Model-based Scheduling for Networked Control Systems

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Abstract— **In this paper, we introduce a model-based scheduling strategy to achieve ultimate boundedness stability in the sensor-actuator networked control systems, where the communication network between the sensor and the network controller is subject to time-varying network induced delays and data-packet dropouts. An estimator and a nominal model of the plant are used explicitly at the controller node to generate control action and schedule control action updates. The data transmissions from the sensor to the network controller are "self-triggered" by imposing the scheduling of the data packet transmissions to meet a soft deadline, while the control action updates generated by the network controller are "event-triggered", with a new measurement of the nominal model's state obtained to update control action whenever a triggering condition is satisfied or whenever the state of the nominal model is reset by the estimator. We also extend this proposed scheduling strategy to the case when signal quantization of the transmitted measurements has to be considered. The approach presented in this paper provides us with a systematic way to design the scheduling strategy for networked control systems by using a model-based approach.**

I. Introduction

Reducing the amount of communication between sensor and controller nodes without compromising the stability of a networked control system has been a popular research topic. A way to address the reduction of communication in a control network is by maximizing the time intervals that the nodes need to send data to each other. Two important approaches that aim at extending the time intervals that a networked control system can operate in open-loop are Model-Based Networked Control Systems (MB-NCSs) and event-triggered control.

MB-NCSs were introduced and described in [1]. This approach makes use of an explicit model of the plant which is added to the actuator/controller node to compute the control input based on the state of the model rather than the plant state. The goal is to operate in open-loop mode (without feedback measurements) for as long as possible by using a state estimate provided by the model of the plant to generate the control input. The measurement updates take place every h seconds in periodic fashion, i.e. h is constant. Conditions for stability provide a range for h that can be used given the plant, the model, and the controller parameters. In an extension, the same authors considered time-varying updates [2]. Two stochastic cases were studied: first, the assumption is that transmission times are identically independent distributed, and second, transmission times are driven by a finite Markov chain. They also considered separately the network induced delays [3] and quantization [4]-[5] problems using periodic updates.

In event-triggered control, sensor measurements are sent to the controller node only when a measure of the local subsystem state error is above a specified threshold. Compared with time-driven control, where constant sampling period is adopted to guarantee stability in the worst case scenario, the possibility of reducing the number of re-computations, and thus of transmissions, while guaranteeing desired levels of performance makes event-based control very appealing in networked control systems (NCSs). A comparison of time-driven and event-driven control for stochastic systems favoring the latter can be found in [8]; a deterministic event-triggered control strategy is introduced in [9]; similar results on deterministic self-triggered feedback control have been reported in [10], [11]; output-based event-triggering control with guaranteed L_∞-gain for linear time-invariant systems has been studied in [14]; an event-triggered real-time scheduling approach for stabilization of passive and output feedback passive (OFP) systems has been proposed in [15], and extensions to more general dissipative systems with time-varying network induced delays have been reported in [16] and [17]; event-triggering stabilization for distributed networked control systems has been studied in [12]; in [13], a self-triggered coordination strategy for optimal deployment of mobile robotics is proposed.

In [6], the authors discarded the periodicity assumption for updating the model that has been used in MB-NCSs. Instead, they embraced a non-periodic approach that is based on events. The estimate of the state given by the model of the plant is used to compare with the actual state. The sensor then transmits the state of the plant to the network controller if the error is above some predefined tolerance. This approach increases the time intervals that we use to update the model with respect to model-based networked control system with periodic sampling by selecting the stabilizing threshold.

When communication networks are used to close the control-loop, we have to consider the possible communication delays and packet dropouts when design the control system. The work on MB-NCSs discussed above is focused on reducing the data transmissions between the sensor and the network controller so that the networked control system can run open-loop for a longer time. The roles of the communication networks discussed in [1]-[6] could be considered as time-delay operators, in general. However, in the presence of time-varying network induced delays, the data packets transmitted by the sensor could arrive at the controller node in a wrong order, which implies that a data packet arriving

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later due to long delay may not contain new information about the plant. Moreover, packet dropouts are very likely to occur due to long delays or data flow congestions. Hence, how to deal with the outdated data received by the network controller and the data-loss in the communication networks are important issues that need be addressed in the context of MB-NCSs. These are the main problems investigated in the present paper. We have derived a systematic model-based scheduling strategy to achieve ultimate boundedness stability in the sensor-actuator networked control systems, which could be applied to both linear and nonlinear networked control systems.

The rest of this paper is organized as follows: we introduce some notations and some basic assumptions that have been used in Section II; our main results are presented in Section III and extensions to signal quantization have been discussed in Section IV; numerical examples are provided in Section V; concluding remarks are made in Section VI.

II. Notations and Basic Assumptions

A. Notations

We shall use the notation $\|x\|_2$ to denote the 2-norm of a vector $x \in \mathbb{R}^n$. A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is said to be locally Lipschitz continuous on a compact set S if there exists a constant $L > 0$ such that $\|f(x) - f(y)\|_2 \leq L\|x - y\|_2$ for every $x, y \in S$. A continuous function $\alpha : [0, a) \to \mathbb{R}_0^+$, is said to be of class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. The symbol Ω_r is used to denote the set $\Omega_r := \{x \in \mathbb{R}^n : V(x) \leq r\}$ where V is a scalar positive definite, continuous differentiable function and $V(0) = 0$. The notation t_0 indicates the initial time instant. The set $\{t_k \geq 0\}$ and $\{\widehat{t}_s \geq 0\}$ denote two sequences of asynchronous time instants such that the interval between two consecutive time instants is not necessary fixed.

B. System Description

Consider control systems described by the following state-space model:

$$\dot{x}(t) = f(x(t), u(t), \omega(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the vector of state variables, $u(t) \in \mathbb{R}^m$ denotes the vector of control input, $\omega(t) \in \mathbb{R}^w$ denotes the vector of disturbance. f is a locally Lipschitiz vector function on $\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^w$ in a compact set S containing the origin and $f(0, 0, 0) = 0$. The disturbance vector is bounded, that is, $\omega(t) \in W$ where:

$$W := \{\omega \in \mathbb{R}^w : \|\omega\|_2 \leq \theta, \theta > 0\}, \tag{2}$$

with θ being a known positive real number. The vector of uncertain variable, $\omega(t)$, is introduced into the model in order to account for the occurrence of uncertainty in the values of the model's parameters and the influence of disturbances.

C. Lyapunov-Based Control

We assume that there exists a feedback control law $u(t) = h(x(t))$ for all x inside a given stability region that renders the origin of the nominal closed-loop system asymptotically stable. Specifically, we assume there exist functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ of class $\mathcal{K}$ and a continuously differentiable Lyapunov function $V(x)$ for the nominal closed-loop system, that satisfy the following inequalities

$$\alpha_1(\|x\|_2) \leq V(x) \leq \alpha_2(\|x\|_2) \tag{3}$$

$$\frac{\partial V(x)}{\partial x} f(x, h(x), 0) \leq -\alpha_3(\|x\|_2) \tag{4}$$

$$\left\| \frac{\partial V(x)}{\partial x} \right\|_2 \leq \alpha_4(\|x\|_2) \tag{5}$$

$$\left\| f(x, u, \omega) - f(x', u, 0) \right\|_2 \leq L_\omega \|\omega\|_2 + L_x \|x - x'\|_2 \tag{6}$$

$$\left\| \frac{\partial V(x)}{\partial x} f(x, u, 0) - \frac{\partial V(x')}{\partial x} f(x', u, 0) \right\|_2 \leq L_{x'} \|x - x'\|_2 \tag{7}$$

for all $x \in \Omega_\rho \subseteq S \subseteq R^n$, where Ω_ρ and S are compact sets containing the origin, and we denote Ω_ρ as the stability region of the nominal closed-loop system under the control $u = h(x)$.

III. Main Results

A. Proposed Set-up

In this section, we introduce our proposed set-up for data transmission scheduling of sensor-actuator networked control systems by using a model-based approach. Generally speaking, in our proposed set-up, one could consider the control action updates generated by the network controller as "event-triggered" while the data transmissions from the sensor to the network controller are "self-triggered". We explain this idea in details in the following sections. The configuration of our proposed set-up is shown in Fig.1.

In Fig.1, the plant dynamics is described in Section II-B, and we assume that there exists a Lyapunov-based control $h(x)$ and a Lyapunov function $V(x)$ which satisfy conditions (3)-(7) in Section II-C. The nominal "*Model* " of the plant is described by

$$\dot{\widehat{x}}(t) = f(\widehat{x}(t), u(t), 0), \tag{8}$$

and the state-space representation of the "*Estimator*" is given by

$$\dot{\widetilde{x}}(t) = f(\widetilde{x}(t), u(t), 0). \tag{9}$$

The estimator is used to estimate the current state of the plant based on the information received from the communication network. The "*Scheduler*" at the plant side is used to schedule the data transmission from the sensor to the network controller. The "*Event-Detector*" is used to monitor the state of the model and update the control actions based on the model's state whenever some event triggering conditions are satisfied, where $\widehat{t}_s$ is the corresponding "*event-time*". t_k represents the time instant of the k_{th} data transmission from the sensor to the network controller and d_k represents the network induced delay associated with the k_{th} data

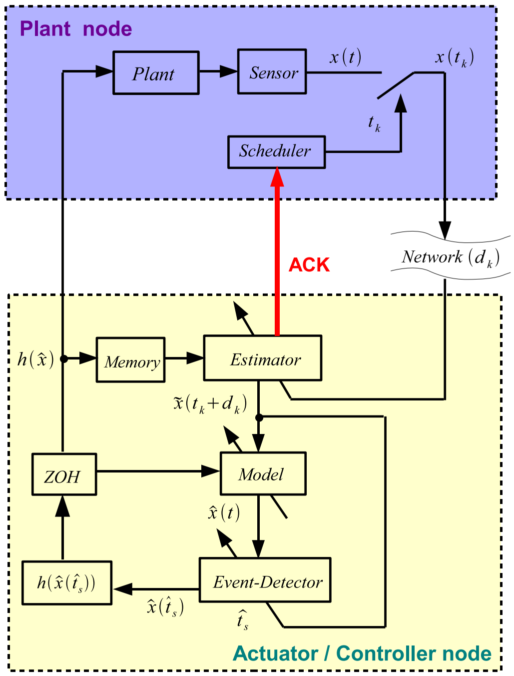

Fig. 1. Proposed Set-up

transmission. We assume that all the network induced delays are upper bounded by a positive constant d_{max}, i.e., $0 \le d_k \le d_{max}$, $\forall k$. This is a reasonable assumption since if the networked control system is running open-loop too long, it will eventually become unstable unless we have a perfect model of the plant at the controller side to estimate the current state of the plant (i.e., the uncertainty variable $\omega(t) = 0$). "ZOH" represents the zero-order hold and "ACK" is short for *acknowledgement*.

B. Model-based Scheduling

Based on the proposed set-up shown in Section III-A, we first explain the control action update strategy implemented in the controller node by using a model-based approach. The proposed strategy is stated as follows:

Step 1: When a new measurement $x(t_k)$ is available to the estimator at time $t_k + d_k$(assuming that the information of the time instant t_k is contained in the k_{th} data packet), the estimator checks whether this measurement provides new information about the plant. Let $x(t_l)$ denote the last measurement accepted by the estimator, if $t_k > t_l$, then go to Step 2. Otherwise, $x(t_k)$ does not contain new information of the plant and is discarded, jump to Step 4.

Step 2: An "ACK" is transmitted back to the *scheduler* at the plant side (see Fig.1) without delay and the estimator uses the information $x(t_k)$ to estimate the current state of the plant (i.e., $x(t_k + d_k)$, where d_k denotes the delay): at the time $t = t_k + d_k$, the estimator resets its state value at the time t_k to be $\widetilde{x}(t_k) = x(t_k)$, then it estimates $x(t_k + d_k)$ based on $\widetilde{x}(t_k)$ and the control input trajectory $u(\tau_p)$, for $\tau_p \in [t_k, t_k + d_k]$. Denote the estimate of $x(t_k + d_k)$ by $\widetilde{x}(t_k + d_k)$. We assume that the computation time for the estimator to get $\widetilde{x}(t_k + d_k)$ is negligible and the "ACK" is implemented with a high priority identifier so that it can be transmitted and be received with negligible delay.

Step 3: At $t_k + d_k$, the state of the model is reset to $\widehat{x}(t_k + d_k) = \widetilde{x}(t_k + d_k)$, an event-time $\widehat{t}_s = t_k + d_k$ is generated, an updated control action $h(\widehat{x}(\widehat{t}_s)) = h(\widehat{x}(t_k + d_k))$ is calculated and is applied to the plant.

Step 4: The event-detector keeps monitoring the state of the model and transmitting the measurements of the model to the controller for control action updates based on a triggering condition(which will be provided later) until a new measurement sent from the plant is received.

Step 5: When a new measurement is received, go to Step 1.

The following two propositions will be used to obtain the stability results under the model-based scheduling strategy proposed in this section.

Proposition 3.1: Consider the systems described in Section II-B

$$\begin{aligned} \dot{x}_1(t) &= f(x_1(t), u(t), \omega(t)) \\ \dot{x}_2(t) &= f(x_2(t), u(t), 0), \end{aligned} \tag{10}$$

where there exists Lyapunov-based control actions such that conditions (3)-(7) are satisfied. With initial condition $x_1(t_0) = x_2(t_0) \in \Omega_\rho$, then we have

$$\left\| x_1(t) - x_2(t) \right\|_2 \le \delta_x(t - t_0) = \frac{L_w \theta}{L_x} \left[e^{L_x(t-t_0)} - 1 \right] \tag{11}$$

as long as $x_1(t), x_2(t) \in \Omega_\rho$, for all $t \ge t_0$.

Proof: Denote the error vector as $e(t) = x_1(t) - x_2(t)$, then we can get

$$\dot{e}(t) = \dot{x}_1(t) - \dot{x}_2(t) = f(x_1(t), u(t), \omega(t)) - f(x_2(t), u(t), 0), \tag{12}$$

in view of (6) we can further get

$$\frac{d}{dt} \left\| e(t) \right\|_2 \le \left\| \dot{e}(t) \right\|_2 \le L_w \theta + L_x \left\| e(t) \right\|_2, \tag{13}$$

for all $x_1(t), x_2(t) \in \Omega_\rho$. Integrating $\left\| e(t) \right\|_2$with initial condition $e(t_0) = 0$, the following bound on the norm of the error vector is obtained

$$\left\| e(t) \right\|_2 \le \frac{L_w \theta}{L_x} \left[e^{L_x(t-t_0)} - 1 \right], \quad \forall t \ge t_0, \tag{14}$$

which completes the proof. ■

Proposition 3.2: [7] Consider the Lyapunov function described in Section II-C, there exists a quadratic function $f_v(\cdot)$ such that:

$$V(x) \le V(x') + f_v\left(\left\| x - x' \right\|_2 \right) \tag{15}$$

for all $x, x' \in \Omega_\rho$, where

$$f_v(s) = \alpha_4(\alpha_1^{-1}(\rho)) s + M_v s^2, \tag{16}$$

with $M_v > 0$.

Proof: Because the Lyapunov function $V(x)$ is continuous and bounded in compact set Ω_ρ, we can find a positive constant M_v such that a Taylor series expansion of V around x' yields

$$V(x) \le V(x') + \frac{\partial V}{\partial x} \left\| x - x' \right\|_2 + M_v \left\| x - x' \right\|_2^2, \ \ \forall x, x' \in \Omega_\rho. \tag{17}$$

Note that the term $M_v\|x-x'\|_2^2$ bounds the high-order terms of the Taylor series of $V(x)$ for all $x, x' \in \Omega_\rho$. Taking into account assumptions (3) and (5), the following bound for $V(x)$ is obtained:

$$V(x) \le V(x')+\alpha_4\big(\alpha_1^{-1}(\rho)\big)\|x-x'\|_2+M_v\|x-x'\|_2^2, \quad (18)$$

$\forall x, x' \in \Omega_\rho$, which completes the proof. ■

In Theorem 3.3 below, we provide sufficient conditions under which our proposed model-based scheduling strategy can guarantee that the state of the closed-loop system (1) is ultimately bounded in a region that contains the origin. To simplify the presentation, we will denote $t_k + d_k$ as the last time instant when a new measurement is accepted by the estimator (where t_k is the time instant at which the packet is transmitted and d_k is the network induced delay). Thus, $t_k + d_k$ is also the last time instant when the scheduler in the plant node receives an ACK from the controller node.

Theorem 3.3: Consider the networked control system shown in Fig.1, where the plant is described in Section II-B, the control law $h(x)$ satisfies the condition (3)-(7) provided in Section II-C. The state measurements of the plant are transmitted to the network controller at asynchronous time instants $\{t_k\}$. The control actions are updated according to the strategy stated in Step 1-Step 5, where the event-time $\{\widehat{t}_s\}$ of updating control actions in the controller node is determined by the time at which

$$L_{x'}\big\|\widehat{x}(t) - \widehat{x}(\widehat{t}_s)\big\|_2 - \alpha_3(\alpha_2^{-1}(\rho_s)) > -\varepsilon \quad (19)$$

for some $\varepsilon > 0$ and $\rho_s > 0$, or whenever the estimator resets the model's state. If a packet containing new information of the plant is accepted by the estimator between $\big[t_k+d_k, t_k+d_k+\tau\big]$, where τ satisfies

$$\varepsilon\tau \ge f_v\Big(\delta_x(d_k)\Big) + f_v\Big(\frac{L_w\theta}{L_x}\big(e^{L_x\tau}-1\big) + \delta_x(d_k)e^{L_x\tau}\Big), \quad (20)$$

$\delta_x(\cdot)$ and $f_v(\cdot)$ are given in Proposition 3.1 and Proposition 3.2, then with $d_0 = 0$ and the initial condition of the plant $x(t_0)$ satisfying

$$V(x(t_0)) + f_v\big(\delta_x(d_{max})\big) \le \rho, \quad \text{where} \quad \rho > \rho_s > 0, \quad (21)$$

the state of the plant $x(t)$ and the state of the model $\widehat{x}(t)$ are ultimately bounded in Ω_ρ.

Proof: For $t \in \big[\widehat{t}_s, \widehat{t}_{s+1}\big]$, $s = 0, 1, 2, \ldots$, in view of (7), we have

$$\begin{aligned}\dot{V}(\widehat{x}(t)) &= \frac{\partial V(\widehat{x}(t))}{\partial \widehat{x}} f\Big(\widehat{x}(t), h(\widehat{x}(\widehat{t}_s)), 0\Big)\\ &= \frac{\partial V(\widehat{x}(t))}{\partial \widehat{x}} f\Big(\widehat{x}(t), h(\widehat{x}(\widehat{t}_s)), 0\Big)\\ &\quad - \frac{\partial V(\widehat{x}(\widehat{t}_s))}{\partial \widehat{x}} f\Big(\widehat{x}(\widehat{t}_s), h(\widehat{x}(\widehat{t}_s)), 0\Big)\\ &\quad + \frac{\partial V(\widehat{x}(\widehat{t}_s))}{\partial \widehat{x}} f\Big(\widehat{x}(\widehat{t}_s), h(\widehat{x}(\widehat{t}_s)), 0\Big)\\ &\le L_x^{'}\big\|\widehat{x}(t) - \widehat{x}(\widehat{t}_s)\big\|_2 - \alpha_3\Big(\big\|\widehat{x}(\widehat{t}_s)\big\|_2\Big)\\ &\le L_x^{'}\big\|\widehat{x}(t) - \widehat{x}(\widehat{t}_s)\big\|_2 - \alpha_3\big(\alpha^{-1}(\rho_s)\big)\end{aligned} \quad (22)$$

for all $\widehat{x}(\widehat{t}_s) \in \Omega_\rho/\Omega_{\rho_s}$. So if

$$L_{x'}\big\|\widehat{x}(t) - \widehat{x}(\widehat{t}_s)\big\|_2 - \alpha_3(\alpha_2^{-1}(\rho_s)) \le -\varepsilon, \quad \text{for some} \quad \varepsilon > 0, \quad (23)$$

then

$$\dot{V}(\widehat{x}(t)) \le -\varepsilon, \quad \text{for} \quad t \in \big[\widehat{t}_s, \widehat{t}_{s+1}\big), \quad (24)$$

which further yields

$$V(\widehat{x}(t)) \le V(\widehat{x}(\widehat{t}_s)) \quad \text{and} \quad V(\widehat{x}(t)) \le V(\widehat{x}(\widehat{t}_s)) - \varepsilon(t-\widehat{t}_s), \quad (25)$$

for $t \in \big[\widehat{t}_s, \widehat{t}_{s+1}\big)$. Note that the triggering condition (19) guarantees that (23) is satisfied.

In our case, the event-time for control action updates is determined by the time when the triggering condition (19) in the controller node is satisfied, or whenever a new measurement is accepted by the estimator to obtain a new estimate of the plant. Assume that at $t = t_k + d_k$, a measurement $x(t_k)$ is received, and the estimator detects that this measurement provides new information about the plant. Hence at $t_k + d_k$, the estimator resets its state at t_k to be $\widetilde{x}(t_k) = x(t_k)$, and estimates the current state of the plant based on the control trajectory $u(\tau_p)$ applied to the plant, for $\tau_p \in [t_k, t_k + d_k]$. One can verify that:

$$\|\widetilde{x}(t_k + d_k) - x(t_k + d_k)\|_2 \le \frac{L_w\theta}{L_x}\Big(e^{L_x d_k} - 1\Big) = \delta_x(d_k), \quad (26)$$

where $\widetilde{x}(t_k+d_k)$ is the estimate of $x(t_k+d_k)$ obtained by the estimator. Since at the same time, $\widehat{x}(t)$ is reset to $\widetilde{x}(t_k+d_k)$, we can get

$$\|\widehat{x}(t_k + d_k) - x(t_k + d_k)\|_2 \le \frac{L_w\theta}{L_x}\Big(e^{L_x d_k} - 1\Big) = \delta_x(d_k). \quad (27)$$

One can further conclude that

$$\begin{aligned}\big\|\widehat{x}(t) - x(t)\big\|_2 \le \frac{L_w\theta}{L_x}\big[e^{L_x(t-t_k-d_k)} - 1\big]\\ + \delta_x(d_k)e^{L_x(t-t_k-d_k)},\end{aligned} \quad (28)$$

for $t \in \big[t_k+d_k, t_{k+j}+d_{k+j}\big)$, where j is an unknown integer such that $t_{k+j} + d_{k+j}$ is the next time instant at which a new measurement is accepted by the estimator (i.e., all the measurements received between $(t_k+d_k, t_{k+j}+d_{k+j})$ do not provide new information about the plant and are discarded). However, the inductions shown in (22)-(28) are all obtained under the assumptions that $x(t), \widehat{x}(t) \in \Omega_\rho$, $\forall t \ge t_0$, which will be proved as follows:

Assume that $x(t), \widehat{x}(t) \in \Omega_\rho$, $\forall t \ge t_0$, for $t \in \big[t_k + d_k, t_{k+j} + d_{k+j}\big)$, based on Proposition 2 and in view of (28), we can get

$$\begin{aligned}V(x(t)) &\le V(\widehat{x}(t)) + f_v\Big(\big\|x(t) - \widehat{x}(t)\big\|_2\Big) \le V(\widehat{x}(t))\\ &\quad + f_v\Big(\frac{L_w\theta}{L_x}\big[e^{L_x(t-t_k-d_k)} - 1\big] + \delta_x(d_k)e^{L_x(t-t_k-d_k)}\Big),\end{aligned} \quad (29)$$

let $\tau = t - t_k - d_k$ and in view of (25), we can rewrite (29) as

$$\begin{aligned} V(x(t)) &\le V(\widehat{x}(t)) + f_v\Big(\frac{L_w\theta}{L_x}\big(e^{L_x\tau}-1\big) + \delta_x(d_k)e^{L_x\tau}\Big) \\ &\le V(\widehat{x}(t_k+d_k)) - \varepsilon\tau \\ &+ f_v\Big(\frac{L_w\theta}{L_x}(e^{L_x\tau}-1) + \delta_x(d_k)e^{L_x}\tau\Big). \end{aligned} \tag{30}$$

Since

$$\begin{aligned} V(\widehat{x}(t_k+d_k)) &\le V(x(t_k+d_k)) \\ &+ f_v\Big(\big\|\widehat{x}(t_k+d_k) - x(t_k+d_k)\big\|_2\Big) \\ &\le V(x(t_k+d_k)) + f_v\Big(\delta_x(d_k)\Big), \end{aligned} \tag{31}$$

replace (31) into (30), we can get

$$\begin{aligned} V(x(t)) &\le V(x(t_k+d_k)) - \varepsilon\tau + f_v\Big(\delta_x(d_k)\Big) \\ &+ f_v\Big(\frac{L_w\theta}{L_x}\big(e^{L_x\tau}-1\big) + \delta_x(d_k)e^{L_x\tau}\Big) \\ &\le V(x(t_k+d_k)) - \varepsilon\tau + f_v\Big(\delta_x(d_k)\Big) \\ &+ f_v\Big(\frac{L_w\theta}{L_x}\big(e^{L_x\tau}-1\big) + \delta_x(d_k)e^{L_x\tau}\Big) \end{aligned} \tag{32}$$

Let $f_1(\tau) = \varepsilon\tau$ and $f_2(\tau) = f_v\Big(\delta_x(d_k)\Big) + f_v\Big(\frac{L_w\theta}{L_x}\big(e^{L_x\tau} - 1\big) + \delta_x(d_k)e^{L_x\tau}\Big)$, with both $f_1(\tau)$ and $f_2(\tau)$ being strictly increasing function of τ, if there exists a non-empty set $\widehat{\tau}$ such that $f_1(\tau) \ge f_2(\tau)$, $\forall \tau \in \widehat{\tau}$, then we can get

$$V(x(t)) \le V(x(t_k+d_k)), \quad \text{for} \quad t \in [t_k+d_k, t_{k+j}+d_{k+j}), \tag{33}$$

with $t_{k+j}+d_{k+j} = t_k+d_k+\tau$, $\tau \in \widehat{\tau}$. This further indicates that if a measurement containing new information of the plant is received by the estimator between $[t_k+d_k, t_k+d_k+\tau]$, where $\tau \in [min\{\widehat{\tau}\}, max\{\widehat{\tau}\}]$, then (33) holds. The typical look of the functions $f_1(\tau)$ and $f_2(\tau)$ is shown in Fig.2.

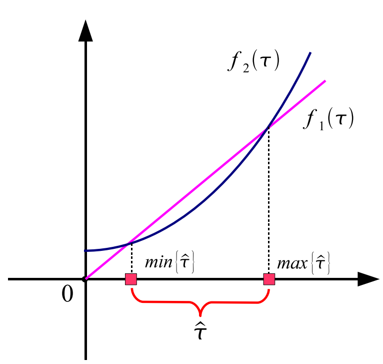

Fig. 2. functions $f_1(\tau)$ and $f_2(\tau)$

By using (33) recursively, we can get

$$V(x(t)) \le V(x(t_0+d_0)). \tag{34}$$

With $d_0 = 0$ (which implies that the initial measurement of the plant is transmitted to the controller node without delay), we have

$$V(x(t)) \le V(x(t_0)), \tag{35}$$

which further yields $x(t) \in \Omega_\rho$, $\forall t \ge 0$. Next, we need to show that $\widehat{x}(t) \in \Omega_\rho$, $\forall t \ge t_0$. Since at $t = t_{k+j} + d_{k+j}$, we have

$$\begin{aligned} V(\widehat{x}(t_{k+j}+d_{k+j})) &\le V(x(t_{k+j})+d_{k+j}) + f_v\Big(\delta_x(d_{k+j})\Big) \\ &\le V(x(t_0)) + f_v\Big(\delta_x(d_{k+j})\Big), \forall k, j \end{aligned} \tag{36}$$

with

$$V(x(t_0)) + f_v\Big(\delta_x(d_{k+j})\Big) \le V(x(t_0)) + f_v\Big(\delta_x(d_{max})\Big) \le \rho$$

(in view of (44)), we can conclude that

$$V(\widehat{x}(t_{k+j}+d_{k+j})) \le \rho, \quad \forall k, j. \tag{37}$$

Moreover, during the time interval $[t_k+d_k, t_{k+j}+d_{k+j})$, the triggering condition (19) guarantees that $\dot{V}(\widehat{x}(t)) \le -\varepsilon$, which indicates that $V(\widehat{x}(t))$ is decreasing for $t \in [t_k + d_k, t_{k+j}+d_{k+j})$, together with (37), we can conclude that $V(\widehat{x}(t)) \le \rho$, for all $t \ge t_0$. Hence, both $x(t)$ and $\widehat{x}(t) \in \Omega_\rho$,$\forall t \ge t_0$, which completes the proof. ■

***Remark** 3.4:* In our proposed scheduling strategy, d_k can be estimated at the plant node if the scheduler remembers the time instant associated with the k_{th} data packet transmission and compares it with the time instant at which the ACK associated with the k_{th} data packet is received.

***Remark** 3.5:* Let us consider the worst case scenario and see how to design the scheduling strategy in order to avoid zeno transmission time from the sensor to the controller node (i.e., to guarantee that there exists a positive constant ζ such that $t_{k+1} - t_k \ge \zeta$, $\forall k$). Assume that the $(k+1)_{th}$ data packet has to be accepted by the estimator after the k_{th} data packet is accepted (we consider the case when $j = 1$ in Theorem 3.3). Then, we need

$$\begin{aligned} \tau &= t_{k+1} + d_{k+1} - t_k - d_k = (t_{k+1} - t_k) + (d_{k+1} - d_k) \\ &\ge \zeta + (d_{k+1} - d_k). \end{aligned} \tag{38}$$

In the worst case scenario, we can assume $\max_k(d_{k+1} - d_k) = d_{max}$. Hence, if there exists a τ satisfying (20) and $\tau \ge \zeta + d_{max}$, then we can achieve non-zeno transmission time. This would require that

$$\begin{aligned} &\varepsilon(\zeta + d_{max}) \ge f_v\Big(\delta_x(d_k)\Big) \\ &+ f_v\Big(\frac{L_w\theta}{L_x}\big(e^{L_x(\zeta+d_{max})} - 1\big) + \delta_x(d_k)e^{L_x(\zeta+d_{max})}\Big), \end{aligned} \tag{39}$$

and in the worst case scenario ($d_k = d_{max}$, $\forall k$), we need

$$\begin{aligned} &\varepsilon(\zeta + d_{max}) \ge f_v\Big(\delta_x(d_{max})\Big) \\ &+ f_v\Big(\frac{L_w\theta}{L_x}\big(e^{L_x(\zeta+d_{max})} - 1\big) + \delta_x(d_{max})e^{L_x(\zeta+d_{max})}\Big). \end{aligned} \tag{40}$$

However, whether there exist non-trivial ζ and d_{max} such that the inequality (40) holds depends on the dynamics of the plant, the dynamics of the stabilizing controllers and the size of the model uncertainty.

***Remark** 3.6:* Let us consider the ideal case when there is no model uncertainties (i.e., $\theta = 0$), then inequality (40) will be reduced to $\varepsilon(\zeta + d_{max}) \geq 0$, which holds for any positive ζ and d_{max}. This is true because if we have a perfect model of the plant in the controller node and the external disturbance to the plant can be neglected, then there is no need to transmit the plant measurements to the network controller through the communication network for control action update as long as the network controller has a good estimate on the initial condition of the plant. In this case, it is sufficient to stabilize the networked control system by applying the control action generated based on the state of the model under the triggering condition (19).

***Remark** 3.7:* One can consider the data transmissions from the sensor to the network controller as "self-triggered" since the scheduling of the data-packet transmissions has to meet a soft deadline implicitly determined by condition (20). The control action updates generated by the network controller could be considered as "event-triggered" , since the control action update is triggered whenever the triggering condition (19) is satisfied or whenever the state of the model is reset by the estimator.

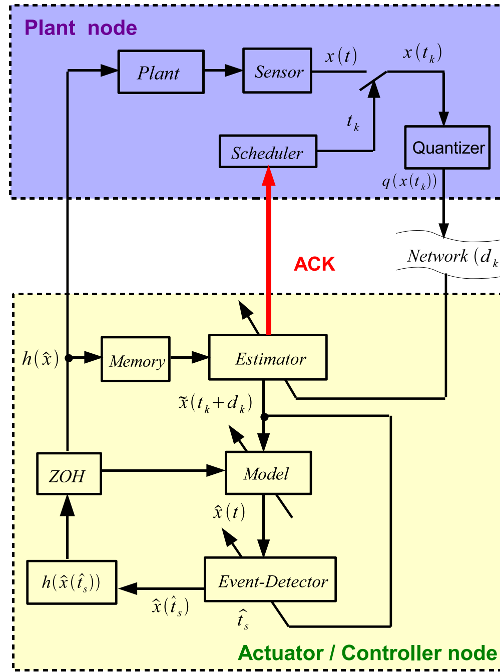

Fig. 3. Proposed Set-up with Signal Quantization

IV. Extension to Signal Quantization

It was assumed in the previous section that the sensor is able to measure the state of the plant with infinite precision. The sensor uses that measurement to send it through the network and the estimator uses it to estimate the current state of the plant. In reality, however, the measured variables have to be quantized in order to be represented by a finite number of bits, then to be used in processor operations and be carried over a digital communication network. It becomes necessary to study the effects of quantization error in our proposed model-based scheduling strategy.

The proposed set-up is shown in Fig.3, which is very similar to the set-up shown in Fig.1, but the measurement $x(t_k)$ has to be quantized first then it can be transmitted through the communication network. We assume that the quantizer implemented at the plant side is with *bounded quantization error*, such that

$$\left\|x(t_k) - q(x(t_k))\right\|_2 \leq \delta_q, \tag{41}$$

where δ_q denotes the quantization error. This assumption applies to most quantizer used in practice.

When considering signal quantization of the transmitted measurements at the plant side, the control action update strategy will be very similar to the strategy without considering signal quantization as stated in Section III-B, with some minor difference briefly described as follows: assume that at $t = t_k + d_k$, a measurement $q(x(t_k))$ is received, the estimator needs to decide whether this measurement provides new information about the plant; if the estimator detects that new information is received, then at $t = t_k + d_k$, the estimator resets its state at t_k to be $\widetilde{x}(t_k) = q(x(t_k))$, and estimates the current state of the plant based on the control trajectory $u(\tau_p)$ applied to the plant, for $\tau_p \in [t_k, t_k + d_k]$. By using the same techniques shown in Proposition 3.1, one can verify that

$$\begin{aligned}
&\left\|\widetilde{x}(t_k + d_k) - x(t_k + d_k)\right\|_2 \\
&\leq \frac{L_\omega \theta}{L_x}\left(e^{L_x d_k} - 1\right) + \left\|x(t_k) - q(x(t_k))\right\|_2 e^{L_x d_k} \\
&= \frac{L_\omega \theta}{L_x}\left(e^{L_x d_k} - 1\right) + \delta_q e^{L_x d_k} = \widetilde{\delta}_x(d_k),
\end{aligned} \tag{42}$$

as long as $\widetilde{x}(t), x(t) \in \Omega_\rho$, $\forall t$.

The techniques to derive the stability results under the model-based scheduling strategy are similar to the previous discussed results in Theorem 3.3. Note that in this case, the function $\delta_x(d_k)$ in (20) should be replaced by $\widetilde{\delta}_x(d_k)$ in (42), and the function $\delta_x(d_{max})$ in (44) should be replaced by $\widetilde{\delta}_x(d_{max})$, where

$$\widetilde{\delta}_x(d_{max}) = \frac{L_\omega \theta}{L_x}\left(e^{L_x d_{max}} - 1\right) + \delta_q e^{L_x d_{max}}.$$

The result is briefly summarized in Theorem 4.1.

***Theorem** 4.1:* Consider the modified set-up shown in Fig.3. The event-time $\{\widehat{t}_s\}$ of updating control actions in the controller node is determined by the same conditions as provide in Theorem 3.3. If a packet containing new information of the plant is accepted by the estimator between $\left[t_k + d_k, t_k + d_k + \tau\right]$, where τ satisfies

$$\varepsilon\tau \geq f_v\left(\widetilde{\delta}_x(d_k)\right) + f_v\left(\frac{L_w \theta}{L_x}\left(e^{L_x \tau} - 1\right) + \widetilde{\delta}_x(d_k) e^{L_x \tau}\right), \tag{43}$$

then with $d_0 = 0$ and the initial condition of the plant $x(t_0)$ satisfying

$$V(x(t_0)) + f_v\left(\widetilde{\delta}_x(d_{max})\right) \leq \rho, \quad \text{where} \quad \rho > \rho_s > 0, \tag{44}$$

the state of the plant $x(t)$ and the state of the model $\widehat{x}(t)$ are ultimately bounded in Ω_ρ.

V. Example

***Example** 5.1:* We now illustrate our results provided in Section III by an example which has also been examined in [9]. Consider the linear control system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u + \begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}\omega, \quad (45)$$

where the external disturbance $\|\omega\|_2 \leq 0.1$. The model of the plant is given by

$$\begin{bmatrix}\dot{\widehat{x}}_1\\ \dot{\widehat{x}}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & 3\end{bmatrix}\begin{bmatrix}\widehat{x}_1\\ \widehat{x}_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u, \quad (46)$$

and it is stabilized by the linear feedback control $u = \widehat{x}_1 - 4\widehat{x}_2$. Using $V = \widehat{x}^T P \widehat{x}$ as a Lyapunov function, we obtain $\frac{\partial V}{\partial \widehat{x}}(A\widehat{x} + BK\widehat{x}) = -\widehat{x}^T Q \widehat{x}$ with P and Q defined by

$$P = \begin{bmatrix}1 & \frac{1}{4}\\ \frac{1}{4} & 1\end{bmatrix}, \quad Q = \begin{bmatrix}\frac{1}{2} & \frac{1}{4}\\ \frac{1}{4} & \frac{3}{2}\end{bmatrix}. \quad (47)$$

Hence, we can get

$$\alpha_1(\|\widehat{x}\|_2) = \lambda_{min}(P)\|\widehat{x}\|_2^2 = 0.75\|\widehat{x}\|_2^2, \quad (48)$$

$$\alpha_2(\|\widehat{x}\|_2) = \lambda_{max}(P)\|\widehat{x}\|_2^2 = 1.25\|\widehat{x}\|_2^2, \quad (49)$$

$$\alpha_3(\|\widehat{x}\|_2) = \lambda_{min}(Q)\|\widehat{x}\|_2^2 = 0.441\|\widehat{x}\|_2^2, \quad (50)$$

$$\left\|\frac{\partial V(\widehat{x})}{\partial \widehat{x}}\right\|_2 = \left\|\begin{bmatrix}2 & \frac{1}{2}\\ \frac{1}{2} & 2\end{bmatrix}\right\|_2 \left\|\begin{bmatrix}\widehat{x}_1\\ \widehat{x}_2\end{bmatrix}\right\|_2 \leq 2.5\|x\|_2 \quad (51)$$
$$\Rightarrow \alpha_4(\|\widehat{x}\|_2) = 2.5\|\widehat{x}\|_2,$$

$L_w = 0.1$, $L_x = 3.7025$, $L_x^{'} = 2.5$, $M_v = \lambda_{max}(P) = 1.25$, $\theta = 0.1$. With $\rho = 100$, we can get $f_v(s) = \alpha_4(\alpha_1^{-1}(\rho))s + M_v s^2 = 28.8675s + 1.25s^2$. Assume that the maximum network induced delay is $d_{max} = 0.05s$, and the external disturbance is given by $\omega(t) = 0.1sin(t)$. The plot of the functions $f_1(\tau)$ and $f_2(\tau)$ with $\varepsilon = 2$ is shown in Fig.4, and one can find that $min\{\widehat{\tau}\} = 0.03s$ and $max\{\widehat{\tau}\} = 0.76s$, which indicates that as long as a packet containing new information of the plant is received by the estimator between $[t_k + d_k, t_k + d_k + \tau](\tau \in [0.03s, 0.76s])$ after the k_{th} packet is accepted by the estimator, the state of the plant is ultimately bounded. In our simulation, we randomly choose τ between $[0.5s, 0.7s]$ for a new packet to be accepted by the estimator. The initial state of the plant is given by $x_1(t_0) = -5$ and $x_2(t_0) = 8$, which satisfies $V(x(t_0)) + f_v(\delta_x(d_{max})) \leq \rho$ and $x(t_0) \in \Omega_\rho$. The simulation results are shown in Fig.5-Fig.7.

Fig.5 shows the time-instant at which the estimator resets the sate of the model. Fig.6 shows the evolution of the state of the plant, and Fig.7 shows the evolution of the state of the model. One can see that both the state of the plant $x(t)$ and the state of the model $\widehat{x}(t)$ are ultimately bounded, and $x(t), \widehat{x}(t) \in \Omega_\rho$, for all $t \geq 0$.

***Example** 5.2:* We now consider signal quantization by examining the same system studied in Example 1, where a uniform quantizer with quantization error $\delta_q = 0.1$ is used to quantize the transmitted measurements. Based on the discussions in Section IV, the plot of the functions $f_1(\tau)$ and $f_2(\tau)$ (note that $\delta_x(d_k)$ should be replaced by $\widetilde{\delta}_x(d_k)$ in this case) with $d_{max} = 0.01s$ and $\varepsilon = 50$ is shown in Fig.8, and one can find that $min\{\widehat{\tau}\} = 0.2s$ and $max\{\widehat{\tau}\} = 0.55s$, which indicates that as long as a packet containing new information of the plant is received by the estimator between $[t_k + d_k, t_k + d_k + \tau](\tau \in [0.2s, 0.55s])$ after the k_{th} packet is accepted by the estimator, the state of the plant is ultimately bounded. In our simulation, we randomly choose τ between $[0.25s, 0.5s]$ for a new packet to be accepted by the estimator. The initial state of the plant is again given by $x_1(t_0) = -5$ and $x_2(t_0) = 8$, which satisfies $V(x(t_0)) + f_v(\delta_x(d_{max})) \leq \rho$ and $x(t_0) \in \Omega_\rho$. The simulation results are shown in Fig.9-Fig.11.

VI. Conclusion

In this paper, we propose a model-based scheduling strategy for sensor-actuator networked control systems. An estimator and a nominal model of the plant have been used explicitly in the controller node to generate control action and schedule data transmissions. The data transmissions from the sensor to the network controller are "self-triggered" since the scheduling of the data-packet transmissions has to meet a soft deadline. The control action updates generated by the network controller are "event-triggered", since a new measurement of the state of the model is sent to the network controller for control action update whenever a triggering condition is satisfied or whenever the state of the model is reset by the estimator. We have derived a systematic scheduling strategy to achieve ultimate boundedness stability of the sensor-actuator networked control system by using a model-based approach, where model uncertainties, time-varying network induced delays, data-packet drop-outs and signal quantization are considered to derive the scheduling strategy. In our future work, we will further consider the delays from the network controller to the actuator.

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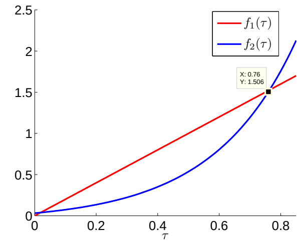

Fig. 4. plot of $f_1(\tau)$ and $f_2(\tau)$ in Example 5.1

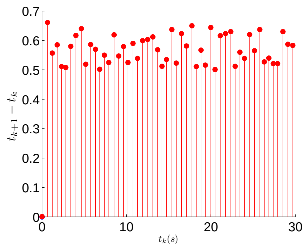

Fig. 5. time-instant at which the estimator resets the sate of the model in Example 5.1

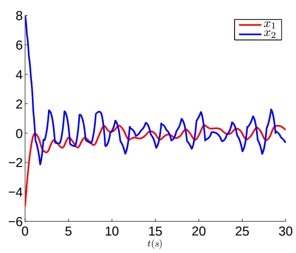

Fig. 6. state of the plant in Example 5.1

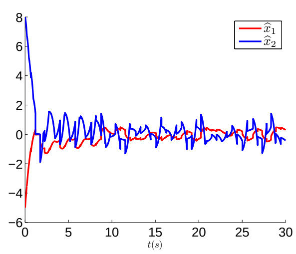

Fig. 7. state of the model in Example 5.1

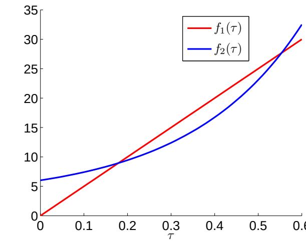

Fig. 8. plot of $f_1(\tau)$ and $f_2(\tau)$ in Example 5.2

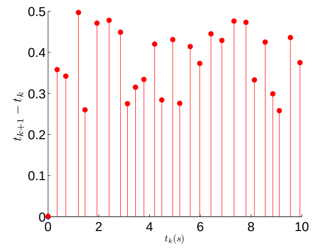

Fig. 9. time-instant at which the estimator resets the sate of the model in Example 5.2

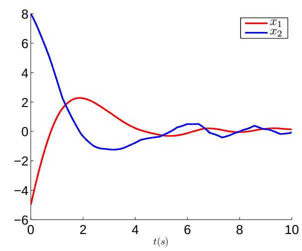

Fig. 10. state of the plant in Example 5.2

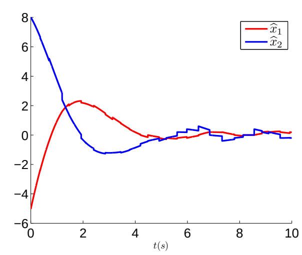

Fig. 11. state of the model in Example 5.2

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