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Robust Stabilizing Output Feedback Nonlinear Model Predictive Control by Using Passivity and Dissipativity

Han Yu, Feng Zhu, Meng Xia and Panos J. Antsaklis

Abstract— **Motivated by the passivity-based nonlinear model predictive control (NMPC) scheme reported in [1], in this paper, we propose a robust stabilizing output feedback NMPC scheme by using passivity and disspativity. Model discrepancy between the nominal model and the real system is characterized by comparing the outputs for the same excitation function, and with this kind of characterization, we are able to compare the supply rate between the nominal model and the real system based on their passivity indices. Then, by introducing specific stabilizing constraint based on the passivity indices of the nominal model into the MPC, we show that our proposed NMPC scheme can stabilize the real system to be controlled.**

I. Introduction

Model predictive control (MPC), as an effective control technique to deal with multi-variable constrained control problems, has been widely adopted in a variety of industrial applications. The success of MPC can be attributed to its effective computational control algorithm and its ability to impose various constrains when optimizing the plant behavior. Unlike the conventional feedback control, MPC allows one to first compute an open-loop optimal control trajectory by using an explicit model over a specified prediction horizon, then only the first part of the calculated control trajectory is actually implemented and the entire process is repeated for the next prediction intervals. For extensive surveys on MPC, one can refer to [2], [3], [4], [5] and the references therein.

Although MPC has many advantages, several issues, such as feasibility, closed-loop stability, nonlinearity and robustness still need to be studied. If either models of the plant or constraints are nonlinear, nonlinear MPC (NMPC) schemes are required to be used. However, it was pointed out in [6] that NMPC does not always guarantee closed-loop stability. Moreover, robustness of MPC is also an issue when model uncertainty or noise appears [5]. Model uncertainty usually exists in MPC because the model used for prediction cannot perfectly match the real dynamics of the plant to be controlled.

On the other hand, passivity theory is a powerful tool in analysis and control of nonlinear systems [7], [8], [9], [10]. Recently, a passivity-based NMPC scheme is proposed in [1], motivated by the relationship between optimal control and passivity as well as by the relationship between optimal control and NMPC. It is shown that close-loop stability and feasibility can be guaranteed by introducing specific passivity-based constraints.

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Motivated by [1], in this paper, we propose a robust stabilizing output feedback NMPC scheme by using passivity and disspativity. Instead of assuming both the nominal model and the plant are passive systems with the same dynamics as reported in the previous work, we assume that the model for prediction and the actual plant dynamics are dissipative (which are more general than passive systems since they could be non-passive), and they do not have to possess the same dynamics. Model discrepancy between the nominal model and the real system is characterized by comparing the outputs for the same excitation function. With this characterization of model discrepancy, we are able to compare the supply rate between the nominal model and the real system based on their passivity indices [11]. Then, by introducing specific stabilizing constraint into the MPC based on the passivity indices of the nominal model, we can show that the control input calculated using the nominal model can guarantee stability of the plant to be controlled.

The rest of this paper is organized as follows: in Section II, background material on passivity and dissipativity is provided; in Section III, we give a brief review on the results of passivity-based NMPC; in Section IV, model discrepancy between the nominal model and the real system is characterized, and conditions under which the supply rate of the real system is bounded above by the supply rate of the nominal model is provided; our proposed stabilizing output feedback NMPC scheme is presented in Section V; simulations are provided to validate our results in Section VI; finally, conclusions are made in Section VII.

II. Background on Dissipative and Passive Systems

We first introduce some basic concepts on passive and dissipative systems. Consider the following control system, which could be linear or nonlinear:

$$H:\begin{cases}\dot{x}(t)=f(x(t))+g(x(t))u(t)\\ y(t)=h(x(t),u(t))\end{cases} \tag{1}$$

where $x(t)\in X\subset\mathbb{R}^n$, $u(t)\in U\subset\mathbb{R}^m$ and $y(t)\in Y\subset\mathbb{R}^m$ are the state, input and output variables, respectively, and X, U and Y are the state, input and output spaces, respectively. The representation $\phi(t,t_0,x_0,u(t))$ is used to denote the state at time t reached from the initial state x_0 at the time t_0 under the control $u(t)$.

For an easy understanding on the concepts of dissipativity and passivity it is convenient to imagine that system H is a physical system with the property that its energy can only be increased through the supply from an external source. The definitions below give the generalizations of such physical properties.

***Definition** 2.1:* (Supply Rate [12]) The *supply rate* $\omega(t) = \omega(u(t), y(t))$ is a real valued function defined on $U \times Y$, such that for any $u(t) \in U$ and $x_0 \in X$ and $y(t) = h(\phi(t, t_0, x_0, u(t)), u(t))$, $\omega(t)$ satisfies

$$\int_{t_0}^{t} |\omega(\tau)| d\tau < \infty. \tag{2}$$

***Definition** 2.2:* (Dissipative System[12]) System H with supply rate $\omega(t)$ is said to be *dissipative* if there exists a nonnegative real function $V : X \to \mathbb{R}^+$, called the storage function, such that, for all $t \geq t_0 \geq 0$, $x_0 \in X$ and $u \in U$,

$$V(x_t) - V(x_0) \leq \int_{t_0}^{t} \omega(\tau) d\tau, \tag{3}$$

where $x_t = \phi(t, t_0, x_0, u(t))$ and $\mathbb{R}^+$ is a set of nonnegative real numbers. If V is $\mathcal{C}^1$, then we have $\dot{V} \leq \omega(t)$, $\forall t \geq 0$.

***Definition** 2.3:* (Passive System [12]) System H is said to be *passive* if there exists a storage function V such that

$$V(x_t) - V(x_0) \leq \int_{t_0}^{t} u^T(\tau) y(\tau) d\tau. \tag{4}$$

If V is $\mathcal{C}^1$, then

$$\dot{V} \leq u^T(t) y(t), \ \forall t \geq 0. \tag{5}$$

***Definition** 2.4:* (IF-OFP systems [13]) System H is said to be *Input Feed-forward Output Feedback Passive*(IF-OFP) if there exists a storage function V such that

$$\begin{aligned} &V(x_t) - V(x_0) \\ &\leq \int_{t_0}^{t} \Big[u^T(\tau) y(\tau) - \rho y^T(\tau) y(\tau) - \nu u^T(\tau) u(\tau) \Big] d\tau. \end{aligned} \tag{6}$$

for some $\rho, \nu \in \mathbb{R}$.
For the rest of this paper, we will denote an $m-$inputs $m-$outputs dissipative system with supply rate (6) by *IF-OFP*$(\nu, \rho)^m$ and we will call (ν, ρ) the *passivity indices* of the system.

***Remark** 2.5:* A positive ν and a positive ρ indicate that the system has an excess of passivity; otherwise, the system is lack of passivity. In the case when $\nu > 0$ or $\rho > 0$, the system is said to be input strictly passive (ISP) or output strictly passive(OSP) respectively; if either ν or ρ is negative, then the system is non-passive. Clearly, if a system is IFP(ν) or OFP(ρ), then it is also IFP($\nu - \varepsilon$), or OFP($\rho - \varepsilon$), $\forall \varepsilon > 0$.

***Lemma** 2.6:* [14] The domain of ρ, ν in IF-OFP system (6) is $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\rho, \nu \in \mathbb{R} | \rho\nu < \frac{1}{4}\}$, $\Omega_2 = \{\rho, \nu \in \mathbb{R} | \rho\nu = \frac{1}{4}; \rho > 0\}$.

Proof: The proof shown in this paper is a little bit different from the one shown in [14], and it is provided in the Appendix I. ■

III. Passivity-based NMPC

Different to many other NMPC schemes, which achieve stability by enforcing a decrease of the control Lyapunov function (CLF) along the solution trajectory, stability is achieved for the passivity-based NMPC scheme by using a nonlinear input-output constraint, which is implemented as an additional condition within the NMPC set-up. The passivity-based NMPC scheme in [1] is given by

$$\begin{aligned} &\min_{u(\cdot)} \int_{t_k}^{t_k + T_p} \Big[q(x(\tau)) + u(\tau)^T u(\tau) \Big] d\tau \\ &s.t. \begin{cases} \dot{x}(t) = f(x(t)) + g(x(t)) u(t) \\ y(t) = h(x(t)) \\ u^T(t) y(t) + y^T(t) y(t) \leq 0, \end{cases} \end{aligned} \tag{7}$$

where t_k denotes the time instant at which state measurement of the controlled system is available to the MPC, $q(x(t))$ is a positive semi-definite function and T_p denotes the finite time horizon for prediction. The passivity-based constraint $u^T(t) y(t) + y^T(t) y(t) \leq 0$ guarantees stability of the plant to be controlled, where the dynamics of the plant is assumed to be passive and zero state-detectable. Feasibility is also guaranteed due to the already known stabilizing output feedback control law $u = -y$. It is also shown in [1] that as $T_p \to 0$, the passivity-based NMPC recovers the known stabilizing output feedback $u = -y$.

Motivated by the work reported in [1], we propose a robust stabilizing output feedback NMPC scheme in this paper. The nominal model and the real plant are assumed to be IF-OFP, and they do not have to possess the same dynamics. Compared to the previous work, our results can also be applied to a class of non-passive systems, and model discrepancy between the real system and the nominal model can be accommodated as well. We will discuss our proposed scheme in the following sections in details.

IV. Characterization of Model Discrepancy

Consider two systems denoted by Σ and $\widehat{\Sigma}$ as shown in Fig.1. One can view $\widehat{\Sigma}$ as an approximation of Σ, and $\widehat{\Sigma}$ describes some behavior of interests of Σ. A common used measure for judging how well $\widehat{\Sigma}$ approximates Σ is to compare the outputs for the same excitation function u. We denote the difference in the output by Δy. The error may be due to the modeling, linearization or model reduction, etc. For a "good" approximation, we require that the "worst" case Δy over all control inputs u be small. Thus $\widehat{\Sigma}$ is a good approximation of Σ if there exists a positive constant $\gamma > 0$ such that

$$\|\Delta y\|_t \leq \gamma \|u\|_t, \ \forall u \text{ and } \forall t \geq 0, \tag{8}$$

where $\|\cdot\|_t$ denotes the truncated $\mathcal{L}_2$-norm update to time t. One can conclude that the value of γ characterizes the model discrepancy between Σ and $\widehat{\Sigma}$.

***Remark** 4.1:* One can verify that for linear systems, γ is an upper bound on the H_∞ norm of the difference in the transfer functions of systems Σ and $\widehat{\Sigma}$.

Assume that system Σ is IF-OFP$(\nu, \rho)^m$, and system $\widehat{\Sigma}$ is an approximation of system Σ which is IF-OFP$(\widehat{\nu}, \widehat{\rho})^m$. It can be shown that under some conditions, the supply rate of system Σ is always bounded above by the supply rate of $\widehat{\Sigma}$. Those conditions are summarized in Lemma 4.2.

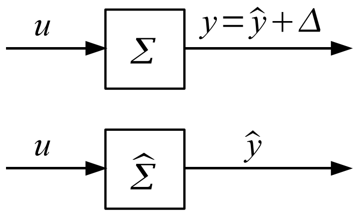

Fig. 1. Model Approximation

***Lemma** 4.2:* Consider system Σ and system $\widehat{\Sigma}$ as shown in Fig.1, where Σ is IF-OFP$(\nu,\rho)^m$ and system $\widehat{\Sigma}$ is IF-OFP$(\widehat{\nu},\widehat{\rho})^m$. If (8) holds and there exists a $\xi>0$ such that

$$\begin{aligned} &\nu-\widehat{\nu}\geq\frac{\gamma^2}{\xi}+\gamma+b\\ &\rho-\widehat{\rho}\geq\xi\widehat{\rho}^2 \end{aligned} \tag{9}$$

where $b=2\max\{0,\widehat{\rho}\gamma^2\}$, then

$$u^Ty-\rho y^Ty-\nu u^Tu\leq u^T\widehat{y}-\widehat{\rho}\widehat{y}^T\widehat{y}-\widehat{\nu}u^Tu. \tag{10}$$

Proof: Let $\omega=u^Ty-\rho y^Ty-\nu u^Tu$, $\widehat{\omega}=u^T\widehat{y}-\rho\widehat{y}^T\widehat{y}-\widehat{\nu}u^Tu$, then

$$\begin{aligned} \widehat{\omega}-\omega&=u^T\widehat{y}-\rho\widehat{y}^T\widehat{y}-\widehat{\nu}u^Tu-(u^Ty-\rho y^Ty-\nu u^Tu)\\ &=-u^T\Delta y-\widehat{\rho}y^Ty+2\widehat{\rho}y^T\Delta y-\widehat{\rho}\Delta y^T\Delta y\\ &-\widehat{\nu}u^Tu+\rho y^Ty+\nu u^Tu, \end{aligned} \tag{11}$$

with $-u^T\Delta y\geq-\gamma\|u\|_2^2$ and $-2\widehat{\rho}y^T\Delta y\geq-\frac{\gamma^2}{\xi}\|u\|_2^2-\xi\widehat{\rho}^2\|y\|_2^2$, based on (11), we can further get

$$\widehat{\omega}-\omega\geq(\nu-\widehat{\nu}-\frac{\gamma^2}{\xi}-\gamma)\|u\|_2^2+(\rho-\widehat{\rho}-\xi\widehat{\rho}^2)\|y\|_2^2-\widehat{\rho}\|\Delta y\|_2^2. \tag{12}$$

So for the case $\widehat{\rho}\leq0$, if $\nu-\widehat{\nu}\geq\frac{\gamma^2}{\xi}+\gamma$ and $\rho-\widehat{\rho}\geq\xi\widehat{\rho}^2$, then $\widehat{\omega}\geq\omega$; for the case $\widehat{\rho}>0$, if $\nu-\widehat{\nu}\geq\frac{\gamma^2}{\xi}+\gamma+\gamma^2\widehat{\rho}$ and $\rho-\widehat{\rho}\geq\xi\widehat{\rho}^2$, then $\widehat{\omega}\geq\omega$. This completes the proof. ■

***Remark** 4.3:* Lemma 4.2 provides conditions under which the supply rate of system Σ is upper bounded by the supply rate of $\widehat{\Sigma}$. Those conditions are related to the bound on the model discrepancy between Σ and $\widehat{\Sigma}$ (which, in our case, is characterized by γ), and is also related to their passivity indices (as indicated by $\nu\geq\widehat{\nu}+\frac{\gamma^2}{\xi}+\gamma+b$ and $\rho\geq\widehat{\rho}+\xi\widehat{\rho}^2$). The conditions on the bound of passivity indices can be relaxed if the bound on the model discrepancy is small (i.e., γ is small, thus $\widehat{\Sigma}$ approximates the behavior of Σ well for the same excitation function).

V. Robust Stabilizing Output Feedback NMPC for IF-OFP Systems by Using Passivity and Dissipativity

In this section, we first study the problem of stabilization of IF-OFP systems by using static output feedback gains. This result is important for us to derive the stabilizing condition in our proposed NMPC scheme. We then propose a NMPC scheme by using passivity and dissipativity when there is no model discrepancy between the nominal model and the real system. Finally, we extend this result to the case when model discrepancy between the nominal model and the real system can be characterized in the way as discussed in Section IV.

A. Stabilization of IF-OFP Systems by Using Static Output Feedback Gain

***Lemma** 5.1:* If system H is IF-OFP(ρ,ν), then there always exists an output feedback stabilizing control $u(t)=r(t)-Ky(t)$, where $K\in\mathbb{R}$, $r(t),y(t)\in\mathcal{L}_2$, such that the closed-loop system is $\mathcal{L}_2$ stable from $r(t)$ to $y(t)$. Moreover, if the system is also zero-state detectable, then with $r(t)=0$, the closed-loop system is asymptotically stable.

Proof: Since $u(t)=r(t)-Ky(t)$, we can get

$$\begin{aligned} &V(x_t)-V(x_0)\leq\int_0^t\Big\{\big[r(\tau)-Ky(\tau)\big]^Ty(\tau)\\ &-\rho y^T(\tau)y(\tau)-\nu\big[r(\tau)-Ky(\tau)\big]^T\big[r(\tau)-Ky(\tau)\big]\Big\}d\tau\\ &=\int_0^t\Big[(1+2\nu K)r^T(\tau)y(\tau)-(K+\rho+K^2\nu)y^T(\tau)y(\tau)\\ &-\nu r^T(\tau)r(\tau)\Big]d\tau, \end{aligned} \tag{13}$$

thus

$$\begin{aligned} V(x_t)-V(x_0)\leq\int_0^t\Big(&|1+2\nu K|\|r(\tau)\|_2\|y(\tau)\|_2\\ &+|\nu|\|r(\tau)\|_2^2-(K+\rho+K^2\nu)\|y(\tau)\|_2^2\Big)d\tau. \end{aligned} \tag{14}$$

If $K+\rho+K^2\nu>0$, then we can obtain

$$\begin{aligned} &V(x_t)-V(x_0)\leq\int_0^t\Big[\Big(\frac{|1+2K\nu|^2}{2(K+\rho+\nu K^2)}+|\nu|\Big)\|r(\tau)\|_2^2\\ &-\frac{K+\rho+\nu K^2}{2}\|y(\tau)\|_2^2\Big]d\tau, \end{aligned} \tag{15}$$

which further yields

$$\begin{aligned} &\int_0^t\frac{K+\rho+\nu K^2}{2}\|y(\tau)\|_2^2d\tau\\ &\leq\int_0^t\Big(\frac{|1+2K\nu|^2}{2(K+\rho+\nu K^2)}+|\nu|\Big)\|r(\tau)\|_2^2d\tau+V(x_0), \end{aligned} \tag{16}$$

which shows that the closed-loop system is $\mathcal{L}_2$ stable from $r(t)$ to $y(t)$. With $r(t)=0$, we have $\int_0^t\frac{K+\rho+\nu K^2}{2}\|y(\tau)\|_2^2d\tau\leq V(x_0)$. With $V(x_0)$ being bounded, we can further conclude that $\lim_{t\to\infty}y(t)=0$. Asymptotic stability follows from that the system H is zero-state detectable. Now it remains to show that there always exists K such that $K+\rho+\nu K^2>0$. Assume that there does not exist a K such that $K+\rho+\nu K^2>0$. This can only happen when $\nu<0$. Let $p(K)=K+\rho+\nu K^2$, then one can find that with $\nu<0$, $p(K)$ has a global maximum at $K=-\frac{1}{2\nu}$, and $\max_K\{p(K)\}=\frac{4\rho\nu-1}{4\nu}$. In view of Lemma 2.6, with $\nu<0$, we have $\rho\in\Omega_1$, which yields $\max_K\{p(K)\}=\frac{4\rho\nu-1}{4\nu}>0$. This implies that there exists K such that $p(K)>0$ when $\nu<0$, which completes the proof. ■

***Remark** 5.2:* It can be shown that:

- if $\nu = 0$, then we can choose $K > -\rho$;
- if $\nu > 0$, then we can choose $K > \frac{-1+\sqrt{1-4\rho\nu}}{2\nu}$ or $K < \frac{-1-\sqrt{1-4\rho\nu}}{2\nu}$;
- if $\nu < 0$, we can choose $\frac{-1+\sqrt{1-4\rho\nu}}{2\nu} < K < \frac{-1-\sqrt{1-4\rho\nu}}{2\nu}$.

So based on the passivity indices (ν, ρ), we can find the range of stabilizing output feedback gains for the system.

B. Stabilizing Output Feedback NMPC for IF-OFP Systems with no Model Discrepancy

Motivated by the passivity-based NMPC scheme reported in [1], we extend this scheme to the more general cases, where the systems to be controlled are IF-OFP. In view of Lemma 5.1, we can conclude that it is always possible to find a range of stabilizing output feedback gains for an IF-OFP system based its passivity indices. We first consider the case when there is no model discrepancy between the system to be controlled and the nominal model being used for prediction. The scheme of stabilizing output feedback NMPC for IF-OFP systems with no model discrepancy is given by:

$$\min_{u(\cdot)} \int_{t_k}^{t_k+T_p} \Big[q(x(\tau)) + u(\tau)^T R u(\tau)\Big] d\tau$$
$$s.t. \begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u(t) \\ y(t) = h(x(t), u(t)) \\ u^T(t)y(t) - \rho y^T(t)y(t) - \nu u^T(t)u(t) \\ \leq -\frac{K+\rho+\nu K^2}{2} y^T(t)y(t), \end{cases} \tag{17}$$

where R is a positive definite matrix, the stabilizing output feedback gain K should be chosen based on the indices (ν, ρ) of the system such that $K + \rho + \nu K^2 > 0$, which has been discussed in Remark 5.2.

***Theorem** 5.3:* The output feedback NMPC scheme proposed in (17) can asymptotically stabilize system (1) if it is IF-OFP$(\nu, \rho)^m$ with a continuously differentiable storage function and is zero-state detectable.

Proof: The proof is very similar to the proof provided in [1]. First, we need to show that the NMPC scheme proposed in (17) for the system (1) which is IF-OFP$(\nu, \rho)^m$ is always feasible; second, we need to show that the NMPC scheme will stabilize the system asymptotically. Actually, feasibility is guaranteed due to the known output feedback stabilizing control law $u(t) = -Ky(t)$. Let V be the storage function of system (1). With the differentiable storage function V and the stability constraint $u^T(t)y(t) - \rho y^T(t)y(t) - \nu u^T(t)u(t) \leq -\frac{K+\rho+\nu K^2}{2} y^T(t)y(t)$, one can obtain $\dot{V} \leq u^T(t)y(t) - \rho y^T(t)y(t) - \nu u^T(t)u(t) \leq -\frac{K+\rho+\nu K^2}{2} y^T(t)y(t)$. Using the fact that system (1) is zero-sate detectable, asymptotic stability follows from the result shown in Lemma 5.1. ∎

C. Stabilizing Output Feedback NMPC for IF-OFP Systems with Model Discrepancy

Consider system Σ and system $\widehat{\Sigma}$ as shown in Fig.1, where $\widehat{\Sigma}$ is an approximation of Σ. In NMPC, Σ represents the real system to be controlled, and $\widehat{\Sigma}$ represents the nominal model used in MPC for prediction. System Σ is IF-OFP$(\nu, \rho)^m$ and system $\widehat{\Sigma}$ is IF-OFP$(\widehat{\nu}, \widehat{\rho})^m$. Since $\widehat{\Sigma}$ is an approximation of Σ, $(\widehat{\nu}, \widehat{\rho})$ is not necessarily equal to (ν, ρ). In this case, we need to rectify the stabilizing output feedback NMPC scheme proposed in Section V-B as:

$$\min_{u(\cdot)} \int_{t_k}^{t_k+T_p} \Big[q(\widehat{x}(\tau)) + u(\tau)^T R u(\tau)\Big] d\tau$$
$$s.t. \begin{cases} \dot{\widehat{x}}(t) = \widehat{f}(\widehat{x}(t)) + \widehat{g}(\widehat{x}(t))u(t) \\ \widehat{y}(t) = \widehat{h}(\widehat{x}(t), u(t)) \\ u^T(t)\widehat{y}(t) - \widehat{\rho}\widehat{y}^T(t)\widehat{y}(t) - \widehat{\nu} u^T(t)u(t) \\ \leq -\frac{K+\widehat{\rho}+\widehat{\nu} K^2}{2} \widehat{y}^T(t)\widehat{y}(t), \end{cases} \tag{18}$$

where

$$\begin{cases} \dot{\widehat{x}}(t) = \widehat{f}(\widehat{x}(t)) + \widehat{g}(\widehat{x}(t))u(t) \\ \widehat{y}(t) = \widehat{h}(\widehat{x}(t), u(t)) \end{cases} \tag{19}$$

is the state-space model of $\widehat{\Sigma}$, and K is chosen based on $(\widehat{\nu}, \widehat{\rho})$ such that $K + \widehat{\rho} + \widehat{\nu} K^2 > 0$ (see Remark 5.2 on how to choose the range of K).

Due to possible model mismatch between Σ and $\widehat{\Sigma}$, the control action generated through the NMPC (18) may not be able to stabilize the real system Σ. Intuitively, stabilization results may still hold if $\widehat{\Sigma}$ is a good approximation of Σ. Theorem 5.4 provides sufficient conditions under which the NMPC scheme provided in (18) can still stabilize the real system Σ.

***Theorem** 5.4:* Consider system Σ and system $\widehat{\Sigma}$ as shown in Fig.1, where Σ is IF-OFP$(\nu, \rho)^m$ with a continuously differentiable storage function V and is zero-state detectable; system $\widehat{\Sigma}$ is IF-OFP$(\widehat{\nu}, \widehat{\rho})^m$ with a continuously differentiable storage function $\widehat{V}$ and is also zero-state detectable. If (8) holds and there exists a $\xi > 0$ such that

$$\begin{aligned} \nu - \widehat{\nu} &\geq \frac{\gamma^2}{\xi} + \gamma + b \\ \rho - \widehat{\rho} &\geq \xi \widehat{\rho}^2 \end{aligned} \tag{20}$$

where $b = 2\max\{0, \widehat{\rho}\gamma^2\}$, then the NMPC scheme provided in (18) is also a stabilizing MPC for the system Σ.

Proof: If (8) and (20) are satisfied, then in view of Lemma 4.2, we can conclude that $u^T(t)y(t) - \rho y^T(t)y(t) - \nu u^T(t)u(t) \leq u^T(t)\widehat{y}(t) - \widehat{\rho}\widehat{y}^T(t)\widehat{y}(t) - \widehat{\nu} u^T(t)u(t)$. With the stabilization condition $u^T(t)\widehat{y}(t) - \widehat{\rho}\widehat{y}^T(t)\widehat{y}(t) - \widehat{\nu} u^T(t)u(t) \leq -\frac{K+\widehat{\rho}+\widehat{\nu} K^2}{2} \widehat{y}^T(t)\widehat{y}(t)$ in the NMPC, we can conclude that

$$\begin{aligned} \dot{\widehat{V}} &\leq u^T(t)\widehat{y}(t) - \widehat{\rho}\widehat{y}^T(t)\widehat{y}(t) - \widehat{\nu} u^T(t)u(t) \\ &\leq -\frac{K+\widehat{\rho}+\widehat{\nu} K^2}{2} \widehat{y}^T(t)\widehat{y}(t) \leq 0, \end{aligned} \tag{21}$$

and

$$\begin{aligned} \dot{V} &\leq u^T(t)y(t) - \rho y^T(t)y(t) - \nu u^T(t)u(t) \\ &\leq u^T(t)\widehat{y}(t) - \widehat{\rho}\widehat{y}^T(t)\widehat{y}(t) - \widehat{\nu} u^T(t)u(t) \\ &\leq -\frac{K+\widehat{\rho}+\widehat{\nu} K^2}{2} \widehat{y}^T(t)\widehat{y}(t) \leq 0, \end{aligned} \tag{22}$$

since $\widehat{V} \geq 0$ and $V \geq 0$, this implies that $\lim_{t\to\infty} \dot{\widehat{V}} = 0$ and $\lim_{t\to\infty} \dot{V} = 0$ (otherwise, if $\lim_{t\to\infty} \dot{\widehat{V}} < 0$

and $\lim_{t\to\infty}\dot{V} < 0$, then V and $\widehat{V}$ will eventually become negative). Observing from (21), we can directly conclude that $\lim_{t\to\infty}\widehat{y}(t) = 0$, and asymptotic stability of the model $\widehat{\Sigma}$ follows from the assumption that $\widehat{\Sigma}$ is zero-state detectable. Since $0 = \lim_{t\to\infty}\dot{\widehat{V}} \leq \lim_{t\to\infty}\{u^T(t)\widehat{y}(t) - \widehat{\rho}\widehat{y}^T(t)\widehat{y}(t) - \widehat{\nu}u^T(t)u(t)\} \leq 0$, with $\lim_{t\to\infty}\widehat{y}(t) = 0$, we can get $\lim_{t\to\infty}u(t) = 0$. With $0 = \lim_{t\to\infty}\dot{V} \leq \lim_{t\to\infty}\{u^T(t)y(t) - \rho y^T(t)y(t) - \nu u^T(t)u(t)\} \leq 0$ and $\lim_{t\to\infty}u(t) = 0$, we can further conclude that $\lim_{t\to\infty}y(t) = 0$. Asymptotic stability of the system Σ follows from the assumption that Σ is zero-state detectable. ■

***Remark** 5.5:* One can see that the basic idea behind this robust stabilizing NMPC scheme is that if the supply rate of the real system is upper bounded by the supply rate of the nominal model, then by introducing the stabilizing condition provided in Theorem 5.4, we are able to stabilize the real system as well. And in view of the discussions provided in Section IV, this bound on the supply rate is related to bound on the model discrepancy between the real systems and the nominal models, and their own passivity indices. This approach may appear to be conservative at the first look, but one should be aware that the nominal model used for prediction can always be adapted in order to meet those conditions.

VI. Example

***Example** 6.1:* In this example, the dynamics of the real system is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-2 & 0.1\\ 0.2 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0.1\\ 1\end{bmatrix}u \qquad (23)$$
$$y = 0.1x_1 + x_2,$$

while the nominal model is given by

$$\begin{bmatrix}\dot{\widehat{x}}_1\\ \dot{\widehat{x}}_2\end{bmatrix} = \begin{bmatrix}-1 & 0.2\\ 0.3 & 0.95\end{bmatrix}\begin{bmatrix}\widehat{x}_1\\ \widehat{x}_2\end{bmatrix} + \begin{bmatrix}0.12\\ 0.96\end{bmatrix}u \qquad (24)$$
$$\widehat{y} = 0.12\widehat{x}_1 + 0.96\widehat{x}_2.$$

In this case, one can verify that $\rho = -1.05$, $\nu = -0.04$, $\widehat{\rho} = -1.2$, $\widehat{\nu} = -0.2$, $\gamma = 0.0451$, which satisfy the conditions provided in Lemma 4.2, and we choose $K = 2.5$ based on Remark 5.2. By using the proposed NMPC scheme provided in Theorem 5.4, we get the simulation results by using JModelica[15], where the state measurements of the plant are sent to the NMPC at every 0.5s, while the prediction period of NMPC is 2s. In the cost function, $q(\widehat{x}) = 0.5\widehat{x}_1^2 + 0.3\widehat{x}_1^2$, and $R = 0.5$. The simulation results are shown in Fig.2-Fig.4.

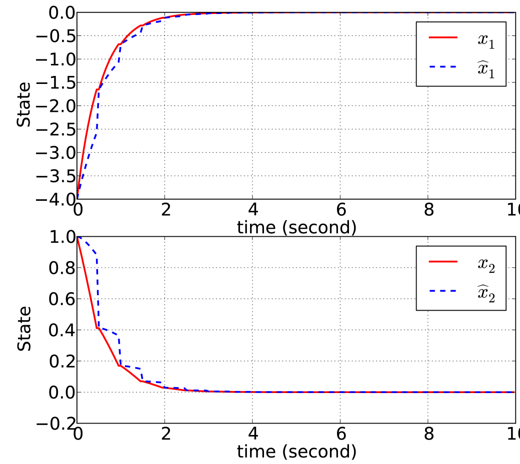

Fig. 2. State of the Plant and State of the Nominal Model

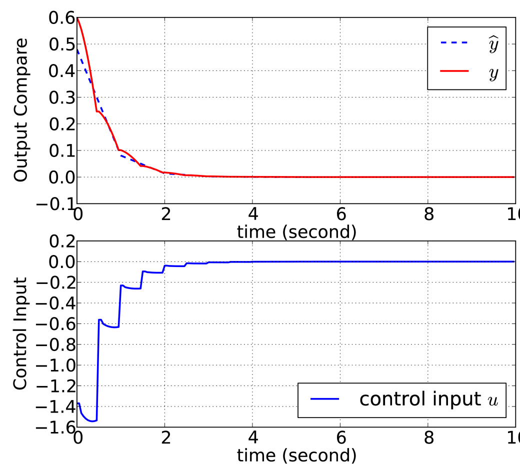

Fig. 3. Output and Control Input

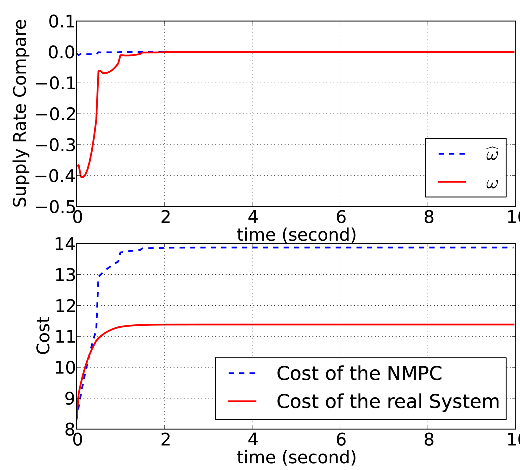

Fig. 4. Supply Rate and Cost Comparison

Fig.2 compares the state of the plant and the state of the nominal model; Fig.3 shows the output and the control input of the plant and the nominal model; Fig.4 compares their supply rate and the cost, where ω denotes the supply rate of the real system, and $\widehat{\omega}$ denotes the supply rate of the nominal model; one can see that ω is always upper bounded by $\widehat{\omega}$.

VII. Conclusion

In this paper, we propose a robust nonlinear model predictive control scheme by using passivity and dissipativity. Compared with the previous results on passivity-based MPC reported in [1], our proposed scheme is more general because it can also be applied to a class of non-passive systems, and model discrepancy can be accommodated as well. By introducing specific stabilizing constraint based on the passivity indices of the nominal model into the MPC, we show that our proposed NMPC scheme can guarantee the stability of the real system to be controlled.

Appendix I
Proof of Lemma 2.6

Proof: If $\rho, \nu \in \bar{\Omega} = \Omega_3 \cup \Omega_4$ with $\Omega_3 = \{\rho, \nu \in \mathbb{R} | \rho\nu \geq \frac{1}{4}; \rho < 0\}$ and $\Omega_4 = \{\rho, \nu \in \mathbb{R} | \rho\nu > \frac{1}{4}; \rho > 0\}$, degenerate cases occur. In case $\rho, \nu \in \Omega_3$, multiplying (6) with $\rho < 0$ and taking the square complement it follows

$$\begin{aligned} &\rho\Big[V(x_t) - V(x_0)\Big] + \int_0^t \Big[\rho^2\|y(\tau)\|_2^2 - \rho u^T(\tau)y(\tau) \\ &+ \frac{1}{4}\|u(\tau)\|_2^2 + (\rho\nu - \frac{1}{4})\|u(\tau)\|_2^2\Big] d\tau \geq 0. \end{aligned} \tag{25}$$

Let $\beta_1 = -\rho \min\{V(x_0)\}$, then $\rho\Big[V(x_t) - V(x_0)\Big] \leq -\rho V(x_0) \leq \beta_1$, in view of (25), we have

$$\beta_1 + \int_0^t \Big[\big\|\rho y(\tau) - \frac{1}{2}u(\tau)\big\|_2^2 + (\rho\nu - \frac{1}{4})\big\|u(\tau)\big\|_2^2\Big] d\tau \geq 0, \tag{26}$$

which is satisfied for any pair of (u, y), since $\rho\nu - \frac{1}{4} \geq 0$ imposing no restriction to the system's input-output behavior. In case $\rho, \nu \in \Omega_4$, multiplying (6) with $\rho > 0$ and taking the square complement it follows

$$\begin{aligned} V(x_t) - V(x_0) \leq &-\frac{1}{\rho}\int_0^t \Big[\big\|\rho y(\tau) - \frac{1}{2}u(\tau)\big\|_2^2 \\ &+ (\rho\nu - \frac{1}{4})\big\|u(\tau)\big\|_2^2\Big] d\tau \leq 0, \end{aligned} \tag{27}$$

which indicates that $V(x_t) \leq V(x_0), \forall t \geq 0$. Thus

$$\begin{aligned} 0 &= \max\big\{V(x_t) - V(x_0)\big\} \\ &\leq -\frac{1}{\rho}\int_0^t \Big[\big\|\rho y(\tau) - \frac{1}{2}u(\tau)\big\|_2^2 + (\rho\nu - \frac{1}{4})\big\|u(\tau)\big\|_2^2\Big] d\tau, \end{aligned} \tag{28}$$

which can be only satisfied for $u(t) = 0$ since $\rho\nu - \frac{1}{4} > 0$. The proof is completed. ■

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