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Output Feedback Networked Control with Persistent Disturbance Attenuation.

Eloy Garcia[a,*] and Panos J. Antsaklis[b]

[a] Infoscitex Corporation, Dayton, OH 45431 USA

[b] Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556 USA

Abstract

This paper presents a model-based control approach for output feedback stabilization and disturbance attenuation of continuous time systems that transmit measurements over a limited bandwidth communication network. Necessary and sufficient conditions for asymptotic stability of the networked system in the presence of persistent external disturbances are given. The results in this paper provide a significant improvement in the performance of the system and provide a considerable reduction of the necessary network bandwidth with respect to similar approaches in the literature.

1. Introduction

In Networked Control Systems (NCS) a digital communication network is used to transfer information among the components of a control system including actuators, controllers, and sensors. This type of implementation differs significantly from classical control systems where all system components are attached directly to the control plant exchanging information using dedicated wiring [1]. The use of a shared communication channel for control systems makes feedback measurements inaccessible to the controller for long intervals of time. An approach followed by different authors is to keep the latest received feedback measurement constant until new information arrives from the controller, i.e. using a Zero-Order-Hold (ZOH), [2]-[4].

The use of a model of the system in the controller node provides better performance in general since an estimate of the state of the system can be used for control during the time intervals that feedback measurements are not available.

Model-based frameworks have been used by different authors for control of networked systems [5]-[14] or for control of systems with limited feedback communication [15]. The work in [5]-[7], [13]-[14]

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*Corresponding author. (e-mail: elgarcia@infoscitex.com).

considers model uncertainties and no external disturbances while the authors of [8]-[9] focus on attenuation of constant input disturbances by considering no plant-model mismatch. Similar model-based frameworks have been studied by different authors, for instance, the work in [11] considers discrete time systems subject to packet dropouts and the model dynamics are used to generate the control inputs when no feedback measurements are successfully received. The authors of [10] and [12] used the model-based framework in [6] for, respectively, stabilization of singularly perturbed systems and stabilization of coupled networked uncertain systems.

In the present paper we address the case of disturbance attenuation and consider the situation of output feedback using state observers. The main contributions of this paper are as follows: first, we introduce an augmented state observer that provides estimates not only of the states of the disturbed system but also estimates of the unknown external disturbance. We provide observability conditions for the augmented system in terms of the original plant parameters. Second, the augmented state observer is implemented in the networked system and we provide stability conditions as a function of the update interval parameter which dictates how often the sensor transmits the observed variables to the controller node. Constant update intervals are used throughout this paper. It is shown that this approach considerable improves the performance of the networked system compared to similar work in [8]-[9].

The paper is organized as follows. Section 2 introduces the model-based architecture for control over networks. Section 3 describes the augmented state observer and provides observability conditions on the augmented system. Section 4 provides stability results for continuous time systems that transmit measurements using a limited bandwidth communication network. Illustrative examples are given in Section 5 and Section 6 concludes the paper.

2. Model-based networked architecture

The model-based networked setup illustrated in Fig. 1 makes use of an explicit model of the plant which is added to the actuator/controller node to compute the control input based on the state of the model rather than on the plant state. The state of the model is updated when the controller receives the measured state of the plant that is sent from the sensor node every h seconds.

Consider a Model-Based Networked Control System (MB-NCS) as shown in Fig. 1. We concern ourselves with continuous time systems that are disturbed by an unknown external disturbance as shown in that figure. Assume that the disturbance is a persistent disturbance; in particular, we assume that it is a constant signal. This approach can be used in practice for piece-wise constant disturbances. In this case the output of the system is not asymptotically stable since there could be infinite number of discrete changes on the disturbance as time goes to infinity. However, bounded stability can be obtained as it is shown in this paper.

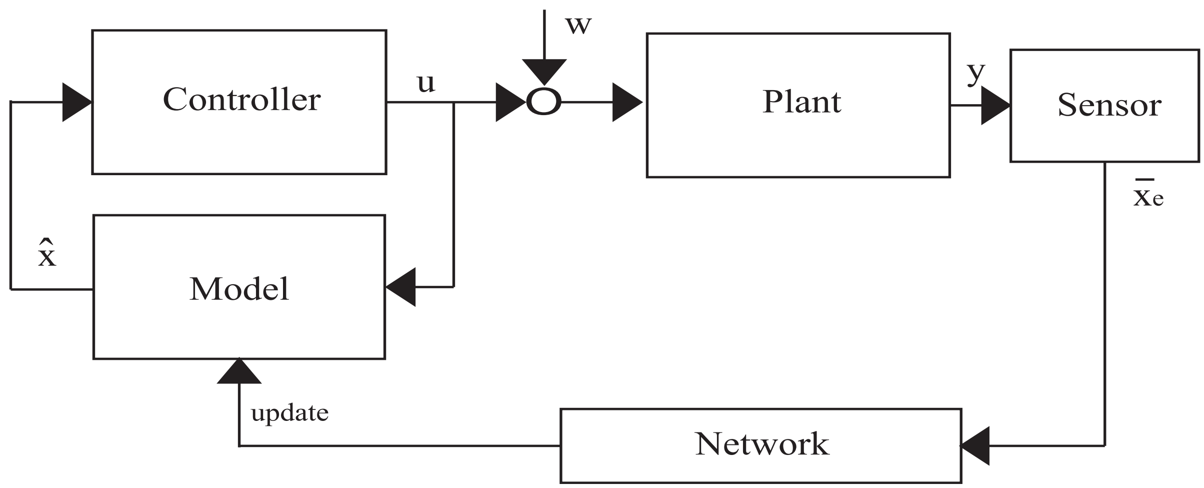

Fig. 1. Model-based networked system with external disturbance.

The dynamics of the system are given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u(t) + w(t)) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$, represent the state, control input, and measurable output of the system, respectively. $w(t) \in \mathbb{R}^m$ represents the unknown constant external disturbance. It is assumed that the disturbance is bounded as follows $\|w(t)\| \le W$. The model dynamics are given by the nominal system dynamics:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) \\ \hat{y}(t) &= C\hat{x}(t). \end{aligned} \tag{2}$$

We do not assume that the entire state vector is available for measurement but only the output $y(t)$. We implement a state observer in the controller node in order to estimate the state of the system and use the observer state to update the state of the model. Since the input disturbance $w(t)$ cannot be measured we augment the system in order to estimate both the states and the disturbance. This approach will be used in Section 4 in order to attenuate the undesired effects of the disturbance when the system is controlled using a networked architecture as shown in Fig. 1.

3. Augmented state observer.

Consider a linear system given by (1). In this section we consider an augmented state observer under the assumption of continuous access to the input and output signals, $u(t)$ and $y(t)$ as in a traditional closed loop control system. This state observer will be used later in the networked implementation represented in Fig. 1. The purpose of this augmented observer is to provide estimates not only of the state of the system $x(t)$ but also to obtain estimates of the unknown disturbance $w(t)$. Define the augmented state vector:

$$x_e(t) = \begin{bmatrix} x(t)^T & w(t)^T \end{bmatrix}^T. \tag{3}$$

The augmented system is given by:

$$\begin{aligned} \dot{x}_e(t) &= \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} x_e(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) = A_e x_e(t) + B_e u(t) \\ y(t) &= \begin{bmatrix} C & 0 \end{bmatrix} x_e(t) = C_e x_e(t). \end{aligned} \tag{4}$$

A state observer is designed for the augmented system (4) and it is given by:

$$\dot{\overline{x}}_e(t) = (A_e - LC_e)\overline{x}_e(t) + B_e u(t) + Ly(t). \tag{5}$$

Theorem 1. Assume the continuous time pair (A,C) is observable, then (A_e,C_e) is observable if $\mathcal{N}(M) = \{0\}$, the nullspace of M contains only the zero vector, where

$$M = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}. \tag{6}$$

Proof. Let λ_i represent the eigenvalues of A for i=1,...,n, with associated eigenvectors v_i, and let λ_i^e represent the eigenvalues of A_e for i=1,...,n+m, with associated eigenvectors v_i^e. Note that

$$\begin{aligned} \lambda_i^e &= \lambda_i \quad \text{for } i = 1,\ldots,n. \\ \lambda_i^e &= 0 \quad \text{for } i = n+1,\ldots,n+m. \end{aligned} \tag{7}$$

For any $\lambda_i^e \neq 0$ we have that:

$$(\lambda_i^e I - A_e)v_i^e = \begin{bmatrix} \lambda_i I - A & -B \\ 0 & \lambda_i I \end{bmatrix} \begin{bmatrix} v_n \\ v_m \end{bmatrix} = 0 \tag{8}$$

since $\lambda_i \neq 0$ then v_m=0 and

$$\begin{aligned} &(\lambda_i I - A)v_n = 0 \;\Rightarrow\; v_n = v_i \\ &\Rightarrow v_i^e = \begin{bmatrix} v_i \\ 0 \end{bmatrix}. \end{aligned} \tag{9}$$

Additionally, assume that $\lambda_i^e \neq 0$ is an unobservable eigenvalue of (A_e,C_e), then by the PBH unobservable eigenvalue condition [16] we have that:

$$\begin{bmatrix} \lambda_i^e I - A_e \\ C_e \end{bmatrix} \hat{v} = 0 \tag{10}$$

for some $\hat{v} \neq 0$. Equation (10) can be expressed as:

$$\begin{bmatrix} \lambda_i I - A & -B \\ 0 & \lambda_i I \\ C & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_n \\ \hat{v}_m \end{bmatrix} = 0 \tag{11}$$

which results in $\hat{v}_m = 0$ and

$$\begin{bmatrix} (\lambda_i I - A)\hat{v}_n \\ C\hat{v}_n \end{bmatrix} = 0 \tag{12}$$

but since (A,C) is observable (12) is only satisfied by $\hat{v}_n = 0$ then we have a contradiction and consequently any $\lambda_i^e \neq 0$ is an observable eigenvalue of (A_e,C_e).

Now consider $\lambda_i^e = 0$ and, similarly, assume that $\lambda_i^e = 0$ is an unobservable eigenvalue of (A_e,C_e), then we have that (10) is satisfied for some $\hat{v} \neq 0$. Equation (10) now results in the following:

$$\begin{bmatrix} -A\hat{v}_n - B\hat{v}_m \\ C\hat{v}_n \end{bmatrix} = 0 \tag{13}$$

but from the condition on M expressed in the theorem, (13) can only be satisfied by $\hat{v} = \begin{bmatrix} \hat{v}_n^T & \hat{v}_m^T \end{bmatrix}^T = 0$ and we have a contradiction, therefore any $\lambda_i^e = 0$ is an observable eigenvalue of (A_e,C_e) and the augmented system (4) is observable.■

4. Continuous time systems with limited output feedback and disturbance estimation

The networked system in Fig. 1 contains a model (2) of the system in order to generate an estimate of the state of the disturbed system (1) and to compute the control input between update intervals. The use of the extended observer described in the previous section provides an estimate of the unknown external disturbance that can be transmitted to the controller node in order to improve the control action by compensating for the effects of the disturbance $w(t)$, that is, the control input is given by:

$$u(t) = K\hat{x}(t) - \hat{w}(t) \tag{14}$$

where $\hat{w}(t)$ represents the latest update of the disturbance estimate received at the controller node, i.e.

$$\hat{w}(t) = \overline{w}(t_\mu) \quad \text{for } t \in [t_\mu, t_{\mu+1}) \tag{15}$$

where t_μ represents the update instants and $t_{\mu+1} - t_\mu = h$ is the update period and it is assumed to be constant, i.e. we use periodic communication. At the update instants the augmented observer state $\overline{x}_e(t_\mu)$ is received at the controller node. The observer state contains estimates of both, the states of the plant and the disturbance. The observed disturbance is kept constant in the controller node during the next update interval as described in (15) and the model state is updated using the part of the observer state corresponding to the estimated plant states, i.e.

$$\hat{x}(t_\mu) = \overline{x}(t_\mu). \tag{16}$$

After a model update takes place the model keeps its normal execution as in (2).

The sensor node contains the augmented observer and it has continuous access to the output of the system. In order to provide the observer with the control input $u(t)$ a copy of the model is implemented in the sensor node which is updated using the observer state at the same time instants that the model in the controller. The purpose is to stabilize the system by maintaining minimum communication between the sensor and controller nodes.

Define the state vector

$$z(t) = \begin{bmatrix} x_e(t)^T & \hat{e}(t)^T & \overline{x}_e(t)^T \end{bmatrix}^T \tag{17}$$

that contains the state of the augmented system (3), the state of the observer, and the state error:

$$\hat{e}(t) = x_e(t) - \begin{bmatrix} \hat{x}(t) \\ \hat{w}(t) \end{bmatrix}. \tag{18}$$

Between model update instants the networked system operates in open loop mode since feedback measurements are not transmitted to the controller node. The evolution of (17) during the intervals $t \in [t_\mu, t_{\mu+1})$ is given by:

$$\dot{z}(t) = \Lambda z(t) \tag{19}$$

where

$$\Lambda = \begin{bmatrix} A_e + B_{KB} & -B_{KB} & 0 \\ 0 & A_e & 0 \\ B_{KB} + LC_e & -B_{KB} & A_e - LC_e \end{bmatrix} \tag{20}$$

$$B_{KB} = \begin{bmatrix} BK & -B \\ 0 & 0 \end{bmatrix}. \tag{21}$$

At the update instants t_μ we have that:

$$\begin{aligned} x_e(t_\mu) &= x_e(t_\mu^-) \\ \hat{e}(t_\mu) &= x_e(t_\mu^-) - \overline{x}_e(t_\mu^-) \\ \overline{x}_e(t_\mu) &= \overline{x}_e(t_\mu^-). \end{aligned} \tag{22}$$

Proposition 2. The system described by (19) with updates (22) and with initial conditions $z_0=z(t_0)$ has the following response:

$$z(t) = e^{\Lambda(t-t_\mu)} \left(\begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \right)^{\mu} z_0 \tag{23}$$

for $t \in [t_\mu, t_{\mu+1})$ where $h = t_{\mu+1} - t_\mu$.

Proof. Without loss of generality consider $t_0=0$. On the interval $t \in [0, t_1)$ the response of the system is given by:

$$z(t) = e^{\Lambda t} z_0. \tag{24}$$

At $t=t_1$ the state of the model is updated using the state of the observer, this can be represented as follows:

$$z(t_1) = \begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} z_0. \tag{25}$$

Continuing with the interval $t \in [t_1, t_2)$ we have:

$$z(t) = e^{\Lambda(t-t_1)} \begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} z_0. \tag{26}$$

At $t=t_2$ the state of the model is updated once again using the state of the observer, and we have the following:

$$\begin{aligned} z(t_2) &= \begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} z_0 \\ &= \left(\begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \right)^2 z_0. \end{aligned} \tag{27}$$

By following the same analysis at every cycle $t \in [t_\mu, t_{\mu+1})$ we obtain the response of the system given by (23).■

Theorem 3. The system described by (19) with updates (22) is globally exponentially stable around the solution

$$z = \begin{bmatrix} 0_n & w & 0_{n+m} & 0_n & w \end{bmatrix}^T \tag{28}$$

if and only if the eigenvalues of

$$\begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \tag{29}$$

are within the unit circle of the complex plane, where w represents the value of the constant disturbance $w(t)$ and 0_n represents a row vector of n zeros.

Proof. Consider the following change of variables:

$$\begin{aligned} & w^c(t) = w(t) - w \\ & \overline{w}^c(t) = \overline{w}(t) - w \\ & z^c(t) = z(t) - \begin{bmatrix} 0_n & w & 0_{n+m} & 0_n & w \end{bmatrix}^T \\ & \Rightarrow \dot{z}^c(t) = \dot{z}(t). \end{aligned} \tag{30}$$

It is also easy to check that:

$$z^c(t) = e^{\Lambda(t-t_\mu)} \left(\begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \right)^\mu z_0^c. \tag{31}$$

Sufficiency. Taking the norm of (31) we have:

$$\begin{aligned} \left\| z^c(t) \right\| &= \left\| e^{\Lambda(t-t_\mu)} \left(\begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \right)^\mu z_0^c \right\| \\ &\leq \left\| e^{\Lambda(t-t_\mu)} \right\| \left\| \left(\begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \right)^\mu \right\| \left\| z_0^c \right\|. \end{aligned} \tag{32}$$

Let us analyze the first term on the right hand side of (32):

$$\begin{aligned} \left\| e^{\Lambda(t-t_\mu)} \right\| &\leq 1 + (t-t_\mu)\overline{\sigma}(\Lambda) + \frac{(t-t_\mu)^2}{2!}\overline{\sigma}(\Lambda)^2 ... = e^{\overline{\sigma}(\Lambda)(t-t_\mu)} \\ &\leq e^{\overline{\sigma}(\Lambda)h} = K_1 \end{aligned} \tag{33}$$

where $\overline{\sigma}(\Lambda)$ is the largest singular value of Λ.

The second term in (32) can be bounded if and only if the eigenvalues of (29) are within the unit circle of the complex plane:

$$\left\| \left(\begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \right)^{\mu} \right\| \le K_2 e^{-\alpha_1 \mu} \tag{34}$$

for some $K_2, \alpha_1 > 0$.

Since μ is a function of time we can bound the right term of (34) in terms of t:

$$K_2 e^{-\alpha_1 \mu} < K_2 e^{-\alpha_1 \frac{t-1}{h}} = K_2 e^{\frac{\alpha_1}{h}} e^{-\frac{\alpha_1}{h} t} = K_3 e^{-\alpha t} \tag{35}$$

for some $K_3, \alpha > 0$.

From (32), (33), and (35) we can conclude that:

$$\left\| z^c(t) \right\| \le K_1 K_3 e^{-\alpha t} \left\| z_0^c \right\| \tag{36}$$

that is, $z^c(t)$ converges exponentially to zero and $z(t)$ to (28).

Necessity. We will now proof the necessity part of the theorem by contradiction. Assume the system is stable and also assume that (29) has at least one eigenvalue outside the unit circle. Since the system is stable, a periodic sample of the response should be stable as well. In other words the sequence product of a periodic sample of the response should converge to zero with time. We will take the sample at times $t_{\mu+1}^-$. We can express the solution $z^c(t_{\mu+1}^-)$ as:

$$z^c(t_{\mu+1}^-) = \xi(\mu) = e^{\Lambda(t_{\mu+1}^- - t_\mu)} \left(\begin{bmatrix} I & 0 & 0 \\ I & 0 & -I \\ 0 & 0 & I \end{bmatrix} e^{\Lambda h} \right)^{\mu} z_0^c \tag{37}$$

we also know that (29) has at least one eigenvalue outside the unit circle. This means that (37) will in general grow with μ. In other words, we cannot ensure that $\xi(\mu)$ will converge to zero for general initial condition z_0^c.

$$\left\| z^c(t_{\mu+1}^-) \right\| = \left\| \xi(\mu) \right\| \to \infty \quad \text{as} \quad \mu \to \infty \tag{38}$$

this clearly means the system cannot be stable, and thus we have a contradiction. ■

Remark 1. Note that the states of the disturbed system (1) are asymptotically stable around the equilibrium point x=0. The non-zero components of the solution (28) correspond to the external disturbance, its value cannot be controlled, and to the elements of the observer state corresponding to the observation of the disturbance.

The model-based approach with disturbance estimation presented in this paper can also be applied when the disturbance is a piece-wise constant signal. This is possible since the observer is able to estimate the new setpoint of the disturbance. In this case the output of the system is not asymptotically stable since there could be infinite number of discrete changes on the disturbance as time goes to infinity. A bound on the state can be obtained assuming that the time between changes of the disturbance is large compared to the time response of the compensated model and the observer. In addition, the update period h_b should be selected smaller than the minimum time between disturbance changes in order to transmit the new estimated disturbance to the controller node, $h_b < \min(h, \tau_w)$ where the period h is a stabilizing period obtained from Theorem 3 and τ_w is the minimum time between changes on the disturbance. Under the previous conditions we can estimate an upper bound on the state of system (1) with model-based control input (14), observer (5), and with periodic updates h_b.

System (1) can be written as:

$$\dot{x}(t) = Ax(t) + B(K\hat{x}(t) - \hat{w}(t_v^-) + w(t_v^+)) \tag{39}$$

where $\hat{w}(t_v^-)$ represents the estimated value of the disturbance before a step change in the disturbance and $w(t_v^+)$ represents the value of the disturbance after a step change has occurred. Since sufficient time has elapsed since the previous step change on the disturbance and the conditions in Theorem 3 are satisfied, there exists positive constants c, $\hat{c}$, and c_w such that: $\|x(t_v^-)\| \le c$, $\|\hat{x}(t_v^-)\| \le \hat{c}$, and $\|\tilde{w}(t_v^-)\| = \|w(t_v^-) - \hat{w}(t_v^-)\| \le c_w$. Let us consider the worst case scenario where the disturbance step change occurs just after an update instant t_μ which means that the new estimated states and disturbance will be received until time $t_{\mu+1} = t_\mu + h$ and the step change is the maximum step change given by $\|w(t_v^+) - w(t_v^-)\| = 2W$. The response of (39) is given by:

$$x(t_{\mu+1}) = e^{Ah_b}x(t_v^-) + \int_0^{h_b} e^{A(h_b - s)}BK\hat{x}(s)ds + \int_0^{h_b} e^{A(h_b - s)}Bw_v ds, \quad t \in [t_\mu, t_{\mu+1}) \tag{40}$$

where $w_v = w(t_v^+) - \hat{w}(t_v^-)$ and we have set $x(t_\mu) = x(t_v^-)$, then an upper bound for the system response (40) is given by:

$$\|x(t)\| \le c \cdot a_{h_b,0} + \hat{c}\|BK\| \int_0^{h_b} a_{h_b,s}\hat{a} \cdot ds + (2W + w_c)\|B\| \int_0^{h_b} a_{h_b,s} \cdot ds \tag{41}$$

where $a_{x,y} = \|e^{A(x-y)}\|$ and $\hat{a} = \|e^{(A+BK)s}\|$.

5. Examples

Example 1. Consider the same example given in [9] where a similar technique to the present paper was applied except that a disturbance observer is not used and the communication is based on events in that example. The main idea of the approach in [9] is the same as in this paper, that is, to use the nominal model to generate estimates of the current state of the system, while the system is disturbed by a an unknown constant disturbance. The results in this example show the significant improvements that can be obtained using the approach described in the present paper in terms of both system performance and network communication.

Consider the continuous time system given by:

$$\dot{x}(t) = \begin{bmatrix} -0.1 & 0.1 \\ \frac{1}{12} & -\frac{1}{8} \end{bmatrix} x(t) + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} w(t)$$

$$y(t) = \begin{bmatrix} 0 & 0.5 \end{bmatrix} x(t).$$

The simulation in [9] has been repeated in Fig. 2. It can be seen that the states of the system are bounded but disturbance rejection is poor. For the framework studied in the present paper, we consider the same initial conditions $x(0) = \begin{bmatrix} 3 & 1 \end{bmatrix}^T$ and constant disturbance $w(t)=0.5$ as in [9]. The initial conditions for the model, the augmented observer, and the disturbance estimate in the controller node are all equal to zero.

Fig. 3 shows the simulation results following the approach described in the present paper. The update period is $h=100$ seconds. The first update takes place at $t_1=20$ seconds, at this instant the model and the estimated disturbance are first updated using the current observer quantities and the response of the system considerably improves in comparison to the first 20 seconds when the model states and the estimated disturbance in the controller $\hat{w}(t)$ were all equal to zero. The performance of the system is much better than the results in [9], which were shown in Fig. 2, as measured by the state norm during the interval shown and we also use fewer communication instants during the same simulation interval as shown by the dark arrows at the bottom of both figures. We are able to achieve a much better disturbance rejection only at the cost of degraded initial response, that is, the initial seconds of the execution of the system which are shown in detail in Fig. 4.

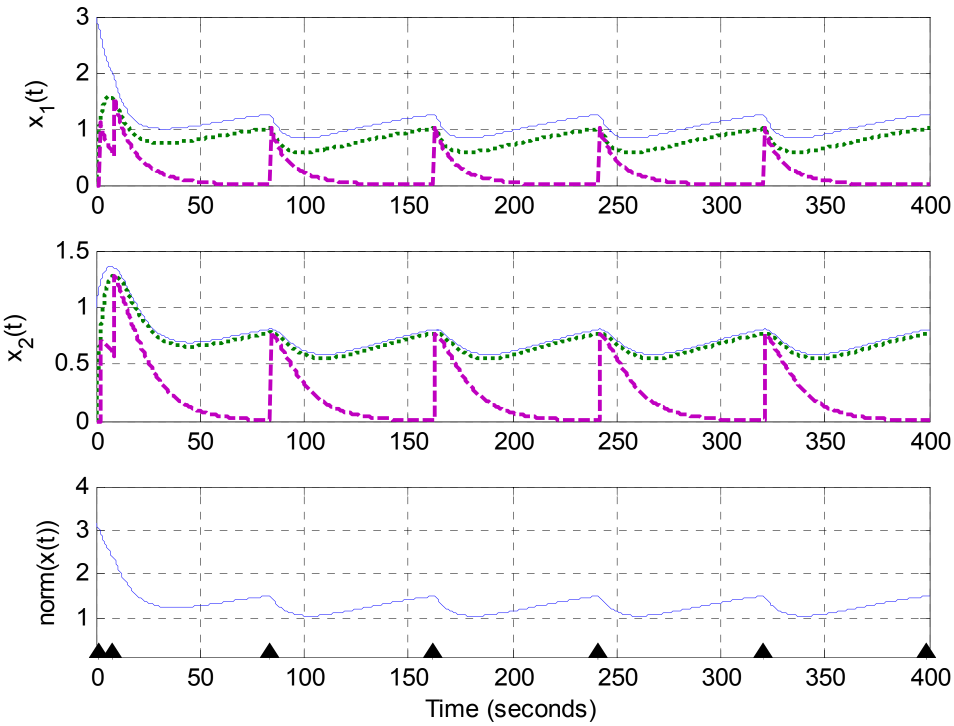

Fig. 2. Example in [9]. Solid lines represent plant states x, dotted lines represent observer states $\bar{x}$, and dashed lines represent model states $\hat{x}$. The dark arrows at the bottom represent the update instants.

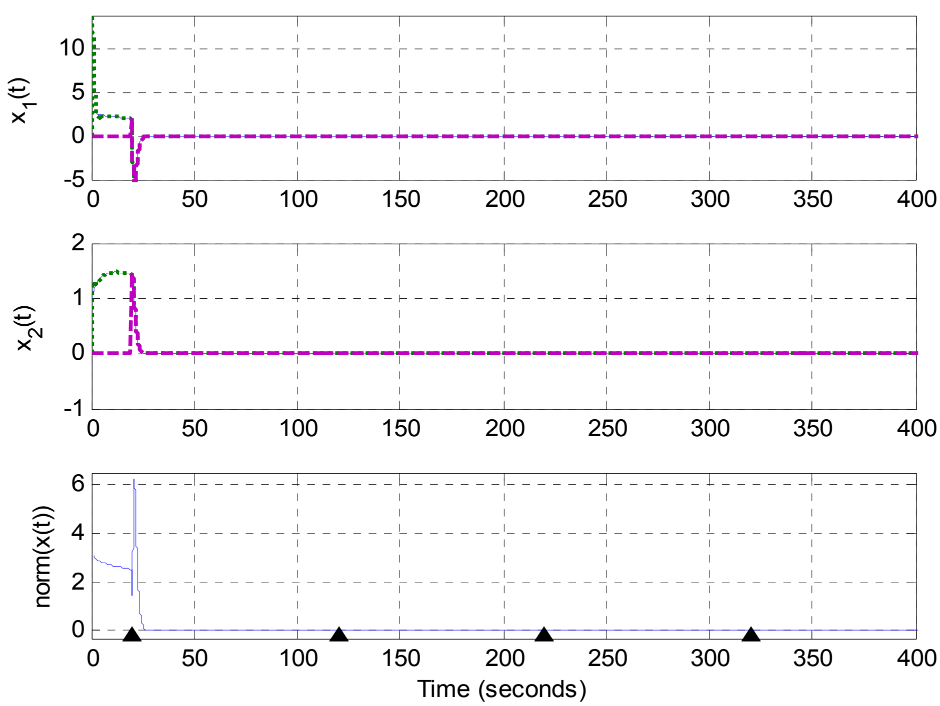

Fig. 3. Response of example 1 using the model-based approach with disturbance observer described in this paper. Solid lines represent plant states x, dotted lines represent observer states $\bar{x}$, and dashed lines represent model states $\hat{x}$. The dark arrows at the bottom represent the periodic update instants.

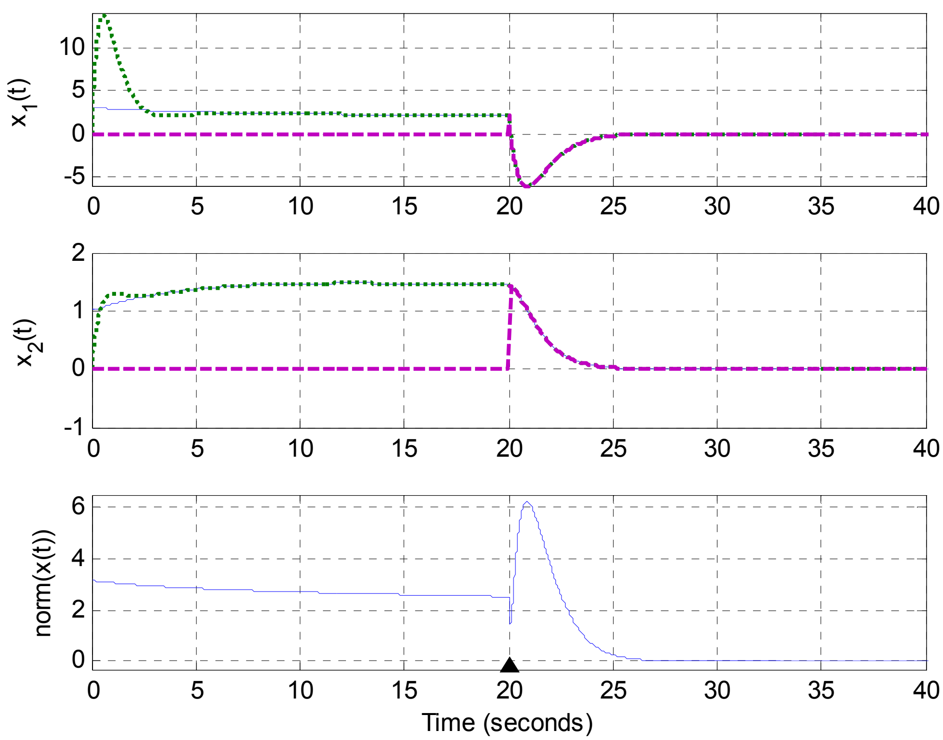

Fig. 4. Response corresponding to the first 40 seconds of Fig. 3. Solid lines represent plant states x, dotted lines represent observer states $\overline{x}$, and dashed lines represent model states $\hat{x}$. The dark arrows at the bottom represent the periodic update instants.

For completeness, we show in Fig. 5 the simulation of the same example using the same disturbance for the case of a traditional networked system where a model-based approach is not used neither a disturbance observer. In this case the received measurements are held constant in the controller, that is, a ZOH model is implemented. The top of Fig. 5 shows the case when the transmission period is equal to 30 seconds, the system becomes unstable for periods larger than about 40 seconds. Good disturbance rejection is not obtained even if we decrease the transmission period as shown at the bottom of Fig. 5, where the transmission period is equal to 5 seconds.

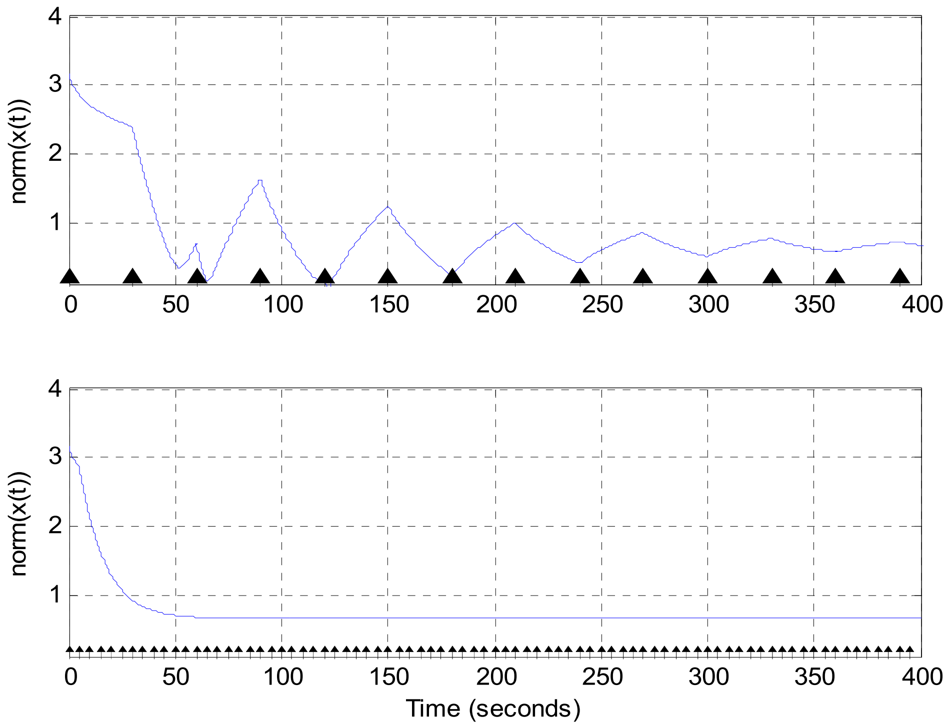

Fig. 5. Example 1 using a ZOH model. Norm of the state for periodic communication h=30seconds (top), and h=5 seconds (bottom).

Example 2. Consider the 2 input-2 output unstable system given by:

$$\dot{x}(t) = \begin{bmatrix} 0.8 & -1.5 & 1.6 \\ 0 & 0.3 & -0.5 \\ -0.5 & 1 & -0.7 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} (u(t) + w(t))$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0.5 \\ 1 & -0.3 & 0 \end{bmatrix} x(t).$$

Fig. 6 shows the response of the system subject to piece-wise constant disturbances for h=10 seconds. Our estimation-control scheme is able to asymptotically stabilize the unstable system and to reject the effects of the external disturbance. The same figure also shows the disturbances that were used in this example. The system is able to react to changes in the value of one or both of the disturbances and to bring the plant states to its equilibrium point after these disturbance variations.

Remark 2. Although in general very large stabilizing update periods can be obtained as given by the conditions in previous sections, it is not recommended using very large update intervals in the case of piece-wise disturbances since the transient response could not be satisfactory. By using very large periodic communication intervals the system may react to the changes in the disturbance set-point values after long time since the disturbance changed. The system can be stabilized again but the system response during that time interval may not be desired in general.

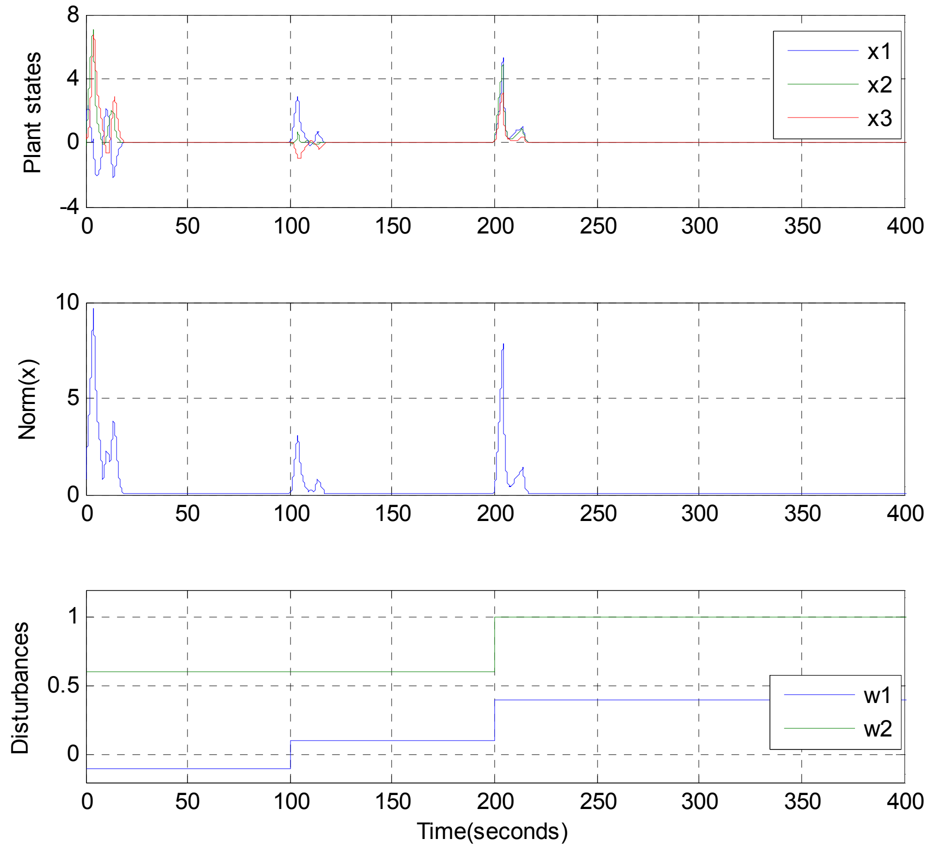

Fig. 6. Response of the system in example 2 showing the states of the plant, the norm of the plant states, and the piece-wise constant disturbance.

6. Conclusion

Output feedback control of networked systems in the presence of persistent disturbances has been addressed in this work. An estimate of the unknown external disturbance is used in the controller node along with the model state in order to stabilize the system and compensate for the disturbance effects on the system output. An augmented state observer was implemented in the sensor node to provide estimates of both the external disturbance and the states of the disturbed system. Observability conditions for the augmented system were also provided in terms of the original system parameters. The overall networked system is shown to be asymptotically stable even in the presence of persistent disturbances in comparison to similar methods that only offer bounded output results. This shows that our approach is able to reject the effects of unknown piece-wise constant disturbances on the networked system. In addition, this approach requires very infrequent communication between sensor and controller which reduces network traffic and releases the network so it can be used by other systems and applications to transmit information. Future research will address dynamic communication schemes in order to improve the transient response of the system when the disturbances present sudden changes since the periodic communication approach does not react instantaneously under that situation.

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