Passivity Analysis and Passivation of Feedback Systems Using Passivity Indices

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***Abstract*—Passivity indices are used to measure the excess or shortage of passivity. While most of the work in the literature focuses on stability conditions for interconnected systems using passivity indices, here we focus on passivity and passivation of the feedback interconnection of two input feed-forward output-feedback (IF-OF) passive systems. The conditions are given to determine passivity indices in feedback interconnected systems. The results can be viewed as the extension of the well-known compositional property of passivity. We also consider the passivation problem which can be used to render a non-passive plant passive using a feedback interconnected passive controller. The passivity indices of the passivated system are also determined. The results derived do not require linearity of the systems as it is commonly assumed in the literature.**

I. Introduction

The notion of dissipativity, and its special case of passivity, are characterizations of system input and output behavior based on a generalized notion of energy. The ideas of passivity first emerged from the phenomenon of dissipation of energy across passive components in the circuit theory field [1], [2]. Passive systems can viewed as systems that do not generate energy, but only store or release the energy which was provided. Dissipativity was introduced and formalized by [3], and it is a generalized notion of passivity. Dissipativity and passivity can be applied to the analysis of chemical, mechanical, electromechanical and electrical systems where the definition of energy has both clear physical meaning and concrete mathematical representation. Over the past decades, dissipativity and passivity have received constantly high attention by the systems and control community with plenty of applications in theory and practice [4], [5], [6]. Recent summaries of dissipativity and passivity theory can be found in [7].

Due to the fact that Lyapunov functions can serve as the candidate energy functions in dissipative and passive systems, dissipativity and passivity theory act as powerful tools for analyzing a large class of systems behavior by utilizing Lyapunov function techniques [5], [8] . Other than stability, the significant benefit of passivity is that when two passive systems are interconnected in parallel or in feedback, the overall system is still passive. Thus passivity is preserved when large-scale systems are combined from components of passive subsystems. Recent results [9], [10] showed its power in the compositional design of cyber-physical systems.

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In order to measure the excess or shortage of passivity, passivity indices [5], [4], [6] were introduced. The indices can be used to render the system passive using feedback and feed-forward gains, and describe the performance of passive systems. Various stability conditions based on passivity indices are derived to assess the stability of interconnected systems [5]. In addition to characterizing stability, passivity indices can also be used in passivity analysis and passivation of interconnected systems. [5] and [11] gave the passivity indices for the closed-loop system when the subsystems are passive. [12], [13] showed the passivity condition for the feedback interconnected linear systems. [14] considered the schemes of altering the passivity indices of a given system using constant feedback and feed-forward interconnection matrices. A passivity measure of system interconnections in series using passivity indices is reported in [15].

The present paper is motivated by the compositional property of passivity and the results in [12], [13], [14]. Although it is well known that the negative feedback interconnection of two passive systems is still passive, the quantitative characterization of passivity for the closed-loop system has not been addressed previously. We propose a measure of passivity indices for the negative feedback interconnection of two input feed-forward output-feedback (IF-OF) passive systems (See Fig. 2). The two systems need to be neither passive nor linear in general. It is shown that passivity, with respect to the full input and output, may be reinforced under feedback interconnection. Then the problem of partial passivation is considered. We present the conditions under which passivity for a desired input and output pair can be guaranteed. The conditions are identical to the conditions in [12], [13] but the linearity assumption is no longer needed. Moreover, a measure of passivity indices for the passivated system is provided in the end.

The paper is organized as follows. In Section II, we introduce some background on dissipativity/passivity theory and passivity indices. The previous work on passivity analysis and passivation using passivity indices is also presented. Section III considers two problems of passivity analysis and passivation using passivity indices for feedback interconnected systems. We first derive the conditions to measure the passivity indices for the negative feedback interconnection of two input feed-forward output-feedback (IF-OF) passive systems. Then partial passivation and its measure of passivity indices are discussed. Two examples are discussed in Section IV. The conclusion is provided in Section V.

II. Preliminaries and Background

A. Dissipativity and Passivity

We first introduce some basic concepts in passive and dissipative system theory. Consider the following nonlinear system G, which is driven by an input $u(t)$ and has an output $y(t)$

$$G : \left\{ \begin{array}{l} \dot{x}(t) = f\left(x(t), u(t)\right) \\ y\left(t\right) = h\left(x(t), u(t)\right) \end{array} \right. \tag{1}$$

where $x\left(t\right) \in \mathcal{X} \subset \mathbb{R}^n$, $u\left(t\right) \in \mathcal{U} \subset \mathbb{R}^m$ and $y\left(t\right) \in \mathcal{Y} \subset \mathbb{R}^p$ are the state, input and output of the system respectively and $\mathcal{X}, \mathcal{U}$ and $\mathcal{Y}$ are the state, input and output spaces, respectively.

The definition of a dissipative system is based on a storage function (energy stored in the system) and a supply function (externally supplied energy). The basic idea behind dissipativity is that the increase of the stored energy is bounded by the supplied energy.

Definition 1. [7] System G is said to be dissipative with respect to the supply rate $\omega(x, u, y)$, if there exists a positive semi-definite storage function $V(x)$ such that the (integral) dissipation inequality

$$V(x(t_1)) - V(x(t_0)) \quad \leq \quad \int_{t_0}^{t_1} \omega(x(t), u(t), y(t))dt \tag{2}$$

is satisfied for all t_0, t_1 with $t_0 \leq t_1$ and all solutions $x = x(t), u = u(t), y = y(t)$, $t \in [t_0, t_1]$. If the storage function is differentiable, then the integral dissipation inequality (2) can be rewritten as

$$\dot{V}(x(t)) \leq \omega(x(t), u(t), y(t)), \forall t \tag{3}$$

As a special case of dissipativity, *QSR-dissipativity* was proposed in [16] . In this case the supply rate is defined as

$$\omega(u, y) = y^T Q y + 2y^T S u + u^T R u \tag{4}$$

where Q, S and R are matrices with proper dimensions. The relation between QSR-dissipativity and $\mathcal{L}_2$ stability has been shown [16].

Theorem 1. *[16] If System G is QSR-dissipative with $Q < 0$, then it is $\mathcal{L}_2$ stable.*

Definition 2. [4] System G with $m = p$ is passive if there exists a positive semi-definite storage function $V(x)$ such that the following inequality holds for all $t_1, t_2 \in [0, \infty)$ such that

$$V(x(t_2)) - V(x(t_1)) \leq \int_{t_1}^{t_2} u^T y dt \tag{5}$$

If the storage function is smooth, then the integral dissipation inequality (5) can be rewritten as $\dot{V}\left(x\left(t\right)\right) \leq u^T y$.

Note that passivity is also a special case of dissipativity, with supply rate $\omega = u^T y$. One useful property of passive systems in systems theory is the fact that the parallel interconnection and the negative feedback interconnection of two passive systems is again a passive system. Consider the parallel interconnection (Fig. 1) and negative feedback interconnection (Fig. 2) of two passive systems. The following theorems show that passivity is preserved under parallel and negative feedback interconnections.

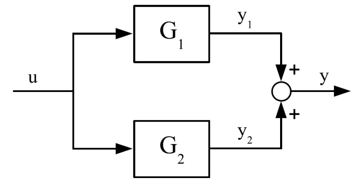

Fig. 1. The parallel interconnection of two systems

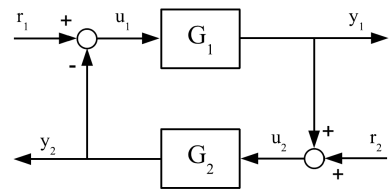

Fig. 2. The negative feedback interconnection of two systems

Theorem 2. *[4] The parallel interconnection of two passive systems (Fig. 1) is passive, with respect to the input u and the output y.*

Theorem 3. *[4] The negative feedback interconnection of two passive systems (Fig. 2) is passive, with respect to the input $\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ and the output $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$.*

It is noted that such compositional property is often used in large-scale network design of nonlinear interconnected systems and related topics [17], [18]. The advantage of using this property is that one can always guarantee passivity of the interconnected passive systems and thus stability of the whole system.

B. Passivity Indices

Passivity indices are used to characterize how passive a system is.

Definition 3. [5] [6] A system is input feed-forward output feedback passive (IF-OFP) if it is dissipative with respect to the supply rate

$$\omega(u, y) = u^T y - \nu u^T u - \rho y^T y, \; \forall t \geq 0,$$

for some $\rho, \nu \in \mathbb{R}$.

Definition 3 is often used in passivity analysis, passivation and passivity-based control [19], [20], [21], [22]. We can denoted an IF-OFP system by IF-OFP$(\nu, \rho)^m$. When $\rho = \nu = 0$ an IF-OFP system is simply a passive system.

Based on Definition 3, one can further have the definitions of input feed-forward (strictly) passive, output feedback (strictly) passive and very strictly passive.

1) When $\rho = 0$ and $\nu \neq 0$, the system is said to be input feed-forward passive (IFP), denoted as IFP(ν). when in

addition $\nu > 0$, the system is input feed-forward strictly passive (ISP).

2) When $\rho \neq 0$ and $\nu = 0$, the system is said to be output feedback passive (OFP), denoted as OFP(ρ). When in addition $\rho > 0$, the system is output feedback strictly passive (OSP).
3) When $\rho > 0$ and $\nu > 0$, the system is said to be very strictly passive (VSP).

Note that positive ρ or ν means that the system has an excess of passivity, such as ISP, OSP and VSP. If either ρ or ν is negative, the system has a shortage of passivity and thus is non-passive.

The valid domain of ρ and ν has been proposed in [23], [24].

Lemma 1. *[24] The domain of ρ and ν in IF-OFP system is $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\rho, \nu \in \mathbb{R} | \rho\nu < \frac{1}{4}\}$ and $\Omega_2 = \{\rho, \nu \in \mathbb{R} | \rho\nu = \frac{1}{4}; \rho > 0\}$.*

With the help of passivity indices, $\mathcal{L}_2$ stability conditions for the interconnected system can be derived.

Theorem 4. *[5] Consider the feedback interconnection of Fig. 2 and suppose each feedback component satisfies the inequality*

$$\dot{V}_i \leq u_i^T y_i - \nu_i u_i^T u_i - \rho_i y_i^T y_i,$$

for some storage function $V_i(x_i)$ where $i = 1, 2$. Then, the closed-loop map from $\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$ to $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ is finite gain $\mathcal{L}_2$ stable if $\nu_1 + \rho_2 > 0$ and $\nu_2 + \rho_1 > 0$.

In this paper, we adopt Definition 3 and assume that ρ and ν are in the domain unless otherwise noted.

C. Passivity Analysis and Passivation using Passivity Indices

In addition to stability conditions, passivity indices can also be used in the problem of passivity analysis and passivation. For nonlinear systems, [5] and [11] gave the passivity indices for the closed-loop system (as in Fig. 2) when G_1 and G_2 are either OSP or ISP.

For linear systems, [12], [13] showed when the closed-loop system (Fig. 2, assuming $r_2 = 0$) is passive with respect to the input r_1 and output y_1 using a slightly different definition of passivity indices. By assuming G_1 and G_2 are both linear systems with (ν_1, ρ_2) and (ν_2, ρ_2) respectively, a sufficient passivity condition on the closed-loop system is given in Theorem 5.

Theorem 5. *[12] Consider the feedback interconnection (Fig. 2) when G_1 is a linear system with a shortage of OFP, i.e. $\rho_1 < 0$ and $\nu_1 \geq 0$. G_2 is also linear and passive with (ν_2, ρ_2). Then this interconnection is passive if $\rho_1 + \nu_2 \geq 0$.*

Another work related to passivity-based design using passivity indices appeared in [14]. It considered the schemes of altering the passivity indices of a given system using constant feedback and feed-forward interconnection matrices. It assumed that G_1 is a diagonal transfer function matrix and G_2 (denoted as H_ρ in [14]) is a constant output feedback matrix. As in [12], [13], the passivity is defined with respect to the input r_1 and output y_1, assuming $r_2 = 0$.

The recent work on the passivity analysis for parallel and series interconnections using passivity indices is reported in [15], [14].

In the present paper, passivity conditions on closed-loop passivity are derived. The results are similar to Theorem 5, but here we do not assume that the systems are linear. Also, Definition 3 is adopted. Moreover, we mainly focus on the feedback interconnection of two IF-OF nonlinear systems and on finding passivity indices for the interconnected system.

III. Main Results

We consider two problems in feedback interconnected systems. The first problem is to determine the passivity indices of the interconnected system if the passivity indices of each individual systems are known. Although it is well known that the negative feedback interconnection of two passive systems is still passive, the quantitative characterization of passivity for the closed-loop system has not been addressed. The interconnection considered here is the negative feedback interconnection of two input feed-forward output-feedback (IF-OF) passive systems, as shown in Fig. 3. It is assumed that the passivity indices of the two systems are known, denoted as (ν_p, ρ_p) for the system G_p and (ν_c, ρ_c) for the system G_c.

The second problem considered is the "partial passivation" problem, which refers to finding the conditions for which the closed-loop system is passive with respect to the input w_1 and output y_p when $w_2 = 0$. The condition can be used to passivate a non-passive plant G_p using a passive controller G_c. This problem has been considered in [12] for linear systems. Here we consider nonlinear systems. Moreover, the passivity indices of the passivated system are also given.

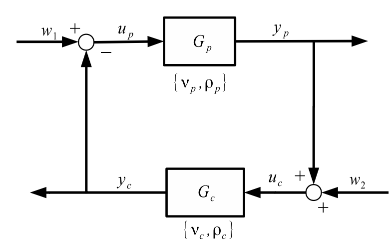

Fig. 3. Feedback connection of two IF-OF systems

A. Measure of Passivity Indices for Feedback Interconnected Systems

We first present the result relating the interconnected system to QSR-dissipative systems.

Lemma 2. *Consider the feedback interconnection of two IF-OF systems with the passivity indices ν_p, ρ_p and ν_c, ρ_c respectively, The interconnected system with the input $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ and output $y = \begin{bmatrix} y_p \\ y_c \end{bmatrix}$ is QSR-dissipative (See*

Fig. 3) with

$$\dot{V} \le y^T Q y + 2w^T S y + w^T R w$$

where $Q = \begin{bmatrix} -(\rho_p+\nu_c)I & 0I \\ 0I & -(\nu_p+\rho_c)I \end{bmatrix}$, $S = \begin{bmatrix} \frac{1}{2}I & \nu_p I \\ -\nu_c I & \frac{1}{2}I \end{bmatrix}$ *and* $R = \begin{bmatrix} -\nu_p I & 0I \\ 0I & -\nu_c I \end{bmatrix}$.

Proof: Since G_p and G_c are IF-OF systems with the passivity indices ν_p, ρ_p, ν_c and ρ_c, there exist V_p and V_c such that

$$\dot{V}_p \le u_p^T y_p - \nu_p u_p^T u_p - \rho_p y_p^T y_p$$

and

$$\dot{V}_c \le u_c^T y_c - \nu_c u_c^T u_c - \rho_c y_c^T y_c.$$

Then we have

$$\begin{aligned} \dot{V} &= \dot{V}_p + \dot{V}_c \\ &\le u_p^T y_p - \nu_p u_p^T u_p - \rho_p y_p^T y_p + u_c^T y_c \\ &\quad -\nu_c u_c^T u_c - \rho_c y_c^T y_c. \end{aligned} \tag{6}$$

Consider that $u_p = w_1 - y_c$ and $u_c = y_p + w_2$. (6) can be rewritten as

$$\begin{aligned} \dot{V} &\le w_1^T y_p + w_2^T y_c + 2\nu_p w_1 y_c - 2\nu_c w_2^T y_p \\ &\quad -\nu_p w_1^T w_1 - \nu_c w_2^T w_2 - (\nu_p+\rho_c) y_c^T y_c \\ &\quad -(\rho_p+\nu_c) y_p^T y_p \\ &= y^T \begin{bmatrix} -(\rho_p+\nu_c)I & 0I \\ 0I & -(\nu_p+\rho_c)I \end{bmatrix} y \\ &\quad + w^T \begin{bmatrix} -\nu_p I & 0I \\ 0I & -\nu_c I \end{bmatrix} w + 2w^T \begin{bmatrix} \frac{1}{2}I & \nu_p I \\ -\nu_c I & \frac{1}{2}I \end{bmatrix} y \\ &= y^T Q y + 2w^T S y + w^T R w \end{aligned} \tag{7}$$

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Remark 1. From Theorem 1, the interconnected system is $\mathcal{L}_2$ stable if $Q < 0$. For this particular system, a sufficient condition for $Q < 0$ is $\nu_p+\rho_c > 0$ and $\nu_c+\rho_p > 0$. Therefore, we can recover the stability condition stated in Theorem 4.

Remark 2. Although we have the stability conditions for the interconnected system, it is not clear how the passivity indices of the closed-loop system can be characterized.

Theorem 6 shows how to determine the passivity indices of the closed-loop system.

Theorem 6. *Consider the feedback interconnected system in Fig. 3. Suppose the passivity indices ν_p, ρ_p, ν_c and ρ_c are known. If we choose ϵ and δ such that*

$$\begin{cases} \epsilon < \min\{\nu_p, \nu_c\} \\ \delta \le \min\left\{\rho_c - \frac{\epsilon\nu_p}{\nu_p-\epsilon}, \rho_p - \frac{\epsilon\nu_c}{\nu_c-\epsilon}\right\} \end{cases}, \tag{8}$$

then the closed-loop system has passivity indices ϵ and δ satisfying

$$\dot{V} \le w^T y - \epsilon w^T w - \delta y^T y \tag{9}$$

where $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ *and* $y = \begin{bmatrix} y_p \\ y_c \end{bmatrix}$.

Proof: From (6) , we have

$$\begin{aligned} \dot{V} &= \dot{V}_p + \dot{V}_c \le w_1^T y_p + w_2^T y_c + 2\nu_p w_1 y_c - 2\nu_c w_2^T y_p \\ &\quad -\nu_p w_1^T w_1 - \nu_c w_2^T w_2 - (\nu_p+\rho_c) y_c^T y_c \\ &\quad -(\rho_p+\nu_c) y_p^T y_p \\ &= w^T y - \begin{bmatrix} w_1^T & y_c^T \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p+\rho_c \end{bmatrix} \begin{bmatrix} w_1 \\ y_c \end{bmatrix} \\ &\quad - \begin{bmatrix} w_2^T & y_p^T \end{bmatrix} \begin{bmatrix} \nu_c & \nu_c \\ \nu_c & \rho_p+\nu_c \end{bmatrix} \begin{bmatrix} w_2 \\ y_p \end{bmatrix}. \end{aligned} \tag{10}$$

Since ϵ and δ are chosen such that (8) is satisfied, (11) holds for the chosen ϵ and δ.

$$\begin{cases} \epsilon \le \nu_p \\ \epsilon \le \nu_c \\ (\nu_p-\epsilon)(\nu_p+\rho_c-\delta) \ge \nu_p^2 \\ (\nu_c-\epsilon)(\rho_p+\nu_c-\delta) \ge \nu_c^2 \\ \nu_p+\rho_c-\delta \ge 0 \\ \rho_p+\nu_c-\delta \ge 0 \end{cases}. \tag{11}$$

(11) further implies that the matrices $M = \begin{bmatrix} \nu_p-\epsilon & -\nu_p \\ -\nu_p & \nu_p+\rho_c-\delta \end{bmatrix}$ and $N = \begin{bmatrix} \nu_c-\epsilon & \nu_c \\ \nu_c & \rho_p+\nu_c-\delta \end{bmatrix}$ are positive semi-definite. Therefore, we have

$$\begin{bmatrix} w_1^T & y_c^T \end{bmatrix} M \begin{bmatrix} w_1 \\ y_c \end{bmatrix} + \begin{bmatrix} w_2^T & y_p^T \end{bmatrix} N \begin{bmatrix} w_2 \\ y_p \end{bmatrix} \ge 0 \tag{12}$$

for $\forall\, w_1$, w_2, y_c and y_p. After re-arranging the terms in (12), one can obtain

$$\begin{aligned} &-\begin{bmatrix} w_1^T & w_2^T \end{bmatrix} E \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} - \begin{bmatrix} y_p^T & y_c^T \end{bmatrix} \Delta \begin{bmatrix} y_p \\ y_c \end{bmatrix} \ge \\ &-\begin{bmatrix} w_1^T & y_c^T \end{bmatrix} O \begin{bmatrix} w_1 \\ y_c \end{bmatrix} - \begin{bmatrix} w_2^T & y_p^T \end{bmatrix} P \begin{bmatrix} w_2 \\ y_p \end{bmatrix}. \end{aligned} \tag{13}$$

where $E = \begin{bmatrix} \epsilon & 0 \\ 0 & \epsilon \end{bmatrix}$, $\Delta = \begin{bmatrix} \delta & 0 \\ 0 & \delta \end{bmatrix}$, $O = \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p+\rho_c \end{bmatrix}$ and $P = \begin{bmatrix} \nu_c & \nu_c \\ \nu_c & \rho_p+\nu_c \end{bmatrix}$.

From (13) and (10), we can finally show that

$$\begin{aligned} \dot{V} &\le w^T y - \begin{bmatrix} w_1^T & y_c^T \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p+\rho_c \end{bmatrix} \begin{bmatrix} w_1 \\ y_c \end{bmatrix} \\ &\quad - \begin{bmatrix} w_2^T & y_p^T \end{bmatrix} \begin{bmatrix} \nu_c & \nu_c \\ \nu_c & \rho_p+\nu_c \end{bmatrix} \begin{bmatrix} w_2 \\ y_p \end{bmatrix} \\ &\le w^T y - \epsilon w^T w - \delta y^T y \end{aligned} \tag{14}$$

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Remark 3. (8) can be used to obtain an estimate of the passivity indices for the closed-loop system. The condition implies that the interconnected system may have smaller passivity indices than each subsystems. Note that the passivity considered here is with respect to the input $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ and output $y = \begin{bmatrix} y_p \\ y_c \end{bmatrix}$.

B. Partial Passivation

Based on Theorem 8, passivity with respect to the full input and output (i.e. input w and output y), may not be guaranteed to be reinforced under feedback interconnection. However, by selecting different inputs and outputs the corresponding passivity may change accordingly. As in Fig. 3, if our goal is to passivate a non-passive plant G_p using a passive controller G_c we only consider whether the closed-loop system is passive with the input w_1 and output y_p by assuming w_2 is zero. Theorem 7 shows that it is possible to guarantee passivity for the desired input and output although passivity for full input and output may not hold.

Theorem 7. *Assume $w_2 = 0$. The closed-loop system is passive with respect to the input w_1 and output y_p if the passivity indices satisfy the conditions*

$$\begin{aligned} \nu_p &\geq 0 & (15)\\ \rho_c &\geq 0 & (16)\\ \rho_p + \nu_c &\geq 0. & (17)\end{aligned}$$

Proof: If $w_2 = 0$, (6) becomes

$$\begin{aligned} \dot{V} &\leq w_1^T y_p + 2\nu_p w_1 y_c - \nu_p w_1^T w_1 \\ &\quad - (\nu_p + \rho_c)\, y_c^T y_c - (\rho_p + \nu_c)\, y_p^T y_p \\ &\leq w_1^T y_p - \begin{bmatrix} w_1^T & y_c^T \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c \end{bmatrix} \begin{bmatrix} w_1 \\ y_c \end{bmatrix} \\ &\quad - (\rho_p + \nu_c)\, y_p^T y_p \end{aligned} \tag{18}$$

Since we have $\nu_p \geq 0$, $\rho_c \geq 0$ and $\rho_p + \nu_c \geq 0$, it can be shown that

$$\begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c \end{bmatrix} \geq 0 \tag{19}$$

$$\rho_p + \nu_c \geq 0. \tag{20}$$

Therefore, we can conclude that

$$\begin{aligned} \dot{V} &\leq w_1^T y_p - \begin{bmatrix} w_1^T & y_c^T \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c \end{bmatrix} \begin{bmatrix} w_1 \\ y_c \end{bmatrix} \\ &\quad - (\rho_p + \nu_c)\, y_p^T y_p \\ &\leq w_1^T y_p. \end{aligned} \tag{21}$$

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Remark 4. When the plant G_p is non-passive (i.e. $\rho_p < 0$), the closed-loop system can be rendered passive by choosing a passive controller G_c with $\rho_c \geq 0$ and $\nu_c \geq -\rho_p$. It is noted that the conditions are identical to the conditions in Theorem 5 but here we do not need to assume linearity of the systems.

Remark 5. (21) shows that closed-loop system is OSP with the OFP index $\rho_p + \nu_c$. We can recover the conditions in [5] where G_p and G_c are assumed to be OSP and ISP, respectively. The conditions (15)-(17) are similar to the conditions in Theorem 4 but are more restrictive since OSP is more conservative than $\mathcal{L}_2$ stable.

We can also obtain an estimate of the passivity indices for the passivated closed-loop system, as shown in Theorem 8.

Theorem 8. *Suppose that the conditions (15)-(17) are satisfied and $\nu_p + \rho_c > 0$. If we choose ϵ and δ such that*

$$\begin{cases} \epsilon \leq \frac{\nu_p \rho_c}{\nu_p + \rho_c} \\ \delta \leq \nu_c + \rho_p \end{cases}, \tag{22}$$

the closed-loop system has the passivity indices ϵ and δ satisfying

$$\dot{V} \leq w_1^T y_p - \epsilon w_1^T w_1 - \delta y_p^T y_p \tag{23}$$

Proof: If the condition (15)-(17), (22) and $\nu_p + \rho_c > 0$ are satisfied, we have

$$\begin{bmatrix} \nu_p - \epsilon & -\nu_p \\ -\nu_p & \nu_p + \rho_c \end{bmatrix} \geq 0 \tag{24}$$

$$\nu_c + \rho_p - \delta \geq 0. \tag{25}$$

Then it implies $\begin{bmatrix} w_1^T & y_c^T \end{bmatrix} \begin{bmatrix} \nu_p - \epsilon & -\nu_p \\ -\nu_p & \nu_p + \rho_c \end{bmatrix} \begin{bmatrix} w_1 \\ y_c \end{bmatrix} + (\rho_p + \nu_c - \delta)\, y_p^T y_p \geq 0$, which can be written as

$$\begin{aligned} -\begin{bmatrix} w_1^T & y_c^T \end{bmatrix} O \begin{bmatrix} w_1 \\ y_c \end{bmatrix} - (\rho_p + \nu_c)\, y_p^T y_p \\ \leq -\epsilon w_1^T w_1 - \delta y_p^T y_p \end{aligned} \tag{26}$$

where $O = \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c \end{bmatrix}$.

Since it is already known that

$$\begin{aligned} \dot{V} &\leq w_1^T y_p - \begin{bmatrix} w_1^T & y_c^T \end{bmatrix} \begin{bmatrix} \nu_p & -\nu_p \\ -\nu_p & \nu_p + \rho_c \end{bmatrix} \begin{bmatrix} w_1 \\ y_c \end{bmatrix} \\ &\quad - (\rho_p + \nu_c)\, y_p^T y_p \end{aligned} \tag{27}$$

we can conclude that

$$\dot{V} \leq w_1^T y_p - \epsilon w_1^T w_1 - \delta y_p^T y_p \tag{28}$$

holds for $\forall w_1$. ■

Remark 6. Because of the conditions (15)-(17) and $\nu_p + \rho_c > 0$, the passivity indices ϵ and δ are upper bounded by positive numbers. For feedback passivation, (22) provides a way to obtain the desired passivity indices of the closed-loop system by choosing a passive G_c with proper indices.

Remark 7. When G_c is a constant feedback with gain K_c where K_c is a positive definite matrix, we can show that G_c is IFP($\underline{\lambda}(K_c)$). Consider that G_p is OFP(ρ_p) with $\rho_p < 0$. If we choose K_c such that $\underline{\lambda}(K_c) + \rho_p \geq 0$, the closed-loop system is passive. Moreover, if $\underline{\lambda}(K_c) + \rho_p > 0$ Theorem 8 shows that the passivated system is OFP($\underline{\lambda}(K_c) + \rho_p$). The result here is consistent with the previous result [4] but can be applied even when G_c is a dynamical controller.

IV. Examples

In this section, two examples are presented to show how Theorem 7 and 8 can be applied for partial passivation. In order to be able to conveniently verify the results, we will focus on linear systems. Note that the methods can be applied to nonlinear systems in the same way.

Both examples consider a feedback system as in Fig. 3 with $w_2 = 0$. The first example assumes a plant $G_p = \frac{s+0.5}{s-0.1}$

and a controller $G_c = \frac{s+4}{s+2}$. It can be calculated (from the Nyquist plots [4]) that the plant is OFP with $\rho_p = -0.2$ and the controller is IFP with $\nu_c = 1$. From Theorem 8, the closed-loop system is OSP with the estimated output feedback passivity index $\delta = \rho_p + \nu_c = 0.8$. We can further verify that the closed-loop transfer function $\frac{s^2+2.5s+1}{2s^2+6.4s+1.8}$ is OSP with actual output feedback passivity index $\delta = 1.8$.

The second example assumes the plant is a 5th-order linear system with

$$A = \begin{bmatrix} -1.8 & 0.1 & 1.2 & 0 & 0 \\ 0.1 & -0.5 & 0 & -0.3 & 0 \\ 1.2 & 0 & -3 & -2 & 0.5 \\ 0 & -0.3 & -2 & -3 & 0.4 \\ 0 & 0 & 0.5 & 0.4 & -1 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 5 \\ 1 \end{bmatrix},$$

$$C = B^T, \ D = 0.2.$$

Here the controller is a 2nd-order system with

$$A = \begin{bmatrix} -2 & -1 \\ -3 & -5 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \ D = 1.$$

We can determine the passivity indices of G_p and G_c to be $(0.18, 0.02)$ and $(0.3, 0.5)$, respectively. From Theorem 8, the closed-loop system has passivity indices $(0.1324, 0.32)$. It can be verified by the KYP lemma that $(0.1324, 0.32)$ are the valid passivity indices for the closed-loop system. This example also shows that it is possible to increase the OFP of the plant by choosing a proper controller, as pointed out in Remark 6.

V. Conclusion

In this paper, we considered the problems in passivity analysis and passivation using passivity indices. A measure of passivity indices for two input feed-forward output-feedback (IF-OF) interconnected system was provided. We also presented conditions for partial passivation and discussed passivity indices for the passivated system. In contrast to previous results, we do not need to assume that the systems are linear.

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