On the problem of reliable stabilization for large power systems

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Abstract

In this paper, we consider the problem of reliable stabilization for large power systems – when some of the controllers are faulty in the sense that they fail to act optimally, or do not function in the way that they were originally intended to function. Specifically, we introduce a solution concept that requires controllers to respond optimally (i.e., in the sense of mutual best-response correspondences) to the non-faulty controllers regardless of the identity or actions of the faulty controllers.At any time, we assume that the non-faulty controllers know only that there can be at most one faulty controller in the system, but they know neither the identity of the faulty controller nor how this faulty controller behaves. We present a design framework using an extended LMI technique for deriving reliable state-feedback gains; while a set of filters whose estimation-error dynamics satisfy certain quadratic integral constraints is used as decentralized observers within the subsystems for extending the result to the output-feedback case. Moreover, a sufficient condition for solvability of the problem is provided in terms of the minimum-phase condition of the subsystems. We also present an application of the results to a practical power system problem.

Index Terms

Decentralized control, extended LMI, filters, integral quadratic constraints, power system, reliable control, stabilization.

I. INTRODUCTION

Over the last decades, the electric power systems such as the Eastern/Western North American grids and European grid have experienced unprecedented changes due to the emergence of deregulation in the sector as well as the development of competitive electricity market. These changes have caused a noticeable uncertainty in the load flows, and moreover pushed the networks to their operational limits. Besides, the integration of land-based/offshore large-scale wind generations into the existing network will bring a significant effect on the system dynamics as well as on the load flow of the system. On the other hand, the transmission grids have seen very little expansion due to environmental restrictions. As a result, available transmission and generation facilities are highly utilized with large

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amounts of power interchanges taking place through tie-lines and geographical regions. It is also expected that this trend will continue in the future and result in more stringent operational requirements to maintain reliable services and adequate system dynamic performances. Critical controls like excitation systems, power system stabilizers, static VAR compensators, and a new class of control devices (driven by modern power electronics) will play increasingly key roles in maintaining adequate system dynamic performance. Moreover, proper design of these control systems that takes into account the continually changing dynamic structure of the network is imperative to ensure/guarantee robustness over wide operating conditions in the system. With the emphasis on the robustness and reliability or system performance, there is a need to analyze and design controls in an integrated manner, taking into effect the interaction between the various subsystems and controllers in the system.

In this paper, we consider the problem of reliable decentralized stabilization for large power systems using multi-controller configurations to enhance system robustness/reliability against some changes in operating conditions and/or possible component failures that may occur in actuators, sensors or controllers. In a multi-channel control configuration (e.g., see [1], [2], [3], [4], [5] and references therein on the problem of reliable stabilization with multi-controller configurations), the main objective is to guarantee stability and/or to maintain certain performance criteria of the closed-loop system both when all of the controllers work together and when some controllers become faulty or deviate from nominal operating conditions. Specifically, we introduce a solution concept that requires controllers to respond optimally (i.e., in the sense of mutual best-response correspondences) to the non-faulty controllers regardless of the identity and/or actions of the faulty controllers. At any time, the non-faulty controllers know only that there can be at most one faulty controller in the system, but they know neither the identity of the faulty controller nor how this faulty controller behaves. Such solution concept (which is also required to be robust to any deviations from the equilibrium solutions) is then linked with the problem of reliable state-feedback stabilization using an extended LMI technique, while a set of filters whose estimation-error dynamics satisfy certain quadratic integral constraints (IQCs) is used as decentralized observers within the subsystems to extend the result to the output-feedback case. As an application of our approach, we present a practical power system problem where model reduction and low pass filters are further utilized.

The outline of this paper is as follows. In Section II, we present a preliminary result on the problem of reliable stability for a multi-channel system using a new class of extended LMIs. Section III presents the main results, where the problem of reliable stabilizing for a general multi-channel system is formally stated. Then, a design method of reliable decentralized state feedback stabilization is derived using the extended LMI technique. The design is further extended to decentralized output feedback case using a set of filters (whose estimation error dynamics satisfy certain IQCs) that is also used as decentralized observers within the subsystems. In Section IV, we also present an application of the results to a power system problem, and Section V contains concluding remarks.

Notation: For a matrix $A \in \mathbb{R}^{n \times n}$, $\operatorname{He}(A)$ denotes a hermitian matrix defined by $\operatorname{He}(A) \stackrel{\text{def}}{=} (A + A^T)$, where A^T is the transpose of A. For a matrix $B \in \mathbb{R}^{n \times p}$ with $r = \operatorname{rank} B$, $B^{\perp} \in \mathbb{R}^{(n-r) \times n}$ denotes an orthogonal complement of B, which is a matrix that satisfies $B^{\perp} B = 0$ and $B^{\perp} B^{\perp T} \succ 0$. $\mathbb{S}^n$ and $\mathbb{S}^n_+$ denote the set of positive definite and strictly positive definite $n \times n$ real matrices, respectively, and $\mathbb{C}^-$ denotes the set of complex numbers with negative real parts, that is $\mathbb{C}^- \overset{\text{def}}{=} \{s \in \mathbb{C} \,|\, \mathrm{Re}\{s\} < 0\}$. $\mathbb{RH}_\infty$ denotes the set of rational functions with real coefficients that are proper and analytic in the closed right-half of the complex plane. $\mathbb{RH}_\infty^{m\times n}$ denotes the set of $m \times n$ matrices whose elements are in $\mathbb{RH}_\infty$. $\mathrm{Sp}(A)$ denotes the spectrum of a matrix $A \in \mathbb{R}^{n\times n}$, i.e., $\mathrm{Sp}(A) \overset{\text{def}}{=} \{\lambda \in \mathbb{C} \,|\, \mathrm{rank}(A - \lambda I) < n\}$ and $\mathrm{GL}_n(\mathbb{R})$ denotes the general linear group consisting of all $n \times n$ real nonsingular matrices.

II. Preliminaries

Consider the following finite-dimensional generalized multi-channel system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i\in\mathcal{N}} B_i u_i(t), \\ y_i(t) &= C_i x(t), \quad x(0) = x_0, \quad t \in [0, +\infty), \end{aligned} \tag{1}$$

where $x(t) \in \mathscr{X} \subset \mathbb{R}^n$ is the state of the system, $u_i(t) \in \mathscr{U}_i \subset \mathbb{R}^{r_i}$ is the control input to the ith-channel of the system, $y_i(t) \in \mathscr{Y}_i \subset \mathbb{R}^{m_i}$ is the output of the i-th channel, $A \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times r_i}$, $C_i \in \mathbb{R}^{m_i\times n}$, and $\mathcal{N} \overset{\text{def}}{=} \{1, 2, \ldots, N\}$ represents the set of controllers in the system.

For the above system, we restrict the set $\mathcal{K}$ to be the set of all linear, time-invariant (reliable) stabilizing state-feedback gains that satisfies

$$\mathcal{K} \subseteq \left\{ (K_1, K_2, \ldots, K_N) \in \prod_{i\in\mathcal{N}} \mathcal{K}_i \subseteq \prod_{i\in\mathcal{N}} \mathbb{R}^{r_i\times n} \,\middle|\, \mathrm{Sp}(A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i) \subseteq \mathbb{C}^-, \; \forall j \in \mathcal{N} \cup \{0\} \right\}, \tag{2}$$

where the sets $\mathcal{N}_{\neg 0} \overset{\text{def}}{=} \mathcal{N}$ and $\mathcal{N}_{\neg j}$ are defined by $\mathcal{N}_{\neg j} \overset{\text{def}}{=} \mathcal{N} \backslash \{j\}$ for $j = 1, 2, \ldots, N$ with cardinality of $|\mathcal{N}_{\neg 0}| = N$ and $|\mathcal{N}_{\neg j}| = N - 1$, respectively.

Remark 1: In this paper, we consider the stability of the closed-loop systems $\big(A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i\big)$ under a nominal operation condition (i.e., when $j = 0$) as well as under a possible single-channel controller failure (i.e., when $j \in \mathcal{N}$). However, following the same discussion, we can also consider at most two or more possible controllers failures in the system.

Let us introduce the following matrices that will be used in the sequel.

Definition 1:

$$E_{\neg 0} \overset{\text{def}}{=} \underbrace{\begin{bmatrix} I_{n\times n} & I_{n\times n} & \cdots & I_{n\times n} \end{bmatrix}}_{(|\mathcal{N}_{\neg 0}|+1)\ times}, \quad \langle X_0, X_{\neg 0}\rangle \overset{\text{def}}{=} \mathrm{block\,diag}\{\underbrace{X_0, \overbrace{X_0, \ldots, X_0}^{=X_{\neg 0}}}_{(|\mathcal{N}_{\neg 0}|+1)\ times}\},$$

$$[A, B]_{U_0, L_{\neg 0}} \overset{\text{def}}{=} \begin{bmatrix} AU_0 & B_1L_1 & B_2L_2 & \cdots & B_NL_N \end{bmatrix},$$

$$\langle U_0, W_{\neg 0}\rangle \overset{\text{def}}{=} \mathrm{block\,diag}\{\underbrace{U_0, \overbrace{W_1, W_2, \ldots, W_N}^{=W_{\neg 0}}}_{(|\mathcal{N}_{\neg 0}|+1)\ times}\},$$

$$B_{\neg 0} \overset{\text{def}}{=} \begin{bmatrix} B_1 & B_2 & \cdots & B_N \end{bmatrix}, \quad C_{\neg 0} \overset{\text{def}}{=} \begin{bmatrix} C_1^T & C_2^T & \cdots & C_N^T \end{bmatrix}^T,$$

and for $i \in \mathcal{N}$

$$E_{\neg i} \overset{\text{def}}{=} \underbrace{\left[\begin{array}{cccccc} I_{n\times n} & \cdots & I_{n\times n} & I_{n\times n} & \cdots & I_{n\times n} \end{array}\right]}_{(|\mathcal{N}_{\neg i}|+1)\ times},$$

$$\langle X_i, X_{\neg i}\rangle \overset{\text{def}}{=} \operatorname{block\,diag}\{\underbrace{X_i, \overbrace{X_i, \ldots, X_i, X_i, \ldots, X_i}^{=X_{\neg i}}}_{(|\mathcal{N}_{\neg i}|+1)\ times}\},$$

$$[A,B]_{U_i, L_{\neg i}} \overset{\text{def}}{=} \left[\begin{array}{cccccccc} AU_j & B_1L_1 & \cdots & B_{i-1}L_{i-1} & B_{j+1}L_{j+1} & \cdots & B_NL_N \end{array}\right],$$

$$\langle U_i, W_{\neg i}\rangle \overset{\text{def}}{=} \operatorname{block\,diag}\{\underbrace{U_i, \overbrace{W_1, \ldots, W_{i-1}, X_{i+1}, \ldots, X_N}^{=W_{\neg i}}}_{(|\mathcal{N}_{\neg i}|+1)\ times}\},$$

$$B_{\neg i} \overset{\text{def}}{=} \left[\begin{array}{ccccc} B_1 & \cdots & B_{i-1}B_{i+1} & \cdots & B_N \end{array}\right],\ C_{\neg i} \overset{\text{def}}{=} \left[\begin{array}{cccccc} C_1^T & \cdots & C_{i-1}^T & C_{i+1}^T & \cdots & C_N^T \end{array}\right]^T.$$

Then, we can characterize the set $\mathcal{K}$ using a new-class of extended LMIs as follows.[1]

Theorem 1: Suppose the pairs $(A, B_{\neg j})$ are stabilizable for all $j \in \mathcal{N} \cup \{0\}$. Then, there exist $X_j \in \mathbb{S}_+^n$, $\epsilon_j > 0$, $U_j \in \mathrm{GL}_n(\mathbb{R})$, $j = 0, 1, \ldots, N$, $W_i \in \mathrm{GL}_n(\mathbb{R})$ and $L_i \in \mathbb{R}^{r_i \times n}$, $i = 1, 2, \ldots, N$ such that

$$\begin{bmatrix} 0_{n\times n} & E_{\neg j}\langle X_j, X_{\neg j}\rangle \\ \langle X_j, X_{\neg j}\rangle E_{\neg j}^T & 0_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \end{bmatrix} + \operatorname{He}\left(\begin{bmatrix} [A, B_{\neg j}]_{U_j, L_{\neg j}} \\ -\langle U_j, W_{\neg j}\rangle \end{bmatrix} \times \begin{bmatrix} E_{\neg j}^T & \epsilon_j I_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \end{bmatrix} \right) \prec 0. \tag{3}$$

Moreover, for any family of $|\mathcal{N}_{\neg 0}|$-tuples $\left(\begin{array}{cccc} L_1, & L_2, & \ldots, & L_N \end{array}\right)$ and $\left(\begin{array}{cccc} W_1, & W_2, & \ldots, & W_N \end{array}\right)$ as above, if we set $K_i = L_i W_i^{-1}$ for each $i = 1, 2, \ldots, N$, then the matrices $\big(A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i\big)$ are Hurwitz for all $j \in \mathcal{N} \cup \{0\}$, i.e., $\mathrm{Sp}\big(A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i\big) \subseteq \mathbb{C}^-,\ \forall j \in \mathcal{N} \cup \{0\}$.

Proof: Sufficiency: Note that $\langle X_j, X_{\neg j}\rangle E_{\neg j}^T = E_{\neg j}^T X_j$ and

$$\begin{bmatrix} [A, B]_{U_j, L_{\neg j}} \\ -\langle U_j, W_{\neg j}\rangle \end{bmatrix}^{\perp} = \begin{bmatrix} I_{n\times n} & [A, B]_{U_j, L_{\neg j}}\langle U_j, W_{\neg j}\rangle^{-1} \end{bmatrix},$$
$$\overset{\text{def}}{=} \begin{bmatrix} I_{n\times n} & \big(A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i\big) \end{bmatrix}, \tag{4}$$

$$\begin{bmatrix} E_{\neg j} \\ \epsilon_j I_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \end{bmatrix}^{\perp} = \begin{bmatrix} \epsilon_j I_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} & -E_{\neg j} \end{bmatrix}, \tag{5}$$

for $j = 0, 1, \ldots, N$.

Then, eliminating $\langle U_j, W_{\neg j}\rangle$ from (3) by using these matrices, we have the following matrix inequalities

$$\begin{bmatrix} I_{n\times n} & [A, B]_{U_j, L_{\neg j}}\langle U_j, W_{\neg j}\rangle^{-1} \end{bmatrix} \begin{bmatrix} 0_{n\times n} & E_{\neg j}\langle X_j, X_{\neg j}\rangle \\ \langle X_j, X_{\neg j}\rangle E_{\neg j}^T & 0_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \end{bmatrix} \times \begin{bmatrix} I_{n\times n} \\ (\langle U_j, W_{\neg j}\rangle^{-1})^T [A, B]_{U_j, L_{\neg j}}^T \end{bmatrix} = \operatorname{He}\left((A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i) X_j \right) \prec 0, \tag{6}$$

[1] Recently, a similar extended LMI condition together with dissipativity-based certifications have been investigated by Befekadu et al. [6] in the context of reliable stabilization for multi-channel systems.

$$\begin{bmatrix} \epsilon_j I_{(|\mathcal{N}_{\neg j}|+1)n\times(N+1)n} & -E_{\neg j} \end{bmatrix} \begin{bmatrix} 0_{n\times n} & E_{\neg j}\langle X_j, X_{\neg j}\rangle \\ \langle X_j, X_{\neg j}\rangle E_{\neg j}^T & 0_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \end{bmatrix}$$
$$\times \begin{bmatrix} \epsilon_j I_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \\ -E_{\neg j}^T \end{bmatrix} = -2\epsilon_j(|\mathcal{N}_{\neg j}|+1)X_j \prec 0. \tag{7}$$

Hence, we see that equations (6) and (7) state the Lyapunov stability condition with $X_j \in \mathbb{S}_+^n$ and state-feedback gains $K_i = L_i W_i^{-1}$ for $i = 1, 2, \ldots, N$.

Necessity: Suppose the system in (1) is stable with state-feedback gains $K_i = L_i W_i^{-1}$ for $W_i \in \mathrm{GL}_n(\mathbb{R})$, $i = 1, 2, \ldots, N$. Then, there exist sufficiently small $\epsilon_j > 0$ for $j = 0, 1, \ldots N$ that satisfy

$$\mathrm{He}\,\Big((A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i) X_j \Big) + \frac{1}{2}\epsilon_j \begin{bmatrix} A, B \end{bmatrix}_{X_j, L_{\neg j}} \langle X_j, X_{\neg j}\rangle \begin{bmatrix} A, B \end{bmatrix}_{X_j, L_{\neg j}}^T \prec 0, \tag{8}$$

where $[A, B]_{X_i, L_{\neg i}} = \begin{bmatrix} AX_i & B_1L_1 & \cdots & B_{i-1}L_{i-1} & B_{i+1}L_{i+1} & \cdots & B_N L_N \end{bmatrix}$ for $i \in \mathcal{N}$ and $[A, B]_{X_0, L_{\neg 0}} = \begin{bmatrix} AX_0 & B_1L_1 & B_2L_2 & \cdots & B_N L_N \end{bmatrix}$.

Note that $\langle X_j, X_{\neg j}\rangle \succ 0$ and $\langle X_j, X_{\neg j}\rangle E_{\neg j}^T = E_{\neg j}^T X_j$, employing the Schur complement for (8), then we have

$$\begin{bmatrix} \mathrm{He}\,\big((A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i) X_j \big) & \epsilon_j \, [A, B]_{X_j, L_{\neg j}} \langle X_j, X_{\neg j}\rangle \\ \epsilon_j \langle X_j, X_{\neg j}\rangle ([A, B]_{X_j, L_{\neg j}})^T & -2\epsilon_j \langle X_j, X_{\neg j}\rangle \end{bmatrix}$$
$$= \begin{bmatrix} 0_{n\times n} & E_{\neg j}\langle X_j, X_{\neg j}\rangle \\ \langle X_j, X_{\neg j}\rangle E_{\neg j}^T & 0_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \end{bmatrix} + \mathrm{He}\left(\begin{bmatrix} [A, B]_{X_j, L_{\neg j}} \langle U_j, W_{\neg j}\rangle^{-1} \\ -I_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \end{bmatrix} \right.$$
$$\left. \times \langle X_j, X_{\neg j}\rangle \begin{bmatrix} E_{\neg j}^T & \epsilon_j I_{(|\mathcal{N}_{\neg j}|+1)n\times(|\mathcal{N}_{\neg j}|+1)n} \end{bmatrix} \right) \prec 0. \tag{9}$$

Thus, the above expression (i.e., equation (9)) implies that (3) holds with $\langle U_j, W_{\neg j}\rangle = \langle X_j, X_{\neg j}\rangle$ for $U_j \in \mathrm{GL}_n(\mathbb{R})$ and $j \in \mathcal{N} \cup \{0\}$. ■

Remark 2: We remark that the above extended LMI framework stated in Theorem 1 is useful in the context of reliable control for systems with generalized multi-channel configurations, since the framework effectively separates design variables such as X_j from the system data $(A, B_{\neg j})$ for all $j \in \mathcal{N} \cup \{0\}$.

Note that Theorem 1 is a generalization of the square-extended LMI technique that has been considered in [7] in the context of reliable stabilization for multi-channel systems (e.g., see [8], [9] and references therein for a review of square extended LMIs). In fact, if we multiply equation (3) from the left side by

$$\Gamma_{\neg j} = \begin{bmatrix} (|\mathcal{N}_{\neg j}|+1) I_{n\times n} & 0_{n\times(|\mathcal{N}_{\neg j}|+1)n} \\ 0_{n\times n} & E_{\neg j} \end{bmatrix}, \tag{10}$$

and from the right side by

$$\Gamma_{\neg j}^T = \begin{bmatrix} (|\mathcal{N}_{\neg j}|+1) I_{n\times n} & 0_{n\times n} \\ 0_{(|\mathcal{N}_{\neg j}|+1)n\times n} & E_{\neg j}^T \end{bmatrix}, \tag{11}$$

make use of the relation $E_{\neg j}E_{\neg j}^T = (|\mathcal{N}_{\neg j}|+1)I$ and set $W_i \to W$ for $i = 1, 2, \cdots, N$ and $U_j \to W$ for $j = 0, 1, \cdots, N$ (which also gives us the condition $\langle W, W_{\neg j}\rangle E_{\neg j}^T = E_{\neg j}^T W$), then (3) reduces to

$$\begin{bmatrix} 0 & (|\mathcal{N}_{\neg j}|+1)X_j \\ (|\mathcal{N}_{\neg j}|+1)X_j & 0 \end{bmatrix} + \mathrm{He}\Bigg(\begin{bmatrix} \left(AW + \sum_{i\in\mathcal{N}_{\neg j}} B_i L_i\right) \\ -W \end{bmatrix} \\ \times \begin{bmatrix} (|\mathcal{N}_{\neg j}|+1)I_{n\times n} & \epsilon_j I_{n\times n} \end{bmatrix} \Bigg) \prec 0, \tag{12}$$

which is basically the square extended LMI condition presented in [7], i.e., if we let further $\epsilon_j \to (|\mathcal{N}_{\neg j}|+1)\epsilon_j'$ for all $j \in \mathcal{N} \cup \{0\}$, we then have

$$\begin{bmatrix} 0 & X_j \\ X_j & 0 \end{bmatrix} + \mathrm{He}\left(\begin{bmatrix} \left(AW + \sum_{i\in\mathcal{N}_{\neg j}} B_i L_i\right) \\ -W \end{bmatrix} \begin{bmatrix} I_{n\times n} & \epsilon_j' I_{n\times n} \end{bmatrix} \right) \prec 0. \tag{13}$$

Moreover, if we set $K_i = L_i W^{-1}$, $i = 1, 2, \ldots, N$, for any $W \in \mathrm{GL}_n(\mathbb{R})$ and a family of $|\mathcal{N}_{\neg 0}|$-tuple $\begin{pmatrix} L_1, & L_2, & \ldots, & L_N \end{pmatrix}$ as above, then the matrices $\left(A + \sum_{i\in\mathcal{N}_{\neg j}} B_i K_i\right)$ are Hurwitz for all $j \in \mathcal{N} \cup \{0\}$.

We remark that equation (3) describes a new-class of extended LMI conditions in terms of $W_i \in \mathrm{GL}_n(\mathbb{R})$, $L_i \in \mathbb{R}^{r_i \times n}$, $i = 1, 2, \ldots, N$, and $U_j \in \mathrm{GL}_n(\mathbb{R})$, $X_j \in \mathbb{S}_+^n$, $j = 0, 1, \ldots, N$. Note also that a common set of $\{W_i,\ L_i\}_{i=1}^N$ matrix variables is used for all failure modes, i.e., for all $j \in \mathcal{N} \cup \{0\}$. This is because we need an $|\mathcal{N}_{\neg 0}|$-tuple state-feedback gain $K \overset{\text{def}}{=} \begin{pmatrix} K_1 & K_2 & \ldots & K_N \end{pmatrix}$ with $K_i \in \mathcal{K}_i$ for $i \in \mathcal{N}$ that ensures stability for all possible closed-loop systems. However, it should be noted that, since we use a new-class of extended LMI framework, we do not require either a common quadratic Lyapunov stability certificate $X \in \mathbb{S}_+^n$ as in the case of quadratic Lyapunov technique or a common $W \in \mathrm{GL}_n(\mathbb{R})$ and $\{L_i\}_{i=1}^N$ that will be needed in the case of square extended LMI technique for all possible failure modes (c.f. equation (13)).

In the following, we assume that the following statement holds for the system in (1).

Assumption 1: There are no unstable decentralized fixed modes (DFMs) with respect to triplets of $(C_{\neg j}, A, B_{\neg j})$ for all $j \in \mathcal{N} \cup \{0\}$.

Remark 3: In the following section, this assumption is required for synthesizing the main results that include decentralized stabilization problems with respect to triplets of $(C_{\neg j}, A, B_{\neg j})$ for all $j \in \mathcal{N} \cup \{0\}$. The necessary and sufficient condition for decentralized stabilization can be characterized in terms of the fixed modes of the system (e.g., see reference [10]). Moreover, if any one of these triplets is decentralized stabilizable, then the triplet $(C_{\neg 0}, A, B_{\neg 0})$ is also decentralized stabilizable.

III. Main Results

In this section, we introduce a solution concept for the problem of reliable stabilization that, at any time, the non-faulty controllers know only that there can be at most one faulty controller in the system, but they know neither the identity of the faulty controller nor how this faulty controller behaves. Note that the framework presented in the preceding section (which is based on the extended LMI technique for deriving reliable stabilizing state-feedback gains) is in fact equivalent to the problem of simultaneously stabilizing $(|\mathcal{N}_{\neg 0}| + 1)$ systems which also requires full-state information from each channel. In the following, we consider a set of filters that is used as decentralized observers within the subsystems. By imposing some restrictions on these filters, i.e., if the corresponding estimation-error dynamics are required to satisfy certain IQCs (e.g., see [11], [12] and references therein for a review of IQC formulation), then we can extend the result to the output-feedback case, where the extension implicitly requires each non-faulty controller to respond optimally (i.e., in the sense of best-response correspondences) to the other non-faulty controllers regardless of the identity or actions of the faulty controllers.

Consider the following set of filters

$$\begin{aligned}\dot{\xi}_i(t) &= A_{\pi_i}\xi_i(t) + B_{\pi_i}u_i(t) + L_{\pi_i}y_i(t),\\ \hat{u}_i(t) &= C_{\pi_i}\xi_i(t) + D_{\pi_i}y_i(t),\end{aligned} \tag{14}$$

where the matrix A_{π_i}, $\mathrm{Sp}\,(A_{\pi_i}) \subseteq \mathbb{C}^-$ and appropriate matrices $B_{\pi_i}, C_{\pi_i}, D_{\pi_i}, L_{\pi_i}$ for $i \in \mathcal{N}$.[2] Let us also introduce the estimation-error $e_i(t)$ as

$$e_i(t) = \xi_i(t) - Z_{\pi_i}x(t), \tag{15}$$

where $Z_{\pi_i} \in \mathbb{R}^{n_i \times n}$ (with $n_i \le n$) for $i \in \mathcal{N}$.

We further assume that the estimation-error dynamics satisfy certain IQCs. To see the idea more precisely, let us rewrite the system in (1) as

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B_i u_i(t) + B_{\neg i}u_{\neg i}(t),\\ y_i(t) &= C_i x(t),\end{aligned}$$

where

$$u_{\neg i}(t) = \begin{bmatrix} u_1^T(t) & \cdots & u_{i-1}^T(t) & u_{i+1}^T(t) & \cdots & u_N^T(t) \end{bmatrix}^T.$$

Then, we have the following two equations

$$\begin{aligned}\dot{e}_i(t) &= \dot{\xi}_i(t) - Z_{\pi_i}\dot{x}(t),\\ &= A_{\pi_i}\xi_i(t) + B_{\pi_i}u_i(t) + L_{\pi_i}y_i(t) - Z_{\pi_i}Ax(t) - Z_{\pi_i}B_i u_i(t) - Z_{\pi_i}B_{\neg j}u_{\neg i}(t),\\ &= A_{\pi_i}e_i(t) + (\underbrace{B_{\pi_i} - Z_{\pi_i}B_i}) u_i(t) + (\underbrace{A_{\pi_i}Z_{\pi_i} - Z_{\pi_i}A + L_{\pi_i}C_i}) x(t) - Z_{\pi_i}B_{\neg i}u_{\neg i}(t),\end{aligned} \tag{16}$$

and

$$\begin{aligned}\tilde{u}_i(t) &= \hat{u}_i(t) - u_i(t),\\ &= C_{\pi_i}\xi_i(t) + (\underbrace{D_{\pi_i}C_i - K_i}) x(t),\\ &= C_{\pi_i}e_i(t) + (\underbrace{C_{\pi_i}Z_{\pi_i} + D_{\pi_i}C_i - K_i}) x(t).\end{aligned} \tag{17}$$

[2] Note that the eigenvalues for these filters are at least assumed to the left of all eigenvalues of the closed-loop systems, i.e.,

$$\max_{i \in \mathcal{N}} \left\{ \mathrm{Re}\left\{ \mathrm{Sp}\,(A_{\pi_i}) \right\}\right\} \le \min_{j \in \mathcal{N}\cup\{0\}} \left\{ \mathrm{Re}\left\{ \mathrm{Sp}\left(A + \sum_{i \in \mathcal{N}_{\neg j}} B_i K_i \right)\right\}\right\}.$$

Suppose that the $|\mathcal{N}_{\neg 0}|$-tuple of state-feedback gains $K \overset{\text{def}}{=} (K_1, K_2, \ldots, K_N) \in \mathcal{K}$ that satisfy the conditions in Theorem 1 are given. Moreover, let the matrices A_{π_i} with $\mathrm{Sp}(A_{\pi_i}) \subseteq \mathbb{C}^-$ and B_{π_i}, C_{π_i}, D_{π_i}, L_{π_i}, Z_{π_i} satisfy the following conditions

$$Z_{\pi_i} A = A_{\pi_i} Z_{\pi_i} + L_{\pi_i} C_i, \tag{18}$$

$$Z_{\pi_i} B_i = B_{\pi_i}, \tag{19}$$

$$K_i = C_{\pi_i} Z_{\pi_i} + D_{\pi_i} C_i, \tag{20}$$

for all $i \in \mathcal{N}$. Then, we can rewrite equations (16) and (17) as

$$\dot{e}_i(t) = A_{\pi_i} e_i(t) - Z_{\pi_i} B_{\neg i} u_{\neg i}(t), \tag{21}$$

$$\tilde{u}_i(t) = C_{\pi_i} e_i(t). \tag{22}$$

Next, for all $i \in \mathcal{N}$, we require that the pairs $(u_{\neg i}(t), \tilde{u}_i(t))$ to satisfy the following IQCs

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{u}_{\neg i}(j\omega) \\ \hat{\tilde{u}}_i(j\omega) \end{bmatrix}^* \Pi_i(j\omega) \begin{bmatrix} \hat{u}_{\neg i}(j\omega) \\ \hat{\tilde{u}}_i(j\omega) \end{bmatrix} d\omega \geq 0, \tag{23}$$

where $\hat{u}_{\neg i}(j\omega)$ and $\hat{\tilde{u}}_i(j\omega)$ are Fourier transforms of $u_{\neg i}(t)$ and $\tilde{u}_i(t)$, respectively.

Note that if the above bounded self-adjoint functions $\Pi_i(j\omega)$ further satisfy the following factorization

$$\Pi_i(j\omega) = \Psi_i^*(j\omega) M_i \Psi_i(j\omega) \in \mathbb{C}^{(r_{\neg i}+r_i)\times(r_{\neg i}+r_i)}, \tag{24}$$

where

$$\Psi_i(j\omega) = C_{\pi_i}\big(j\omega I - A_{\pi_i}\big)^{-1} B_{\neg i} \in \mathbb{RH}_{\infty}^{(r_{\neg i}+r_i)\times(r_{\neg i}+r_i)} \quad \text{and} \quad M_i = \begin{bmatrix} Q_i & S_i^T \\ S_i & R_i \end{bmatrix} \in \mathbb{R}^{r_{\neg i}+r_i},$$

with symmetric matrices Q_i and R_i for $i \in \mathcal{N}$. Then, it is easy to see that the following time-domain integral constraints will hold

$$\begin{aligned} \int_0^{T_k} \begin{bmatrix} u_{\neg i}(t) \\ \tilde{u}_i(t) \end{bmatrix}^T M_i \begin{bmatrix} u_{\neg i}(t) \\ \tilde{u}_i(t) \end{bmatrix} dt &= \int_0^{T_k} \left\{ u_{\neg i}^T(t) Q_i u_{\neg i}(t) + 2 u_{\neg i}^T(t) S_i^T \tilde{u}_i(t) + \tilde{u}_i^T(t) R_i \tilde{u}_i(t) \right\} dt, \\ &\geq 0, \end{aligned} \tag{25}$$

for $i \in \mathcal{N}$ and every $T_k > 0$ (see [13], [14]).[3]

In the following, we provide conditions on the constant matrices M_i (i.e., the matrices Q_i, S_i and R_i) under which the asymptotic estimate for $u_i(t)$ can be guaranteed, i.e.,

$$\begin{aligned} \lim_{t\to\infty} \|\tilde{u}_i(t)\| &= \lim_{t\to\infty} \|\hat{u}_i(t) - u_i(t)\|, \\ &= 0, \end{aligned} \tag{27}$$

[3] We remark that the estimation-error dynamics satisfy the passivity property when $Q_i = 0$, $R_i = 0$ and

$$\int_0^{T_k} u_{\neg i}^T(t) S_i^T \tilde{u}_i(t) dt \geq 0, \quad \forall i \in \mathcal{N}, \tag{26}$$

for every $T_k > 0$ and for all square-integrable $u_{\neg i}(t)$.

for all $i \in \mathcal{N}$. This further implies that the set of filters in (14) will work well as a decentralized estimator for $u_i(t)$, $i \in \mathcal{N}$, even if the inputs of the other channels $u_{\neg i}(t)$ are nonzero or unknown (but assumed to be square-integrable on $[0, +\infty)$).[4]

Then, we have the following result which completely characterizes this set of filters within the subsystems.

Theorem 2: Suppose that the $|\mathcal{N}_{\neg 0}|$-tuple of state-feedback gains $K = (K_1, K_2, \ldots, K_N) \in \mathcal{K}$ satisfying the conditions in Theorem 1 are given. Let the matrices A_{π_i} with $\mathrm{Sp}(A_{\pi_i}) \subseteq \mathbb{C}^-$ and B_{π_i}, C_{π_i}, D_{π_i}, L_{π_i}, Z_{π_i}, $i \in \mathcal{N}$ satisfy conditions (18)–(20). Then, there exist $P_i \in \mathbb{S}_+^n$, $Q_i = Q_i^T$, $S_i \in \mathbb{R}^{r_i \times r_{\neg i}}$ and $-R_i \in \mathbb{S}^{r_i}$ for $i \in \mathcal{N}$ that satisfy

$$\begin{bmatrix} P_i A_{\pi_i} + A_{\pi_i}^T P_i - C_{\pi_i}^T Q_i C_{\pi_i} & P_i Z_{\pi_i} B_{\neg i} - C_{\pi_i}^T S_i \\ B_{\neg i}^T Z_{\pi_i}^T P_i - S_i^T C_{\pi_i} & -R_i \end{bmatrix} \preceq 0, \tag{28}$$

where $r_{\neg i} \overset{\text{def}}{=} \sum_{k \in \mathcal{N}_{\neg i}} r_k$.

Proof: To prove the above theorem, we require that the estimation-error dynamics

$$\begin{aligned} \dot{e}_i(t) &= A_{\pi_i} e_i(t) - Z_{\pi_i} B_{\neg i} u_{\neg i}(t), \\ \tilde{u}_i(t) &= C_{\pi_i} e_i(t), \end{aligned}$$

to satisfy certain dissipativity property for all $i \in \mathcal{N}$. To this end, consider the following supply rate functions

$$w_i(u_{\neg i}(t), \tilde{u}_i(t)) \overset{\text{def}}{=} \begin{bmatrix} u_{\neg i}(t) \\ \tilde{u}_i(t) \end{bmatrix}^T \begin{bmatrix} Q_i & S_i^T \\ S_i & R_i \end{bmatrix} \begin{bmatrix} u_{\neg i}(t) \\ \tilde{u}_i(t) \end{bmatrix}, \tag{29}$$

that satisfy $w_i(0, \tilde{u}_i(t)) \le -\alpha_i \|\tilde{u}_i(t)\|^2$ for all $\tilde{u}_i(t) \in \mathbb{R}^{r_i}$, for some constants $\alpha_i > 0$ and for all $i \in \mathcal{N}$ with $w_i(0,0) = 0$.

Since all of A_{π_i} for $i \in \mathcal{N}$ are assumed to be Hurwitz matrices, then we clearly see that the dissipative property is characterized by the following *dissipation inequality*

$$V_i(x(0)) + \int_0^{T_k} w_i(u_{\neg i}(t), \tilde{u}_i(t)) dt \ge V_i(x(T_k)), \tag{30}$$

for every $T_k \ge 0$ and non-negative quadratic storage functions $V_i(x(t)) = x^T(t) P_i x(t)$ with $P_i \in \mathbb{S}_+^n$ satisfying $V_i(0) = 0$ for $i \in \mathcal{N}$ (e.g., see reference [15]).

Hence, the set of conditions in (30) together with (29) further imply that there exist $P_i \in \mathbb{S}_+^n$ for $i \in \mathcal{N}$ such that the following LMI conditions hold

$$\begin{bmatrix} P_i A_{\pi_i} + A_{\pi_i}^T P_i - C_{\pi_i}^T Q_i C_{\pi_i} & P_i Z_{\pi_i} B_{\neg i} - C_{\pi_i}^T S_i \\ B_{\neg i}^T Z_{\pi_i}^T P_i - S_i^T C_{\pi_i} & -R_i \end{bmatrix} \preceq 0.$$

[4]Note that such frameworks within the subsystems will allow controllers to respond optimally (i.e., in the sense of best-response correspondences) to the non-faulty controllers regardless of the identity and actions of the faulty controllers. For example, for any single-channel failure that belongs to the set of channels $\mathcal{N}_{\neg i}$, then the i-th controller always responds with an optimal value, i.e., $u_i(t) = K_i x(t)$, to the other $(|\mathcal{N}_{\neg 0}| - 2)$ non-faulty controllers regardless of the identity or action of the faulty controller. This fact is further clarified in Section IV (see also footnotes in Section IV).

On the other hand, suppose the inputs and outputs of the estimation-error dynamics in equations (21) and (22), i.e., $u_{\neg i}(t)$ and $\tilde{u}_i(t)$ for $i \in \mathcal{N}$, satisfy the following IQCs

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{u}_{\neg i}(j\omega) \\ \hat{\tilde{u}}_i(j\omega) \end{bmatrix}^* \Pi_i(j\omega) \begin{bmatrix} \hat{u}_{\neg i}(j\omega) \\ \hat{\tilde{u}}_i(j\omega) \end{bmatrix} d\omega \geq 0,$$

where $\hat{u}_{\neg i}(j\omega)$ and $\hat{\tilde{u}}_i(j\omega)$ are Fourier transforms of $u_{\neg i}(t)$ and $\tilde{u}_i(t)$, respectively, and $\Pi_i \colon j\mathbb{R} \to \mathbb{C}^{(r_{\neg i}+r_i)\times(r_{\neg i}+r_i)}$ are bounded self-adjoint functions for $i \in \mathcal{N}$.

Then, there exist uniformly-bounded rational functions $\Psi_i(j\omega) \in \mathbb{RH}_{\infty}^{(r_{\neg i}+r_i)\times(r_{\neg i}+r_i)}$ and constant matrices $M_i \in \mathbb{R}^{r_{\neg i}+r_i}$ for $i \in \mathcal{N}$ that satisfy the following factorization

$$\Pi_i(j\omega) = \Psi_i^*(j\omega) M_i \Psi_i(j\omega),$$

where

$$\Psi_i(j\omega) = C_{\pi_i}\big(j\omega I - A_{\pi_i}\big)^{-1} B_{\neg i}, \quad \text{and} \quad M_i = \begin{bmatrix} Q_i & S_i^T \\ S_i & R_i \end{bmatrix},$$

with $\mathrm{Sp}(A_{\pi_i}) \subseteq \mathbb{C}^-$, $Q_i = Q_i^T$ and $-R_i \in \mathbb{S}^{r_i}$ (e.g., see [16]); moreover, the LMI conditions in (28) will hold for all $i \in \mathcal{N}$. ■

Remark 4: We remark that, for every pair of $(u_{\neg i}(t), \tilde{u}_i(t))$, the estimation-error dynamics in equations (21) and (22) with restriction (25) further imply that there exist positive constants δ_i such that

$$\|e_i(t)\| \leq \delta_i \|e_i(0)\|,$$

for every $t \geq 0$ and for all $i \in \mathcal{N}$.

Next, let us define the following matrices that will be used in Theorem 3 below.

Definition 2:

$$\begin{aligned}
A_{D_{\neg 0}}^{\pi} &= \mathrm{block\,diag}\{A_{\pi_1}, A_{\pi_2}, \ldots, A_{\pi_N}\}, & B_{D_{\neg 0}}^{\pi} &= \mathrm{block\,diag}\{B_{\pi_1}, B_{\pi_2}, \ldots, B_{\pi_N}\}, \\
C_{D_{\neg 0}}^{\pi} &= \mathrm{block\,diag}\{C_{\pi_1}, C_{\pi_2}, \ldots, C_{\pi_N}\}, & D_{D_{\neg 0}}^{\pi} &= \mathrm{block\,diag}\{D_{\pi_1}, D_{\pi_2}, \ldots, D_{\pi_N}\}, \\
L_{D_{\neg 0}}^{\pi} &= \mathrm{block\,diag}\{L_{\pi_1}, L_{\pi_2}, \ldots, L_{\pi_N}\}, & Z_{\neg 0}^{\pi} &= \begin{bmatrix} Z_{\pi_1}^T & Z_{\pi_2}^T & \cdots & Z_{\pi_N}^T \end{bmatrix}^T,
\end{aligned}$$

and for $i \in \mathcal{N}$

$$\begin{aligned}
A_{D_{\neg i}}^{\pi} &= \mathrm{block\,diag}\{A_{\pi_1}, \ldots, A_{\pi_{i-1}}, A_{\pi_{i+1}}, \ldots, A_{\pi_N}\}, & B_{D_{\neg i}}^{\pi} &= \mathrm{block\,diag}\{B_{\pi_1}, \ldots, B_{\pi_{i-1}}, B_{\pi_{i+1}}, \ldots, B_{\pi_N}\}, \\
C_{D_{\neg i}}^{\pi} &= \mathrm{block\,diag}\{C_{\pi_1}, \ldots, C_{\pi_{i-1}}, C_{\pi_{i+1}}, \ldots, C_{\pi_N}\}, & D_{D_{\neg i}}^{\pi} &= \mathrm{block\,diag}\{D_{\pi_1}, \ldots, D_{\pi_{i-1}}, D_{\pi_{i+1}}, \ldots, D_{\pi_N}\}, \\
L_{D_{\neg i}}^{\pi} &= \mathrm{block\,diag}\{L_{\pi_1}, \ldots, L_{\pi_{i-1}}, L_{\pi_{i+1}}, \ldots, L_{\pi_N}\}, & Z_{\neg j}^{\pi} &= \begin{bmatrix} Z_{\pi_1}^T & \cdots & Z_{\pi_{i-1}}^T & Z_{\pi_{i+1}}^T & \cdots & Z_{\pi_N}^T \end{bmatrix}^T.
\end{aligned}$$

Theorem 3: Suppose that Theorem 2 holds, then there exist F_i, G_i, H_i, J_i, $i \in \mathcal{N}$ such that

$$\mathrm{Sp}\left(\begin{bmatrix} A + B_{\neg j} J_{D_{\neg j}} C_{\neg j} & B_{\neg j} H_{D_{\neg j}} \\ G_{D_{\neg j}} C_{\neg j} & F_{D_{\neg j}} \end{bmatrix}\right) \subseteq \mathbb{C}^-, \tag{31}$$

for all $j \in \mathcal{N} \cup \{0\}$. Moreover, these matrices (i.e., the decentralized output-feedback controllers $\widetilde{C}_i(s) = H_i(sI - F_i)^{-1}G_i + J_i$ for $i \in \mathcal{N}$) that achieve reliable stabilization are given by

$$\begin{aligned} F_i &= A_{\pi_i} + B_{\pi_i} C_{\pi_i}, & G_i &= L_{\pi_i} + B_{\pi_i} D_{\pi_i}, \\ H_i &= C_{\pi_i}, & J_i &= D_{\pi_i}, \end{aligned} \tag{32}$$

for all $i \in \mathcal{N}$.

Proof: For $j \in \mathcal{N} \cup \{0\}$, let us rewrite equations (18)–(20) as

$$\begin{aligned} Z^{\pi}_{\neg j} A &= A^{\pi}_{D_{\neg j}} Z^{\pi}_{\neg j} + L^{\pi}_{D_{\neg j}} C_{\neg j}, \\ Z^{\pi}_{\neg j} B_{\neg j} &= B^{\pi}_{D_{\neg j}}, \\ K_{\neg j} &= C^{\pi}_{D_{\neg j}} Z^{\pi}_{\neg j} + D^{\pi}_{D_{\neg j}} C_{\neg j}, \end{aligned}$$

and also equation (32) as

$$\begin{aligned} F_{D_{\neg j}} &= A^{\pi}_{D_{\neg j}} + B^{\pi}_{D_{\neg j}} C^{\pi}_{D_{\neg j}}, & G_{D_{\neg j}} &= L^{\pi}_{D_{\neg j}} + B^{\pi}_{D_{\neg j}} D^{\pi}_{D_{\neg j}}, \\ H_{D_{\neg j}} &= C^{\pi}_{D_{\neg j}}, & J_{D_{\neg j}} &= D^{\pi}_{D_{\neg j}}. \end{aligned}$$

Then, the rest of the proof follows a standard state-space transformation of the closed-loop systems for all $j \in \mathcal{N} \cup \{0\}$.

To this end, let us introduce the following state-space transformations

$$\begin{bmatrix} x(t) \\ \xi_{\neg j} \end{bmatrix} \mapsto \Gamma^{\pi}_{\neg j} \begin{bmatrix} x(t) \\ \xi_{\neg j} \end{bmatrix}, \tag{33}$$

with nonsingular matrices

$$\Gamma^{\pi}_{\neg j} = \begin{bmatrix} I & 0 \\ -Z^{\pi}_{\neg j} & I \end{bmatrix} \quad \text{for} \quad j \in \mathcal{N} \cup \{0\},$$

where $\xi_{\neg 0}(t) = \left[\xi_1^T(t)\ \xi_2^T(t)\ \cdots\ \xi_N^T(t)\right]^T$ and $\xi_{\neg i}(t) = \left[\xi_1^T(t)\ \cdots\ \xi_{i-1}^T(t)\ \xi_{i+1}^T(t)\ \cdots\ \xi_N^T(t)\right]^T$ for $i \in \mathcal{N}$.

Then, we obtain the following set of transformed systems

$$\begin{aligned} &\Gamma^{\pi}_{\neg j} \begin{bmatrix} A + B_{\neg j} J_{D_{\neg j}} C_{\neg j} & B_{\neg j} H_{D_{\neg j}} \\ G_{D_{\neg j}} C_{\neg j} & F_{D_{\neg j}} \end{bmatrix} (\Gamma^{\pi}_{\neg j})^{-1} \\ &\quad = \begin{bmatrix} I & 0 \\ -Z^{\pi}_{\neg j} & I \end{bmatrix} \begin{bmatrix} A + B_{\neg j} D^{\pi}_{D_{\neg j}} C_{\neg j} & B_{\neg j} C^{\pi}_{D_{\neg j}} \\ (L^{\pi}_{D_{\neg j}} + B^{\pi}_{D_{\neg j}} D^{\pi}_{D_{\neg j}}) C_{\neg j} & A^{\pi}_{D_{\neg j}} + B^{\pi}_{D_{\neg j}} C^{\pi}_{D_{\neg j}} \end{bmatrix} \begin{bmatrix} I & 0 \\ Z^{\pi}_{\neg j} & I \end{bmatrix}, \\ &\quad = \begin{bmatrix} A + B_{\neg j} K_{\neg j} & B_{\neg j} C^{\pi}_{D_{\neg j}} \\ 0 & A^{\pi}_{D_{\neg j}} \end{bmatrix}, \end{aligned}$$

for all $j \in \mathcal{N} \cup \{0\}$. Since all of $(A + \sum_{i \in \mathcal{N}_{\neg j}} B_i K_i)$ for $j \in \mathcal{N} \cup \{0\}$ and A_{π_i} for $i \in \mathcal{N}$ are Hurwitz matrices, then we immediately see that the statement of the theorem holds. ■

We remark that the solvability condition of Theorem 2 is given by the following lemma, i.e., the minimum-phase condition of each subsystem.[5]

Lemma 1: The set of equations (18)–(20), together with conditions in (28), are solvable with Hurwitz matrices A_{π_i} for all $i \in \mathcal{N}$, if the following relative-degree and minimum-phase conditions hold

$$\begin{aligned} \operatorname{rank} C_i B_{\neg i} &= \operatorname{rank} B_{\neg i}, \\ &= r_{\neg i} \ \left(\overset{\text{def}}{=} \sum_{k \in \mathcal{N}_{\neg i}} r_k \right), \end{aligned} \tag{34}$$

$$\operatorname{rank} \begin{bmatrix} A - sI & B_{\neg i} \\ C_i & 0 \end{bmatrix} = n + r_{\neg i}, \tag{35}$$

for all $i \in \mathcal{N}$ and for all $s \in \mathbb{C}_0^- \overset{\text{def}}{=} \{ s \in \mathbb{C} \,|\, \mathrm{Re}\{s\} \le 0 \}$.

Remark 5: Note that the conditions of Lemma 1 implicitly require $r_{\neg i} \le m_i$ for all $i \in \mathcal{N}$, i.e., the number of outputs of each channel is bounded below. For example, if all of the system channels have single input, then we can see that two outputs are at least required for three-channel system, while one output can be allowed for two-channel system.

IV. Modeling of Power Systems and Simulation Results

In this section, for the sake of completeness, we briefly discuss about modeling of power system with respect to an industrial-scale Power System Dynamics (PSD) simulation software [18] since this software is used primarily in this paper for analysis and simulation of power systems. This section also presents a four machine two area test system which is used for all simulation studies in this paper. Detail information about this test system including the controllers and their parameter values can be found in the appendix part of the paper.

A. Nonlinear modeling of power systems

Modern power systems are characterized by complex dynamic behaviors owing to their size and complexity. As the size of power systems increases, the dynamical processes are becoming more challenging for analysis as well as understanding the underlying physical phenomena. Power systems, even in their simplest form, exhibit nonlinear and time-varying behaviors. Moreover, there are numerous equipment found in todays power systems.[6] Though these equipment found in todays power systems are well-established and quite uniform in design, their precise modeling plays important role for analysis and simulation studies of the whole system. To obtain a meaningful model of power systems, each equipment or component of the power system should be described by appropriate algebraic and/or differential equations. Combining the dynamic models of these individual components together

[5] This is a direct interpretation of Theorem 2, where the existence condition for this class of decentralized estimators/observers entails strong* detectability conditions (e.g., see reference [17]).

[6] Such as synchronous generators, loads, reactive-power control devices like capacitor banks and shunt reactors, power electronically switched devices such as static Var Compensators (SVCs), and the newly developed flexible AC transmission systems (FACTS) devices, series capacitors and other equipments.

with the associated algebraic constraints leads to the dynamic model of power systems. In general, the dynamic model of power systems can be formulated by the following nonlinear differential-algebraic equations:

$$\begin{aligned} \dot{x}(t) &= f(x(t), y(t), u(t), p(t)), \\ 0 &= g(x(t), y(t), u(t), p(t)), \end{aligned} \tag{36}$$

where $x(t)$, $y(t)$ and $u(t)$ are the state output and input variables of the power system, respectively. The parameters $p(t)$ represent parameters and/or effects of control at particular time in the system.

In the following, a brief explanation of the PSD environment, which includes the main model components and their interaction or implementation (see Fig. 1), is given:

- The block in the middle of Fig. 1 is used to describe the dynamics of synchronous machines. Their overall dynamics involve the full scale of energy-storing elements from mechanical masses to electric and magnetic fields, all driven by prime mover, normally turbines and under direct primary controls. Synchronous machines provide virtually all power generations in all todays power system. Moreover, synchronous machines have major influence on the overall dynamic performance of power systems due to their characteristics. A reduced 5th-order model, where stator transient dynamics are neglected [18] and [19], is used for all synchronous machines in this study. The model consists of a set of differential and a set of algebraic equations. Input variables to the models are the complex terminal voltage $\underline{v}_i$, the mechanical turbine torque m_{Ti} and the excitation voltage E_{fd_i}. Moreover, the injected currents into the network which depend on the corresponding state variables of the synchronous machines are used as input to the algebraic network equations.
- The nodal voltages shown at the bottom of the right-side are computed by solving the algebraic network equations of the nodal admittance matrix. Moreover, nonlinear voltage dependent loads are incorporated in the system where the solutions for updating injection currents are carried out iteratively.
- The blocks in the left of Fig. 1 represent the voltage and governor controllers. The governor control block contains, in addition to the direct primary control of the turbine torque (i.e., the governor mechanism), the mechanical dynamics of the equipment, such as the turbine or boiler that tie to the system dynamically through the governor control valve. Similarly, the voltage control block typically includes voltage regulators and exciters; and their dynamics depend on the nature of the feedback control arrangement and the nature of the source of DC voltage. Moreover, additional supplementary controllers with special structures can be easily incorporated either through voltage or governor controller sides and such options give greater flexibility in analysis and simulation studies.

The PSD performs nonlinear simulation for large power systems by using efficient numerical algorithms. Moreover, the PSD contains functional units for numerically linearizing the nonlinear differential-algebraic equations of the system, and then based on the modified Arnoldi's algorithm, it determines a set of eigenvalues and eigenvectors of the linearized system matrix near a given point on the complex plane.

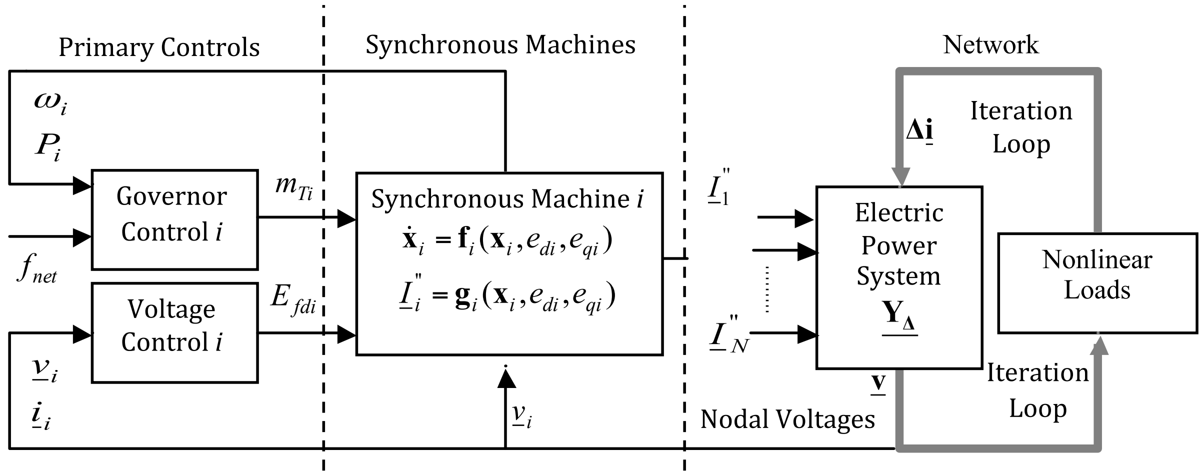

Fig. 1. Nonlinear modeling and simulation of large power system (see [19])

B. Power systems modeling for small-signal analysis

The starting model for small-signal analysis in power system is derived by linearizing the general nonlinear dynamic model of (36) around an operating (or equilibrium) point (x_0, y_0, u_0, p_0) and given as follows:

$$\dot{x}(t) = Ax(t) + B_1 u(t) + B_2 p(t), \tag{37}$$

where $x(t) = \tilde{x}(t) - x_0$, $u(t) = \tilde{u}(t) - u_0$ and $p(t) = \tilde{p}(t) - p_0$. Here the tilde stands for the actual values of states $\tilde{x}(t)$ outputs $\tilde{y}(t)$, inputs $\tilde{u}(t)$ and parameters $\tilde{p}(t)$. Moreover, the matrices A, B_1 and B_2 are evaluated at the operating point (x_0, y_0, u_0, p_0) and given as follows:

$$A = \left[\frac{\partial f(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{x}} - \left[\frac{\partial g(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{y}}\right]^{-1}\left[\frac{\partial g(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{x}}\right]\right]\Bigg|_{(x_0,y_0,u_0,p_0)} \tag{38}$$

$$B_1 = \left[\frac{\partial f(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{u}} - \left[\frac{\partial g(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{y}}\right]^{-1}\left[\frac{\partial g(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{u}}\right]\right]\Bigg|_{(x_0,y_0,u_0,p_0)} \tag{39}$$

and

$$B_2 = \left[\frac{\partial f(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{p}} - \left[\frac{\partial g(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{y}}\right]^{-1}\left[\frac{\partial g(\tilde{x},\tilde{y},\tilde{u},\tilde{p})}{\partial \tilde{p}}\right]\right]\Bigg|_{(x_0,y_0,u_0,p_0)} \tag{40}$$

Depending on how detailed the model in (36) is used; the resulting linearized model (37) may or may not be applicable to study particular physical phenomena in power system. To start with, any disturbance affects all system states, and their exact changes are complex and can only be analyzed by using the full-order model. In a large power system consisting of weakly connected subsystems, it is possible to derive a relatively low-order model relevant for understanding the interactions among the subsystems (inter-area dynamics), as well as detailed models relevant for understanding the dynamics inside each subsystem (intra-area dynamics) [18], [21], [22], [23].[7] Once the models are introduced, the small-signal stability analysis of these models is straightforward.

[7]The reduced model of the subsystem should accurately captures the system dynamics in the frequency range (which lies usually between 0.1 Hz and 10.0 Hz) under considerations and forcing inputs, for instance, the frequency range for the electromechanical dynamic studies of power systems.

Basic analysis uses the elementary result that, given $u(t) = 0$ and $p(t) = 0$, the system of time-invariant linear differential equations (37) will have a stable response to initial conditions $x(0) = 0$ when all eigenvalues of system matrix A are in the left-half plane. Moreover, the robustness of system dynamics can be analyzed using the more involved sensitivity techniques with respect to parameter uncertainties.

C. Simulation results

In this section, we present simulation results to a practical problem in power systems. The system, which is shown in Figure 2, has been specifically designed to study the fundamental behavior of large-interconnected power systems including inter-area oscillations in power systems [20]. This system has four generators and each generator is equipped with the IEEE standard exciter (i.e., IEEE Type DC1A Excitation System) and governor controllers (i.e., Thermal Type Governors). In the simulation studies, the parameters for the exciter and governor controllers were taken from [19], while the generators for all simulations were represented by their 5th-order models with a rated terminal voltage of $15.75\,\mathrm{kV}$. Moreover, the following base-loading condition was assumed: Area-1 at node-1 a load of $P_{L1} = 1600\,\mathrm{MW}$, $Q_{L1} = 150\,\mathrm{Mvar}$ and Area-2 at node-2 a load of $P_{L2} = 2400\,\mathrm{MW}$, $Q_{L2} = 120\,\mathrm{Mvar}$. Detail information about this system including controllers and their parameter values can be found in the appendix part of the paper (see Tables II–VI).

In the actual design, the deviation of real-power ΔP_G from generators G_2 and G_3 were used for decentralized stabilization control through the excitation submodule of a two-channel system. Notice that the corresponding linearized system around the nominal operating, i.e., the base-loading condition, is described by a 36th-order model. We further considered the absolute rotor angle of the first-generator, i.e., $\delta_1(t)$, as a reference frame and, with this setting, we obtained a two-channel model of 35th-order system (c.f. equation (1)), where A is a 35×35 matrix, B_1 and B_2 are 35×1 matrices, and C_1 and C_2 are 1×35 matrices. Furthermore, we performed a model reduction, since a direct design approach for such a system will likely lead to undesirable high-order controllers. With the Hankel model reduction for the minimal realization of the system, we in fact have a 2nd-order model with the following system matrices

$$A_r = \begin{bmatrix} -0.5077 & 6.4908 \\ -1.5679 & -0.5077 \end{bmatrix}, \quad B_{r_1} = \begin{bmatrix} -0.8810 \\ -0.2435 \end{bmatrix}, \quad B_{r_2} = \begin{bmatrix} -1.2060 \\ -0.2922 \end{bmatrix},$$
$$C_{r_1} = \begin{bmatrix} 0.6277 & -0.4804 \end{bmatrix}, \qquad C_{r_2} = \begin{bmatrix} 0.8933 & -0.5763 \end{bmatrix}.$$

Moreover, for $\epsilon_j = 1$, $j = 0, 1, 2$, if we solve the set of extended LMIs feasibility problems that are stated in Theorem 1, then we have the following set of solutions

$$\{X_j\}_{j=0}^{2} = \left\{ \begin{bmatrix} 45.1129 & 1.0916 \\ 1.0916 & 19.9119 \end{bmatrix}, \begin{bmatrix} 50.8752 & 1.6663 \\ 1.6663 & 17.2309 \end{bmatrix}, \begin{bmatrix} 50.4210 & 1.2997 \\ 1.2997 & 17.3699 \end{bmatrix} \right\},$$

$$\{U_j\}_{j=0}^2 = \left\{ \begin{bmatrix} 8.4638 & 7.2871 \\ -5.9527 & 3.0610 \end{bmatrix}, \begin{bmatrix} 8.2312 & 8.0194 \\ -6.2452 & 2.4340 \end{bmatrix}, \begin{bmatrix} 8.3024 & 7.9405 \\ -6.3351 & 2.3713 \end{bmatrix} \right\},$$
$$\{W_i\}_{i=1}^2 = \left\{ \begin{bmatrix} 29.4321 & -0.0476 \\ -6.8470 & 19.6245 \end{bmatrix}, \begin{bmatrix} 28.9167 & 0.0079 \\ -6.0835 & 19.9291 \end{bmatrix} \right\},$$
$$\{L_i\}_{i=1}^2 = \left\{ \begin{bmatrix} 13.2190 & 19.9114 \end{bmatrix}, \begin{bmatrix} 10.9075 & 13.1214 \end{bmatrix} \right\}.$$

Therefore, the corresponding reliable decentralized state-feedback gains $K_i = L_i W_i^{-1}$ for $i = 1, 2$ are, respectively, given by $K_1 = \begin{bmatrix} 0.6856 \; 1.0163 \end{bmatrix}$ and $K_2 = \begin{bmatrix} 0.5157 \; 0.6582 \end{bmatrix}$. To design the corresponding output-feedback controllers, we need to solve simultaneously the set of equations (18)–(20) and the LMI conditions of (28) (c.f. Theorem 2).[8] With Hurwitz matrices A_{π_i} for $i = 1, 2$, we can determine a set of candidate matrices that satisfies the conditions of (18)–(20) and (28), namely, they are given by[9]

$$\begin{aligned} &A_{\pi_1} = -3.9115, && B_{\pi_1} = 0.0079, && C_{\pi_1} = -7.2117, \\ &D_{\pi_1} = 1.8225, && L_{\pi_1} = 1, && Z_{\pi_1} = \begin{bmatrix} 0.0636 & -0.2623 \end{bmatrix}, \\ &A_{\pi_2} = -3.9220, && B_{\pi_2} = -0.0152, && C_{\pi_2} = -3.3645, \\ &D_{\pi_2} = 0.9449, && L_{\pi_2} = 1, && Z_{\pi_2} = \begin{bmatrix} 0.0992 & -0.3574 \end{bmatrix}. \end{aligned}$$

Using Equation (32) of Theorem 3, then the reliable decentralized output-feedback controllers, i.e., $\widetilde{C}_{r_i}(s) = H_i(sI - F_i)^{-1}G_i + J_i$ for $i = 1, 2$, are given by

$$\widetilde{C}_{r_1}(s) = \frac{1.8230s - 0.0823}{s + 3.9690} \qquad \text{and} \qquad \widetilde{C}_{r_2}(s) = \frac{0.9449s - 0.3415}{s + 3.8710}.$$

Note that, since we have employed a model reduction, any high-frequency residual modes of the original system may affect the stability and/or performance of the closed-loop system, which is composed of the original system as well as controllers that may have direct-feedthrough terms (e.g., see [25]). Therefore, we used a first-order low pass filter $\widetilde{H}(s) = 20/(s + 20)$ in both channels.

For a short circuit of $150\,\mathrm{ms}$ duration near to the $380\,\mathrm{kV}$ high-voltage side of the generator's G_2 transformer (i.e., a bus-fault occurred at "Fault" in Figure 2), the transient responses of this generator with/without the reliable stabilization controllers in the system are shown in Figure 3. We remark that any fault conditions corresponding to failure in controller, actuator or sensor in the system are realized by removing the controller from the corresponding excitation submodule, which essentially makes the supplementary control-input signal of the corresponding controller to zero. We also remark that the reliable decentralized state-feedback controllers guaranteed the stability of all closed-loop systems, i.e., $\mathrm{Sp}(A_r + B_{r_1}K_1) = \{-0.9148 \pm j3.1216\} \subseteq \mathbb{C}^-$, $\mathrm{Sp}(A_r + B_{r_2}K_2) = \{-0.9334 \pm j3.1105\} \subseteq \mathbb{C}^-$ and $\mathrm{Sp}(A_r + B_{r_1}K_1 + B_{r_2}K_2) = \{-1.3405 \pm j2.9831\} \subseteq \mathbb{C}^-$

[8] Here we remark that such equilibrium solutions are obtained by simultaneously resolving the $|\mathcal{N}_{\neg 0}|$-system problems (i.e., the set of equations (18)–(20) together with (28)). Note that the behavior of such solutions as a result of changes in the system problem is always a concern when, in particular, robustness is assessed for those essential equilibrium solutions.

[9] Note that the reduced-order system, which is a 2nd-order model, satisfies the minimum-phase conditions of Lemma 1 and, moreover, the invariant (or transmission) zeros of the triplet $(C_{r_i}, A_r, B_{r_{\neg i}})$ for $i = 1, 2$ are -3.9115 and -3.9220, respectively. Hence, we can set the values of A_{π_i} for $i = 1, 2$ to these invariant zeros (e.g., see reference [24]).

that correspond to controller failure at channel-one, channel-two and without any controller failure for the reduced-order system, respectively. Implementing further these reliable output-feedback controllers in the original system, the maximum value for $\mathrm{Re}(\mathrm{Sp})$, which corresponds to the output-feedback controller failure at G_2, G_3 or without any controller failure, is at least less than -0.1225 with damping ratio greater than $25\,\%$.

Moreover, to assess the effectiveness of the approach with respect to transient performances for different loading conditions in the system, we computed the transient performance indices for the generator real-powers P_{G_i}, generator terminal-voltages V_{t_i} and excitation-voltages E_{fd_i} following a short circuit of $150\,\mathrm{ms}$ at the bus-fault location of "Fault" using the following indices[10]

$$I^{P_G} = \sum_{i=1}^{N_G} \int_{t_0}^{t_f} \mid P_{G_i}(t) - P_{G_i}^0 \mid dt, \tag{41}$$

$$I^{V_t} = \sum_{i=1}^{N_G} \int_{t_0}^{t_f} \mid V_{t_i}(t) - V_{t_i}^0 \mid dt, \tag{42}$$

$$I^{E_{fd}} = \sum_{i=1}^{N_G} \int_{t_0}^{t_f} \mid E_{fd_i}(t) - E_{fd_i}^0 \mid dt, \tag{43}$$

where $P_{G_i}^0$, $V_{t_i}^0$ and $E_{fd_i}^0$ are the *pre-fault* generator real-power, terminal and excitation voltages for the i-th generator, respectively, and N_G is the number of generators in the system, while the time-interval $(t_f - t_0)$ is usually 10 to 15 seconds for such transient analyses.

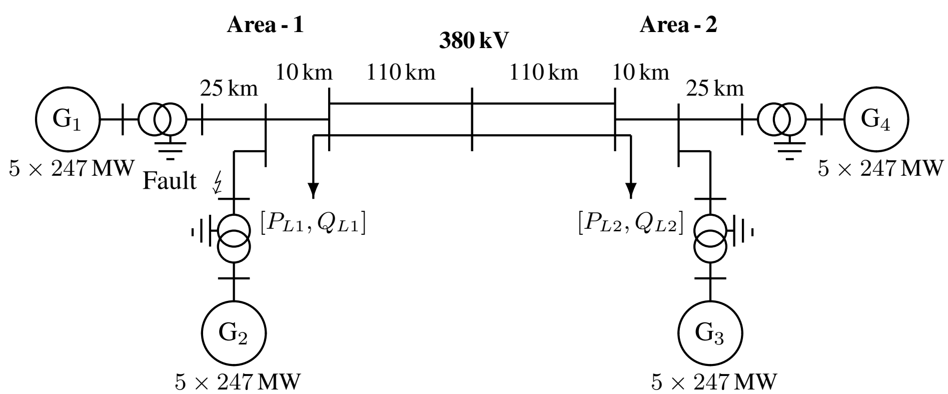

Fig. 2. One-line diagram of four machine two area system.

Notice that these transient performance indices, which are used to investigate the behavior of the system for possible failure modes and/or sudden-load changes, are further normalized to the corresponding transient performance indices of the base-operating condition at which the controller design has been carried-out. That is, the normalized index I_{norm} is computed as: $I_{\mathrm{norm}} = I_{\mathrm{doc}}/I_{\mathrm{boc}}$, where I_{boc} is the transient performance index for the base-operating condition (i.e., Area-1 at node-1 a load of $P_{L1} = 1600\,\mathrm{MW}$, $Q_{L1} = 150\,\mathrm{Mvar}$ and Area-2 at node-2 a load of $P_{L2} = 2400\,\mathrm{MW}$, $Q_{L2} = 120\,\mathrm{Mvar}$), while I_{doc} is the transient performance index for different operating conditions in Area-1 and Area-2. For different loading conditions, the computed normalized transient performance indices are also given in Table I, and it can be seen from this table that these indices for $I_{\mathrm{norm}}(P_G)$,

[10]Notice that, for transient performance analysis, we first validate the feasibility of load flow analysis for each load profile, while the total load in the system, i.e., $[P_{L_{\mathrm{total}}}, Q_{L_{\mathrm{total}}}] = [4000\,\mathrm{MW},\ 270\,\mathrm{Mvar}]$, is kept constant. Then, we perform the fault analysis (and/or the load-switching) in the system.

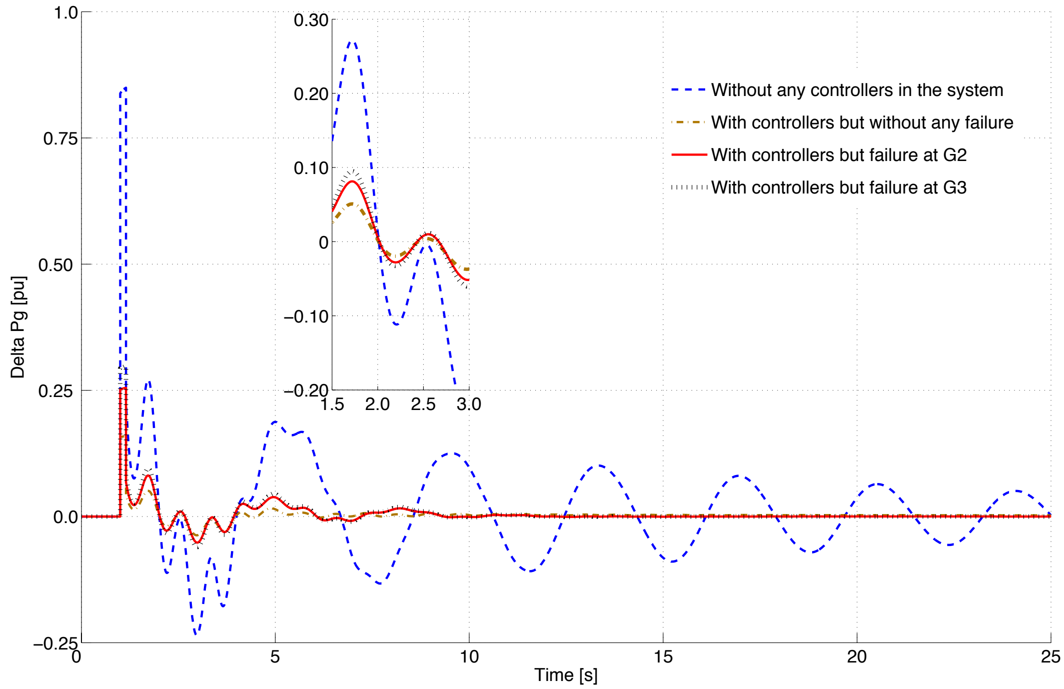

Fig. 3. Transient responses of Generator G_2 to a short circuit of duration 150 ms.

$I_{\text{norm}}(V_t)$ and $I_{\text{norm}}(E_{fd})$ are either near to unity or less than unity for wide-operating conditions. This shows that the transient responses of the generators are significantly damped for different operating conditions – and which clearly indicates the robustness of the system behavior for all loading conditions and possible failure modes.

TABLE I

THE NORMALIZED TRANSIENT PERFORMANCE INDICES FOR P_G, V_t AND E_{fd}

$[P_{L1},\ Q_{L1}]$	$[P_{L2},\ Q_{L2}]$	$I_{\text{norm}}(P_G)$	$I_{\text{norm}}(V_t)$	$I_{\text{norm}}(E_{fd})$
100.0 %	0.0 %	0.8153	0.7677	0.6560
87.5 %	12.5 %	0.8714	0.8561	0.8015
75.0 %	25.0 %	0.9290	0.9328	0.9259
62.5 %	37.5 %	0.9760	0.9862	1.0054
50.0 %	50.0 %	0.9993	1.0080	1.0269
37.5 %	62.5 %	0.9994	0.9967	0.9893
25.0 %	75.0 %	0.9904	0.9622	0.9033
12.5 %	87.5 %	0.9721	0.9179	0.7847
0.0 %	100.0 %	0.9541	0.8669	0.6478

Remark 6: Note that these normalized indices provide a "qualitative" measure on the behavior of the system for possible failure modes and/or sudden-load changes. A value much greater than one implies the system behaves poorly as compared to the base-operating condition.

We remark that the main features of the reliable decentralized output-feedback controllers that have been implemented in this test system are as follows.

(i) The steady-state tie-lines power exchange and the frequency deviation are reduced to zero in a short-time for disturbances such as

- three-phase faults with normal fault-clearing time and single-line to ground faults with delayed fault-clearing time and/or
- load-switchings during light and peak loading conditions with or without failure in either channels.

(ii) The reliable decentralized output-feedback controllers are all linear (with fixed-order, c.f. equation (15)) and use local accessible or measurable information such as deviation of real-power ΔP_G from the generators. Here we remark that there is, in general, no upper-bound on the order of the reliable controllers in terms of the multi-channel system's order. This is a direct consequence of strong stabilization which is essentially involved in reliable stabilization problem.

V. Concluding Remarks

In this paper, we considered the problem of reliable stabilization for power systems using multi-controller configurations. A sufficient condition for the solvability of the problem are also derived in terms of a set of extended LMI conditions, while a set of filters whose estimation-error dynamics satisfy certain quadratic integral constraints is used as decentralized estimators within the subsystems for extending the result to the output-feedback case. This advantage has been confirmed through practical problems in power systems, where we use model reduction to capture some of the relevant system dynamics, i.e., the frequency range which normally lies between 0.1 Hz and 10 Hz for power system small-signal stability analysis.

References

[1] Šiljak DD. Reliable control using multiple control systems. *International Journal of Control.* 1980; **31**(2):303–329.

[2] Šiljak DD. *Decentralized control of complex systems.* Academic Press, 1991.

[3] Vidyasagar M, Viswanadham N. Reliable stabilization using a multi-controller configuration. *Automatica,* 1985; **21**(5):599–602.

[4] Veillette RJ, Medanić JV, Perkins WR. Design of reliable control systems. *IEEE Transactions on Automatic Control.* 1992; **37**(3):290–304.

[5] Tan XL, Šiljak DD, Ikeda M. Reliable stabilization via factorization methods. *IEEE Transactions on Automatic Control.* 1992; **37**(11):1786–1791.

[6] Befekadu GK, Gupta V, Antsaklis PJ. On reliable stabilization via rectangular dilated LMIs and dissipativity-based certifications. *IEEE Transactions on Automatic Control.* 2012; **57**(12), DOI:10.1109/TAC.2012.2211443.

[7] Fujisaki Y, Befekadu GK. Reliable decentralized stabilization of multi-channel systems: A design method via dilated LMIs and unknown disturbance observers," *International Journal of Control.* 2009; **82**(11):2040–2050.

[8] Geromel JC, De Oliveira MC, Hsu L. LMI characterization of structural and robust stability. *Linear Algebra and Its Applications.* 1998; **285**(1-3):69–80.

[9] Fujisaki Y, Befekadu GK. Reliable decentralised stabilization via dilated LMIs and unknown disturbance observers. *Proceedings of 2007 IEEE Conference onControl Applications,* Singapore, 2007; 1583–1588.

[10] Wang SH, Davidson EJ. On stabilization of decentralized control systems. *IEEE Transactions on Automatic Control.* 1973; **18**(5):473–478.

[11] Megretski A, Rantzer F. System analysis via integral quadratic constraints. *IEEE Transactions on Automatic Control.* 1997; **42**(6):819–830.

[12] Yakubovich VA. Quadratic criterion for absolute stability. *Doklady Mathematick.* 1998; **58**(1):169–172.

[13] Willems JC. Least squares stationary optimal control and the algebraic Riccati equation. *IEEE Transactions on Automatic Control.* 1971; **16**(6):621–634.

[14] Yakubovich VA. A frequency theorem for the case in which the state and control spaces are Hilbert spaces, with an application to some problems in the synthesis of optimal controls. *Siberian Mathematical Journal.* 1974; **15**(3):457–476.

[15] Willems JC. Dissipative dynamical systems, part I: General theory. *Archive for Rational Mechanics and Analysis.* 1971; **45**:321–351.

[16] Youla DC. On the factorization of rational matrices. *IRE Transactions on Information Theory.* 1961; **7**(3):172–189.

[17] Hautus MLJ. Strong detectability and observers. *Linear Algebra and Its Applications.* 1983; **50**:353–368.

[18] Erlich I. *Analyse und Simulation des dynamischen Verhaltens von Elektroenergiesystemen.* Habilitation Dissertation, Technical University of Dresden, 1995.

[19] Befekadu GK. *Robust decentralized control of large power systems: a matrix inequalities approach.* PhD Dissertation, University of Duisburg-Essen, Germany, 2006.

[20] Klein M, Rogers GJ, Kundur P. A fundamental study of inter-area oscillations in power systems. *IEEE Transactions on Power Systems.* 1991; **6**(3):914–921.

[21] Chow JH (ed). *Time-Scale modeling of dynamic networks with applications to power systems.* vol. 46, Springer-Verlag, NY, 1982.

[22] Kokotovic P, Khalil HK, Reilly JO. *Singular perturbation methods in control: analysis and design.* Orland, FL Academic Press, 1986.

[23] Chow JH, Winkelman JR, Pai MA, Sauer P. "Application of singular perturbations theory to power system modeling and stability analysis." In *Proc. Am. Control Conf.,* pp. 1401–1407, 1985.

[24] Kudva P, Viswanadham N, Ramakrishna A. Observers for linear systems with unknown inputs. *IEEE Transactions on Automatic Control.* 1980; **25**:113–115.

[25] Khalil HK. A further note on the robustness of output feedback control methods to modeling errors. *IEEE Transactions on Automatic Control.* 1984; **29**(9):861–862.

Appendix – Four Machine Two Area Test System Data

TABLE II

Synchronous Machine Parameters

S_r/MVA	247	x''_q/p.u.	0.24
U_r/kV	15.75	x'_q/p.u.	-
T_m/s	7.0	x''_q/p.u.	0.24
r_s/p.u.	0.002	T'_d/s	0.93
x_s/p.u.	0.19	T''_d/s	0.11
x_d/p.u.	2.49	T'_q/s	-
x_q/p.u.	2.49	T''_q/s	0.2
x'_d/p.u.	2.49	x_{fDd}/p.u.	-

TABLE III

Transmission Lines Data

Single Lines:	$Z_{11} = 0.0309 + j0.266\Omega/km$	$C_b = 0.0136F/km$
Double Lines:	$Z_{11} = 0.0155 + j0.1358\Omega/km$	$C_b = 0.0267F/km$

TABLE IV

Two Winding Transformers Data

S_r/MVA	235	r_{ps}	0.246	z_{ps}	14.203

TABLE V

IEEE DC1A Type Exciter

T_C	0.0173 s	B_{EX}	1.55	T_B	0.06 s	K_F	0.05
K_A	187	T_F	0.62 s	T_A	0.89 s	V_{RMAX}	1.7
T_E	1.15 s	V_{RMIN}	-1.7	A_{EX}	0.014		

TABLE VI

Thermal Type Governor

T_R	0.167 s	T_1	1.0 s	T_G	0.25 s	T_2	0.9 s