OPTIMAL STABILIZATION OF DISCRETE EVENT SYSTEMS

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Abstract

We utilize a class of not necessarily finite state "logical" discrete event system (DES) models which can also model the costs for events to occur. Let P and A denote two such models. Suppose that P characterizes the valid behavior of a dynamical system and A represents certain design objectives which specify the allowable DES behavior which is "contained in" the valid behavior. An optimal control problem for P and A is how to choose the sequence of inputs to P so that the DES behavior lies in A (i.e., it is allowable) and so that a performance index defined in terms of the costs of the events is minimized. We provide two solutions to an optimal stabilization problem, i.e., how to find a sequence of inputs that results in an optimal state trajectory which cycles in a pre-specified set. The results here are based on those in [11, 13].

1. INTRODUCTION

Recently there have been investigations into the foundations of the optimal control of DES. Extensions to the theory of heuristic search in [11, 13] showed that the $A^*$ algorithm [4] can be adapted to provide a computationally efficient method to find solutions to several optimal control problems for DES. These results were applied to various manufacturing and planning system problems in [9–11]. In [12] the authors developed a heuristic search approach to find solutions to various near-optimal control problems for DES. The results in [11, 13] have been extended and applied to flexible manufacturing systems and other examples in [14]. In this paper we show that the approach in [14] can be extended to solve optimal stabilization problems.

2. AN OPTIMAL STABILIZATION PROBLEM FOR DISCRETE EVENT SYSTEMS

We consider DES that can be accurately modelled with

$$P=(X,Q,P,X_0)$$

where (i) $X$ is the possibly infinite set of plant states, (ii) $Q$ is the possibly infinite set of plant inputs, (iii) $P:Q+X$ is the plant transition function, (iv) $X*X+R^+$ is the event cost function, (v) $P_0$ is the initial plant state, and (vi) $X_0:C$ is the non-empty finite set of final states. $R^+$ denotes the set of positive reals and $R_+=[0,\infty)$.

The set

$$E(P)=\{(x_0)x\in X\times X:x'=\delta(q,x(1))\}$$

denotes the (possibly infinite) set of events for the DES P $(x_0)$ is a dummy state, and $(x_0)$ is a dummy event added for convenience. The event cost function $(x,x')$ is defined for all $(x,x')E(EP)$; it specifies the "cost" for each event (state transition) to occur and it is required that there exist a $\delta>0$ such that $\delta(x,x')>0$ for all $(x,x')E(EP)$. Finally, we require that for each $x\in X$, $\{(x,x':x)eQ\}$ is finite. The mathematical notation in this paper is as follows: Let Z be an arbitrary set. $Z^*$ denotes the set of all finite strings over Z including the empty string $\emptyset$. For any string $s^*$ such that $s^*=x^*y^*$, where $s$ is used to indicate that $t$ is a substring of $s$, i.e., $s^*=x^*y^*$. A (finite directed) cycle is a string $s^*Z^*$ such that $s^*=x^*z^*$ has the same first and last element $xeZ$. A string $s^*Z^*$ is cyclic if it contains a cycle (for $t,s,s^*,t$).

A string $s^*Z^*$ is called a state trajectory or state path of P if for all successive states $xx'e$, $x'=(x',x'')$, $x''(x',x'')$ is found by simply forming the pairs $(x',x''),(x'',x''')...$. An input sequence $s^*$ that produces a state trajectory $s^*$ is constructed by concatenating $eq$ for all $xeX$, $\delta(q,x)eX_2$ for all $qE(Q)$. Let $X_2:C$ then

$$\gamma_2(x_2)cX^+$$

denotes the set of all state trajectories $s=xx'...x'$ of P beginning with $x$ and ending with $x'eX$.

Then, for instance, $\gamma_2(x_2)cX$ denotes the set of all state trajectories of P that begin with the initial state $x_0$ and end in a cycle whose elements are all contained in $X_2$. A plant P is said to be $(x,X_2)$-stabilizable if there exists a sequence of inputs $s^*$ that produces an state trajectory $s^*$.

There are other ways to define $(x,X_2)$-stabilizability. For instance we could have said that P is $(x,X_2)$-stabilizable if there exists a sequence of inputs $s^*$ that produces an state trajectory $s=xx'...x'$.

Let $P=(X,Q,P,X_0)$ specify the valid behavior of the plant and $A=(X_0,\delta_0,\delta_0,\delta_0,\delta_0,\delta_0,\delta_0,\delta_0)$ specify the allowable behavior of A. For other studies of stability and stabilizability of logical DES see, for instance, [1, 7, 8, 15].

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To specify optimal control problems let the performance index be

$$J:Z^+R_+$$

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where the cost of a state trajectory is defined by

$$J(s) = \sum_{(x, x') \in E(A)} X(x, x')$$

for all $x \in X_a$ and $s \in \mathbb{N}(A, x, x_a)$. By definition, $J(s) = 0$ if and only if $x, x' \in X_a$. Let $A$ describe the allowable behavior for a plant $P$ such that $A[P]$ then we have:

The Optimal Stabilization Problem (OSP): Assume that $X_{af}$ is an invariant set for $A$ and that $A$ is $(x_0, X_{af})$-stabilizable where $X_{af} \subseteq X_a$. Find the $Q_s^*$ that drives $A$ along an optimal state trajectory $s^*$, i.e.,

$$s^* \in \mathbb{N}(A, x_0, X_{af}) \text{ such that } J(s^*) = \inf_i J(s) : s \in \mathbb{N}(A, x_0, X_{af}).$$

Here, we focus on the case where $X_{af}$ is composed of states that form a single cycle. It is not required that $X_{af}$ be an invariant set. Since we require that use $Q_s^*$ and $s \in X_a$, the solutions to the OSP will achieve not only optimal but also allowable DES behavior. There may, in general, be more than one optimal state trajectory, i.e., the solution to the OSP is not necessarily unique. The set of optimal state trajectories for $A$, beginning at state $x \in X_a$ and ending at $i$ in a cycle in $X_a$, where $X_a \subseteq X_a$, is denoted by

$$\mathbb{S}_{i}^*(A, x, x_a) \subseteq \mathbb{N}(A, x, x_a).$$

In this paper we are concerned with finding only one optimal state trajectory for the OSP and finding it in a computationally efficient manner.

3. SOLUTIONS TO THE OPTIMAL STABILIZATION PROBLEM

First, it is important to examine the constraints on the OSP to determine what type of solution is sought. Consider the case where $X_{af} = X_{af}$, $s^* \in \mathbb{S}_{i}^*(A, x_0, X_{af})$ is a solution to the OSP, and $s^* = s^*$ where $s^*$ is a cycle in $X_{af}$. Ultimately, the state of $P$ will cycle on $s^*$ at a cost of $J(s^*)$ per cycle; hence it is of primary importance to minimize $J(s^*)$. The minimization of $J(s^*)$ or even $J(s^*)$ is of secondary importance (this treatment of the definition of optimality is similar to that in [6] where the authors consider systems with an infinite time horizon). It is for these reasons that we choose to split the OSP into two problems: (i) finding an optimal cost cycle of all those in $X_{af}$, and (ii) finding an optimal cost path from $x_0$ to that cycle. We use the following approach:

1. Pick some optimal cost cycle $s^*_c$ in $X_{af}$.
2. Let $X_{af} = \{x \in \mathbb{S}_{i}^*(A, x, x_a) \}.
3. Assume that $A$ is $(x_0, X_{af})$-stabilizable and use the adapted $A^*$ algorithm in [14] to find an optimal path from $x_0$ to $X_{af}$ and name it $s^*_p$.
4. Combine $s^*_c$ and $s^*_p$ to obtain a solution to the OSP.

Note also that some desired cycle may be known for some DES applications then Steps (1) and (2) can be avoided. In this paper we briefly describe two solutions to the OSP which both rely on the results in [14].

#### kth Shortest Paths Methods

First, notice that several kth shortest paths methods [3] can be adapted to find the cycle $s^*_c$ in $X_{af}$. For instance, Dantzig's algorithm will not escape $X_{af}$ since it is invariant and hence will result in a shortest cycle $s^*_c$ in $X_{af}$ (one that minimizes the sum of the costs along the arcs in the cycle traversing it just once).

Dantzig's algorithm has complexity $O(\|X_{af}\|^3)$ where $X_{af}$ is normally relatively small. From the results in [14] Step (3) will have quadratic complexity in the worst case so via kth shortest path methods the overall complexity is polynomial. It is for this reason that using kth shortest path methods in conjunction with the $A^*$ approach developed in [14] provides a practical solution procedure for the OSP.

#### Minimum Ratio Methods

Next we show how a minimum ratio method [3] can be used in conjunction with $A^*$ to find a solution to an OSP. Assume that for each $x \in X_{af}$ there exists $s_x \in \mathbb{S}_{i}^*(A, x, x_a)$ for all $x \in X_{af}$. Let $m = \{x \in X_a \times x \in X_{af} \}$. Use Karp's algorithm (complexity $O(m \|X_{af}\|^2)$) [3, 5] to find the minimum average cost cycle in $X_{af}$ and name it $s_c^*$. (Karp's algorithm is also used in [2] to compute eigenvalues and critical circuits.) Let $X_{af}(s_0, s_c^*)$ and use $A^*$ to find an optimal state trajectory to $X_{af}$. The result is a solution to an OSP with polynomial complexity.

Note that the meaning of "optimal" with our two solution approaches is different (i.e., $X_{af}$ may be different depending on which approach is used). For both cases there are many application areas. For instance, for the kth shortest path method the resulting $s_c^*$ could represent a processing cycle in a manufacturing system that takes a minimum number of steps while $s_c^*$ could represent the steps that must be taken to reach the processing cycle so that resource consumption is minimized. For the minimum ratio method the resulting $s_c^*$ could represent the processing cycle that is completed in minimum average time and $s_c^*$ could represent the steps taken to reach $s_c^*$ in a minimum amount of time.

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