

Relationships Among Passivity, Positive Realness, and Dissipativity with an Application to Passivity Based Pairing

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Abstract

The notions of passivity and positive realness are fundamental concepts in classical control theory, but the use of the terms has varied. For LTI systems, these two concepts capture the same essential property of dynamical systems, that is, a system with this property does not generate its own energy but only stores and dissipates energy supplied by the environment. This paper summarizes the connection between these two concepts for continuous and discrete time LTI systems. Beyond that, the paper summarizes relationships between classes of strictly passive systems and classes of positive real systems. The more general framework of dissipativity is introduced to connect passivity and positive realness. An application is included to demonstrate how these results can be applied to input-output pairing in MIMO control systems. Two case studies are provided to demonstrate the performance of the proposed methods.

1 INTRODUCTION

In our recent research we have pursued constructive techniques based on passivity theory to design networked-control systems which can tolerate time delay and data loss, see e.g. Kottenstette and Antsaklis (2007b) and McCourt and Antsaklis (2012). As a result we have had to rediscover and clarify key relationships between three classes of systems. The first class is passive and strictly passive systems, which are characterized by a time-based input-output relationship, see e.g. Zames (1966a,b) and Desoer and Vidyasagar (1975). The second class is stable dissipative systems, which satisfy a time-based property that relates an input-output energy supply function to a state-based storage function, see e.g. Willems (1972a), Willems (1972b), Hill and Moylan (1980), and Goodwin and Sin (1984). The third class is that of positive real and strictly positive real systems, which are characterized by a frequency-based input-output relationship, see e.g. Anderson (1967), Hitz and Anderson (1969), Tao and Ioannou (1990), Wen (1988b), and Haddad and Bernstein (1994). It is noted in Willems (1972b) that, for the continuous time case, these relationships “are all derivable from the same principles and are part of the same scientific discipline”. However, it is not clear that such connections have been fully exploited, although recently Haddad and Chellaboina (2008) provided an excellent exposition of some such connections. The goals of this paper are to (1) review the classical notions of passivity, dissipativity, and positive realness; (2) summarize existing relationships between these classes of systems with appropriate references; and (3) provide original results to clarify these relationships. Rather than attempting to survey all major contributions to these fields, this paper instead reviews literature that addresses the relationships between these concepts in order to identify discrepancies and provide clarifying results and remarks.

Classical Results: The notion of passivity originated in electrical circuit theory where circuits made up of only passive components were known to be stable. It was also known that any two passive circuits could be interconnected in feedback or in parallel and the resulting circuit would still be passive, see e.g. Anderson and Vongpanitlerd (1973). This compositionality property greatly reduces the analysis required to analyze a network of circuits and assess stability. The property of passivity itself is an energy-based characterization of the input-output behavior of dynamical systems. A passive system is one that stores and dissipates energy without generating its own. The notion of stored energy can be either a traditional physical notion of energy, as it is with many physical systems, or a generalized energy, see Anderson and Vongpanitlerd (1973) and Desoer and Vidyasagar (1975). Passivity and dissipativity were formalized for general nonlinear state space systems in Willems (1972a,b). These papers provided results for passivity, specifically that passive systems were stable and that the passivity property was preserved when systems were combined in feedback or parallel. Specific forms of dissipativity for nonlinear control affine systems were studied further in Hill and Moylan (1976), Hill and Moylan (1977), and Hill and Moylan (1980). These notions were studied for more general nonlinear systems in continuous time in Lin (1995) and Lin (1996) and in discrete time in Lin (1996) and Lin and Byrnes (1994).

As the focus of this survey is on the relationship between passive systems and positive real systems, the Positive Real Lemma is of special importance. This is also known as the KYP Lemma which originated in Kalman (1963) which used results from Yakubovich (1962) and Popov (1961). Later this lemma would be used to develop linear matrix inequality (LMI) methods to demonstrate passivity for linear systems in Boyd et al. (1994).

There are two particularly valuable survey papers, Ortega et al. (2001) and Kokotovic and Arcak (2001), that cover the history of dissipativity theory in control. Both papers make a case for analyzing systems using dissipativity due to its strong connection to physics and conservation of energy. A more recent paper highlighting new advances in energy-based analysis is Ebenbauer et al. (2009). In Willems (2007), the classical work in dissipativity was reassessed from a modern perspective. Strong introductions to passivity can be found in the textbooks Khalil (2002) and van der Schaft (1999). The more general framework of dissipativity is thoroughly covered in Bao and Lee (2007), Haddad and Chellaboina (2008), and Brogliato et al. (2007).

Recent Progress: For passivity and dissipativity, progress has been made recently in numerous areas. While passivity based control has traditionally been applied to electrical circuits, see e.g. Anderson and Vongpanitlerd (1973), and robotic manipulators, see e.g. Spong et al. (2006), recently this approach has been expanded to chemical processes, where passivity can be used to design robust controllers as in Bao et al. (2003) and Bao and Lee (2007). Passivity has also been used as a design tool for coordination in multi-agent systems in Chopra and Spong (2006b) and Arcak (2007).

One particular application area that has seen recent growth is in telemanipulation systems where a human user operates a robotic arm remotely and is aided by tactile feedback. The use of passivity in this field began with the work in Anderson and Spong (1988) using the wave variable transformation in Fettweis (1986). This approach was greatly expanded through numerous papers, see e.g. Niemeyer and Slotine (1991, 2004), Stramigioli et al. (2002b), Secchi et al. (2003), Chopra et al. (2008), Hirche and Buss (2012). The study of telemanipulation has led to promising approaches for control of passive systems over a network, see e.g. Chopra and Spong (2006a), Kottenstette and Antsaklis (2007b), Kottenstette et al. (2011), and Hirche et al. (2009).

Another area that has seen much growth in recent years is the study of passivity and dissipativity for switched or hybrid systems. Passivity has been considered for continuous time in Zefran et al. (2001) and discrete time in Bemporad et al. (2005) and Bemporad et al. (2008) switched systems. These notions were studied for the more general framework of dissipativity for switched systems in continuous time in Zhao and Hill (2008) and discrete time in Liu and Hill (2011). The related notion of passivity indices for switched systems was studied in McCourt and Antsaklis (2010). Dissipativity was considered for a class of hybrid systems in Teel (2010) and a class of left continuous systems in Haddad and Hui (2009).

Lastly, it should be mentioned that there has been much recent work on passivity for sampled data systems. This work in this area has taken two distinct approaches. The first approach is to study conditions under which passivity is guaranteed when a continuous time system is discretized by the application of the ideal sampler and zero-order hold in de la Sen (2000) and Oishi (2010). The second approach is to compensate for a potential loss of passivity due to the zero-order hold as in Stramigioli et al. (2002a), Costa-Castello and Fossas (2006), and Kottenstette and Antsaklis (2007b).

Main Results of the Paper: While passivity and dissipativity are typically applied to general nonlinear systems, we choose to focus on the linear time invariant (LTI) case to emphasize the connection to positive real systems, as this notion only applies to LTI systems. Some of the basic results covered in this paper are summarized in Fig. 1. The foundational relationship is that, for LTI systems, the property of passivity is equivalent to the property of positive realness. Under mild technical assumptions, these systems are Lyapunov stable. For LTI systems, *strict passivity* is equivalent to strict positive realness. For asymptotically stable systems, strongly positive real is equivalent to strictly input passive (*SIP*). This will be covered in Section 3. Other relationships will be covered that relate *SIP*, strictly output passive (*SOP*), and very strictly

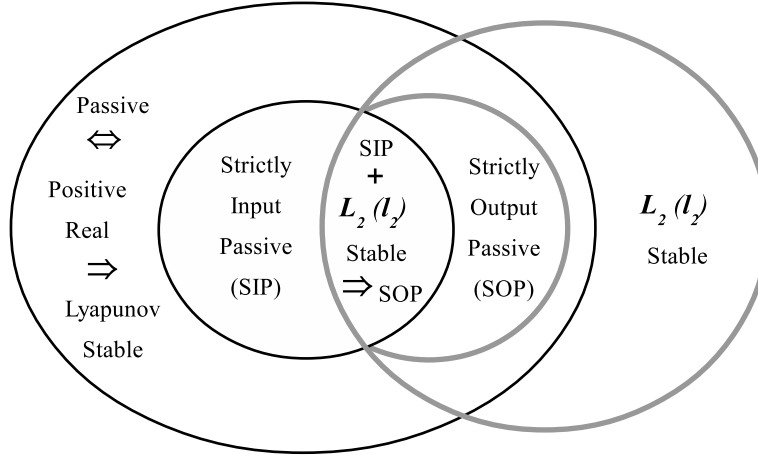


Figure 1: This Venn Diagram shows relationships between passivity, positive realness, and L_2 stability for continuous and discrete time LTI systems.

passive (VSP) to notions of stability and of state strict passivity. While the figure shows that SOP systems are passive and L_2^m (l_2^m) stable it should be noted that this relationship is sufficient only. Systems that are passive and L_2^m (l_2^m) stable are not necessarily SOP . This fact will be demonstrated in Section 4 with a counterexample. Also covered in that section is another connection from Fig. 1, that systems that are both SIP and L_2^m (l_2^m) stable must be SOP . Some preliminary results from this paper were presented in Kottenstette and Antsaklis (2010). The current paper expands on those results and presents additional clarifying results. A more complete version of this paper Kottenstette et al. (2014) includes more results related to energy-based control. The current paper covers a subset of these results in order to present an original application in passivity based control.

Before the main results of the paper are presented, definitions of the relevant properties are provided in Section 2. This section begins with some mathematical preliminaries and then moves on to define passivity, dissipativity, and positive realness. Section 3 includes some fundamental results involving passive and positive real systems. The main results of the paper are given in Section 4. Section 5 demonstrates how to use some of the results discussed in this paper by applying them to a passivity-based input-output pairing framework. Some preliminaries for this approach are covered and then two case studies are provided to demonstrate achievable performance. Concluding remarks are provided in Section 6.

2 Defining the Properties of Passivity, Dissipativity and Positive Realness

2.1 Mathematical Preliminaries

This paper covers both the continuous time and discrete time cases. When it is clear which time series is relevant or results apply to both continuous and discrete time, the time series is denoted \mathcal{T} . In continuous time this is $\mathcal{T} = \mathbb{R}^+$, while for discrete time $\mathcal{T} = \mathbb{Z}^+$. The space of signals of dimension m with finite energy in continuous time is L_2^m and l_2^m in discrete time. When the context is clear, the general space \mathcal{H} will be used to denote either. A continuous time signal $x : \mathcal{T} \rightarrow \mathbb{R}^m$ is in \mathcal{H} ($x \in \mathcal{H}$) if the signal has finite L_2^m -norm,

$$\|x\|_2^2 = \int_0^\infty x^T(t)x(t)dt < \infty. \quad (1)$$

Likewise, a discrete time signal $x : \mathcal{T} \rightarrow \mathbb{R}^m$ is in \mathcal{H} ($x \in \mathcal{H}$) if the signal has finite l_2^m -norm,

$$\|x\|_2^2 = \sum_{i=0}^{\infty} x^\top(i)x(i) < \infty. \quad (2)$$

The extended signal spaces, L_{2e}^m and l_{2e}^m , can be defined by introducing the truncation operator. The truncation of a continuous time signal $x(t)$ to time T is indicated $x_T(t)$,

$$x_T(t) = \begin{cases} x(t), & t < T, \\ 0, & t \geq T \end{cases}$$

The truncation operator is

$$x_T(i) = \begin{cases} x(i), & i < T, \\ 0, & i \geq T \end{cases}$$

in discrete time. A continuous time signal $x : \mathcal{T} \rightarrow \mathbb{R}^m$ is in \mathcal{H}_e if

$$\|x_T\|_2^2 = \int_0^T x^\top(t)x(t)dt < \infty, \quad \forall T \in \mathcal{T}. \quad (3)$$

Likewise, a discrete time signal $u : \mathcal{T} \rightarrow \mathbb{R}^m$ is in \mathcal{H}_e if

$$\|x_T\|_2^2 = \sum_{i=0}^{T-1} x^\top(i)x(i) < \infty, \quad \forall T \in \mathcal{T}. \quad (4)$$

The inner product of signals y and u over the interval $[0, T]$ in continuous time is denoted

$$\langle y, u \rangle_T = \int_0^T y^\top(t)u(t)dt. \quad (5)$$

Similarly the inner product over the discrete time interval $\{0, 1, \dots, T-1\}$ is denoted

$$\langle y, u \rangle_T = \sum_0^{T-1} y^\top(i)u(i). \quad (6)$$

A system H is a relation on \mathcal{H}_e . For $u \in \mathcal{H}_e$, the symbol Hu denotes an image of u under H (Zames (1966a)). Furthermore $Hu(t)$ denotes the value of Hu at continuous time t while $Hu(i)$ denotes the value of Hu at discrete time i . The following two definitions cover L_2^m stability in continuous time and l_2^m stability in discrete time.

Definition 1 A continuous time dynamical system $H : \mathcal{H}_e \rightarrow \mathcal{H}_e$ is L_2^m stable if

$$u \in L_2^m \implies Hu \in L_2^m.$$

Definition 2 A discrete time dynamical system $H : \mathcal{H}_e \rightarrow \mathcal{H}_e$ is l_2^m stable if

$$u \in l_2^m \implies Hu \in l_2^m.$$

For both continuous and discrete time finite-gain $L_2^m (l_2^m)$ stability can be defined by the following input-output condition. For all time $T \in \mathcal{T}$ and for all inputs $u \in \mathcal{H}$, a system H is finite-gain $L_2^m (l_2^m)$ stable if there exist $\gamma > 0$ and β such that

$$\|(Hu)_T\|_2 \leq \gamma \|u_T\|_2 + \beta. \quad (7)$$

The notion of finite-gain stability can be used to show stability of feedback interconnections using the small gain theorem, see e.g. van der Schaft (1999) or Isidori (1999). The small gain theorem has an important relationship to the passivity theorem for feedback interconnections that was first written in Anderson (1972). There has been some effort recently to combine the benefits of the passivity theorem and small gain theorem, see e.g. Griggs et al. (2007) or Forbes and Damaren (2010).

Another notion related to finite-gain is that of a system being *non-expansive* (van der Schaft (1999)). A system is *non-expansive* if there exist constants $\hat{\gamma} > 0$ and $\hat{\beta}$ such that

$$\|(Hu)_T\|_2^2 \leq \hat{\gamma}^2 \|u_T\|_2^2 + \hat{\beta}. \quad (8)$$

Remark 1 ((van der Schaft 1999, p. 4), (Kottenstette and Antsaklis 2007b, Remark 1)) A continuous time (discrete time) system H is non-expansive iff it is finite-gain $L_2^m (l_2^m)$ stable.

For the remainder of the paper, when results involving *non-expansive* or *finite-gain* $L_2^m (l_2^m)$ -stability arise, the notion of *finite-gain* $L_2^m (l_2^m)$ -stability will be used without loss of generality.

This paper focuses on *LTI* systems that are real and causal with m inputs and m outputs. A system in continuous time can be described by a proper square ($m \times m$) transfer function matrix $H(s)$. This system can be equivalently described by a minimal state space representation $\Sigma \triangleq \{A, B, C, D\}$, with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^m$, that can be written

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (9)$$

$$y(t) = Cx(t) + Du(t) \quad (10)$$

where

$$H(s) = C(sI - A)^{-1}B + D. \quad (11)$$

Remark 2 A proper continuous time *LTI* system $H(s)$ is L_2^m stable if and only if all poles have negative real part (Antsaklis and Michel 2006, Theorem 9.5 p.488). This is referred to as uniform *BIBO* stability. Equivalently, the minimal state space realization Σ is asymptotically stable (Antsaklis and Michel 2006, Theorem 9.4 p.487).

A discrete time *LTI* system can be described by a proper square ($m \times m$) transfer function matrix $H(z)$. This system has an equivalent minimal state space realization $\Sigma_z \triangleq \{A, B, C, D\}$, with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^m$, that can be written

$$x(k+1) = Ax(k) + Bu(k), \quad (12)$$

$$y(k) = Cx(k) + Du(k) \quad (13)$$

where

$$H(z) = C(zI - A)^{-1}B + D. \quad (14)$$

Remark 3 A discrete time *LTI* system $H(z)$ is l_2^m stable if and only if all poles have magnitude less than one (i.e. they are inside the unit circle of the complex plane) (Antsaklis and Michel 2006, Theorem 10.17 p.508). Again, this result is known as uniform *BIBO* stability. Equivalently, the corresponding minimal state space realization Σ_z is asymptotically stable (Antsaklis and Michel 2006, Theorem 10.16 p.508).

2.2 Passive Systems

A system is passive if it only stores and dissipates energy without generating its own energy. This is captured by an inequality where the energy supplied to the system by its environment, $\langle Hu, u \rangle_T$, is an upper bound on the loss of stored energy, $-\beta$. From an alternative perspective, the maximum energy that can be extracted from a system, $-\langle Hu, u \rangle_T$, is bounded above by the constant β that represents initially stored energy.

Definition 3 Consider a continuous or discrete time LTI system $H : \mathcal{H}_e \rightarrow \mathcal{H}_e$. Considering all inputs $u \in \mathcal{H}_e$ and all times $T \in \mathcal{T}$, H is

i) passive if $\exists \beta$ such that

$$\langle Hu, u \rangle_T \geq -\beta, \quad (15)$$

ii) strictly input passive (SIP) if $\exists \delta > 0$ and $\exists \beta$ such that

$$\langle Hu, u \rangle_T \geq \delta \|u_T\|_2^2 - \beta, \quad (16)$$

iii) strictly output passive (SOP) if $\exists \epsilon > 0$ and $\exists \beta$ such that

$$\langle Hu, u \rangle_T \geq \epsilon \|(Hu)_T\|_2^2 - \beta, \quad (17)$$

iv) very strictly passive (VSP) if $\exists \epsilon > 0, \delta > 0$ and $\exists \beta$ such that

$$\langle Hu, u \rangle_T \geq \delta \|u_T\|_2^2 + \epsilon \|(Hu)_T\|_2^2 - \beta, \quad (18)$$

Remark 4 There have been many subtle differences in the naming of these definitions in the literature. In some references (Desoer and Vidyasagar (1975), for example) strictly input passive was referred to as strictly passive. This will be avoided as strictly passive often refers to state strictly passive. Other references (e.g. Khalil (2002)) use the terms input strictly passive and output strictly passive, however, these are equivalent to the definitions of strictly input passive and strictly output passive provided here.

Remark 5 If H is linear and initial conditions are assumed to be zero, then β can be set equal to zero without loss of generality in regards to passivity. When initial conditions are not zero, β is a generalized measure of initially stored energy. If H is causal and finite-gain L_2^m (l_2^m) stable then the notion of positive given in (Desoer and Vidyasagar 1975, p.174) is equivalent to passive given here (assuming $Hu(0) = 0$).

Passivity is preserved when two passive systems are combined in either feedback or parallel, see Khalil (2002) or van der Schaft (1999). This provides valuable stability results for small and large interconnections of dynamical systems. An important related problem is to determine conditions under which a system can be made passive so that these stability results may be applied. The necessary conditions for passivating a nonlinear system can be found for continuous time in Byrnes et al. (1991) and for discrete time in Byrnes and Lin (1994).

2.3 Dissipative Systems

The property of dissipativity is a generalization of passivity that relates internally stored energy of a system to a generalized energy supply function, $s(u, y)$. The internally stored energy is measured by an energy storage function $V(x)$ that is analogous to a Lyapunov function. As a measure of energy, $V(x)$ must be non-negative, $V(x) \geq 0, \forall x$. Without loss of generality, it is assumed that $x = 0$ is an equilibrium and $V(x) = 0$ at this point. As with passivity, the discussion of dissipativity can be generalized to non-linear

systems, however for simplicity we will focus on the linear time invariant case. For *LTI* systems it can be assumed that $V(x)$ has a quadratic form, see Khalil (2002),

$$V(x) = x^\top P x, \quad (19)$$

where $P = P^\top > 0$. The following definitions cover dissipativity and (Q, S, R) -dissipativity in continuous time and discrete time.

Definition 4 (Willems (1972a)) *A continuous time system Σ is dissipative with respect to the energy supply rate $s(u, y)$ if there exists a non-negative storage function $V(x)$ (19), such that for all input signals $u \in \mathbb{R}^m$, all trajectories $x \in \mathbb{R}^n$, and all $t_2 \geq t_1$ the following inequality holds*

$$V(x(t_2)) \leq V(x(t_1)) + \int_{t_1}^{t_2} s(u(t), y(t)) dt. \quad (20)$$

Additionally, the system Σ is (Q, S, R) -dissipative if it is dissipative with respect to

$$s(u, y) = y^\top Q y + 2y^\top S u + u^\top R u, \quad (21)$$

where $Q = Q^\top$ and $R = R^\top$.

Dissipativity can be defined in discrete time with supply rate $s(u, y)$ and energy storage function $V(x)$, such that $V(x) \geq 0$ for all x and $V(x) = 0$ for $x = 0$,

$$V(x) = x^\top P x. \quad (22)$$

Definition 5 (Goodwin and Sin 1984, Appendix C) *A discrete time system Σ_z is dissipative with respect to the supply rate $s(u, y)$ iff there exists a matrix $P = P^\top > 0$, such that for all $x \in \mathbb{R}^n$, all times $l, j \in \mathcal{T}$ s.t. $l > j \geq 0$, and all input functions $u \in \mathcal{H}_e$*

$$V(x[l]) \leq V(x[j]) + \sum_{i=j}^{l-1} s(u[i], y[i]), \text{ holds.} \quad (23)$$

Additionally, the system Σ is (Q, S, R) -dissipative if it is dissipative with respect to supply rate (21) where $Q = Q^\top$ and $R = R^\top$.

Passivity and some related definitions can be given with respect to the definition of (Q, S, R) -dissipativity.

Lemma 1 (Kottenstette and Antsaklis (2010)) *Consider a minimal continuous time system Σ or a discrete time system Σ_z that is (Q, S, R) -dissipative. This system*

i) *is passive iff the system is*

$$(0, \frac{1}{2}I, 0)\text{-dissipative,} \quad (24)$$

ii) *is strictly input passive iff $\exists \delta > 0$ such that the system is*

$$(0, \frac{1}{2}I, -\delta I)\text{-dissipative,} \quad (25)$$

iii) *is strictly output passive iff $\exists \epsilon > 0$ such that the system is*

$$(-\epsilon I, \frac{1}{2}I, 0)\text{-dissipative,} \quad (26)$$

iv) is very strictly iff $\exists \epsilon > 0, \delta > 0$ such that the system is

$$(-\epsilon I, \frac{1}{2}I, -\delta I)\text{-dissipative,} \quad (27)$$

v) is finite-gain L_2^m (l_2^m) stable iff $\exists \hat{\gamma} > 0$ such that the system is

$$(-I, 0, \hat{\gamma}^2 I)\text{-dissipative.} \quad (28)$$

Remark 6 The reason that these conditions are necessary and sufficient is that the systems Σ and Σ_z are minimal realizations of $H(s)$ and $H(z)$ respectively. This implies they are controllable and observable and therefore satisfy either (Hill and Moylan 1976, Theorem 1) or (Hill and Moylan 1980, Theorem 16).

From the above discussion the following two corollaries can be stated in continuous and discrete time. These results represent a generalization of the Positive Real Lemma (KYP Lemma) from necessary and sufficient conditions for passivity to necessary and sufficient conditions for (Q,S,R)-dissipativity.

Lemma 2 For continuous time LTI systems (9)-(10), a necessary and sufficient test for Definition 1 to hold is that $\exists P = P^T > 0$ such that the following LMI is satisfied:

$$\begin{bmatrix} A^T P + PA - \hat{Q} & PB - \hat{S} \\ (PB - \hat{S})^T & -\hat{R} \end{bmatrix} \leq 0, \quad (29)$$

in which

$$\hat{Q} = C^T Q C \quad (30)$$

$$\hat{S} = C^T S + C^T Q D \quad (31)$$

$$\hat{R} = D^T Q D + (D^T S + S^T D) + R. \quad (32)$$

Lemma 3 (Goodwin and Sin 1984, Lemma C.4.2) For discrete time LTI systems (12)-(13), a necessary and sufficient test for Definition 1 to hold is that $\exists P = P^T > 0$ such that the following LMI is satisfied:

$$\begin{bmatrix} A^T P A - P - \hat{Q} & A^T P B - \hat{S} \\ (A^T P B - \hat{S})^T & -\hat{R} + B^T P B \end{bmatrix} \leq 0, \quad (33)$$

in which \hat{Q} , \hat{S} , and \hat{R} are specified by (30), (31), and (32), respectively.

The matrix inequalities covered in this paper are linear in the decision variable (P) so they can be solved using traditional LMI optimization methods in Boyd et al. (1994).

2.4 Positive Real Systems

The property of positive realness is a condition on the transfer function of a LTI system. A minimal transfer function with this property must be BIBO stable, minimum phase, and have relative degree of zero or one. Positive realness can be shown by an equivalent frequency based condition.

Definition 6 ((Anderson and Vongpanitlerd 1973, p.51)(Tao and Ioannou 1988, Definition 1.1)(Haddad and Chellaboina 2008, Definition 5.18)) Consider a continuous time LTI system represented by an $m \times m$ rational and proper transfer function matrix $H(s)$. This system is positive real (PR) if the following conditions are satisfied:

- i) All elements of $H(s)$ are analytic in $\text{Re}[s] > 0$.
- ii) $H(s)$ is real for all real positive values of s .
- iii) $H^\top(s^*) + H(s) \geq 0$ for $\text{Re}[s] > 0$.

Furthermore $H(s)$ is strictly positive real (SPR) if $\exists \epsilon > 0$ s.t. $H(s - \epsilon)$ is positive real. Finally, $H(s)$ is strongly positive real if $H(s)$ is strictly positive real and $D + D^\top > 0$ where $D \triangleq H(\infty)$.

It should be noted that the definition of *PR* implies that the poles of $H(s)$ are in the closed left-half plane, i.e. a minimal internal realization of the system is Lyapunov stable. The definition of *SPR* implies that the poles of $H(s)$ are in the open left-half plane, i.e. the system is L_2^m stable with a minimal internal realization that is asymptotically stable. The conditions for *PR* and *SPR* can be verified directly or the test can be simplified to a frequency domain condition.

Theorem 1 ((Willems 1972b, Theorem 1)(Anderson and Vongpanitlerd 1973, p.216)(Haddad and Chellaboina 2008, Theorem 5.11)) Let $H(s)$ be a square, proper, and real rational transfer function. $H(s)$ is positive real iff the following conditions hold:

- i) All elements of $H(s)$ are analytic in $\text{Re}[s] > 0$.
- ii) $H^\top(-j\omega) + H(j\omega) \geq 0$, $\forall \omega \in \mathbb{R}$ for which $j\omega$ is not a pole for any element of $H(s)$.
- iii) Any pure imaginary pole $j\omega_o$ of any element of $H(s)$ is a simple pole, and the associated residue matrix $H_o \triangleq \lim_{s \rightarrow j\omega_o} (s - j\omega_o)H(s)$ is nonnegative definite Hermitian (i.e. $H_o = H_o^* \geq 0$).

A similar test is given for strict positive realness.

Theorem 2 (Tao and Ioannou 1988, Theorem 2.1) Let $H(s)$ be a $m \times m$, real rational transfer function and suppose $H(s)$ is non-singular. Then $H(s)$ is strictly positive real iff the following conditions hold:

- i) All elements of $H(s)$ are analytic in $\text{Re}[s] \geq 0$.
- ii) $H(j\omega) + H^\top(-j\omega) > 0$ for $\forall \omega \in \mathbb{R}$.
- iii) Either $\lim_{\omega \rightarrow \infty} [H(j\omega) + H^\top(-j\omega)] = D + D^\top > 0$ or if $D + D^\top \geq 0$ then $\lim_{\omega \rightarrow \infty} \omega^2 [H(j\omega) + H^\top(-j\omega)] > 0$.

To finish the discussion on continuous time positive real systems, we state the Positive Real Lemma and the Strict Positive Real Lemma.

Lemma 4 ((Anderson 1967, Theorem 3), (Anderson and Vongpanitlerd 1973, p.218)) Let $H(s)$ be an $m \times m$ matrix of real proper rational functions of a complex variable s . Let Σ be a minimal realization of $H(s)$. Then $H(s)$ is positive real iff there exists $P = P^\top > 0$ s.t.

$$\begin{bmatrix} A^\top P + PA & PB - C^\top \\ (PB - C^\top)^\top & -(D^\top + D) \end{bmatrix} \leq 0 \quad (34)$$

Lemma 5 (Sun et al. 1994, Lemma 2.3) Let $H(s)$ be an $m \times m$ matrix of real proper rational functions of a complex variable s . Let Σ be a minimal realization of $H(s)$. Then $H(s)$ is strongly positive real iff there exists $P = P^\top > 0$ s.t. Σ is asymptotically stable and

$$\begin{bmatrix} A^\top P + PA & PB - C^\top \\ (PB - C^\top)^\top & -(D^\top + D) \end{bmatrix} < 0. \quad (35)$$

This section up to this point covered continuous time positive real systems. A similar presentation can be made for discrete time systems.

Definition 7 (Hitz and Anderson (1969), Xiao and Hill (1999), (Haddad and Chellaboina 2008, Definition 13.16) (Tao and Ioannou 1990, Definition 2.4, 2.5)) A square transfer function matrix $H(z)$ of real rational functions is a positive real matrix if:

- i) all the entries of $H(z)$ are analytic in $|z| > 1$ and
- ii) $H_o = H(z) + H^\top(z^*) \geq 0, \forall |z| > 1$.

Furthermore $H(z)$ is strictly positive real if $\exists \rho (0 < \rho < 1)$ s.t. $H(\rho z)$ is positive real.

Remark 7 For the discrete time case, there is no need to define strongly positive real. The definition of strictly positive real implies that $(D + D^\top) > 0$ where $D \triangleq H(\infty)$. This satisfies the analogous definition for strongly positive real for discrete time systems, see (Lee and Chen 2000, Remark 4). The terms “strictly positive real” and “strongly positive real” may be used interchangeably for discrete time systems.

The test for a discrete time positive real system can be simplified to a frequency test as follows:

Theorem 3 ((Hitz and Anderson 1969, Lemma 2), (Haddad and Chellaboina 2008, Theorem 13.26)) Let $H(z)$ be a square, real rational $m \times m$ transfer function matrix. $H(z)$ is positive real iff the following conditions hold:

- i) No entry of $H(z)$ has a pole in $|z| > 1$.
- ii) $H(e^{j\theta}) + H^\top(e^{-j\theta}) \geq 0, \forall \theta \in [0, 2\pi]$, in which $e^{j\theta}$ is not a pole of any entry of $H(z)$.
- iii) If $e^{j\hat{\theta}}$ is a pole of any entry of $H(z)$ it is at most a simple pole, and the residue matrix $H_o \triangleq \lim_{z \rightarrow e^{j\hat{\theta}}} (z - e^{j\hat{\theta}})G(z)$ is nonnegative definite.

The test for a strictly positive real system can be simplified to a frequency test as follows:

Theorem 4 (Tao and Ioannou 1990, Theorem 2.2) Let $H(z)$ be a square, real rational $m \times m$ transfer function matrix in which $H(z) + H^\top(z^*)$ has rank m almost everywhere in the complex z -plane. $H(z)$ is strictly positive real iff the following conditions hold:

- i) No entry of $H(z)$ has a pole in $|z| \geq 1$.
- ii) $H(e^{j\theta}) + H^\top(e^{-j\theta}) \geq \epsilon I > 0, \forall \theta \in [0, 2\pi], \exists \epsilon > 0$.

Finally, we state the Positive Real Lemma and the Strictly Positive Real Lemma for the discrete time case.

Lemma 6 (Hitz and Anderson 1969, Lemma 3) Let $H(z)$ be an $n \times n$ matrix of real, proper, and rational transfer functions and let Σ_z be a minimal stable realization of $H(z)$. Then $H(z)$ is positive real iff there exists $P = P^\top > 0$ s.t.

$$\begin{bmatrix} A^\top P A - P & A^\top P B - C^\top \\ (A^\top P B - C^\top)^\top & -(D^\top + D) + B^\top P B \end{bmatrix} \leq 0. \quad (36)$$

Lemma 7 ((Lee and Chen 2000, Corollary 2)(Haddad and Bernstein 1994, Lemma 4.2)) Let $H(z)$ be an $n \times n$ matrix of real, proper, and rational transfer functions and let Σ_z be an asymptotically stable realization of $H(z)$. Then $H(z)$ is strictly positive real iff there exists $P = P^\top > 0$ s.t.

$$\begin{bmatrix} A^\top P A - P & A^\top P B - C^\top \\ (A^\top P B - C^\top)^\top & -(D^\top + D) + B^\top P B \end{bmatrix} < 0. \quad (37)$$

3 Preliminary Results for Passivity, Dissipativity, and Positive Realness

Preliminary results related to the properties of passivity and positive realness are covered in this section. The following result from Desoer and Vidyasagar (1975) summarizes a series of frequency-based conditions that are equivalent to passivity or strict input passivity.

Theorem 5 (Desoer and Vidyasagar 1975, p.174-175) Consider a LTI system H which has a minimal realization Σ (Σ_z) that is asymptotically stable.

(i) If H is a continuous time system then

(a) H is passive iff $H(j\omega) + H^\top(-j\omega) \geq 0, \forall \omega \in \mathbb{R}$.

(b) H is strictly input passive iff $\exists \delta > 0$ s.t.

$$H(j\omega) + H^\top(-j\omega) \geq \delta I, \forall \omega \in \mathbb{R}. \quad (38)$$

(ii) If H is a discrete time system then

(a) H is passive iff $H(e^{j\theta}) + H^\top(e^{-j\theta}) \geq 0, \forall \theta \in [0, 2\pi]$.

(b) H is strictly input passive iff $\exists \delta > 0$ s.t.

$$H(e^{j\theta}) + H^\top(e^{-j\theta}) \geq \delta I, \forall \theta \in [0, 2\pi]. \quad (39)$$

While there are existing results for frequency based conditions for passivity and strict input passivity, there isn't an established test for strict output passivity. One such condition is proposed in the following theorem.

Theorem 6 Consider a single-input single-output LTI strictly output passive system with transfer function $H(s)$ ($H(z)$), real impulse response $h(t)$ ($h(k)$), and corresponding frequency response:

$$H(j\omega) = \text{Re}\{H(j\omega)\} + j\text{Im}\{H(j\omega)\} \quad (40)$$

in which $\text{Re}\{H(j\omega)\} = \text{Re}\{H(-j\omega)\}$ for the real part of the frequency response and $\text{Im}\{H(j\omega)\} = -\text{Im}\{H(-j\omega)\}$ for the imaginary part of the frequency response. If H is SOP then the constant ϵ in the definition may be found by the following inequality:

$$0 < \epsilon \leq \inf_{\omega \in [0, \infty)} \frac{\text{Re}\{H(j\omega)\}}{\text{Re}\{H(j\omega)\}^2 + \text{Im}\{H(j\omega)\}^2} \quad (41)$$

for the continuous time case. Similarly for discrete time case,

$$H(e^{j\theta}) = \text{Re}\{H(e^{j\theta})\} + j\text{Im}\{H(e^{j\theta})\} \quad (42)$$

in which $\text{Re}\{H(e^{j\theta})\} = \text{Re}\{H(e^{-j\theta})\}$ in which $0 \leq \theta \leq \pi$ for the real part of the frequency response and $\text{Im}\{H(e^{j\theta})\} = -\text{Im}\{H(e^{-j\theta})\}$ for the imaginary part of the frequency response. The constant ϵ for (17) satisfies:

$$0 < \epsilon \leq \min_{\theta \in [0, \pi]} \frac{\text{Re}\{H(e^{j\theta})\}}{\text{Re}\{H(e^{j\theta})\}^2 + \text{Im}\{H(e^{j\theta})\}^2} \quad (43)$$

for the discrete time case.

Proof: Since a strictly output passive system has a finite integrable (summable) impulse response (i.e. $\int_0^\infty h^2(t)dt < \infty$ ($\sum_{i=0}^\infty h^2[i] < \infty$)) then the condition for SOP (17) can be written as

$$\int_{-\infty}^{\infty} H(j\omega)|U(j\omega)|^2 d\omega \geq \epsilon \int_{-\infty}^{\infty} |H(j\omega)|^2 |U(j\omega)|^2 d\omega \quad (44)$$

for the continuous time case or

$$\int_{-\pi}^{\pi} H(e^{j\theta})|U(e^{j\theta})|^2 d\theta \geq \epsilon \int_{-\pi}^{\pi} |H(e^{j\theta})|^2 |U(e^{j\theta})|^2 d\theta \quad (45)$$

for the discrete time case. (44) can be written in the following simplified form:

$$\begin{aligned} \int_{-\infty}^{\infty} \operatorname{Re}\{H(j\omega)\}|U(j\omega)|^2 d\omega &\geq \\ \epsilon \int_{-\infty}^{\infty} (\operatorname{Re}\{H(j\omega)\}^2 + \operatorname{Im}\{H(j\omega)\}^2)|U(j\omega)|^2 d\omega &\end{aligned} \quad (46)$$

in which (41) clearly satisfies (46). Similarly (45) can be written in the following simplified form:

$$\begin{aligned} \int_{-\pi}^{\pi} \operatorname{Re}\{H(e^{j\theta})\}|U(e^{j\theta})|^2 d\theta &\geq \\ \epsilon \int_{-\pi}^{\pi} (\operatorname{Re}\{H(e^{j\theta})\}^2 + \operatorname{Im}\{H(e^{j\theta})\}^2)|U(e^{j\theta})|^2 d\theta &\end{aligned} \quad (47)$$

in which (43) clearly satisfies (47). ■

The frequency based conditions for passivity and strict input passivity (Theorem 5) appear to be closely related to the frequency based conditions for positive realness and strong positive realness.

Remark 8 *It is important to note that the value ϵ in (41) or (43) corresponds to the output feedback passivity (OFP) index ρ , see e.g. Bao and Lee (2007) or McCourt and Antsaklis (2009). In (Bao and Lee 2007, p.29), an alternative method of calculating the OFP index is given for minimum phase linear systems. We did not pose such constraints on the system when calculating this value using (41) or (43).*

Lemma 8 *Let $H(s)$ (with a corresponding minimal realization Σ) be a $m \times m$, real rational transfer function that is non-singular. Then the following are equivalent:*

- i) $H(s)$ is strongly positive real
- ii) Σ is asymptotically stable and strictly input passive s.t.

$$H(j\omega) + H^T(-j\omega) \geq \delta I > 0, \forall \omega \in \mathbb{R} \quad (48)$$

Proof: ii \implies i:

Since Σ is asymptotically stable then all poles are in the open left half plane, therefore Theorem 2-i is satisfied. Next (48) clearly satisfies Theorem 2-ii. Also, (48) implies that $D + D^T > \delta I > 0$ which satisfies 2-iii which satisfies the final condition to be strictly positive real and also strongly positive real as noted in Definition 6.

i \implies ii:

First we note that Theorem 2-i implies Σ will be asymptotically stable. Next, from Definition 6 we note that $\exists \delta_1 > 0$ s.t.

$$H^T(-j\infty) + H(j\infty) = D^T + D \geq \delta_1 I > 0$$

Lastly, we assume that $\exists \delta_2 \leq 0$ s.t.

$$H^\top(-j\omega) + H(j\omega) \geq \delta_2 I, \forall \omega(-\infty, \infty) \quad (49)$$

however this contradicts Theorem 2-ii therefore $\exists \delta_2 > 0$ s.t. (49) is satisfied which implies (48) is satisfied in which $\delta = \min\{\delta_1, \delta_2\} > 0$. ■

Remark 9 Note that Lemma 8-ii is equivalent to Σ being asymptotically stable and $H(s)$ being strictly input passive as stated in Theorem 5-ib.

The previous development will be given for discrete time systems. Recall that the definition for strictly positive real and strongly positive real are equivalent in discrete time.

Lemma 9 Let $H(z)$ (with a corresponding minimal realization Σ_z) be a square, real rational $m \times m$ transfer function matrix in which $H(z) + H^\top(z^*)$ has rank m almost everywhere in the complex z -plane. Then the following are equivalent:

- i) $H(z)$ is strictly positive real
- ii) Σ_z is asymptotically stable and strictly input passive s.t.

$$H(e^{j\theta}) + H^\top(e^{-j\theta}) \geq \delta I, \forall \theta \in [0, 2\pi] \quad (50)$$

Proof: ii \implies i:

Since Σ_z is asymptotically stable then all poles are strictly inside the unit circle, therefore Theorem 4-i is satisfied. Next (50) clearly satisfies Theorem 4-ii.

i \implies ii:

First we note that Theorem 4-i implies Σ_z will be asymptotically stable. Finally Theorem 4-ii clearly satisfies (50). ■

4 Main Results

4.1 Connection Between Passive and Positive Real

This section covers the important relationships presented in this paper. This first part focuses on the relationships between the various definitions of passive and positive real. The following lemma covers the connection between passive and positive real for continuous time *LTI* systems. Recall that positive real is defined for square transfer functions that are assumed to have zero initial conditions so the connection will be shown for zero initial conditions. The next result is the connection between strongly positive real and strictly input passive for asymptotically stable systems. The relationship between strictly passive and strictly positive real will not be covered but the reader is directed to Haddad and Chellaboina (2008) or Khalil (2002) for more details. The remainder of this subsection cover these connections for the discrete time case.

Lemma 10 Let $H(s)$ be an $m \times m$ matrix of real, proper, and rational transfer functions of a complex variable s . Let Σ be a minimal realization of $H(s)$. Denote $h(t)$ as the $m \times m$ impulse response matrix of $H(s)$ from which the output $y(t)$ can be computed by,

$$y(t) = \int_0^t h(t - \tau)u(\tau)d\tau.$$

Then the following statements are equivalent:

i) The transfer function $H(s)$ is positive real.

ii) There exists $P = P^\top > 0$ to satisfy the Positive Real Lemma (34).

iii) The system Σ is $(0, \frac{1}{2}I, 0)$ -dissipative, i.e. $\exists P = P^\top > 0$ s.t. (29) is satisfied.

iv) The system is passive, i.e. $\forall T$

$$\int_0^T y^\top(t)u(t)dt \geq 0,$$

for zero initial conditions.

Proof: i) \Leftrightarrow ii): Stated in Lemma 4.

iii) \Leftrightarrow iv): Remark 5 states that iv) is an equivalent test for passivity and Corollary 2 states that iii) is an equivalent test for passivity when $(Q, S, R) = (0, \frac{1}{2}I, 0)$.

ii) \implies iii): A passive system $H(s)$ is also passive iff $kH(s)$ is passive for $\forall k > 0$. Therefore (29) for $kH(s)$ in which $\Sigma = \{A, B, kC, kD\}$ and $(Q, S, R) = (0, \frac{1}{2}I, 0)$, $\hat{Q} = 0$, $\hat{S} = \frac{k}{2}C^\top$, $\hat{R} = \frac{k}{2}(D^\top + D)$:

$$\begin{bmatrix} A^\top P + PA^\top & PB - \frac{k}{2}C^\top \\ (PB - \frac{k}{2}C^\top)^\top & -\frac{k}{2}(D^\top + D) \end{bmatrix} \leq 0, \quad (51)$$

which for $k = 2$ satisfies (34).

iii) \implies ii): The converse argument can be made in which a positive real system $H(s)$ is positive real iff $kH(s)$ is positive real $\forall k > 0$ in which we choose $k = \frac{1}{2}$. ■

Remark 10 *The key to the proof was connecting the work of Anderson and Vongpanitlerd (1973), Desoer and Vidyasagar (1975) and Hill and Moylan (1980). Doing so highlights the connection between positive real system theory and dissipative system theory. This connection was partially made previously in (Willems 1972b, Theorem 1) and Desoer and Vidyasagar (1975). Similar connections are discussed recently in (Haddad and Chellaboina 2008, Theorem 5) which relied on Parseval's Theorem. The benefit of the approach in the current paper is that it does not rely on Parseval's Theorem which cannot be applied to systems with poles on the imaginary axis. As a result, the connection between passive systems and positive real systems holds for systems with poles on the imaginary axis. Finally, it should be noted that this result was given previously with a different proof in Brogliato et al. (2007).*

Lemma 11 *Let $H(s)$ be an $m \times m$ matrix of real, proper, and rational transfer functions of a complex variable s , with $H(\infty) < \infty$. Let Σ be a minimal realization of $H(s)$. Furthermore we denote $h(t)$ as an $m \times m$ impulse response matrix of $H(s)$ in which the output $y(t)$ is computed as follows:*

$$y(t) = \int_0^t h(t - \tau)u(\tau)d\tau$$

Then the following statements are equivalent:

i) The transfer function $H(s)$ is strongly positive real.

ii) There exists $P = P^\top > 0$ to satisfy the strict Positive Real Lemma (35).

iii) Σ is asymptotically stable and $(0, \frac{1}{2}, -\delta I)$ -dissipative, i.e. $\exists P = P^\top > 0$ such that (29) is satisfied, i.e. the system is strictly input passive and L_2^m stable.

iv) Σ is asymptotically stable, and for zero initial conditions ($y(0) = 0$),

$$\int_0^\infty y^\top(t)u(t) \geq \delta \|u(t)\|_2^2$$

in which $\delta = \inf_{-\infty \leq \omega \leq \infty} \operatorname{Re}\{H(j\omega)\}$ for the single input single output case.

Furthermore, iii) implies that for $(Q, S, R) = (-\epsilon I, \frac{1}{2}I, 0)$ there $\exists P = P^\top > 0$ s.t. (29) is also satisfied (strictly output passive). Thus if $y(0) = 0$ then

$$\int_0^\infty y^\top(t)u(t)dt \geq \epsilon \|y(t)\|_2^2.$$

Remark 11 In order for the equivalence between strongly positive real and strictly input passive to be stated, the strictly input passive system must also have finite gain (i.e. Σ is asymptotically stable). For example the realization for $H(s) = 1 + \frac{1}{s}$, $\Sigma = \{A = 0, B = 1, C = 1, D = 1\}$, $\delta = 1$ is strictly input passive but is not asymptotically stable. However $H(s) = \frac{s+b}{s+a}$, $\Sigma = \{A = -a, B = (b-a), C = D = 1\}$, $\delta = \min\{1, \frac{b}{a}\}$ is both strictly input passive and asymptotically stable for all $a, b > 0$.

Proof: i) \Leftrightarrow ii): Stated in Lemma 5.

ii) \Leftrightarrow iv): Stated in Lemma 8.

iii) \Leftrightarrow iv): Stated in Definition 1. ■

Remark 12 It is known that if an L_2^m (l_2^m) stable system is strictly input passive then it is also strictly-output passive (van der Schaft 1999, Remark 2.3.5), the converse however, is not always true (i.e. $\inf_{\forall \omega} \operatorname{Re}\{H(j\omega)\}$ is zero for strictly proper (strictly output passive) systems). It has been shown for the continuous time case (van der Schaft 1999, Theorem 2.2.14) and discrete time case ((Kottenstette and Antsaklis 2007b, Theorem 1) and (Goodwin and Sin 1984, Lemma C.2.1-(iii))) that a strictly output passive system is passive and L_2^m (l_2^m) stable but it remains to be shown if the converse is true or not true. Indeed, we can show that an infinite number of continuous time and discrete time linear time invariant systems do exist which are both passive and L_2^m (l_2^m) stable and are neither strictly output passive nor strictly input passive.

Theorem 7 Let $H : \mathcal{H}_e \rightarrow \mathcal{H}_e$ (in which $y = Hu$, $y(0) = 0$, and for the case when a state-space description exists for H that it is zero-state observable ($y = 0$ implies that the state $x = 0$) and there exists a positive definite storage function $\beta(x) > 0, x \neq 0, \beta(0) = 0$) have the following properties:

a) $\|y_T\|_2 \leq \gamma \|u_T\|_2$

b) $\langle y, u \rangle_T \geq -\delta \|u_T\|_2^2$

c) There exists a non-zero norm input u such that $\langle y, u \rangle_T = -\delta \|u_T\|_2^2$ and $\|y_T\|_2^2 > \delta^2 \|u_T\|_2^2$ for $\delta < \gamma$.

Then the following system H_1 , in which the output y_1 is computed using $y_1 = y + \delta u$ has the following properties:

I. H_1 is passive.

II. H_1 is L_2^m (l_2^m) stable.

III. H_1 is not strictly output passive (also not strictly input passive)

Proof: 7-I: Solving for the inner-product between y_1 and u we have

$$\langle y_1, u \rangle_T = \langle y, u \rangle_T + \delta \|u_T\|_2^2 \geq (-\delta + \delta) \|u_T\|_2^2 = 0$$

7-II: Solving for the extended-two-norm for y_1 we have

$$\begin{aligned} \|(y_1)_T\|_2^2 &= \|(y + \delta u)_T\|_2^2 \leq \|y_T\|_2^2 + \delta^2 \|u_T\|_2^2 \\ \|(y_1)_T\|_2^2 &\leq (\gamma^2 + \delta^2) \|u_T\|_2^2 \end{aligned}$$

7-III: From 7-I, the solution for the inner-product between y_1 and u can be substituted in Assumption c) to give, $\langle y_1, u \rangle_T = (-\delta + \delta) \|u_T\|_2^2 = 0$.

It is obvious that no constant $\delta > 0$ exists such that

$$\langle y_1, u \rangle_T = 0 \geq \delta \|u_T\|_2^2 = 0$$

since it is assumed that $\|u_T\|_2^2 > 0$, hence H_1 is not strictly-input passive. In a similar manner, noting that the added restriction holds $\|y_T\|_2^2 = \delta^2 \|u_T\|_2^2$ for the same input function u when $\langle y, u \rangle_T = -\delta \|u_T\|_2^2$, it is obvious that no constant $\epsilon > 0$ exists such that

$$\begin{aligned} \langle y_1, u \rangle_T = 0 &\geq \epsilon \|(y_1)_T\|_2^2 = 0 \\ 0 &\geq \epsilon (\|y_T\|_2^2 - \delta^2 \|u_T\|_2^2) \end{aligned}$$

holds. ■

Remark 13 *Theorem 7 shows that a system that is passive and L_2^m stable is not necessarily SOP. The continuous time system $H(s)$ given by*

$$H(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}, \quad (52)$$

for $\omega_n > 0$ satisfies the assumptions of the theorem required of a system H in which $\delta = \frac{1}{8}$ and an input-sinusoid $u(t) = \sin(\sqrt{3}\omega_n t)$ is a null-inner-product sinusoid such that

$$H_1(s) = \frac{1}{8} + H(s) = \frac{1}{8} + \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} \quad (53)$$

is both passive and L_2^m stable but neither strictly-output passive nor strictly-input passive.

This section will be finished with the connections between passivity and positive real in discrete time. The proofs are omitted because they closely follow the continuous time case.

Lemma 12 *Let $H(z)$ be an $m \times m$ matrix of real rational transfer functions of variable z . Let Σ_z be a minimal realization of $H(z)$ which is Lyapunov stable. Furthermore we denote $h[k]$ as an $m \times m$ impulse response matrix of $H(z)$ in which the output $y[k]$ is computed as follows:*

$$y[k] = \sum_{i=0}^k h[k-i]u[i]$$

Then the following statements are equivalent:

- i) $H(z)$ is positive real.

ii) There exists $P = P^\top > 0$ to satisfy the discrete time Positive Real Lemma (37).

iii) With $Q = R = 0$, $S = \frac{1}{2}I$ there $\exists P = P^\top > 0$ s.t. (33) is satisfied.

iv) For zero initial conditions ($y[0] = 0$), $H(z)$ is passive

$$\sum_{i=0}^{\infty} y^\top(i)u(i) \geq 0.$$

Lemma 13 Let $H(z)$ be an $m \times m$ matrix of real rational transfer functions of variable z . Let Σ_z be a minimal realization of $H(z)$ which is Lyapunov stable. Furthermore we denote $h[k]$ as an $m \times m$ impulse response matrix of $H(z)$ in which the output $y[k]$ is computed as follows:

$$y[k] = \sum_{i=0}^k h[k-i]u[i]$$

Then the following statements are equivalent:

i) $H(z)$ is strictly positive real.

ii) There exists $P = P^\top > 0$ to satisfy the discrete time Strict Positive Real Lemma (37).

iii) Σ_z is asymptotically stable, and for $Q = 0$, $R = -\delta I$, $S = \frac{1}{2}I$, $\exists P = P^\top > 0$, and $\exists \delta > 0$ s.t. (33) is satisfied.

iv) Σ_z is asymptotically stable, and for zero initial conditions ($y[0] = 0$), $H(z)$ is strictly input passive s.t.

$$\sum_{i=0}^{\infty} y^\top(i)u(i) \geq \delta \|u(i)\|_2^2.$$

4.2 Passivity Based Pairing Methods

In control systems, pairing methods are used to pair inputs and outputs for control of multi-input multi-output (MIMO) systems. Passivity methods have been used for pairing as in Bao et al. (2007). Lemma 13 of this paper can be used to improve the performance of existing passivity based pairing methods.

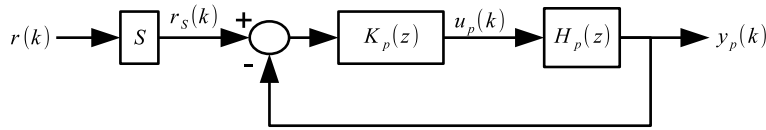


Figure 2: Asymptotically stable feedback structure if $K_p(z)$ is passive and $H_p(z)$ strictly positive real ($r_s(k) = Sr(k)$).

The passivity based pairing method scales and pairs inputs $u(k) \in \mathbb{R}^m$ to outputs $y(k) \in \mathbb{R}^m$ of a stable discrete time plant $H(z)$ and augments its output so that resulting system $H_p(z)$ with input vector $u_p(k) \in \mathbb{R}^m$ and output vector $y_p(k) \in \mathbb{R}^m$ is asymptotically stable and strictly input passive (strictly positive real). $H_p(z)$ is then integrated into the feedback structure depicted in Fig. 2 in which the controller $K_p(z)$ is passive and the matrix S is a permutation matrix resulting from the passivity based pairing method. As depicted in Fig. 3 the plant is scaled, paired and rendered strictly positive real. The procedure to render $H_p(z)$ to be strictly positive real is as follows:

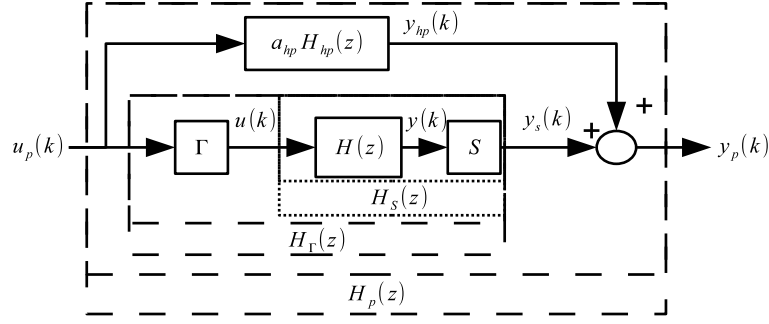


Figure 3: Stable plant $H(z)$ paired, scaled and rendered passive such that $H_p(z)$ is strictly positive real.

- 1) Determine $H_S(z) = SH(z)$. Select one of the $m!$ permutation matrices S such that $y_S(k) = Sy(k)$.
- 2) Determine $H_\Gamma(z) = H_S(z)\Gamma$. The matrix $\Gamma = \text{diag}\{\gamma_1, \dots, \gamma_m\}$ and the corresponding relationship $u(k) = \Gamma u_p(k)$ are used to ensure that the steady state gain from $u_p(k)$ to $y_S(k)$ is positive by setting the coefficients to be

$$\gamma_i = \text{sgn}(H_{S(i,i)}(1)). \quad (54)$$

- 3) Determine $H_p(z) = H_\Gamma(z) + a_{hp}H_{hp}(z)$. $H_{hp}(z) = \text{diag}\{H_{hp(1)}(z), \dots, H_{hp(m)}(z)\}$ in which $H_{hp(i)}(z)$ is an identical highpass filter. The highpass filter is synthesized by applying a passivity preserving sample and hold transform Kottenstette et al. (2011) to the continuous time filter $H_{hp}(s) = \frac{s}{s+\omega_o}$ in which ω_o is a positive real coefficient. The real coefficients a_{hp} and ω_o are determined by minimizing $R(\omega_o, a_{hp}) = -\frac{\omega_o}{a_{hp}}$ while satisfying (37) for the state space realization of $H_p(z)$ to be rendered strictly positive real.

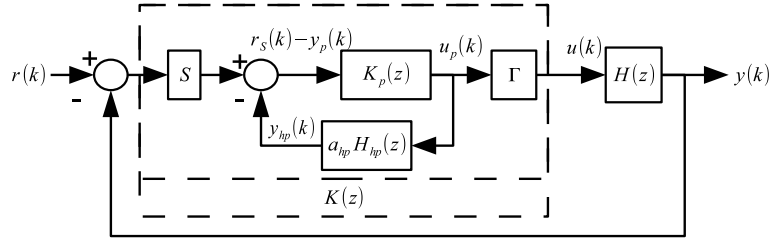


Figure 4: Final control structure in which $K(z)$ is realized for the plant $H(z)$.

The controller $K(z)$ is implemented after $H_p(z)$ is determined. As depicted in Fig. 4 the controller gain $K(z)$ is determined as follows

$$K(z) = \Gamma K_p(z) \times [I + a_{hp}H_{hp}(z)K_p(z)]^{-1} S. \quad (55)$$

The *final control law* $K(z)$ selected will be the one which results from the S which minimizes $R(\omega_o, a_{hp})$ over all possible permutation matrices.

5 Case Studies Demonstrating Performance in Passivity Based Pairing

The passivity based pairing method presented in Section 4.2 can lead to improved performance when compared to controllers derived using the scaled passivity index pairing method presented in Bao et al. (2007). The reason being is that by minimizing $R(\omega_o, a_{hp})$ we are able to explicitly minimize the steady-state gain

a_{hp} and maximizing the pole ω_o of the discrete time high-pass filter $a_{hp}H_{hp}(z)$ used to realize a strictly positive real system $H_p(z)$. In addition solving the feasibility of (37) is a necessary and sufficient test to realize that $H_p(z)$ is strictly positive real instead of requiring the permutation matrix S and filter $a_{hp}H_{hp}(z)$ are chosen such that the frequency dependent scaled passivity index is rendered equal to zero (a sufficient test for the system to be positive real). This improvement in performance will be demonstrated by comparing our final controller performance to two of the illustrative case examples studied in Bao et al. (2007). Case 1 involving the 3×3 distillation column will be considered in which additional feasible pairings are determined possible which includes a better performing 1/2-2/3-3/1 pairing. Case 2 involving a 2×2 process is considered in which we can compare the improved step tracking response. The remaining subsections are as follows: i) Subsection 5.1 presents the discrete time high pass filter realization; ii) Subsection 5.2 presents our discrete time proportional-integral control realization; iii) Subsection 5.3 presents Case 1 results; and iv) Subsection 5.4 presents Case 2 results.

5.1 High Pass Filter Synthesis

In this section we derive the discrete time passive filter $H_{hp}(z)$ from the continuous time filter $H_{hp}(s)$ by using the inner product equivalent sample and hold transform as presented in (Kottenstette and Antsaklis 2007a, Section-IV-A). By preserving the inner product of input and output from continuous time to discrete time, the passivity property is guaranteed. The passivity preserving bilinear transform could be applied as well (Hitz and Anderson (1969)); however, frequency prewarping would also be required. The continuous time filter $H_{hp}(s) = \frac{s}{s+\omega_o}$ used to synthesize $H_{hp(i)}(z)$ has the following state-space realization:

$$\begin{aligned}\dot{x}_{hp(i)}(t) &= -\omega_o x_{hp(i)}(t) + u_{p(i)}(t) \\ y_{hp(i)}(t) &= -\omega_o x_{hp(i)}(t) + u_{p(i)}(t)\end{aligned}$$

The output $y_{hp(i)}(t)$ is then cascaded with an integrator such that:

$$\begin{aligned}\dot{x}_{I-hp(i)}(t) &= A_o x_{I-hp(i)}(t) + B_o u_{p(i)}(t) \\ y_{I-hp(i)}(t) &= C_o x_{I-hp(i)}(t)\end{aligned}$$

in which $A_o = \begin{bmatrix} -\omega_o & 0 \\ -\omega_o & 0 \end{bmatrix}$, $B_o = [1 \ 1]^T$, $C_o = [0 \ 1]$. The final passive discrete time state space realization for $H_{hp(i)}(z)$ with sampling rate T_s seconds is

$$\begin{aligned}x_{hp(i)}(k+1) &= \Phi_{hp(i)-o} x_{hp(i)}(k) + \Gamma_{hp(i)-o} u_{p(i)}(k) \\ y_{hp(i)}(k) &= C_{hp(i)-p} x_{hp(i)}(k) + D_{hp(i)-p} u_{p(i)}(k)\end{aligned}$$

in which

$$\begin{aligned}\Phi_{hp(i)-o} &= e^{A_o T_s} \\ &= \begin{bmatrix} e^{-\omega_o T_s} & 0 \\ (e^{-\omega_o T_s} - 1) & 1 \end{bmatrix} \\ \Gamma_{hp(i)-o} &= \int_0^{T_s} e^{A_o \eta} d\eta B_o \\ &= \frac{1 - e^{-\omega_o T_s}}{\omega_o} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ C_{hp(i)-p} &= \frac{1}{T_s} C_{hp(i)-o} (\Phi_{hp(i)-o} - I)\end{aligned}$$

$$= \frac{1}{T_s} [(e^{-\omega_o T_s} - 1) \quad 0]$$

$$D_{hp(i)-p} = \frac{1}{T_s} C_o \Gamma_o = \frac{1 - e^{-\omega_o T_s}}{T_s \omega_o}.$$

All matrices are diagonalized in order to implement the m identical highpass filters in which we denote $x_{hp} = [x_{hp(1)}^\top, \dots, x_{hp(m)}^\top]^\top$, $u_p = [u_{p(1)}, \dots, u_{p(m)}]^\top$, $y_{hp} = [y_{hp(1)}, \dots, y_{hp(m)}]^\top$, the m dimensional identity matrix as $I_{m \times m}$ and the Kronecker tensor product as \otimes such that

$$x_{hp}(k+1) = \Phi_{hp-o} x_{hp}(k) + \Gamma_{hp-o} u_p(k)$$

$$y_{hp}(k) = C_{hp-p} x_{hp}(k) + D_{hp-p} u_p(k)$$

in which $\Phi_{hp-o} = I_{m \times m} \otimes \Phi_{hp(1)-o}$, $\Gamma_{hp-o} = I_{m \times m} \otimes \Gamma_{hp(1)-o}$, $C_{hp-p} = I_{m \times m} \otimes C_{hp(i)-p}$, $D_{hp-p} = I_{m \times m} \otimes D_{hp(1)-p}$.

5.2 Proportional-Integral Controller Synthesis

In Kottenstette et al. (2012) we presented a multi input multi output passive discrete time proportional integral (PI) control law. We shall consider the simplified case when the positive definite matrices K_i and K_p used in (Kottenstette et al. 2012, eqn. (36)) are of the following form $K_i = k_I I_{m \times m}$ and $K_p = k_P I_{m \times m}$ in which k_I and k_P are positive real numbers. The resulting passive discrete time control law $K_p(z) = \text{diag}\{K_{p(1)}(z), \dots, K_{p(m)}(z)\}$ in which $K_{p(i)}(z)$ is a discrete time PI-controller derived from the application of the inner product equivalent sample and hold transform to the continuous time PI-controller $K_p(s) = k_P + \frac{k_I}{s}$. Denote the: discrete time controller state vector $x_p(k) \in \mathbb{R}^m$; controller input $e_p(k) = S(r(k) - y(k)) \in \mathbb{R}^m$; and the passive controller output as $u_p(k) \in \mathbb{R}^m$. The discrete time state space implementation of $K_p(z)$ is

$$x_p(k+1) = x_p(k) + T_s e_p(k)$$

$$u_p(k) = k_I x_p(k) + \left[\frac{T_s}{2} k_I + k_P \right] e_p(k).$$

The final control gains k_I and k_P used to compute $K_p(z)$ will be determined by minimizing the following performance measure

$$J(N) = \sum_{k=0}^N (k+1) \sqrt{e^\top(k) e(k)} \quad (56)$$

in which the tracking error $e(k) = r(k) - y(k)$. The control gains k_I and k_P used to minimize $J(N)$ were found using the simplex search method detailed in Lagarias et al. (1998)¹. In order to minimize system overshoot an additional low-pass filter is applied to the reference set-point $r(t)$ in which the analog filter $H_{\text{traj}}(s) = \frac{\omega_{\text{traj}}^2}{s^2 + 2\zeta_{\text{traj}}\omega_{\text{traj}}s + \omega_{\text{traj}}^2}$. The discrete time filter $H_{\text{traj}}(z) = \text{diag}\{H_{\text{traj}(1)}(z), \dots, H_{\text{traj}(m)}(z)\}$ in which $H_{\text{traj}(i)}(z)$ ($i = 1, \dots, m$) results from application of the inner product equivalent sample and hold to $H_{\text{traj}}(s)$.

5.3 Case 1: Distillation Column

The 3×3 distillation column has the following transfer function matrix

$$H(s) = \begin{bmatrix} \frac{-1.986e^{-0.71s}}{66.67s+1} & \frac{5.24e^{-60s}}{400s+1} & \frac{5.984e^{-2.24s}}{14.29s+1} \\ \frac{0.0204e^{-4.199s}}{5s+1} & \frac{-0.33e^{-1.883s}}{3.904s+1} & \frac{2.38e^{-1.143s}}{10s+1} \\ \frac{0.374e^{-7.75s}}{22.22s+1} & \frac{-11.3e^{-14.78s}}{35.66s+1} & \frac{-9.881e^{-1.59s}}{11.35s+1} \end{bmatrix} \quad (57)$$

¹Mathwork's fminsearch was used.

A periodic T_s second zero order hold (ZOH) is applied to (57) in order to derive $H(z)$. In order to compute feasibility of (37) for $H_p(z)$ and minimize a_{hp} we used CVX (Grant et al. (2006)), in order to initially bound the range for ω_o in $H_{hp}(z)$ we used a modified golden section search to determine $\omega_o \in [\omega_{o-\min}, \omega_{o-\max}]$, and then proceeded to use Brent's Method to minimize $R(\omega_o, a_{hp})$ (Brent 1973, pp. 79–80). The resulting controller parameters and corresponding performance measures for the top three configurations are summarized in Table 1. The resulting closed loop tracking responses are plotted in Fig. 5 in which it is clear that the 1-2/2-3/3-1 pairing ($\Gamma = \text{diag}\{1, -1, 1\}$) leads to best tracking performance. In addition the 1-1/2-3/3-2 pairing ($\Gamma = \text{diag}\{-1, -1, 1\}$) is the one identified in Bao et al. (2007) by using the frequency dependent passivity index in which we are able to synthesize a feasible controller for. Finally the 1-1/2-2/3-3 pairing ($\Gamma = \text{diag}\{-1, -1, -1\}$) has similar tracking performance to the 1-1/2-3/3-2 pairing.

Table 1: Summary of Controller Results for 3×3 Distillation Column ($T_s=1.0$ seconds, $\zeta_{\text{traj}}=.9$, $\omega_{\text{traj}}=\pi/1000$).

Pairing	$R(\omega_o, a_{hp})$	a_o	ω_o	$J(3e4)$	k_P	k_I
1-2/2-3/3-1	-.0081	2.728	.0222	10,206	1.07	.055
1-1/2-3/3-2	-.0014	6.051	.0084	26,690	1e3	1e6
1-1/2-2/3-3	-.0012	5.719	.0068	29,662	1e3	1e6

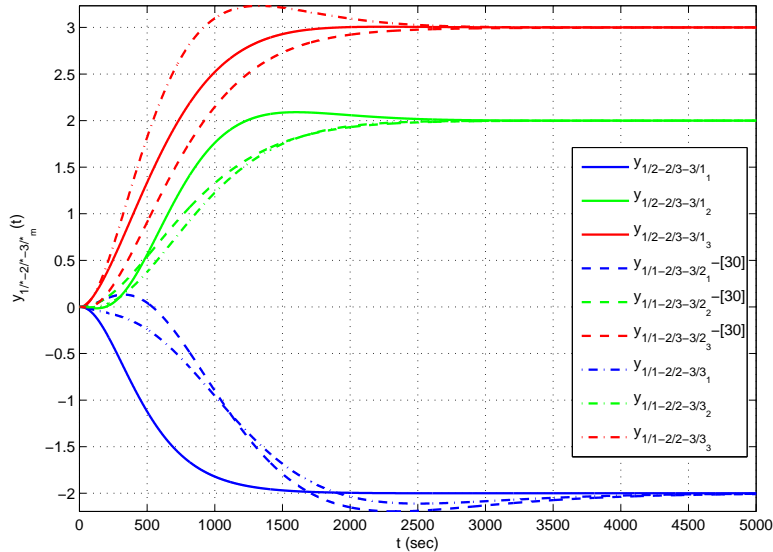


Figure 5: Case 1: closed loop filtered step response $r(t) = [-2, 2, 3]$, ($T_s = 1.0$ seconds, $\zeta_{\text{traj}} = .9$, $\omega_{\text{traj}} = \pi/1000$).

5.4 Case 2: 2×2 Process

The 2×2 process to be considered is as follows

$$H(s) = e^{-s} \begin{bmatrix} \frac{-2}{10s+1} & \frac{1.5}{s+1} \\ \frac{1.5}{s+1} & \frac{-2}{10s+1} \end{bmatrix}. \quad (58)$$

A periodic T_s second zero order hold (ZOH) is applied to (57) in order to derive $H(z)$. Feasibility and optimal solutions for $H_{hp}(z)$ were computed in the same manner as discussed in Section 5.3.

An optimal solution was computed for the two possible input-output pairings in which the diagonal pairing 1-1/2-2 ($\Gamma = -I_{2 \times 2}$) results in the smallest $R(\omega_o, a_{hp})$. The results are summarized in Table 2. The

pairing in Bao et al. (2007) was 1-1/2-2 and for simplicity of discussion we assume the same $H_{hp}(z)$ was used to render $H_p(z)$ strictly positive real. The key improvement is that the simplex search optimization step resulted in a significant improvement in tracking performance in which the cost $J(2686)$ was reduced from 7,458 to 1,307 and is evident in the step responses depicted in Fig. 6. The second pairing 1-2/2-1 ($\Gamma = I_{2 \times 2}$) performed as poorly as indicated in Bao et al. (2007).

Table 2: Summary of Controller Results for 2×2 Process ($T_s = .2$ seconds, $\zeta_{\text{traj}} = .9$, $\omega_{\text{traj}} = \pi/75$).

Pairing	$R(\omega_o, a_{hp})$	a_o	ω_o	$J(2500)$	k_P	k_I
1-1/2-2	-.0868	1.649	.1432	1,177	1.66	.10
Bao(2007)	-.0868	1.649	.1432	6,730	.02	.02

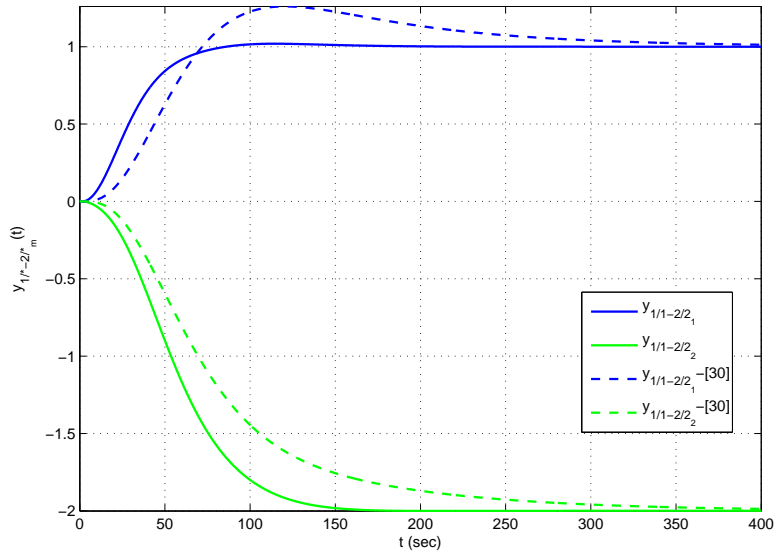


Figure 6: Case 2: closed loop step response $r(t) = [1, -2]$ ($T_s = .2$ seconds, $\zeta_{\text{traj}} = .9$, $\omega_{\text{traj}} = \pi/75$).

6 Conclusions

This paper provided relationships between various energy-based properties for LTI systems. Since an entire survey could be written on classical results from passivity and dissipativity theory, the current paper focuses instead on results that (1) demonstrate relationships between frameworks and (2) provide new insight into energy-based theory. The fundamental connections between definitions of passive and positive real, and their stability results, were summarized in the Venn Diagram in Fig. 1. These connections are valid for continuous time or discrete time LTI systems. The connection between the two was demonstrated using dissipativity theory. While the notions of passivity or positive realness may be restrictive for some application areas, dissipativity is a more general concept that can be applied to a large class of systems, but it may be difficult to apply without a previously defined notion of energy. The paper also surveys the energy-based frameworks of passivity index theory and conic systems theory. As was shown, for systems with a state space representation, the frameworks are identical. Either can be used as a framework that is more general than passivity theory but more easily applied than dissipativity theory.

Lastly, the Case Study section included the application of some of these results to a passivity-based input-output pairing method for *MIMO* control. The method was covered in detail by working through some

of the application details and then covering two case studies from a previous paper. Improved performance, in the sense of reduced tracking error, was demonstrated in the examples.

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