter-event time of its neighbors (τ_m^j, $j \in \mathcal{N}_i$). Based on this information, agent i can estimate the maximal distance from agent j (which is $l_{ij} + \upsilon_m^j\tau_m^j$) before agent i receives the next state information from agent j, $\forall j \in \mathcal{N}_i$.

A triggering condition to achieve distance-based formation is stated in Theorem 1.

***Theorem** 1:* Consider a group of N agents with dynamics given by (1) and control laws given by (6). Assume that at the initial time (t_0), each agent broadcasts the initial state to its neighboring agents and we have

$$\left\|p_i(t_0) - \widehat{p}_j\right\|_2 + \upsilon_m^j\tau_m^j = \left\|p_i(t_0) - p_j(t_0)\right\|_2 + \upsilon_m^j\tau_m^j < \delta_i, \tag{7}$$

$\forall (i,j) \in E(G)$. If each agent transmits its current state information (p_i) to its neighboring agents whenever

$$\left\|e_{pi}(t)\right\|_2 > \gamma_1 \frac{\left\|\sum_{j\in\mathcal{N}_i}\varphi_{ij}[p_i(t)-\widehat{p}_j]\right\|_2}{\left\|\sum_{j\in\mathcal{N}_i}\varphi_{ij}\right\|_2}, \ \forall i, \tag{8}$$

where $e_{pi}(t) = p_i(t) - \widehat{p}_i$, $\gamma_1 \in (0,1)$, or when

$$t - t_k^i = \tau_m^i, \tag{9}$$

where t_k^i is the last event-time of agent i, then under assumption **A.**, the networked agents will achieve distance-based formation asymptotically.

Proof: The total tension energy of the entire networked system can be defined as $V(\delta,p) = \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} V_{ij}(\delta_i, p_i)$, and we have

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\Big(\frac{\partial V_{ij}}{\partial p_i}\Big)^T\dot{p}_i = \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\Big(\frac{\partial V_{ij}}{\partial p_i}\Big)^T u_i \\ &= -\sum_{i=1}^N\Big(\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big)^T\frac{1-\widetilde{sgn}(h_i)}{2}\sum_{j\in\mathcal{N}_i}\varphi_{ij}(\widehat{p_i}-\widehat{p}_j) \\ &\quad -\sum_{i=1}^N\Big(\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big)^T\frac{1+\widetilde{sgn}(h_i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}sgn\Big(\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big), \end{aligned} \tag{10}$$

which further yields

$$\begin{aligned} \dot{V} = &-\sum_{i=1}^N\frac{1-\widetilde{sgn}(h_i)}{2}\Big(\sum_{j\in\mathcal{N}_i}\varphi_{ij}(p_i-\widehat{p}_j)\Big)^T\sum_{j\in\mathcal{N}_i}\varphi_{ij}(\widehat{p_i}-\widehat{p}_j) \\ &-\sum_{i=1}^N\frac{1+\widetilde{sgn}(h_i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}\Big(\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big)^T sgn\Big(\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big) \end{aligned} \tag{11}$$

thus

$$\begin{aligned} \dot{V} = &-\sum_{i=1}^N\frac{1-\widetilde{sgn}(h_i)}{2}\Big(\sum_{j\in\mathcal{N}_i}\varphi_{ij}(p_i-\widehat{p}_j)\Big)^T\sum_{j\in\mathcal{N}_i}\varphi_{ij}(\widehat{p_i}-\widehat{p}_j) \\ &-\sum_{i=1}^N\frac{1+\widetilde{sgn}(h_i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}\Big\|\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big\|_1. \end{aligned} \tag{12}$$

With $e_{pi} = p_i - \widehat{p}_i$, we can get

$$\varphi_{ij}(\widehat{p_i}-\widehat{p}_j) = \varphi_{ij}(p_i - \widehat{p}_j - e_{pi}) = \varphi_{ij}(p_i-\widehat{p}_j) - \varphi_{ij}e_{pi}, \tag{13}$$

and we can rewrite (12) as

$$\begin{aligned} \dot{V} = &-\sum_{i=1}^N\frac{1+\widetilde{sgn}(h_i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}\Big\|\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big\|_1 \\ &-\sum_{i=1}^N\frac{1-\widetilde{sgn}(h_i)}{2}\sum_{j\in\mathcal{N}_i}\Big[\varphi_{ij}(p_i-\widehat{p}_j)-\varphi_{ij}e_{pi}\Big]^T\Big[\sum_{j\in\mathcal{N}_i}\varphi_{ij}(p_i-\widehat{p}_j)\Big] \\ = &-\sum_{i=1}^N\frac{1-\widetilde{sgn}(h_i)}{2}\Big\|\sum_{j\in\mathcal{N}_i}\varphi_{ij}(p_i-\widehat{p}_j)\Big\|_2^2 \\ &+\sum_{i=1}^N\frac{1-\widetilde{sgn}(h_i)}{2}\Big(\sum_{j\in\mathcal{N}_i}\varphi_{ij}e_{pi}\Big)^T\sum_{j\in\mathcal{N}_i}\varphi_{ij}(p_i-\widehat{p}_j) \\ &-\sum_{i=1}^N\frac{1+\widetilde{sgn}(h_i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}\Big\|\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big\|_1, \end{aligned} \tag{14}$$

which further yields

$$\begin{aligned} \dot{V} \le &\sum_{i=1}^N\frac{1-\widetilde{sgn}(h_i)}{2}\left\|e_{pi}\right\|_2\Big\|\sum_{j\in\mathcal{N}_i}\varphi_{ij}\Big\|_2\Big\|\sum_{j\in\mathcal{N}_i}\varphi_{ij}(p_i-\widehat{p}_j)\Big\|_2 \\ &-\sum_{i=1}^N\frac{1-\widetilde{sgn}(h_i)}{2}\Big\|\sum_{j\in\mathcal{N}_i}\varphi_{ij}(p_i-\widehat{p}_j)\Big\|_2^2 \\ &-\sum_{i=1}^N\frac{1+\widetilde{sgn}(h_i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}\Big\|\sum_{j\in\mathcal{N}_i}\frac{\partial V_{ij}}{\partial p_i}\Big\|_1. \end{aligned} \tag{15}$$

So if

$$\left\|e_{pi}\right\|_2 \le \frac{\left\|\sum_{j\in\mathcal{N}_i}\varphi_{ij}(p_i-\widehat{p}_j)\right\|_2}{\left\|\sum_{j\in\mathcal{N}_i}\varphi_{ij}\right\|_2}, \quad \forall i, \tag{16}$$

then $\dot{V} \le 0$, $\forall t \ge 0$. Note that the triggering condition (8) guarantees that (16) is satisfied. Under the triggering condition (8), we have $V(\delta,p) \le V(\delta,p_0)$, $\forall t \ge t_0$. This indicates that $\left\|p_i - \widehat{p}_j\right\|_2 + \upsilon_m^j\tau_m^j$ will never approach δ_i, $\forall (i,j) \in E(G)$, otherwise we might have $V(\delta,p) \gg V(\delta,p_0)$ since the initial deployment of the agents (7) guarantees that $V(\delta,p_0)$ is finite. This further indicates that if the initial deployment of the agents are within the communication radius of neighboring agents, then connectivity is preserved over time because

$$\left\|p_i - p_j\right\|_2 \le \left\|p_i - \widehat{p}_j\right\|_2 + \upsilon_m^j\tau_m^j < \delta_i, \quad \forall (i,j) \in E(G). \tag{17}$$

Moreover, since $\left\|p_i - \widehat{p}_j\right\|_2 + \upsilon_m^j\tau_m^j$ will never approach δ_i, with $V(\delta,p) \ge 0$ and $\dot{V} \le 0$, we can conclude that $\lim_{t\to\infty} V(\delta,p)$ exists and is finite, and furthermore $\lim_{t\to\infty}\dot{V}(\delta,p) = 0$, thus in view of (12), we can get $\lim_{t\to\infty}\varphi_{ij} = 0$, $\forall (i,j) \in E(G)$, which further yields

$$\lim_{t\to\infty}\left\|p_i - \widehat{p}_j\right\|_2 = d_{ij}, \quad \forall (i,j) \in E(G). \tag{18}$$

In view of the triggering condition (8), $\lim_{t\to\infty}\varphi_{ij} = 0$, $\forall (i,j) \in E(G)$ further indicates that

$$\lim_{t\to\infty}\left\|p_i-\widehat{p_i}\right\|_2=\lim_{t\to\infty}\left\|e_{pi}\right\|_2=0,\quad \forall i. \tag{19}$$

(18) and (19) together imply that

$$\lim_{t\to\infty}\left\|p_i-p_j\right\|_2=d_{ij},\quad \forall (i,j)\in E(G), \tag{20}$$

which completes the proof. ■

V. Formation Control with Connectivity Preservation: Double Integrators

We next consider the case when the agents can be modeled as double integrators with constraints on the second order dynamics given by

$$\dot{p}_i(t)=\begin{cases} q_i(t), & \text{if } \left\|q_i(t)\right\|_2\le \upsilon_m^i \\ \frac{\upsilon_m^i}{\sqrt{2}}sgn(q_i(t)), & \text{if } \left\|q_i(t)\right\|_2> \upsilon_m^i \end{cases} \tag{21}$$
$$\dot{q}_i(t)=u_i(t)$$

where $q_i(t),\ p_i(t),\ u_i(t)\in\mathbb{R}^2,\ i=1,2,\dots,N$. We can also rewrite (21) as

$$\begin{cases} \dot{p}_i=\frac{1-\widetilde{sgn}(\|q_i\|_2-\upsilon_m^i)}{2}q_i+\frac{1+\widetilde{sgn}(\|q_i\|_2-\upsilon_m^i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}sgn(q_i) \\ \dot{q}_i=u_i, \quad \text{for} \quad i=1,2,\dots,N. \end{cases} \tag{22}$$

We still use an edge-tension function between agent i and agent j as defined in (2), the control input to agent i is given by

$$\begin{aligned} u_i(t)=&-K_p\sum_{j\in\mathcal{N}_i}\varphi_{ij}[p_i(t)-\widehat{p}_j]\Bigg(\frac{1-\widetilde{sgn}(\|q_i\|_2-\upsilon_m^i)}{2} \\ &-\frac{1+\widetilde{sgn}(\|q_i\|_2-\upsilon_m^i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}\frac{sgn(q_i)^T sgn(q_i)}{\|q_i\|_1}\Bigg) \\ &+K_d\sum_{j\in\mathcal{N}_i}\left[\widehat{q}_j-\widehat{q}_i\right], \end{aligned} \tag{23}$$

where $K_p, K_d>0$ are designed control gains, $\widehat{q}_i=q_i(t_k^i)$, for $t\in[t_k^i,t_{k+1}^i]$, and $\widehat{q}_j=q_j(t_{k'}^j)$, for $t\in[t_{k'}^j,t_{k'+1}^j]$. A triggering condition to achieve distance-based formation is stated in Theorem 2.

***Theorem** 2:* Consider a group of N agents with dynamics given by (22) and control laws given by (23). Assume that at the initial time (t_0), each agent broadcasts its initial state information to the neighboring agents and the initial deployment of the agents satisfies (7), $\forall(i,j)\in E(G)$. If each agent transmits its current state information ($q_i(t)$ and $p_i(t)$) to its neighboring agents whenever

$$\left\|e_{qi}(t)\right\|_2>\gamma_2\frac{\sum_{j\in\mathcal{N}_i}\left\|\widehat{q}_j-\widehat{q}_i\right\|_2^2}{\left\|\sum_{j\in\mathcal{N}_i}\left[\widehat{q}_j-\widehat{q}_i\right]\right\|_2},\ \forall i, \tag{24}$$

where $e_{qi}(t)=q_i(t)-\widehat{q}_i$, $\gamma_2\in(0,0.5)$, or when

$$t-t_k^i=\tau_m^i, \tag{25}$$

where t_k^i is the last event-time of agent i, then under assumption **A.**, the networked agents will achieve distance-based formation asymptotically.

Proof: Let the total tension function for the entire networked system be defined as

$$V_t(\delta,p)=\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}V_{ij}. \tag{26}$$

Define the energy function for the entire networked system as

$$V(\delta,p,q)=K_pV_t+\sum_{i=1}^N\frac{1}{2}\left\|q_i(t)\right\|_2^2, \tag{27}$$

then we have

$$\begin{aligned} \dot{V}=&K_p\dot{V}_t+\sum_{i=1}^N q_i^T(t)\dot{q}_i(t)=K_p\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\dot{V}_{ij}+\sum_{i=1}^N q_i^T(t)u_i(t) \\ =&\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_p\varphi_{ij}\left[p_i(t)-\widehat{p}_j\right]^T\dot{p}_i(t)+\sum_{i=1}^N q_i^T(t)u_i(t) \\ =&\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_p\varphi_{ij}\left[p_i(t)-\widehat{p}_j\right]^T\frac{1-\widetilde{sgn}(\|q_i(t)\|_2-\upsilon_m^i)}{2}q_i(t) \\ &+\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_p\varphi_{ij}\left[p_i(t)-\widehat{p}_j\right]^T\frac{1+\widetilde{sgn}(\|q_i(t)\|_2-\upsilon_m^i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}sgn(q_i(t)) \\ &-\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_p\varphi_{ij}\left[p_i(t)-\widehat{p}_j\right]^T\frac{1-\widetilde{sgn}(\|q_i(t)\|_2-\upsilon_m^i)}{2}q_i(t) \\ &-\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_p\varphi_{ij}\left[p_i(t)-\widehat{p}_j\right]^T\frac{1+\widetilde{sgn}(\|q_i(t)\|_2-\upsilon_m^i)}{2}\frac{\upsilon_m^i}{\sqrt{2}}sgn(q_i(t)) \\ &+\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_d\left[\widehat{q}_j-\widehat{q}_i\right]^Tq_i(t)=\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_d\left[\widehat{q}_j-\widehat{q}_i\right]^Tq_i(t). \\ =&\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_d\left[\widehat{q}_j-\widehat{q}_i\right]^T\left[e_{qi}(t)+\widehat{q}_i\right] \\ =&\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_d\left[\widehat{q}_j-\widehat{q}_i\right]^Te_{qi}(t)+\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_d\left[\widehat{q}_j-\widehat{q}_i\right]^T\widehat{q}_i, \end{aligned} \tag{28}$$

since the underlying information exchange graph is balanced, we have

$$\begin{aligned} &\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_d\left[\widehat{q}_j-\widehat{q}_i\right]^T\widehat{q}_i=\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_d\widehat{q}_j^T\widehat{q}_i-\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\frac{K_d}{2}\widehat{q}_i^T\widehat{q}_i \\ &-\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\frac{K_d}{2}\widehat{q}_j^T\widehat{q}_j=-\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\frac{K_d}{2}\left\|\widehat{q}_j-\widehat{q}_i\right\|_2^2, \end{aligned} \tag{29}$$

replace (29) into (28), we can get

$$\begin{aligned} \dot{V}=&\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}K_d\left[\widehat{q}_j-\widehat{q}_i\right]^Te_{qi}(t)-\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\frac{K_d}{2}\left\|\widehat{q}_j-\widehat{q}_i\right\|_2^2 \\ \le&\sum_{i=1}^N\left\|e_{qi}(t)\right\|_2\Big\|\sum_{j\in\mathcal{N}_i}K_d(\widehat{q}_j-\widehat{q}_i)\Big\|_2-\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\frac{K_d}{2}\left\|\widehat{q}_j-\widehat{q}_i\right\|_2^2, \end{aligned} \tag{30}$$

so if

$$\left\|e_{qi}(t)\right\|_2\le\frac{0.5\sum_{j\in\mathcal{N}_i}\left\|\widehat{q}_i-\widehat{q}_j\right\|_2^2}{\left\|\sum_{j\in\mathcal{N}_i}\left[\widehat{q}_j-\widehat{q}_i\right]\right\|_2},\quad\forall i, \tag{31}$$

then $\dot{V}\le 0$, $\forall t\ge 0$. Note that the triggering condition (24) will guarantee that (31) holds. Under the triggering condition (24), we have $V(\delta,p)\le V(\delta,p_0)$, $\forall t\ge t_0$. This indicates that $\left\|p_i(t)-\widehat{p}_j\right\|_2+\upsilon_m^j\tau_m^j$ will never approach δ_i, $\forall(i,j)\in E(G)$, otherwise we might have $V(\delta,p)\gg V(\delta,p_0)$ since the initial deployment of the agents (7) guarantees that $V(\delta,p_0)$ is finite. This further indicates that if the initial deployment of the agents are within the communication radius of neighbors, then connectivity is preserved over time since

$$\left\|p_i(t)-p_j(t)\right\|_2 \le \left\|p_i(t)-\widehat{p}_j\right\|_2 + \upsilon_m^j \tau_m^j < \delta_i, \quad \forall (i,j) \in E(G). \tag{32}$$

Moreover, since $\left\|p_i(t)-\widehat{p}_j\right\|_2 + \upsilon_m^j \tau_m^j$ will never approach δ_i, with $V(\delta,p) \ge 0$ and $\dot{V} \le 0$, we can conclude that $\lim_{t\to\infty} V(\delta,p)$ exists and is finite, and furthermore $\lim_{t\to\infty} \dot{V}(\delta,p) = 0$. Under the triggering condition (24), we can get

$$0 = \lim_{t\to\infty} \dot{V} \le -\lim_{t\to\infty} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} K_d(0.5-\gamma_2)\left\|\widehat{q}_j - \widehat{q}_i\right\|_2^2 \le 0,$$

thus $\lim_{t\to\infty} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} K_d(0.5-\gamma_2)\left\|\widehat{q}_j - \widehat{q}_i\right\|_2^2 = 0$, which indicates that

$$\lim_{t\to\infty} \widehat{q}_j = \lim_{t\to\infty} \widehat{q}_i, \quad \forall (i,j) \in E(G). \tag{33}$$

In view of (31), (33) further yields

$$\lim_{t\to\infty} e_{q_i}(t) = \lim_{t\to\infty} (q_i(t) - \widehat{q}_i) = 0, \quad \forall i. \tag{34}$$

Based on (33) and (34), we can conclude that

$$\lim_{t\to\infty} q_i(t) = \lim_{t\to\infty} \widehat{q}_i = \lim_{t\to\infty} \widehat{q}_j = \lim_{t\to\infty} q_j(t), \quad \forall (i,j) \in E(G). \tag{35}$$

Furthermore, with $\lim_{t\to\infty} V$ exists, $V, V_t \ge 0$ and $\sum_{i=1}^{N} \left\|q_i(t)\right\|_2^2 \ge 0$, we can conclude that $\lim_{t\to\infty} V_t$ and $\lim_{t\to\infty} \sum_{i=1}^{N} \left\|q_i(t)\right\|_2^2$ exist; with $\lim_{t\to\infty} \dot{V} = 0$, in view of (28), we can further conclude that $\lim_{t\to\infty} \sum_{i=1}^{N} \sum_{j\in\mathcal{N}_i} \dot{V}_{ij} = 0$ and $\sum_{i=1}^{N} q_i^T u_i = 0$. Thus, the solutions of the dynamical system should converge to the set

$$S = \{p_i(t), q_i(t) \in \mathbb{R}^m | \; q_i(t) = 0 \cup \widehat{q}_j = \widehat{q}_i \cup \varphi_{ij} = 0, \forall (i,j) \in E(G)\},$$

which further implies that

$$\lim_{t\to\infty} q_j(t) = \lim_{t\to\infty} q_i(t) = 0, \text{ and } \lim_{t\to\infty} \|\widehat{p}_j - p_i(t)\|_2 - d_{ij} = 0, \tag{36}$$

$\forall (i,j) \in E(G)$. Assume $t_{k_f}^j$ is an event time of agent j at time $t \to \infty$, then at time $t_{k_f}^j$, based on (36), we have

$$\left\|p_j(t_{k_f}^j) - p_i(t_{k_f}^j)\right\|_2 = d_{ij}, \quad \text{where} \quad j \in \mathcal{N}_i. \tag{37}$$

Since $\lim_{t\to\infty} p_j(t) = p_j(t_{k_f}^j) + \lim_{t\to\infty} \int_{t_{k_f}^j}^{t} q_j(\tau)d\tau$ and $\lim_{t\to\infty} p_i(t) = p_i(t_{k_f}^j) + \lim_{t\to\infty} \int_{t_{k_f}^j}^{t} q_i(\tau)d\tau$, $\forall t \ge t_{k_f}^j$, we can further get

$$\begin{aligned}\lim_{t\to\infty} p_j(t) - \lim_{t\to\infty} p_i(t) &= p_j(t_{k_f}^j) - p_i(t_{k_f}^j) \\ &+ \lim_{t\to\infty} \int_{t_{k_f}^j}^{t} q_j(\tau)d\tau - \lim_{t\to\infty} \int_{t_{k_f}^j}^{t} q_i(\tau)d\tau.\end{aligned} \tag{38}$$

Since $\lim_{t\to\infty} q_j(t) = \lim_{t\to\infty} q_i(t) = 0$, $\forall j \in \mathcal{N}_i$, with $t_{k_f}^j \to \infty$, we have

$$\lim_{t\to\infty} \int_{t_{k_f}^j}^{t} q_j(\tau)d\tau = \lim_{t\to\infty} \int_{t_{k_f}^j}^{t} q_i(\tau)d\tau = 0, \tag{39}$$

thus

$$\lim_{t\to\infty} \left\|p_j(t) - p_i(t)\right\|_2 = \left\|p_j(t_{k_f}^j) - p_i(t_{k_f}^j)\right\|_2 = d_{ij}, \; \forall j \in \mathcal{N}_i, \tag{40}$$

which completes the proof. ■

VI. Simulation Study

Example: Consider a group of 3 agents trying to establish a equilateral triangle formation in a 2D space, with each side length equal to 10m. Each agent can be modeled as a double integrators with constraints on the second order dynamics as described in Section V. Let $p_{ix}(t)$ denotes agent i's position on x-axis and $p_{iy}(t)$ denotes agent i's position on y-axis; $q_{ix}(t)$ denotes agent i's velocity on x-axis and $q_{iy}(t)$ denotes agent i's velocity on y-axis, the dynamics of each agent are given by

$$\begin{cases} \dot{p}_{ix}(t) = \dfrac{1-\widetilde{sgn}(\|q_{ix}\|_2 - \upsilon_m^i)}{2} q_{ix} \\ \qquad + \dfrac{1+\widetilde{sgn}(\|q_{ix}\|_2 - \upsilon_m^i)}{2} \dfrac{\upsilon_m^i}{\sqrt{2}} sgn(q_{ix}) \\ \dot{p}_{iy}(t) = \dfrac{1-\widetilde{sgn}(\|q_{iy}\|_2 - \upsilon_m^i)}{2} q_{iy} \\ \qquad + \dfrac{1+\widetilde{sgn}(\|q_{iy}\|_2 - \upsilon_m^i)}{2} \dfrac{\upsilon_m^i}{\sqrt{2}} sgn(q_{iy}) \\ \dot{q}_{ix}(t) = u_{ix}(t) \\ \dot{q}_{iy}(t) = u_{iy}(t), \quad i = 1,2,3, \end{cases} \tag{41}$$

The initial conditions of agents are given by

$$\begin{aligned} &p_1(0) = [-2, \; -3]^T, \quad q_1(0) = [1, \; -2]^T, \\ &p_2(0) = [5, \; -1]^T, \quad q_2(0) = [2, \; -4]^T, \\ &p_3(0) = [1, \; -2]^T, \quad q_3(0) = [3, \; 2]^T. \end{aligned} \tag{42}$$

The communication radius of agent 1 is $\delta_1 = 80m$, the maximal allowable magnitude of the velocity of agent 1 is $\upsilon_m^1 = 10m/s$, and the maximal inter-event time of agent 1 is $\tau_m^1 = 6s$; the communication radius of agent 2 is $\delta_2 = 100m$, the maximal allowable magnitude of the velocity of agent 2 is $\upsilon_m^2 = 5m/s$, and the maximal inter-event time of agent 2 is $\tau_m^2 = 10s$; the communication radius of agent 3 is $\delta_3 = 90m$, the maximal allowable magnitude of the velocity of agent 3 is $\upsilon_m^3 = 15m/s$, and the maximal inter-event time of agent 3 is $\tau_m^3 = 4s$.

The Laplacian matrix of the underlying information exchange graph is given by

$$L = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \tag{43}$$

which satisfies assumption **A**. Choose $\gamma_1 = 0.45$, by applying the results in Theorem 2, we get the simulation results shown in Fig.1-Fig.3.

In Fig.1, the x-axis shows the event-time of each agent (t_k^i) and the y-axis shows the evolutions of inter-event time [$t_{k+1}^i - t_k^i$]; Fig.2 shows the evolution of the distances between agent 1 and agent 2 (d_{12}), agent 2 and agent 3 (d_{23}), and agent 1 and agent 3 (d_{13}), and one can observe that agents are kept within the communication radius of their neighboring agents; Fig.3 shows the evolution of the formation among the three agents, where "squares"denote the initial positions and "circles"denote the final positions.

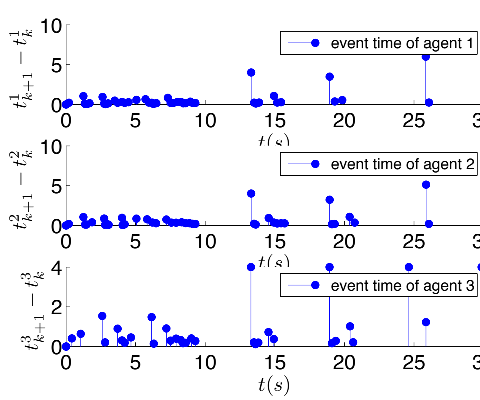

Fig. 1: Event-time of each agent

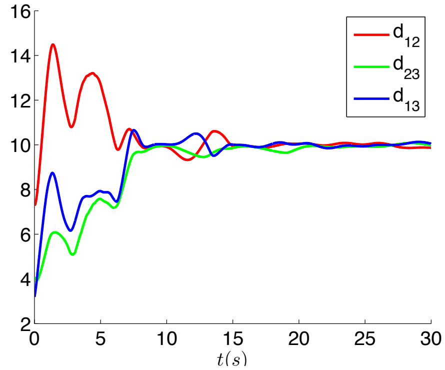

Fig. 2: Distance Evolution

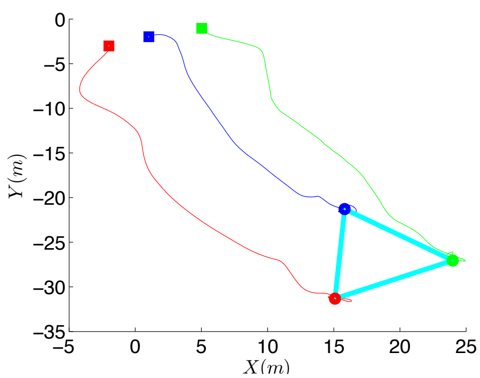

Fig. 3: Formation Evolution

VII. Conclusion

In this paper, distributed control algorithms and event-based communication strategies to achieve formation control with connectivity preservation among a group of networked mobile agents are proposed. Each agent transmits its current state information to its neighbors whenever it satisfies its triggering condition or whenever the time elapsed from its last event time is going to exceed the agent's maximal admissible inter-event time. Distributed control algorithms and distributed triggering conditions for data transmissions are derived to establish formation among the mobile agents with connectivity preservation, provided that the initial deployment of the agents is within the communication radius of their neighbors.

VIII. Acknowledgement

The support of the National Science Foundation under Grant No. CNS-1035655 is gratefully acknowledged.

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