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DYNAMICS OF AIR AND WALL TEMPERATURES IN MULTIROOM BUILDINGS

Mihir Sen[1,*], **Ishmael Amegashie**[2], **Emilio Cecconi**[3]
Department of Aerospace and Mechanical Engineering
University of Notre Dame
Notre Dame, Indiana 46556
Emails: [1]Mihir.Sen.1@nd.edu, [2]iamegash@nd.edu,
[3]emiliocecconi@gmail.com

Panos Antsaklis
Department of Electrical Engineering
University of Notre Dame
Notre Dame, Indiana 46556
Email: Antsaklis.1@nd.edu

ABSTRACT

Temperatures have been recorded every five minutes in a number of rooms in a student dormitory. The experimental data are presented as an example of room temperature dynamics in a large multi-room building. Statistical analyses of the data have been carried out to provide auto-correlations of individual room temperatures, its power spectral density, histogram, and the cross-correlations between the temperatures in neighboring rooms. A fairly complicated behavior is observed. A parallel, simplified mathematical model is set up for air and wall temperatures in multiple rooms to explain the temperature dynamics. Each room has an independent heater that switches *on* and *off* at certain lower and upper limits, respectively. The governing equations are based on energy balances with heat exchanges occurring between the air in a room and the walls surrounding it, and with the exterior. The instants at which heaters switch *on* or *off* are determined by the temperatures, leading thus to a non-linear set of equations. The governing equations are non-dimensionalized to provide the significant non-dimensional groups of the system: there are two which characterize the problem. Limiting solutions for large and small values of these groups provide physical explanation for the effect of the walls. Numerical solutions of this set of first-order ordinary differential equations are easily obtained, and examples of this are shown. The results show that there are still other physical effects to be considered before theory and experiments can be reasonably compared.

*Address all correspondence to this author.

NOMENCLATURE

A heat transfer area between room air and wall
A_∞ heat transfer area between room air and outside
c specific heat
h convective heat transfer coefficient between air and wall
K non-dimensional thermal coupling between rooms
m non-dimensional ratio of thermal capacities of walls and room air
M mass
Q heating
t time
T temperature
T_L lower thermostat temperature
T_U upper thermostat temperature
U_∞ overall heat transfer coefficient between air and outside

Subscripts and superscripts
a room air
i room number
$i \pm \frac{1}{2}$ wall number
w wall
∞ outside
$*$ dimensional quantity

1 Introduction

Air temperatures in buildings are determined by a number of dynamical factors: external thermal loading from the sun, heat transfer from and to the environment, ventilation from the outside, air exchange between rooms and the corridors, presence of heat sources such as people, computers and electronic appliances, condensation and evaporation of humidity in the air, heaters and air-conditioning units in rooms, and heat exchange between the air and the walls, floors and ceilings. All these factors are time-dependent so that the resulting temperatures are also dynamic. In addition, there is a spatial variation of the air temperature within a given room depending on location, and also a difference between the temperature of the air and that of the surrounding walls and building material which is also a function of position.

With few exceptions, the time variation of room temperatures in a building and their statistics [1] have not been reported in any detail in the literature. On the other hand, there do not exist mathematical models of the air temperature that can faithfully reproduce the local temperatures in the different rooms [2]. It is important to be able to relate field measurements to models and thus understand the complete physics of the processes that determine the dynamics of room temperatures.

2 Experimental motivation

Experimental data, their analysis and a search for an explanatory mathematical model is the main reason behind this research. Temperature data from the third floor of a four-storied student dormitory at the *University of Notre Dame* was collected in 5 minute intervals for 3.04 days, beginning at 6:00 AM on March 20, 2010; room heating was on at that time of year. Each room in the dormitory has a temperature sensor and the data were recorded at a central location at the university. There are 50 rooms on each floor, and the rooms were still in use by students during the measurements, with a maximum number of two students per room. The impact of the continued use of the rooms by students is thus a contributor to the fluctuations observed in the time-dependent temperatures measured.

Fig. 1 shows the measured temperatures in every fifth room of the third floor, though all rooms on that floor were measured. T_r denotes the instantaneous temperature of a room, T_a the temporal average of the specific room under consideration, while T_v is the spatial average of all rooms. An *auto-correlation* of an individual room temperature is shown in Fig. 2. It is, of course, symmetric with respect to the delay. It is interesting to note that there is a "shoulder" for a delay of about 0.8 days. This suggests that there is some temperature memory at this delay. There is also a negative correlation beyond one day, with the negative peak being around two days. A *power spectral density* of the same room temperature is in Fig. 3. It is relatively flat up to a frequency of about 1 day^{-1}, and with two later peaks roughly around 2 and 3 day^{-1}. This is consistent with a 24-hour cycle of thermal loading from the outside, whereas the other peaks appear to be influenced by the prevailing thermal control system, perhaps by nighttime setback. *Histograms* of the room temperature and its time derivative are shown in Fig. 4. The ordinates are normalized to make the areas under the curves unity. The probability density of the temperature in Fig. 4(a) seems to be tri-modal, a behavior that is also observed for many of the other rooms; the temperature spends more time at low, medium or high, and less at the ranges in between. It is interesting to see also that the time derivative of the temperature, as Fig. 4(b) shows, is much more in the negative range than it is in the positive; the room heats relatively quickly but then cools slowly. The *cross-correlation* between two neighboring rooms is in Fig. 5. It is high for delays of -2, 1 and 1.5 days; notice also that there is a strong negative cross-correlation for zero delay.

The purpose of the present work is to see if some headway can be made into understanding the dynamic behavior of the temperatures through mathematical models that capture the fundamental physical processes that determine the local temperatures in the building.

3 Mathematical model of n rooms

Though the air temperature determines the comfort level, the walls store heat. The presence of the walls and their interaction with room air thus strongly affects the dynamics of thermal energy in a building, even though it is only the air that is generally heated. It is important to include both air and wall and to understand the effect of heat transfer between them [3, 4].

First-order lumped parameter models have been postulated for the analysis of temperature oscillations in rooms [5, 6]. However, these studies have not taken the presence of walls into account. The configuration of the rooms in the dormitory that was experimentally studied is the reason for the in-line model that is developed here. Many multiroom buildings share a similar arrangement of rooms making this much more practical than models that have been analyzed previously, such as the ring configuration [5, 6]. A paper parallel to the present includes the effect of the walls for a ring configuration [4], while this is for an in-line geometry. The configuration is important because the ring has rotational symmetry, while the in-line has ends that must be considered to be different from the intermediate rooms. Though buildings are usually much more complicated than this, an in-line geometry allows some general conclusions to be reached.

We will consider a line of n rooms in which each room has a heater that, due to a thermostat, goes *on* or *off* depending on the temperature of the air in the room. Though the cooling problem is similar, we will assume that there is only heating. A little hysteresis, i.e. a dead band, is built into the thermostat in the form of a difference in temperatures at which the heater goes *on* and that at which it goes *off*.

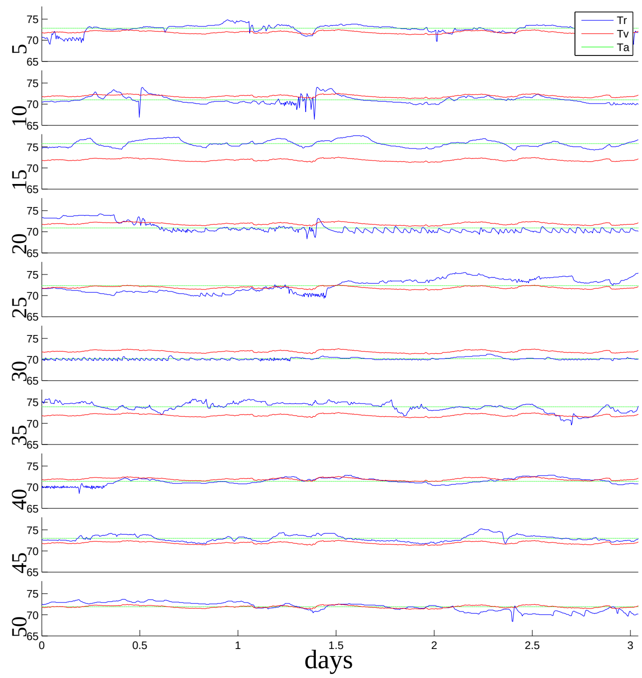


FIGURE 1: Measured and average temperatures during the interval [°F]. The room number is indicated on the ordinate of each trace.

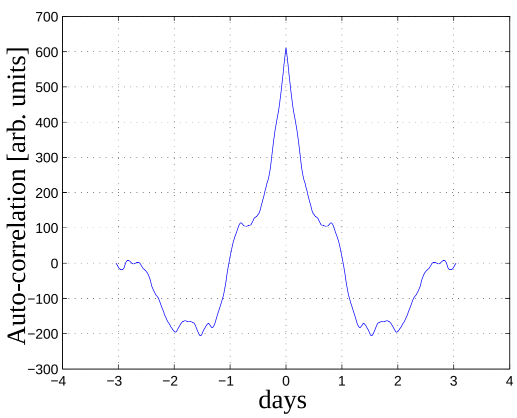


FIGURE 2: Auto-correlation for Room 1 temperature.

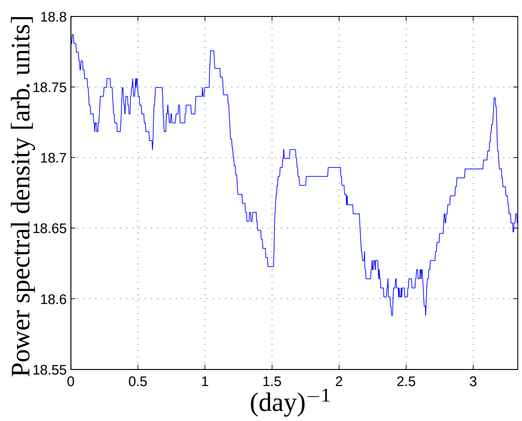


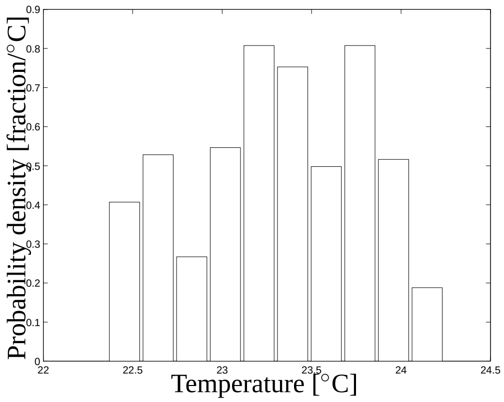
FIGURE 3: Power spectral density for Room 1 temperature.

3.1 Without walls

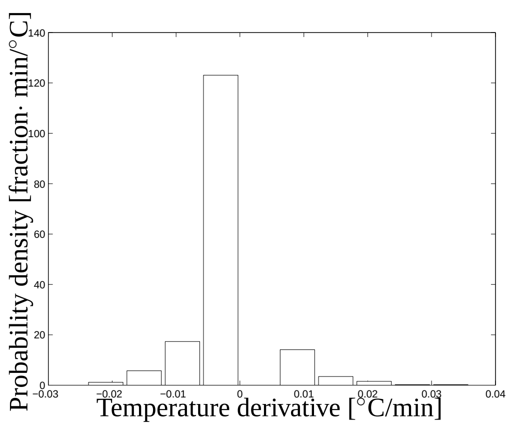
A schematic of the rooms is shown in Fig. 6(a). The dimensional air temperature in room i is $T_i^*(t^*)$, where t^* is dimensional time; integer subscripts are used for the air. There is convective heat transfer from the air in each room to the next. There is also heat exchange from each room air to the exterior which is at temperature T_∞.

The end rooms are special in that they interact with only one other room. For the intermediate rooms, however, energy balance gives

$$M_a c_a \frac{dT_i^*}{dt^*} = Q_i^* + U_\infty A_\infty \left(T_\infty - T_i^*\right) + hA\left(T_{i-1}^* - T_i^*\right) + hA\left(T_{i+1}^* - T_i^*\right), \qquad (1a)$$



(a) Temperature.



(b) Temperature derivative.

FIGURE 4: Histograms for Room 1 with 10 bins.

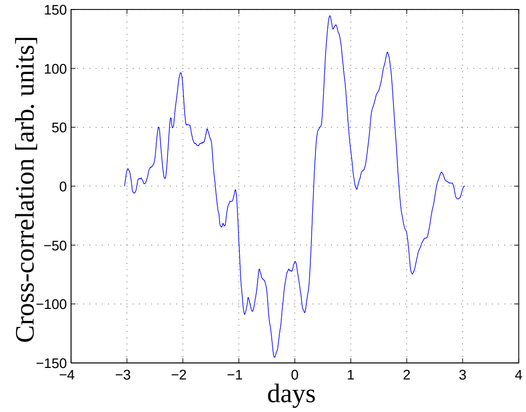
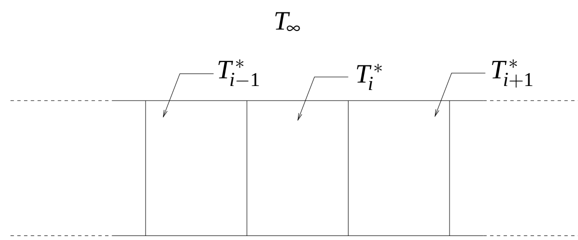
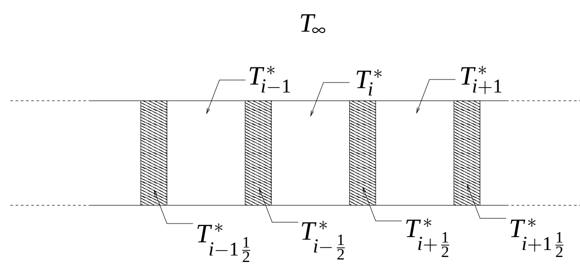


FIGURE 5: Cross-correlation between Rooms 1 and 2 temperatures.



(a) Without walls.



(b) With walls.

FIGURE 6: n rooms.

where M_a is the mass of the air, c_a is the specific heat of air at constant pressure, h and A are the heat transfer coefficient and area for convective heat transfer between the rooms, respectively, and U_∞ and A_∞ are the overall heat transfer coefficient and area for heat exchange with the outside, respectively. The heat input in each room is $Q_i^*(t^*)$, where

$$Q_i^* = \begin{cases} Q^* & \text{heater } \textit{on} \\ 0 & \text{heater } \textit{off} \end{cases}. \tag{1b}$$

The limits of operation of the thermostat must also be prescribed. It has an upper limit T_U^* at which the heater switches *off*, and a lower limit T_L^* at which it comes *on*.

The unknowns in Eqs. (1) are $T_i^*(t^*)$ and $Q_i^*(t^*)$ for $i = 1, 2, \ldots, n$, with initial conditions $T_i^*(0)$ and $Q_i^*(0)$. It must be emphasized that the equations are nonlinear because the instants at which the heater switches from *on* to *off*, or vice versa, are not *a priori* known, but depend on $T_i^*(t^*)$.

3.2 With walls

Fig. 6(b) shows a schematic of a line of rooms with walls between them. The line of rooms ends on either side in walls, and these end walls interact with only one room. The dimensional wall temperatures on either side of an intermediate room i are $T^*_{i-\frac{1}{2}}$ and $T^*_{i+\frac{1}{2}}$. Fractional subscripts are used for the walls.

From an energy balance for an intermediate room

$$M_a c_a \frac{dT_i^*}{dt^*} = Q_i^* + U_\infty A_\infty \left(T_\infty - T_i^*\right) + hA\left(T^*_{i-\frac{1}{2}} - T_i^*\right) + hA\left(T^*_{i+\frac{1}{2}} - T_i^*\right), \tag{2a}$$

$$Q_i^* = \begin{cases} Q^* & \text{heater } on \\ 0 & \text{heater } off \end{cases}. \tag{2b}$$

Energy balance for the walls gives

$$M_w c_w \frac{dT^*_{i+\frac{1}{2}}}{dt^*} = hA\left(T_i^* - T^*_{i+\frac{1}{2}}\right) + hA\left(T^*_{i+1} - T^*_{i+\frac{1}{2}}\right). \tag{2c}$$

4 Non-dimensional equations

The dependent and independent variables in Eqs. (1) are non-dimensionalized for the purpose of generalization. There are two time scales of interest, one for the wall and another for the air. The smaller one, that of the air, has been chosen as the characteristic time. The temperature is scaled by the heat gained from the outside and the heater rate, as these are the most significant contributors to the change in internal energy. Thus, the following independent and dependent variables

$$t = \frac{U_\infty A_\infty}{M_a c_a} t^*, \tag{3a}$$

$$T_i = \frac{U_\infty A_\infty}{Q^*}\left(T_i^* - T_\infty\right), \tag{3b}$$

$$Q_i = \frac{Q_i^*}{Q^*}, \tag{3c}$$

are defined. Eqs. (1) become

$$\frac{dT_i}{dt} = Q_i - T_i + K\left(T_{i-1} - T_i\right) + K\left(T_{i+1} - T_i\right), \tag{4a}$$

$$Q_i = \begin{cases} 1 & \text{heater } on \\ 0 & \text{heater } off \end{cases}, \tag{4b}$$

For Eqs. (2), an additional non-dimensional variable

$$T_{i+\frac{1}{2}} = \frac{U_\infty A_\infty}{Q^*}\left(T^*_{i+\frac{1}{2}} - T_\infty\right), \tag{5}$$

is needed, so that they become

$$\frac{dT_i}{dt} = Q_i - T_i + K\left(T_{i-\frac{1}{2}} - T_i\right) + K\left(T_{i+\frac{1}{2}} - T_i\right), \tag{6a}$$

$$Q_i = \begin{cases} 1 & \text{heater } on \\ 0 & \text{heater } off \end{cases}, \tag{6b}$$

$$m\frac{dT_{i+\frac{1}{2}}}{dt} = K\left(T_i - T_{i+\frac{1}{2}}\right) + K\left(T_{i+1} - T_{i+\frac{1}{2}}\right), \tag{6c}$$

Appropriate initial conditions $T_i(0)$, $T_{i+\frac{1}{2}}(0)$, and $Q_i(0)$ must be prescribed for each dynamical system.

The only non-dimensional group in Eq. (4) is

$$K = \frac{hA}{U_\infty A_\infty}, \tag{7a}$$

while Eqs. (6) have the additional group

$$m = \frac{M_w c_w}{M_a c_a}. \tag{7b}$$

5 Non-dimensional groups

The group K, Eq. (7a), represents the ratio of the thermal resistance between the air in the room and the outside to that between the air and the walls. The value depends greatly on the geometry of the design, the materials used, and the prevailing wind and atmospheric conditions. Nonetheless, one has to have some idea for the value of K to search for solutions in that range. The two areas, A and A_∞, may go from being roughly equal to A being much larger than A_∞. The convective heat transfer coefficient h is generally small because it is mostly due to natural convection. U_∞ is affected by insulation in the outer walls which tends to reduce its value, and by the winds blowing on the exterior which will increase it. The other group m, Eq. (7b), is the ratio of the heat capacity of a wall compared to that of the air in the room.

To get an idea of the numerical values of the non-dimensional groups, let the volume of air = 27 m^3 per room, the mass of each wall = 6210 kg. We also take $A = 9$ m^2, $A_\infty = 24$ m^2, and $c_w = 0.75$ kJ/kg·K. This gives $m = 142.4$. The two heat transfer coefficients, one internal and the other external, are much more problematic [3, 7–13]. There are some values available in the literature for this, but the numbers obviously depend strongly on other factors. If we take $h = 5$ W/m^2·K, $U_\infty = 10$ W/m^2·K, we get $K = 0.188$. The heat rate Q^* provided by the heater is another value that must be chosen. Assuming that the heater, if it is always *on*, can produce maximum air and wall temperatures of $T^*_{i,max} = T^*_{i+\frac{1}{2},max} = (40 + T_\infty)$ °C, then $Q^* = 9.6$ kW. If we take $T^*_U = 20$ °C and $T^*_L = 10$ °C, then their non-dimensional counterparts are $T_U = 0.44$ and $T_L = 0.22$.

For this study we will take $K = 0.1$, $m = 1$, $T_L = 0.2$ and $T_U = 0.7$. The initial conditions are taken to be T_L for all the rooms, $T = T_U$ for the walls, and the heater is initially assumed to be on.

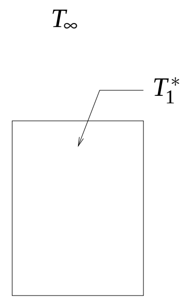
6 Specific configurations

6.1 Without walls

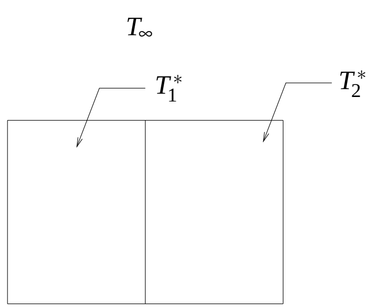
Fig. 7(a) shows a single room, for which the governing equation is

$$\frac{dT_1}{dt} = Q_1 - T_1. \tag{8}$$

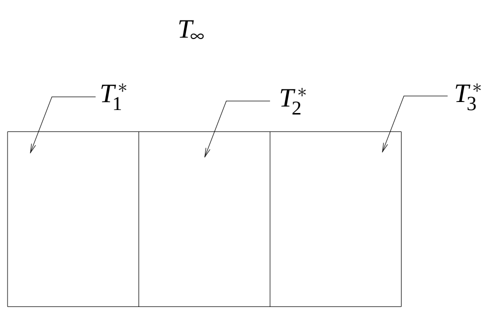
with unit time constant. This problem is easily solved, and Fig. 8 shows the temperature variation.



(a) Single room.



(b) Two rooms.



(c) Three rooms.

FIGURE 7: Rooms without walls.

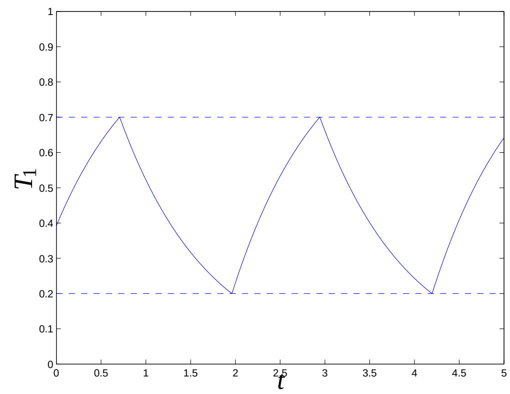


FIGURE 8: Temperature of single room without walls. Dashed lines are lower and upper bounds of thermostat $T_L = 0.2$ and $T_U = 0.7$, respectively.

Figs. 7(b) and 7(c) show two and three rooms, respectively. The corresponding equations for these are

$$\frac{dT_1}{dt} = Q_1 - T_1 + K(T_2 - T_1), \tag{9a}$$

$$\frac{dT_2}{dt} = Q_2 - T_2 + K(T_1 - T_2), \tag{9b}$$

and

$$\frac{dT_1}{dt} = Q_1 - T_1 + K(T_2 - T_1), \tag{10a}$$

$$\frac{dT_2}{dt} = Q_2 - T_2 + K(T_1 - T_2) + K(T_3 - T_2), \tag{10b}$$

$$\frac{dT_3}{dt} = Q_3 - T_3 + K(T_2 - T_3), \tag{10c}$$

respectively.

6.2 With walls

Figs. 9(a), 9(b) and 9(c) show one, two, and three rooms, respectively, with walls. The governing equations are indicated below.

6.3 Single room

$$m\frac{dT_{\frac{1}{2}}}{dt} = -T_{\frac{1}{2}} + K\left(T_1 - T_{\frac{1}{2}}\right), \tag{11a}$$

$$\frac{dT_1}{dt} = Q_1 - T_1 + K\left(T_{\frac{1}{2}} - T_1\right) + K\left(T_{1\frac{1}{2}} - T_1\right), \tag{11b}$$

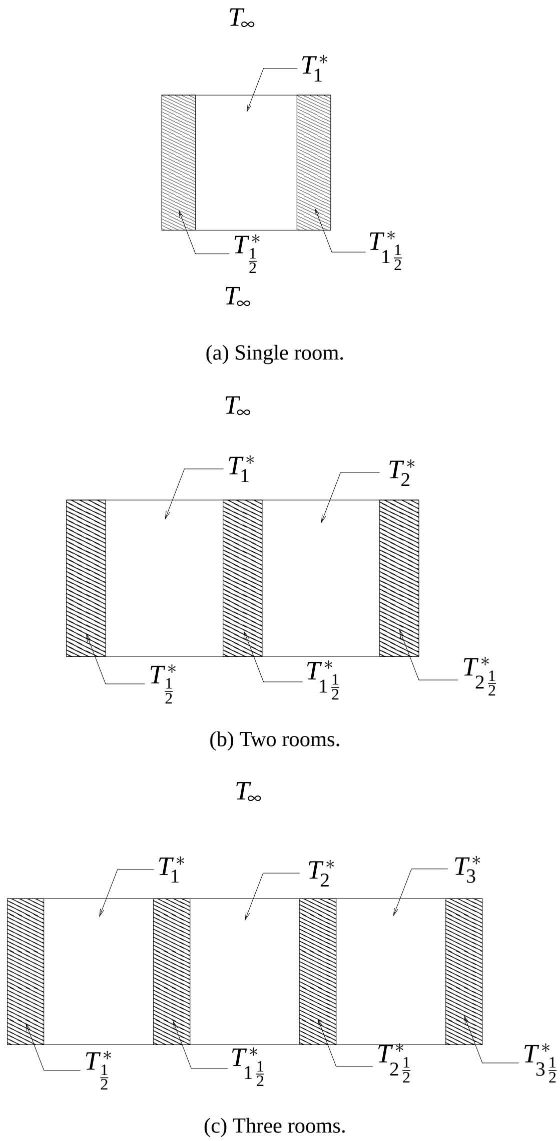


FIGURE 9: Rooms with walls.

$$m\frac{dT_{1\frac{1}{2}}}{dt} = -T_{1\frac{1}{2}} + K\left(T_1 - T_{1\frac{1}{2}}\right). \tag{11c}$$

6.4 Two rooms

$$m\frac{dT_{\frac{1}{2}}}{dt} = -T_{\frac{1}{2}} + K\left(T_1 - T_{\frac{1}{2}}\right), \tag{12a}$$

$$\frac{dT_1}{dt} = Q_1 - T_1 + K\left(T_{\frac{1}{2}} - T_1\right) + K\left(T_{1\frac{1}{2}} - T_1\right), \tag{12b}$$

$$m\frac{dT_{1\frac{1}{2}}}{dt} = K\left(T_1 - T_{1\frac{1}{2}}\right) + K\left(T_2 - T_{1\frac{1}{2}}\right), \tag{12c}$$

$$\frac{dT_2}{dt} = Q_2 - T_2 + K\left(T_{1\frac{1}{2}} - T_2\right) + K\left(T_{2\frac{1}{2}} - T_2\right), \tag{12d}$$

$$m\frac{dT_{2\frac{1}{2}}}{dt} = -T_{2\frac{1}{2}} + K\left(T_2 - T_{2\frac{1}{2}}\right). \tag{12e}$$

6.5 Three rooms

$$m\frac{dT_{\frac{1}{2}}}{dt} = -T_{\frac{1}{2}} + K\left(T_1 - T_{\frac{1}{2}}\right), \tag{13a}$$

$$\frac{dT_1}{dt} = Q_1 - T_1 + K\left(T_{\frac{1}{2}} - T_1\right) + K\left(T_{1\frac{1}{2}} - T_1\right), \tag{13b}$$

$$m\frac{dT_{1\frac{1}{2}}}{dt} = K\left(T_1 - T_{1\frac{1}{2}}\right) + K\left(T_2 - T_{1\frac{1}{2}}\right), \tag{13c}$$

$$\frac{dT_2}{dt} = Q_2 - T_2 + K\left(T_{1\frac{1}{2}} - T_2\right) + K\left(T_{2\frac{1}{2}} - T_2\right), \tag{13d}$$

$$m\frac{dT_{2\frac{1}{2}}}{dt} = K\left(T_2 - T_{2\frac{1}{2}}\right) + K\left(T_3 - T_{2\frac{1}{2}}\right), \tag{13e}$$

$$\frac{dT_3}{dt} = Q_3 - T_3 + K\left(T_{2\frac{1}{2}} - T_3\right) + K\left(T_{3\frac{1}{2}} - T_3\right), \tag{13f}$$

$$m\frac{dT_{3\frac{1}{2}}}{dt} = -T_{3\frac{1}{2}} + K\left(T_3 - T_{3\frac{1}{2}}\right). \tag{13g}$$

7 Limiting analyses

7.1 No energy storage in walls, $m = 0$

Taking $m = 0$ is equivalent to ignoring the heat capacity of the wall. For $n = 1$, Eqs. (11) give

$$T_{\frac{1}{2}} = \frac{K}{1+K}T_1, \tag{14a}$$

$$\frac{dT_1}{dt} = Q_1 - \frac{1+3K}{1+K}T_1, \tag{14b}$$

$$T_{1\frac{1}{2}} = \frac{K}{1+K}T_1. \tag{14c}$$

Eq. (14b) is similar to Eq. (8), behaving thus like a single room without walls but with a time constant of $(1+3K)/(1+K)$. The wall temperatures algebraically follow the room air temperature.

For $n = 2$, Eqns. (12) give

$$T_{\frac{1}{2}} = \frac{K}{1+K}T_1, \tag{15a}$$

$$\frac{dT_1}{dt} = Q_1 - \frac{1}{2}\left(\frac{2+5K+K^2}{1+K}\right)T_1 + \frac{K}{2}T_2, \tag{15b}$$

$$T_{1\frac{1}{2}} = \frac{1}{2}\left(T_1 + T_2\right), \tag{15c}$$

$$\frac{dT_2}{dt} = Q_2 - \frac{1}{2}\left(\frac{2+5K+K^2}{1+K}\right)T_2 + \frac{K}{2}T_1, \tag{15d}$$

$$T_{2\frac{1}{2}} = \frac{K}{1+K} T_2. \tag{15e}$$

Again, Eqs. (15b) and (15d) decouple from the others and become similar to Eqs. (9), but with different time constants. The wall temperatures also algebraically follow the room air temperatures.

For $n = 3$, Eqns. (13) become

$$T_{\frac{1}{2}} = \frac{K}{1+K} T_1, \tag{16a}$$

$$\frac{dT_1}{dt} = Q_1 - \frac{1}{2}\left(\frac{2+5K+K^2}{1+K}\right) T_1 + \frac{K}{2} T_2, \tag{16b}$$

$$T_{1\frac{1}{2}} = \frac{1}{2}\left(T_1 + T_2\right), \tag{16c}$$

$$\frac{dT_2}{dt} = Q_2 - (1+K)T_2 + \frac{K}{2} T_3 + \frac{K}{2} T_1, \tag{16d}$$

$$T_{2\frac{1}{2}} = \frac{1}{2}\left(T_2 + T_3\right), \tag{16e}$$

$$\frac{dT_3}{dt} = Q_3 - \frac{1}{2}\left(\frac{2+5K+K^2}{1+K}\right) T_3 + \frac{K}{2} T_2, \tag{16f}$$

$$T_{3\frac{1}{2}} = \frac{K}{1+K} T_3. \tag{16g}$$

The room air temperatures equations, Eqs. (16b), (16d) and (16f), decouple from the rest, and the wall temperatures follow them algebraically.

Equations for $n > 3$ follow the same pattern. The room air temperature equations decouple from the wall temperatures, and form a system of coupled ODEs. The end rooms are governed by equations of the form of Eqs. (16b) and (16f), while the intermediate rooms are like (16d). The wall temperatures algebraically follow both neighboring rooms if they are interior walls, but follow their single neighbor if they are at the end.

7.2 Infinitely massive walls, $m \to \infty$

With $m \to \infty$, the wall temperature equations for any wall shows that

$$\frac{dT_{i+\frac{1}{2}}}{dt} = 0. \tag{17}$$

The walls are so massive that their temperatures remain constant with time, and equal to the initial values $T_{i+\frac{1}{2}}(0)$. The room air temperature equations then become similar to those for a single room, Eq. (8). For example, taking the middle room in a three-room building, Eq. (13d) becomes

$$\frac{dT_2}{dt} = Q_2 - (1+2K)\,T_2 + K\left(T_{1\frac{1}{2}}(0) + T_{2\frac{1}{2}}(0)\right). \tag{18}$$

The last term in this equation is a constant, and the problem is similar to that for a single room without a wall. The temperature in each room decouples from those of its neighbors.

7.3 No heat exchange with wall, $K = 0$

Assuming $K = 0$

$$\frac{dT_i}{dt} = Q_i - T_i, \tag{19a}$$

$$m\frac{dT_{i+\frac{1}{2}}}{dt} = 0. \tag{19b}$$

Curiously enough, this is basically the same as taking $m \to \infty$ above. The wall temperature is constant in time, while the room air temperatures decouple from each other.

7.4 Infinite heat exchange with wall, $K \to \infty$

Looking at Eqs. (11), (12) and (13), one can see that in this limit, one obtains an algebraic set of equations for the room air and wall temperatures. There are no dynamics in the temperatures.

8 Simulation results

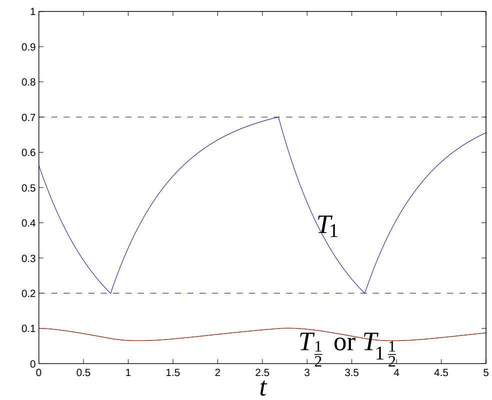
Numerical results were obtained using an an explicit Euler method with a step size in time of 0.001. Integration was performed up to 50,000 time steps. Only the last 5 time units are shown to enable the transients in the system to die out. The steady-state dynamics of the system are strongly dependent on the parameters and initial conditions chosen. The values of the non-dimensional groups are taken to be $K = 0.2$, $m = 1$, $T_L = 0.2$, $T_U = 0.7$. Initially, the air in all the rooms is at T_L with the heater on, and the walls are at T_U. At this stage a comprehensive study of all possibilities and combinations has not been carried out.

8.1 Single room with walls

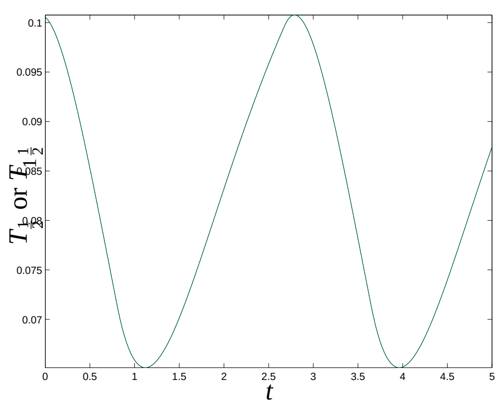
Fig. 10 shows the temperature variation of the room air and that of the two walls. For clarity, the wall temperatures are shown amplified in the lower figure; the two walls have identical temperatures. The room air and the wall temperatures have the same frequency, but the latter are much smaller in amplitude. The existence of temperature oscillations in the wall, however, could in the long term lead to synchronization of the temperature oscillation between rooms.

8.2 Two rooms with walls

Fig. 11 shows the temperature variation of two rooms and that of the three walls.



(a) Air and wall temperatures.



(b) Wall temperatures only.

FIGURE 10: Air and wall temperatures in single room with walls.

8.3 Three rooms with walls

Fig. 12 shows the temperature variation of three rooms and that of the four walls.

9 Conclusions

The dynamics of temperature variation in an actual building is observed to be very complicated, and at this stage it is best studied using statistical tools. Auto-correlations and power-spectral densities show that there is a spectrum of time scales and frequencies present in the temperature dynamics. Probability density analysis indicates that, on the average, the room heats more quickly than it cools. Mathematical models of the thermal dynamics, including heat transfer between room air, the wall, and the exterior in the presence of thermostatic control with hysteresis produces periodic behavior. There are many aspects of the mathematical model that needs to be explored since there are, at present, major qualitative differences between the measurements and the theoretical results.

Further work will also have to sweep the range of possible parameters and initial conditions to see if the non-linear dynamical system produces bifurcations or chaotic solutions, and to determine the effect of variation of the external temperature, T_∞. Experimentally, it will be interesting to place sensors on the walls and observe its dynamics.

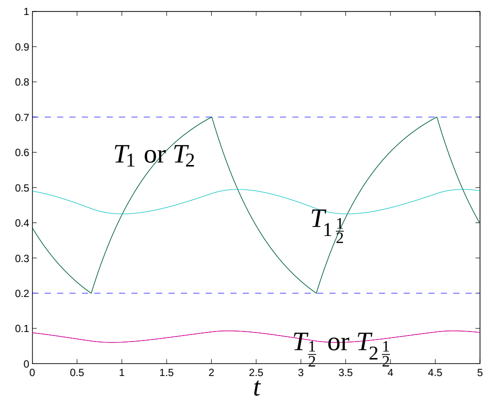


FIGURE 11: Air and wall temperatures in two rooms with walls.

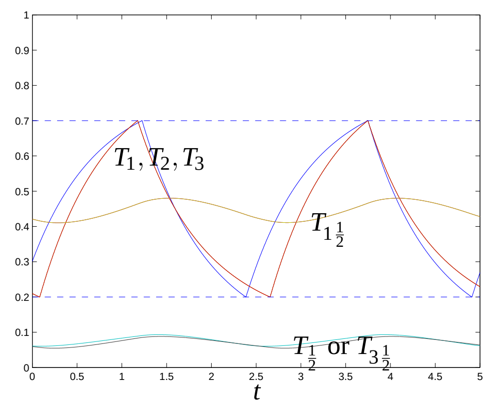


FIGURE 12: Air and wall temperatures in three rooms with walls.

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