Z. Gao and P. J. Antsaklis, "Reconfigurable Control System Design via Perfect Model-Following," Proceedings of the 1991 AIAA Guidance, Navigation and Control Conference, pp 239-246, New Orleans, LA, August 12-14, 1991.

AIAA-91-2625-CP

RECONFIGURABLE CONTROL SYSTEMS DESIGN VIA PERFECT MODEL-FOLLOWING

Zhiqiang Gao*
Cleveland State University
Cleveland, Ohio

Panos J. Antsaklis**
University of Notre Dame
Notre Dame, Indiana

Abstract

In this paper, a Model-Following approach is used to design Reconfigurable Control Systems. The conventional state-space linear model-following approach to control is first reexamined with emphasis on the conditions for Perfect Model-Following(PMF) and its applications to the Reconfigurable Control Systems design. New frequency domain necessary and sufficient conditions for the PMF are then obtained and they are used to gain insight into the selection of the reference model and to develop a new design approach by extending the PMF approach. This novel design approach yield fewer constraints on the reference model than before, and provide much greater flexibility in specifying the state trajectories of the impaired system.

I. Introduction

The linear model-following (LMF) approach in control system design has been studied by various researchers and this subject can be found in many text books in control, see for example, [Landau79]. The objective of such systems is to make the trajectories of the output of a physical plant close to that of a reference model, which exhibits desired behavior. The design process of such systems is straightforward and the resulting control systems possess simple structures with only the constant gains to be implemented in the feedback and feedforward path. This type of control systems has been widely used and it is the foundation of the well-known model-reference adaptive control systems. Note that the LMF approach has been mainly studied in the state-space domain [Erzberger68, Chen73, Landau79].

In the context of reconfigurable control systems (RCS), the idea of controlling the impaired system so that it is "close", in some sense, to the nominal system, has been explored in numerous papers [Caglayan88, Huber84, Ostroff85, Rattan85, Gao90,90a]. The RCS are control systems that possess the ability to accommodate system failures automatically based upon a-priori assumed conditions. The research in this area is largely motivated by the control problems encountered in the aircraft control system design. In that case, the ideal goal is to achieve the so called "fault-tolerant", or, "self-repairing" capability in the flight control systems, so that the unanticipated failures in the system can be accommodated and the airplane can be, at least, landed safely whenever possible. Due to the time constraints in many failure scenarios, the control law redesign process must be automated and the algorithms used should be as numerically efficient as possible.

A well-known approach in RCS is the Pseudo-Inverse Method (PIM), which has been used quite successfully in flight control simulations [Caglayan88, Huber84, Ostroff85, Rattan85]. The idea behind this method is to adjust the constant feedback gain, assuming such gain is used in the nominal system, so that the reconfigured system approximates the nominal system in some sense. The PIM is a typical pragmatic approach; it is attractive because of its simplicity in computation and implementation. A measure of closeness between the systems before and after the failure is the Frobenius norm of the difference in closed-loop 'A' matrices. It was described in [Gao89,90a,90b] that by minimizing this norm, the bound on the variations of the closed-loop eigenvalues due to the failures is minimized. Note that a drawback of this method is that the stability of the impaired system is not guaranteed and this may lead to undesirable behavior in certain failure scenarios. A modified version of PIM (MPIM) was proposed to address the stability issue, by which the difference of the closed-loop 'A' matrices is minimized subject to the stability constraints [Gao90a, 91].

There is a link between the design objectives of the LMF and the PIM in terms of making one system, the plant or the impaired system, imitate the reference model (LMF) or the nominal system (PIM). The difference in the two approaches is that in LMF the plant approximates the reference model in terms of output trajectory, while in the PIM and MPIM the impaired system imitates the nominal system in terms of the closeness of the closed-loop 'A' matrices in their state space model. It is shown in this paper that the PIM is only a special case of the LMF.

A key problem in the LMF control system design is whether the plant can follow the reference model exactly, which is referred to as perfect model-following (PMF). PMF is desirable since it enables us to completely specify the behavior of a system. Without achieving PMF, the LMF approach can not specify how close the trajectory of the plant to that of the reference model and this arbitrariness may not be acceptable in certain control applications such as the RCS. On the other hand, in conventional LMF, the conditions for PMF put severe constraints on the reference model and therefore make it sometimes impractical to obtain the PMF.

In this paper, a new frequency domain approach is applied to analyze the LMF control system. In particular, necessary and sufficient conditions for the PMF are derived, which make the selection of the reference model simple and intuitive. Furthermore, based on the newly developed conditions of the PMF in frequency domain, a new design approach is developed to achieve the PMF with much fewer constraints on the reference model and the plant; it utilized the dynamic compensators instead of the static compensators which were used to achieve the PMF. More specifically in Section 2, the standard state-space Linear Model-Following approaches are discussed and the conditions for PMF are analyzed. In Section 3, a new design approach is developed to achieve the PMF with fewer constraints on the reference model. This new approach is shown to provide better performance for the reconfigured system. Illustration examples are included. Finally concluding remarks are given.

II. The Standard Linear Model Following Methods

LMF is a state-space design methodology by which a control system is designed to make the output of the plant follow the output of a model system with desired behavior. In this approach the design objectives are incorporated into the reference model and the feedback and feedforward controllers are used, which are usually of zero order. By using a reference model to specify the design objectives, a difficulty in control system design is avoided, namely that the design specifications must be expressed directly in terms of the controller parameters. As in any control design approaches, there are limits in the attainable control specifications because of physical limitations and the allowable complexity of dynamic compensation. It is not always clear, however, how the system specifications should be chosen so that they are within those limits. In LMF, this is reflected in the constraints in the reference model for which the PMF can be achieved.

* Assistant Professor, Electrical Engineering
** Professor, Electrical Engineering

Z. Gao and P. J. Antsaklis, "Reconfigurable Control System Design via Perfect Model-Following,"
Proceedings of the 1991 AIAA Guidance, Navigation and Control Conference, pp 239-246, New Orleans,
LA, August 12-14, 1991.

Assume that the plant and the reference model are of the same order. Let the reference model be given as:

$$\dot{x}_m = A_m x_m + B_m u_m \qquad (1)$$
$$y_m = C_m x_m$$

and the plant be represented by

$$\dot{x}_p = A_p x_p + B_p u_p \qquad (2)$$
$$y_p = C_p x_p$$

where $x_m, x_p \in R^n$, $u_m, u_p \in R^m$, $A_m, A_p \in R^{n\times n}$, $B_m, B_p \in R^{n\times m}$, $C_m, C_p \in R^{p\times n}$. The corresponding transfer function matrices of the reference model and the plant are:

$$T_m(s) = C_m(sI - A_m)^{-1}B_m \qquad (3)$$
$$P(s) = C_p(sI - A_p)^{-1}B_p. \qquad (4)$$

Let e(t) represent the difference in the state variables,

$$e(t) = x_m(t) - x_p(t). \qquad (5)$$

To achieve the PMF, one must insure that for any u_m, piecewise continuous, and $e(0) = 0$, we shall have $e(t) \equiv 0$ for all $t > 0$.

Next, we will discuss under what conditions the PMF is possible and how to find the feedback and feedforward controllers to achieve the PMF. In the cases when the PMF cannot be achieved, it is shown how the error can be minimized. These results are described in [Landau79].

2.1 The Implicit LMF

In the control system configuration of implicit LMF, Figure 1, the reference model does not appear explicitly. Instead, the model is used to obtain the control parameters, k_u and k_p. From (1) and (2), by simple manipulation we have

$$\dot{e} = A_m e + (A_m - A_p)x_p + B_m u_m - B_p u_p \qquad (6)$$

From the control configuration in Figure 1, the control input u_p has the form of

$$u_p = k_p x_p + k_u u_m \qquad (7)$$

The PMF is achieved if the control parameter k_u and k_p are chosen such that

$$\dot{e} = A_m e \qquad (8)$$

or equivalently

$$(A_m - A_p)x_p + B_m u_m - B_p u_p \equiv 0 \qquad (9)$$

Note that if a solution u_p of (9) exists, it will take the form

$$u_p = B_p^+(A_m - A_p)x_p + B_p^+ B_m u_m \qquad (10)$$

where B_p^+ represents the pseudo-inverse of the matrix B_p. From (10) k_u and k_p can be found as $k_p = B_p^+(A_m - A_p)$, and $k_u = B_p^+ B_m$. By substituting (10) in (6), a sufficient condition for the existence of the solution of (9) is

$$\begin{cases} (I - B_p B_p^+)(A_m - A_p) = 0 \\ (I - B_p B_p^+)B_m = 0 \end{cases} \qquad (11)$$

Note that (11) is known as Erzberger's conditions [Erzberger64]. Clearly, these are rather restrictive conditions, since most systems have more states than inputs, $B_p B_p^+ \neq I$. Thus (11) can only be fulfilled when $(I - B_p B_p^+)$ is in both the left null spaces of $(A_m - A_p)$ and B_m. It seems for an arbitrary plant, it is rather difficult to find an appropriate reference model such that it represents the desired dynamics and, at the same time, satisfies (11).

It should be noted that even when the conditions for the PMF in (11) are not fulfilled, the solution in (10) still minimizes the 2-norm of the last three terms in the right side of (6). i.e. $\|(A_m - A_p)x_p + B_m u_m - B_p u_p\|_2 = \|\dot{e} - A_m e\|_2$.

This particular method of choosing u_p has the advantage of not involving x_m in the feedback thus eliminating the need for running the model on line. Therefore the complexity of the control system is relatively low. One of the disadvantages of this method is that when the PMF is not achievable, the trajectory of *e* may not be desirable since we don't have control over the location of the poles in the system. Another disadvantage with this approach is that, when conditions in (11) are not satisfied, the solution (10) may result in an unstable system. This drawback will be further discussed in the next section when the relationship between the implicit LMF and the PIM method is explored.

2.2 The Explicit LMF

A typical configuration of the explicit LMF is as illustrated in Figure 2. In the configuration of the explicit LMF, the reference model is actually implemented as part of the controller. To compare with the implicit LMF, let $e = x_m - x_p$, or equivalently, assume $C_m = I$ and $C_p = I$. By manipulating (1) and (2), $\dot{e}$ can also be written as

$$\dot{e} = A_p e + (A_m - A_p)x_m + B_m u_m - B_p u_p. \qquad (12)$$

Let the control input be

$$u_p = u_1 + u_2 = (k_e e) + (k_m x_m + k_u u_m) \qquad (13)$$

where $u_1 = k_e e$ is the stabilizing gain, and $u_2 = k_m x_m + k_u u_m$ is to be determined to minimize $\|(A_m - A_p)x_p + B_m u_m - B_p u_2\|_2 = \|\dot{e} - (A_p - B_p k_e)e\|_2$. From (12), it can be easily shown that the sufficient conditions for the PMF is exactly the same as in (11) and the corresponding control gains are:

$$k_m = B_p^+(A_m - A_p)$$
$$k_u = B_p^+ B_m$$

with k_e any stabilizing gain. Substituting (13) in (12), the equation of error is

$$\dot{e} = (A_p - B_p k_e)e + (I - B_p B_p^+)(A_m - A_p)x_m + (I - B_p B_p^+)B_m u_m. \qquad (14)$$

When the conditions in (11) are met, we have

$$\dot{e} = (A_p - B_p k_e)e \qquad (15)$$

In this approach if the plant is stabilizable, we can guarantee the stability of the closed-loop system by choosing k_e appropriately, regardless of whether the conditions in (11) are met or not. This can be illustrated as follows. Since $\{A_p, B_p\}$ is stabilizable, k_e can be chosen such that $(A_p - B_p k_e)$ has all its eigenvalues in the left half plane; furthermore, let $f(t) = (I - B_p B_p^+)(A_m - A_p)x_m(t) + (I - B_p B_p^+)B_m u_m(t)$, then (14) can be expressed as: $\dot{e} = (A_p - B_p k_e)e + f(t)$. Because the reference model in (1) is stable, x_m will be bounded and therefore f(t) will be bounded for any bounded u_m. This implies that (14) is bounded-input bounded-output (BIBO) stable.

A challenging problem in the LMF approach is to choose the reference model appropriately. It not only must reflect the desired system behavior, but also must be reasonably chosen so that the plant can follow its trajectory closely. The Erzberger's condition gives indications on the constraints of the reference model for the PMF. It can be used to check whether the existing reference model satisfies the PMF conditions. However, as we can see in (11), it does not give much information on how to select (A_m, B_m, C_m). In the design process, what is needed is a guideline that can be used to select the reference model so that it will satisfy the Erzberger's condition. We shall look into frequency domain for the explanations of the PMF conditions to gain additional insight to the problem.

<u>Example 1</u> Let the nominal plant be

$$A = \begin{bmatrix} -.0507 & -3.861 & 0. & -32.17 \\ -.0012 & -.5164 & 1.0 & 0. \\ -.0001 & 1.4168 & -.4932 & 0. \\ 0. & 0. & 1.0 & 0. \end{bmatrix}$$

Z. Gao and P. J. Antsaklis, "Reconfigurable Control System Design via Perfect Model-Following,"
P r oceedings o f t he 1 991 A IAA G uidance, N avigation a nd C ontrol C o nference , pp 239-246, New Orleans,
LA, August 12-14, 1991.

$$B = \begin{bmatrix} 0. \\ -.0717 \\ -1.645 \\ 0. \end{bmatrix} \quad C = \begin{bmatrix} 0. \\ 0. \\ 1.0 \\ 0. \end{bmatrix}^T \quad D = 0$$

and the closed-loop nominal system be $\{A\text{-}Bk, B, C, D\}$, where

$k = [-.0043, -3.872, -.7186, -.0988]$

For a hypothetical impaired plant $\{A_f, B_f, C_f, D_f\}$, assume

$$A_f = A, \quad B_f = \begin{bmatrix} 0. \\ -.0717 \\ -.1645 \\ 0. \end{bmatrix}, \quad C_f = C, D_f = D$$

In failure accommodation, the reference model $\{A_m, B_m, C_m, D_m\}$ is chosen as the nominal closed-loop system, or, $A_m = A\text{-}Bk$, $B_m = B$, $C_m = C$, and $D_m = D$. Now the remaining task is to assign the feedback and feedforward gain matrices $\{k_e, k_u, k_m\}$. In this example the open-loop plant is unstable, thus the stabilizing gain, k_e, must be implemented. Such stabilizing gain was obtained using the LQR control design approach; where $k_e = [.2925\ -8.83\ -13.86\ -16.74]$ is such stabilizing gain. The feedforward gain matrices, k_u and k_m are determined by $k_m = B_p^+(A_m - A_p)$ and $k_u = B_p^+ B_m$ as in (13) and they are $k_m = [.0367\ 33.16\ 6.15\ .85]$, and $k_u = 8.56$. To simulate the closed-loop system in Figure 2, its state-space description is derived which has the form:

$$\begin{bmatrix} \dot{x}_m \\ \dot{x}_p \end{bmatrix} = \begin{bmatrix} A_m & 0 \\ B_p(k_m+k_e) & A_p\text{-}B_p k_e \end{bmatrix} \begin{bmatrix} x_m \\ x_p \end{bmatrix} + \begin{bmatrix} B_m \\ B_p k_u \end{bmatrix} u_m$$

The closed-loop system shown in Figure 2 is simulated using the initial conditions of $x_p(0) = [0\ 0\ 0\ 0.1]$ and $x_m(0) = [0\ 0\ 0]$, and zero input. The outputs of the reference model and the plant are very close to each other as is shown in Figure 3. Note that the performance of the reconfigured system achieved here is much superior that the one obtained by using the MPIM approach under exactly the same conditions [Gao90,91]. The PIM approach, in this example, result in an unstable closed-loop system. On the other hand, however, there is no guarantee that the explicit LMF can always achieve the performance as good as this one. In fact, it is shown in the simulations for different failures that the system response is very much dependent on the types of the failures that occurred.

It seems that the explicit LMF is more computationally efficient than the MPIM since the stability issue is resolved directly by using the stabilizing gain k_e. In contrast, to guarantee the stability of reconfigured systems using MPIM when the PIM solution cannot stabilize the system, a stabilizing gain must be determined first. Then, the stability bounds of the parameters in the gain matrix must be found, which is quite time consuming. Finally these bounds are used to adjust the feedback gains to obtain better performance. In general, it seems that the explicit LMF approach is well suited for the reconfigurable control problem.

2.3 Properties of the LMF Control Systems

The state-space model-following approaches shown above seems to have a simple control structure and their design philosophy seems to fit in the framework of reconfigurable control quite well. For these approaches, simple constant feedback controllers are used to regulate the state trajectories of the plant so that they follow the state of the reference model. To use it effectively in reconfigurable control, we need to gain better understanding of the LMF method. One of the vital properties of such systems is the system stability: is the system stability always guaranteed ? does it have the robust stability properties with respect to uncertainties in the model of the plant ?

It is also of interest to see how the perfect match can be achieved between the plant and the reference model. The PMF is important because it enables us to completely specify the system behavior. In this section, the frequency domain interpretation of the conditions of the PMF is studied. It is shown that, in contrast to the Erzberger's conditions, the conditions in frequency domain can be used directly in choosing the reference model in the design process. The stability robustness of the linear model following system is also analyzed.

Lemma 1: Assume $C_m = I$, $C_p = I$. A necessary and sufficient condition for the PMF is that the reference model T_m be obtained from the model of the plant using constant state feedback $u_p = fx_p + gr$.

Proof: In Figure 2, it is straight forward to find that the transfer function matrix between u_m and e is:

$$e = (I + Pk_e)^{-1}(T_m - P(k_m T_m + k_u))u_m \tag{16}$$

where T_m and P are defined in (3) and (4). This is derived as follows. From Figure 2

$$e = x_m - x_p = T_m u_m - P(k_e e + k_m T_m u_m + k_u u_m),$$

therefore

$$\begin{aligned}(I + Pk_e)e &= T_m u_m - P(k_m T_m u_m + k_u u_m) \\ &= (T_m - P(k_m T_m + k_u))u_m\end{aligned}$$

thus giving (16).

From (16), clearly $e \equiv 0$ for all u_m if and only if

$$T_m - P(k_m T_m + k_u) = 0 \tag{17}$$

which implies

$$T_m = (I - Pk_m)^{-1} P k_u \tag{18}$$

and we have $f = k_m$ and $g = k_u$. □

Compare Lemma 1 to Erzberger's condition (11), Lemma 1 gives necessary and sufficient conditions for the PMF which show exactly how to select the reference model. It makes good practical sense in that if a plant is designed to follow an artificial system exactly via constant state feedback, the artificial system must have the same basic structure as that of the plant. In fact, the Erzberger conditions can be seen as a special case of Lemma 1 since they are simply the sufficient conditions for $A_m = A_p + B_p k_p$, and $B_m = B_p k_u$. It also shows the limitation of this state-space approach in that the reference models a plant can follow exactly in this configuration are those which have the same zero structures as of the plant. This is because the system zeros cannot be changed by state feedback unless they are cancelled by the closed-loop poles.

From Figure 2, when $C_p \neq I$, we have $e(t) = x_m(t) - y_p(t)$. That is, the system is designed so that the output of the plant follows the state of the reference model. In this case, the conditions for the PMF can be derived similarly as above, that is T_m must be obtained from the model of the plant using the *output* feedback $u_p = fy_p + gr$. In this case, the transfer matrix of the reference model, T_m, still satisfies (18) with $f = k_m$ and $g = k_u$. Here the PMF implies that the output of the plant follows the state of the reference model exactly.

For a successful implementation of PMF, it is important that k_e provides robust stability with respect to plant parameter uncertainty. Note that k_e is the only design parameter that affects the stability robustness as shown below. Here robust stability means that if the real plant is $\hat{P}$ instead of P, where $\hat{P} = P + \Delta P$ for some small ΔP, the closed-loop system should still be stable. In the following, the stability robustness is examined for the closed-loop system designed by using the explicit LMF to achieve the PMF.

Lemma 2: For the control gains $\{k_e, k_m, k_u\}$ obtained by using the explicit LMF described in (13), the closed-loop system from u_m to e is stable if, for the real plant $\hat{P} = P + \Delta P$, $(I - \hat{P}k_e)^{-1}\Delta P$ is stable.

Proof:

$$\begin{aligned} e &= (I + \hat{P}k_e)^{-1}(T_m - \hat{P}(k_m T_m + k_u))u_m \\ &= (I + \hat{P}k_e)^{-1}((I - \hat{P}k_m)T_m - \hat{P}k_u))u_m \\ &= (I + \hat{P}k_e)^{-1}((I - Pk_m)T_m - \Delta P k_m T_m - \hat{P}k_u))u_m \\ &= (I + \hat{P}k_e)^{-1}(Pk_u - \Delta P k_m T_m - \hat{P}k_u))u_m \end{aligned}$$

Z. Gao and P. J. Antsaklis, "Reconfigurable Control System Design via Perfect Model-Following," Proceedings of the 1991 AIAA Guidance, Navigation and Control Conference, pp 239-246, New Orleans, LA, August 12-14, 1991.

$$= (I + Pk_e)^{-1}(-\Delta P)(k_m T_m + k_u)u_m \tag{19}$$

if $(I + Pk_e)^{-1}\Delta P$ is stable, then the transfer function matrix from u_m to e is stable. □

Note that when there is no uncertainty in the plant, that is $\Delta P = 0$, (19) shows that the transfer matrix from u_m to e is zero, which agrees with the original PMF design objective. When there is an uncertainty in the model of the plant, Lemma 2 shows that, although the PMF is no longer valid, the system stability will be maintained if k_e provides robust stability. This implies that for bounded u_m, the error e will always be bounded; if u_m is a constant, then will go to zero. Equation (19) also shows that to minimize the effect of system uncertainty, the error $(I + Pk_e)^{-1}\Delta P$ should be made small .

When the Erzberger condition is not met, an extra term will be added to $\dot{e}$ besides the homogeneous part. Since we have two different expressions for $\dot{e}$ and u_p in the derivation of the implicit and explicit LMF, $\dot{e}$ takes two different forms. For the implicit LMF:

$$\dot{e} = A_m e + g(t) \tag{20}$$

where $g(t) = (I - B_pB_p^+)(A_m - A_p)x_p + (I - B_pB_p^+)B_m u_m$ and for explicit LMF:

$$\dot{e} = (A_p - B_p k_e)e + f(t) \tag{21}$$

where $f(t) = (I - B_pB_p^+)(A_m - A_p)x_m + (I - B_pB_p^+)B_m u_m$.

Clearly, when the Erzberger's conditions are satisfied, $f(t) = g(t) = 0$, and we have the perfect model-following. When the condition is not met, f(t) and g(t) are nevertheless minimized in term of the Frobenius norm and e will diminish in steady state if the system is stable. The key difference between these two formulations is that, for the closed-loop system to be stable, it is required that the plant be stable in the implicit LMF while in the explicit LMF, it only requires the plant be stabilizable and k_e be the stabilizing gain. This is because for e to be bounded for a bounded input u_m under the conditions that A_m and $(A_p - B_pk_e)$ are stable matrices, g(t) and f(t) must be bounded. Since the reference model is a stable system, x_m is always bounded for a bounded input u_m, therefore f(t) is always bounded. On the other hand, g(t) is bounded only when x_p is bounded which requires that the plant be stable. It has also been shown [Chen73] that the upper bound on absolute value of e, |e|, is minimized by the explicit LMF when the Erzberger condition is not fulfilled. It seems that the explicit LMF has a clear advantage over the implicit LMF in this sense. In the following it is shown that the PIM is only a special case of the implicit LMF.

Linear Model Following and Its Relation to the Pseudo-Inverse Method (PIM)

The PIM is a method that can be used to accommodate system failures that are formulated as follows. Let the nominal plant be

$$\dot{x} = Ax + Bu$$
$$y = Cx. \tag{22}$$

Assume that the nominal closed-loop system is designed by using the state feedback $u = kx$, and the closed-loop system is

$$\dot{x} = (A+Bk)x$$
$$y = Cx \tag{23}$$

where k is the state feedback gain. Suppose that the model of the system, in which failures have occurred, is given as

$$\dot{x}_f = A_f x_f + B_f u_f$$
$$y_f = C_f x_f \tag{24}$$

and the new closed-loop system is

$$\dot{x}_f = (A_f + B_f k_f)x_f$$
$$y_f = C_f x_f, \tag{25}$$

where k_f is the new feedback gain to be determined. In the PIM, the objective is to find a k_f so that the system A-matrix in (25) approximates in some sense the one in (21). For this A+Bk is equated to $A_f + B_f k_f$ and an approximate solution for k_f is given by

$$k_f = B_f^+(A - A_f + Bk) \tag{26}$$

where B_f^+ denotes the pseudo-inverse of B_f, and the resulting input to the impaired plant is

$$u_f = B_f^+(A - A_f + Bk)x_f. \tag{27}$$

Clearly, this is just a special case of (10) in the implicit LMF where u_m is set to zero.

Since the PIM and the implicit LMF are essentially the same, our main interests are in the case of the explicit LMF. From the discussion above, the explicit LMF has the advantages of the guaranteed stability and the pre-specified error trajectory. Note that in terms of control reconfiguration, the reference model in Figure 2 can be selected as the nominal closed-loop system while the plant is the impaired open-loop system. Once the failures are identified and the new model obtained, the control gains $\{k_e, k_m, k_u\}$ can be immediately reconfigured according to the new model of the plant and the explicit LMF algorithm. The stabilizing gain k_e can be found via many available computer algorithms such as the pole placement or the LQ routine; the only non-trivial part in the determination of k_u and k_m is the calculation of the pseudo-inverse of B_p which can be found via various methods in the numerical analysis literature.

In general, it is felt that the explicit LMF is a practical approach that can be utilized in the control reconfiguration. It seems that the explicit LMF offers better tracking than the implicit LMF and the PIM since it is the difference of the states that are being minimized. A disadvantage of it is that the closed-loop system is more complex since the reference model must be implemented on-line.

A common shortcoming of all the reconfigurable control methods discussed so far, including the PIM and the LMF approaches, is the severe constraints needed to be satisfied for the perfect match between the nominal system and the impaired one. Although the output of the reconfigured system can be made very close to that of the nominal system as shown in Example 1, there are always cases, at least mathematically, that the explicit LMF cannot achieve satisfactory performance for the reconfigured system. This is because the feedforward gains, k_u and k_m are determined using a pseudo-inverse type of approach to minimize a cost function, $f(t) = (A_m - A_p)x_m + B_m u_m - B_p u_2$. The cost function f(t) will be made small if $(I - B_pB_p^+)$ is close to zero matrix, as it is shown in (14), or, $B_pB_p^+$ is close to an identity matrix. Therefore, we cannot prespecify how close the nominal and the impaired systems should be because we do not have any control over B_p, which is a part of the model of the plant.

III. Explicit LMF with Dynamic Compensators

In the RCS, the ideal goal is to develop a control system that is able to accommodate a large class of different system impairments so that the reconfigured systems behave exactly as prespecified. The explicit LMF approach described above is an approach that makes the output of a plant follow that of a reference model to a certain extent using constant feedback and feedforward gains. The exact match only happens for a particular class of the reference models which have been chosen to satisfy the severe constraints illustrated in Lemma 1. In the control reconfiguration to accommodate system failures, these

Z. Gao and P. J. Antsaklis, "Reconfigurable Control System Design via Perfect Model-Following," P r oceedings o f t he 1 991 A IAA G uidance, N avigation a nd C ontrol C o nference , pp 239-246, New Orleans, LA, August 12-14, 1991.

constraints sometimes seem to be too restrictive. In this section we will investigate the use of dynamic compensators, instead of constant ones, to loosen the restrictions on the reference models.

In the proof of Lemma 1, it is shown that for PMF, a necessary and sufficient condition is that the transfer function matrix from u_m to e is zero. Lemma 1 applies only to the reference models and plants where $C_m = C_p = I$. This is rather restrictive in design especially when the reference model is often chosen as the nominal closed-loop system in the control reconfiguration. In the following, the transfer function matrix is derived for the general case where C_m and C_p are not necessarily identities. From Figure 2,

$$\begin{aligned} e &= y_m - y_p \\ &= T_m u_m - P u_p \\ &= T_m u_m - P(k_e e + k_m x_m + k_u u_m) \\ &= - P k_e e + T_m u_m - P[k_m(sI - A_m)^{-1} B_m u_m + k_u u_m] \\ &= (I + P k_e)^{-1}[\, T_m - P[k_m(sI - A_m)^{-1} B_m + k_u]] u_m. \end{aligned} \quad (28)$$

Clearly, a necessary and sufficient condition for the PMF is that

$$[\, T_m - P[k_m(sI - A_m)^{-1} B_m + k_u] = 0. \quad (29)$$

Now it remains to solve (29) with respect to k_m and k_u. Note that there are many solutions of (29). A simple solution is

$$[k_m, k_u] = [0, P_{ri} T_m] \quad (30)$$

where P_{ri} is defined as the right inverse of P, i.e. $PP_{ri} = I$, assuming it exits. It was shown in [Gao89a] that the conditions for k_u to be proper and stable is that the reference model T_m is chosen such that it is 'more proper' than the plant and it has as its zeros all the RHP zeros of P together with their zero structures. It was also shown that the right-inverse of P can be calculated using a state-space algorithm which has good numerical properties. Note that the complexity of the compensator k_u is dependent on how different the reference model T_m is from the open-loop plant, P. This can be seen clearly from (30), where $k_u = P_{ri} T_m$. For example, if only a pair of open-loop poles are undesirable, that is, the poles and zeros of P and T_m are the same except one pair of poles, then k_u is a second order compensator since all the poles and zeros of P_{ri} and T_m are canceled except one pair of zeros of P_{ri} and one pair of poles of T_m. In case of failures, perhaps all open-loop zeros and poles will be shifted. However, only the unstable poles and dominant poles are of major concern in surviving the failures since their locations dominate how the system will behave in general. Therefore, in order to produce a fast and simple solution to keep the system running, T_m should be chosen close to the impaired plant P with exceptions of only a few critical poles.

The main advantage of this approach is that there are fewer restrictions on the reference model than before. The only restrictions are on the zeros of the reference model which are much more manageable than before. If the plant does not have RHP zeros, or its RHP zeros are unchanged after the failure, then the reference model is almost arbitrary except that it should be at least 'as proper' as P so that the compensator k_u is proper. The disadvantages of this approach is the increased complexity of the control system due to the higher order compensators required. This is a trade-off between the performance and complexity of the control system.

Note that there are many control configurations, other than that of Figure 2, that can be used for dynamic compensators. Here the same configuration is used for both constant and dynamic compensators because it is felt that the dynamic compensator can be used in conjunction with the constant compensators for the reconfigurable control purposes. As is mentioned earlier, there are two steps in the accommodation of failures. First, the impaired system must be stabilized. In the explicit LMF approach, this is accomplished via the implementation of the stabilizing gain k_e. This must be executed quickly to prevent catastrophic results from happening. Once the system is stabilized, it gives time to the control reconfiguration mechanism to manipulate the compensators to obtain better system performance. Assuming, by this time, that the model of the impaired system is available, a reference model should be chosen which has the desired behavior for the system under the specific system failure. Once the reference model is chosen, either the constant or the dynamic compensators can be computed and implemented as explained above. The choice of the types of the compensator depends on the performance requirements and the limitations on the complexity of the compensators.

Next, the stability robustness is analyzed. Let the real plant be $\hat{P}$ and the nominal transfer matrix of the plant be P, where $\hat{P} = P + \Delta P$ for some small ΔP.

Lemma 3: For the control gains $\{k_e, k_m, k_u\}$ obtained by using the explicit LMF with dynamic compensator described above from (28) to (30), the closed-loop system from u_m to e is stable if, for the real plant $\hat{P} = P + \Delta P$, $(I + \hat{P}k_e)^{-1}\Delta P$ is stable.

Proof: From (28) and (30)

$$e = (I + \hat{P}k_e)^{-1}(-\Delta P)k_u u_m \quad (31)$$

If $(I + \hat{P}k_e)^{-1}\Delta P$ is stable, then the transfer function matrix from u_m to e is stable. □

Note that when there is no uncertainty in the plant, that is $\Delta P = 0$, (30) shows that the transfer matrix from u_m to e is zero, which agrees with the original design objective. When there is an uncertainty in the model of the plant, Lemma 3 shows that the system stability will be maintained if k_e provides robust stability. This is because, by design, k_u is a stable compensator, therefore the closed-loop system is stable if $(I - \hat{P}k_e)^{-1}\Delta P$ is stable.

Example 2 To show the effectiveness of the new approach, we use the same nominal system as in Example 1 except that the C matrix is changed to [.0241 -135.0 -103.9 21.0]. C is chosen as such for the convenience of the simulation, since now the output stabilizing gain is simply $k_e = 1$. The impaired plant $\{A_f, B_f, C_f, D_f\}$ is as follows:

$$A_f = A, \quad B_f = \begin{bmatrix} 0. \\ -.0717 \\ -.1645 \\ -1.0 \end{bmatrix}, \quad C_f = C, D_f = D$$

The dynamic compensator obtained from (30) is $k_m = 0$, and

$$k_u = 31.2\backslash f(s^7 + 2.7s^6 +.6s^5 - 1.9s^4 -.09s^3 -1.4s^2 - .0016s - .0025,\ s^7 + 6.46s^6 +21.1s^5 + 39.0s^4 + 35.6s^3 + 11.8s^2\ .51s + .1)$$

The impulse responses of both the reference model, which is the nominal closed-loop system, and the reconfigured system are shown in Figure 4. They match exactly as expected. This is compared to the response of the system reconfigured with the standard explicit LMF method.

Note that the exact match is attained at the expense of having a seventh-order compensator. This compensator can be implemented, together with the reference model, in the real-time aircraft control environment via flight control computers. The complexity of these compensators may or may not be an issue in the implementation depending on the capacity of the computers. If it is, then the reference model has to be chosen close to the open-loop plant, as is discussed above, so that the poles and zeros of P_{ri} and T_m cancel each other except the critical poles. The standard explicit LMF has a very simple system structure where only the constant gain matrices are to be adjusted for different failures. It should be used whenever the performance of the reconfigured system is acceptable. However, difficulty may arise when it does not provide satisfactory performance and the dynamic compensator cannot be used due to the limitations on the system complexity. Such problems will be investigated in future research.

IV. Concluding Remarks

The linear model-following methods in control system design were studied in the context of reconfigurable control. The necessary and sufficient conditions for perfect model-following were obtained using a transfer function approach,

Z. Gao and P. J. Antsaklis, "Reconfigurable Control System Design via Perfect Model-Following," Proceedings of the 1991 AIAA Guidance, Navigation and Control Conference, pp 239-246, New Orleans, LA, August 12-14, 1991.

which yield simple and intuitive constraints on the reference model. A new approach was developed to design the reconfigurable control system so that the state trajectories of the reconfigured system to follow those of the reference model exactly. This was achieved at the expense of increased system complexity due to the use of dynamic compensators.

The advantages of using the LMF control methodology in reconfigurable control system design can be summarized as follows. First, the widely used pseudo-inverse method is only a special case of the LMF. By examining the PIM in the context of LMF, it helped us to understand the characteristics and the limitations of the PIM. Secondly, like the PIM, the LMF control system is simple in terms of design and implementation. More importantly, it guarantees the stability of the reconfigured system assuming the impaired plant is stabilizable. Thirdly, the new design approach proposed in this paper enable us to achieve the PMF, and thus completely specify the behavior of the reconfigured system with much fewer constraints on the performance specifications. Finally, since the adaptive model reference control systems have been developed based on the LMF systems, it is possible to extend the results in this paper to adaptive control systems. This may be necessary in certain cases since it could make the reconfigurable control systems less dependent on the fault detection and identification systems.

References

[Antsaklis86] Antsaklis P.J. "On Finite and Infinite Zeros in the Model Matching Problem," Proc. of the 25th IEEE Conf. on Decision and Control, Athens Greece, December 10-12, 1986.

[Chen73] Chen, Y.T., "Perfect Model Following with a Real Model," Proc. of JACC, pp.287-293, 1973.

[Dittmar88] Dittmar C.J., "A Hyperstable Model-Following Flight Control System Used for Reconfiguration Following Aircraft Impairment", Proc. of American Control Conference, pp.2219-2224, June 1988.

[Erzberger68] Erzberger, H., "Analysis and Design of Model Following Control Systems by State Space Techniques," Proc. of JACC, pp.572-581, 1968.

[Gao87] Gao, Z., "The Model Matching and Related Control Problems: Conditions and Algorithms," M.S. Thesis, Dept. of Electrical and Computer Engineering, Univ. of Notre Dame, August 1987.

[Gao89] Gao Z., and Antsaklis P.J., "On the Stability of the Pseudo-Inverse Method for Reconfigurable Control Systems", Proc. of the IEEE 1989 National Aerospace and Electronics Conference, pp.333-337, May 22-26, 1989.

[Gao89a] Gao, Z., and P.J. Antsaklis, "On Stable Solutions of the One- and Two-Sided Model Matching Problems", IEEE Transactions on Automatic Control, Vol. 34, No. 9, pp. 978 - 982, September 1989.

[Gao90] Gao Z., and Antsaklis P.J., "Pseudo-Inverse Method for Reconfigurable Control with Guaranteed Stability", Proc. of the 11th IFAC World Congress, August 1989.

[Gao90a] Gao Z., "Reconfigurable Control: Theory and Applications", Ph.D. Dissertation, August 1990.

[Gao91] Gao Z., and Antsaklis P.J., "On the Stability of Pseudo-Inverse Method for Reconfigurable Control Systems", to appear in International Journal of Control.

[Gavito87] Gavito V.F., and Collins D.J., "Application of Eigenstructure Assignment to Self-Reconfiguring Aircraft MIMO Controllers", Proc. of AIAA Guidance, Navigation and Control Conference, pp.1-12, Aug. 1987.

[Gross88] Gross H.W. and Migyanko B.S., "Application of Supercontroller to Fighter Aircraft Reconfiguration", Proc. of American Control Conference, pp.2232-2237, June 1988.

[Handl89] Handlman D.A. and Stengel R.F., "Combining Expert System and Analytic Redundancy Concepts for Fault Tolerance Flight Control", J. of Guidance, Control, and Dynamics, vol.12, pp.39-45, Jan.-Feb. 1989.

[Howell83] Howell W.E., et al., "Restructurable Controls for Aircraft", Proc. of AIAA Guidance, Navigation and Control Conference, pp.646-651, 1983.

[Huber84] Huber R.R., and McCulloch B., "Self-Repairing Flight Control System", SAE Technical Paper Series, 841552, Aerospace Congress & Exposition, Long Beach, California, Oct. 15-18, 198

[Joshi87] Joshi S.M. "Design of Failure - Accommodating Multiloop LQG - Type Controllers", IEEE Transactions on Automatic Control, AC-32, pp.740-741, August 1987.

[Landau79] Landau Y.D., Adaptive Control: The Model Reference Approach, Marcel Dekker Inc. New York, 1979.

[Looze85] Looze D.P., et al., "An Automatic Redesign Approach for Restructurable Control Systems", IEEE Control Systems Magazine, May 1985.

[McMahan78] McMahan J., "Flight 1080, Air Line Pilot, July 1978.

[Moerder86] Moerder D.D., Caglayan A.K. and Halyo N., "Reconfiguration in Stages, Part 2: Precomputed Control Laws for Force and Moment Redundancy in a Self-Repairing Flight Control System", Proc. of the IEEE 1984 National Aerospace and Electronics Conference, May 1986.

[NTSB79] "National Transportation Safety Board Accident Report of the American Airlines DC10 Crash at Chicago -- O'Hare International Airport, May 25, 1979," NTAB-AAR-79-17, Dec. 21, 1979.

[Ostroff84] Ostroff A.J., and Hueschen R.M., "Investigation of Control Law Reconfigurations to Accommodate a Control Element Failure on a Commercial Airplane", Proc. of American Control Conference, pp.1746-1754, 198

[Ostroff85] Ostroff A.J, "Techniques for Accommodating Control Effector Failures on a Mildly Statically Unstable Airplane", Proc. of American Control Conference, pp.906-903, 1985.

[Patel82] Patel, R.V., "Construction of Stable Inverses for Linear Systems", Int. J. Systems Sci., Vol.13, No.5, pp. 499-515, 1982.

[Potts81] Potts D.W., and D'Azzo J., "Direct Digital Design Method for Reconfigurable Multivariable Control Laws for the A-7D Digital II Aircraft", Proc. of the IEEE 1981 National Aerospace and Electronics Conference, Vol.3, pp.1284-91, 1981.

[Rattan85] Rattan K.S., "Evaluation of Control Mixer Concept for Reconfiguration of Flight Control System", Proc. of the IEEE 1985 National Aerospace and Electronics Conference, pp.560-569, 1985.

[Raza85] Raza S.J., and Silverthorn J.T., "Use of Pseudo - Inverse for Design of a Reconfigurable Flight Control System", Proc. of AIAA Guidance, Navigation and Control Conference, pp.349-356, 1985.

[Sadeghi84] Sadeghi T., and Rosenthal G., "A Control Law for the Self-Repairing/Digital Flight Control System", Proc. of the IEEE 1984 National Aerospace and Electronics Conference, 198

[Weiss86] Weiss J.L., Looze D.P., et al., "Initial Design and Evaluation of Automatic Restructurable Flight Control System Concepts", NASA

Z. Gao and P. J. Antsaklis, "Reconfigurable Control System Design via Perfect Model-Following,"
P r oceedings o f t he 1 991 A IAA G uidance, N avigation a nd C ontrol C o nference , pp 239-246, New Orleans, LA, August 12-14, 1991.

Contractor Report 178064, Alphatech, Inc., Burlington, Massachusetts, June 1986.
[Wolovich74] Wolovich, W.A., Linear Multivariable Systems, Springer-Verlag, New York. 1974.

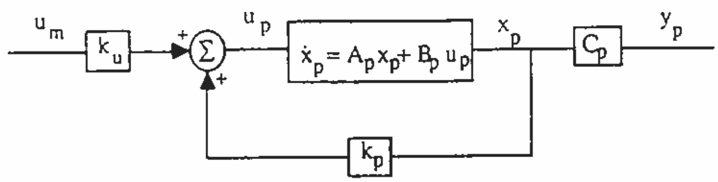

Figure 1 The control configuration of the implicit LMF.

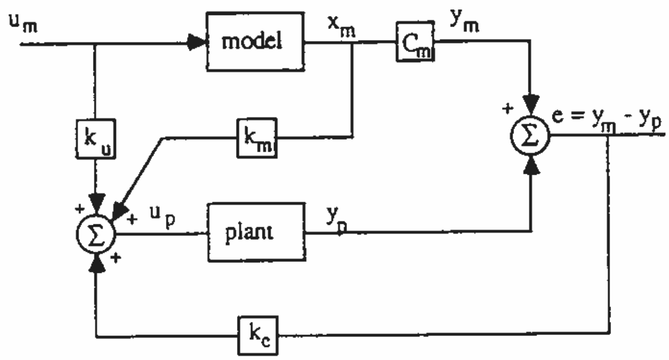

Figure 2 The control configuration of the explicit LMF.

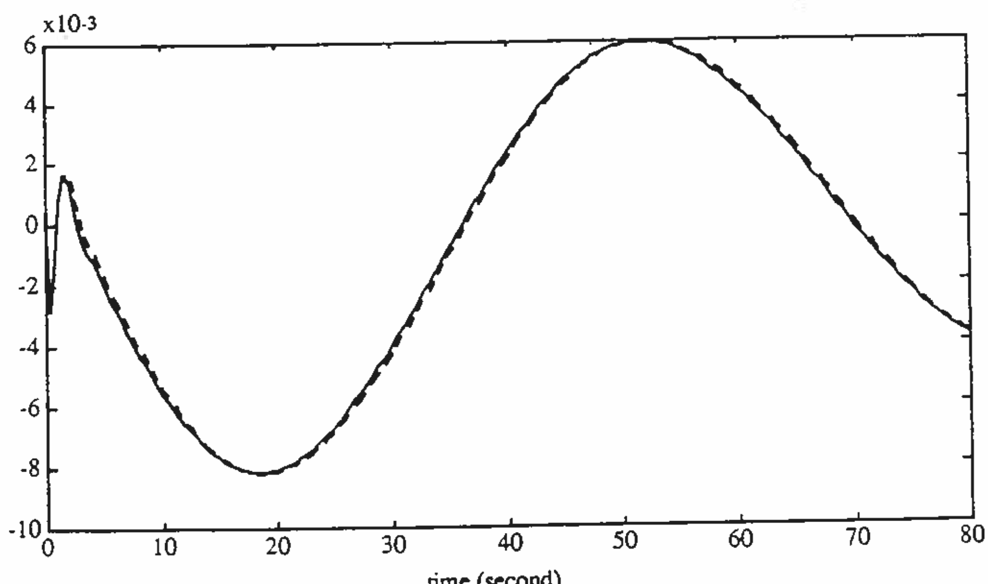

Figure 3 The outputs of the nominal and reconfigured closed-loop system using the explicit LMF approach

Z. Gao and P. J. Antsaklis, "Reconfigurable Control System Design via Perfect Model-Following,"
P r oceedings o f t he 1 991 A IAA G uidance, N avigation a nd C ontrol C o nference , pp 239-246, New Orleans, LA, August 12-14, 1991.

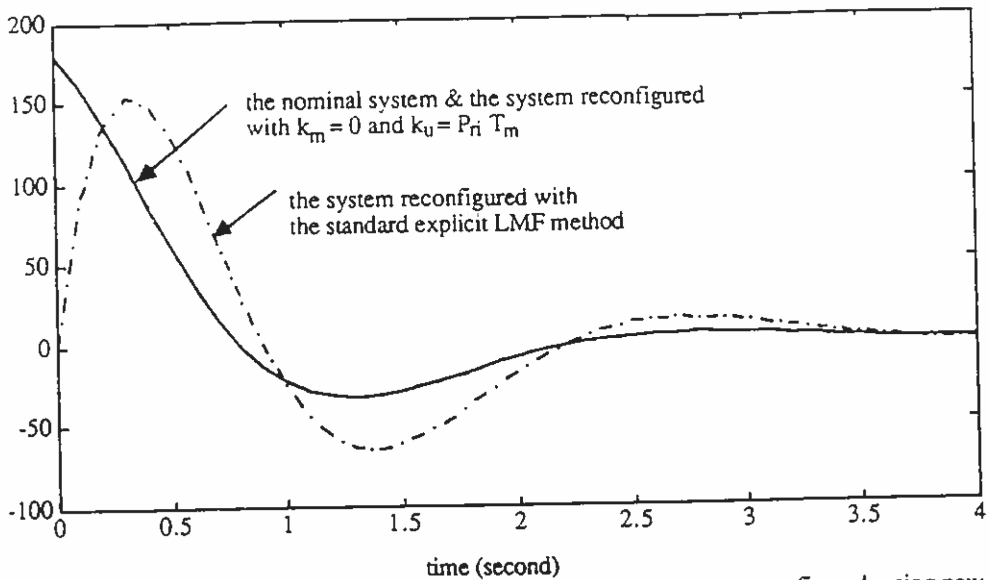

Figure 4 The impulse responses of the reference model and the system reconfigured using new and standard explicit LMF methods