On Passivity of Networked Nonlinear Systems with Packet Drops

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Abstract

We analyze passivity for a class of discrete-time switched nonlinear systems that switch between two modes - an uncontrolled mode in which the system evolves open loop, and a controlled mode in which a control input is applied to the system. Such a model has recently been used for a network controlled system in the presence of packet drops introduced by a communication channel. For the case when the open loop system is non-passive, classical passivity theory considers the switched system to be non-passive as well. We give a new generalized definition of passivity for such a system and show that if the ratio of the time steps for which the system evolves open loop versus the time steps for which the system evolves closed loop is bounded below a critical ratio, then the nonlinear system is locally passive in this sense. Moreover, we show that this generalized definition is useful since it preserves two important properties of the classical passivity concept - that passivity implies asymptotic stability for zero state detectable systems using feedback and that passivity is preserved in parallel and feedback interconnections.

I. Introduction

Networked control systems is now an established area of research [1]. In this paper, we consider a process being controlled across a communication channel that drops control packets in a non-deterministic fashion [2], [3]. In particular, we are interested in a system that is open loop non-passive, but is passive when in closed loop. Because of the control packets being dropped by the communication channel, the system switches between two modes, in one of which the increase in storage function is not bounded by the energy supplied to the system at each time step.

Passivity is one of the most useful forms of dissipativity and is widely used for analyzing the stability of interconnected dynamical systems [4]–[7]. Two of the properties that make passivity particularly useful

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are that (i) passivity implies asymptotic stability for zero state detectable (ZSD) system using feedback [7], and (ii) both negative feedback and parallel interconnections of passive systems are passive. Due to its importance, the classical notion of passivity has been extended to consider systems with delays [8], [9], event-triggered systems [10], discrete-time piecewise affine hybrid systems [11], general nonlinear hybrid systems [12] and switched systems [13].

Nevertheless, the above literature considers systems in which all the modes of the system are individually passive. Since in our application, the open loop mode of the system is not passive; hence, this framework does not hold. The main contribution of this paper is to extend the passivity concept to this case and to show that if the frequency of the time steps at which the system is in open loop (and hence non-passive) is bounded, the switched system remains passive. The closest work to our presentation is [13] from which we borrow the concept of allowing the increase in storage function to be not necessarily bounded by the energy supplied at every time step. However, unlike [13], we do not assume each mode of the system to be passive. Also related is [14] that considered the asymptotic stability analysis of continuous time systems where the Lyapunov function is non-increasing only on certain unbounded discrete time sets. However, unlike [14], the passivity analysis is complicated by the fact that passivity is an input-output property and both the inputs and the outputs are time varying. Due to this difficulty, we analyze the passivity properties of a switched system based on its zero dynamics ([6], [15], [16], and in particular, [17]) which is the internal dynamics of the system that is consistent with constraining the system output to zero.

The remainder of the paper is organized as follows. In Section II, we define the problem framework. Section III-A analyzes the passivity properties of the zero dynamics of the controlled mode of the switched systems. In Section III-B, the passivity of the original switched system is investigated based on the results from its zero dynamics. Numerical examples are provided in Section IV. We conclude the paper with a summary and a list of future work in Section V. Some background on classical passivity theory in discrete-time setting [18] is provided in the Appendix.

Notation: An m-dimensional real vector is denoted by $\mathbb{R}^m$. The space of positive integers is denoted by $\mathbb{Z}^+$. By a smooth vector field, we mean a field that is in C^∞. Bold-face symbols are used for vectors. In particular, if a scalar m has value zero, we denote $m = 0$; while if a vector $\mathbf{m}$ has value zero, we denote $\mathbf{m} = \mathbf{0}$. The Kronecker delta function is denoted by δ_{rs}, which is 0 if $r \neq s$ and 1 otherwise.

II. Problem Formulation

Consider a discrete-time nonlinear system described by the equation

$$\begin{aligned} \mathbf{x}(k+1) &= f(\mathbf{x}(k), \mathbf{u}(\mathbf{x}(k))) \\ \mathbf{y}(k) &= h(\mathbf{x}(k), \mathbf{u}(\mathbf{x}(k))), \end{aligned} \tag{1}$$

where $k \in \mathbb{Z}^+$ is the time index, $\mathbf{x}(k) \in \mathbb{R}^n$ is the state, $\mathbf{u}(\mathbf{x}(k)) \in \mathbb{R}^m$ is the control input generated by a given state feedback controller, $\mathbf{y}(k) \in \mathbb{R}^m$ is the output, and both $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $h : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^m$ are smooth vector fields. We will assume that the system has relative degree zero [6], i.e., $\frac{\partial h(\mathbf{x},\mathbf{u})}{\partial \mathbf{u}}$ is non-singular. The system is also assumed to be locally zero state detectable (ZSD) [19], i.e., there exists a neighborhood $\mathbf{N}$ of the origin such that $\forall \mathbf{x}(0) = \mathbf{x}_0 \in \mathbf{N}$,

$$\mathbf{y}(k)|_{\mathbf{u}(k)=\mathbf{0}} = h(\phi(k; \mathbf{x}_0; \mathbf{0})) = \mathbf{0}, \ \forall k \in \mathbb{Z}^+ \ \text{ implies } \ \lim_{k\to+\infty} \phi(k; \mathbf{x}_0; \mathbf{0}) = \mathbf{0},$$

where $\phi(k; \mathbf{x}_0; \mathbf{0})$ is a trajectory of the uncontrolled system $\mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{0}))$ from $\mathbf{x}(0) = \mathbf{x}_0$.

We consider such a system being controlled across a communication channel that drops packets. At the instants at which the control packet is transmitted successfully, the system evolves as in (1). At the instants at which the control packet is dropped, we assume for concreteness that zero control is applied and the system evolves as

$$\begin{aligned} \mathbf{x}(k+1) &= f(\mathbf{x}(k), \mathbf{0}) \\ \mathbf{y}(k) &= h(\mathbf{x}(k), \mathbf{0}). \end{aligned} \tag{2}$$

Denote the switched system evolving as in (1) and (2) by $\mathcal{S}$. We make no conditions on the packet drops (i.e., whether they are stochastic or periodic). We refer to system evolution according to (1) as Mode 1 and according to (2) as Mode 2 of the switched system. The mode switching sequence for the system is defined as the specification of the value $d(k)$ for every $k \in \mathbb{Z}^+$, where $d(k) \in \{1, 2\}$ is the mode active at time k. We assume that the closed-loop system (1) is passive while the open loop system (2) is non-passive. Passivity of a switched system with at least one non-passive mode has not been defined in the literature. We propose such a definition in this paper. We will assume without loss of generality that at time $k = 1$, the system is in Mode 1.

Clearly, any passivity property of the switched system will depend on the relative frequency with which the two modes are active. Consider the system evolution over T time steps. Let $\tau(T)$ denote the total number of uncontrolled (open loop) time steps when the system is in Mode 2 during this time period, and $T-\tau(T)$ the total number of controlled (closed-loop) time steps when the system is in Mode 1. Let the ratio between the controlled time steps and the uncontrolled time steps be $r(T)=\frac{T-\tau(T)}{\tau(T)}$. When the context is clear, we will abuse the notation and suppress the dependence of τ and r on T. We consider the following definition of passivity.

Definition 2.1: A nonlinear system $\mathcal{S}$ is said to be *globally passive* if there exists a positive semidefinite storage function $\tilde{\tilde{V}}(\mathbf{x}(.))\geq 0$ ($\tilde{\tilde{V}}(\mathbf{x}(.))=0$ if and only if $\mathbf{x}(.)=\mathbf{0}$) such that for any $\mathbf{x}(k)\in\mathbb{R}^n$, $\mathbf{u}(k)\mathbb{R}^m$, and any given $T\in\mathbb{Z}^+$, the following passivity inequality holds:

$$\tilde{\tilde{V}}(\mathbf{x}(T))-\tilde{\tilde{V}}(\mathbf{x}(1))\leq\sum_{k=1}^{T-1}\mathbf{u}^{\mathrm{T}}(k)\mathbf{y}(k). \tag{3}$$

The system is said to be *locally passive* if there exists a neighborhood of the equilibrium point $(\mathbf{x}^*(k),\mathbf{u}^*(k))$ such that for any $(\mathbf{x}(k),\mathbf{u}(k))$ in the neighborhood, the inequality (3) holds.

Note that this definition reduces to the classical passivity definition for a non-switched system. However, for a switched system that can operate for some time in a non-passive mode, Definition 2.1 allows the increase in storage function $\tilde{\tilde{V}}$ to be greater than the supplied energy at particular time steps, as long as the overall increase over the period $[1,T]$ is bounded by the total supplied energy within that period.

With this definition, we answer two questions in this paper. First, we show that this definition is useful for cases such as system $\mathcal{S}$ in the sense that it can be used to show intuitive results such as if the open loop system is active only infrequently, the switched system should be expected to remain passive. More precisely, we prove that there is a ratio $r^\star$, such that if for every T, $r(T)>r^\star$, then the system is passive. Secondly, we show that this definition preserves the following two properties of classical passivity definition:

- A passive system can achieve asymptotic stability using feedback if it is ZSD.
- Parallel or negative feedback interconnections of passive systems are passive.

III. Main Results

A. Passivity Analysis for Zero Dynamics

We begin by considering the zero dynamics of the system $\mathcal{S}$. Since (1) has relative degree zero, the implication function theorem [17], [20] implies that for any given bounded vector sequence $\mathbf{v}(k)\in\mathbb{R}^m$, there exists a control law $\bar{\mathbf{u}}^{\mathbf{v}(k)}(\mathbf{x}(k))$ (that depends on both $\mathbf{v}(k)$ and $\mathbf{x}(k)$) such that the resulting output $\mathbf{y}(k)$ is identically equal to $\mathbf{v}(k)$ and the corresponding inputs are bounded. With this control law, the system in Mode 1 evolves as the transformed system

$$\begin{aligned} \mathbf{x}(k+1) &= f(\mathbf{x}(k), \bar{\mathbf{u}}^{\mathbf{v}(k)}(\mathbf{x}(k)) \triangleq \bar{f}^{\mathbf{v}(k)}(\mathbf{x}(k)) \\ \mathbf{y}(k) &= \mathbf{v}(k). \end{aligned} \tag{4}$$

The evolution in Mode 2 is still governed by (2). Denote the switched system defined by (4) and (2) by $\mathcal{S}_1$. For simplicity and without loss of generality, we assume that the origin $(\mathbf{x}(k), \mathbf{v}(k)) = (\mathbf{0}, \mathbf{0})$ is one equilibrium state of the process (4), i.e., $\bar{f}^{\mathbf{v}(k)}(\mathbf{x}(k))\Big|_{\mathbf{x}(k)=\mathbf{0},\mathbf{v}(k)=\mathbf{0}} = \mathbf{0}$.

In the particular case when $\mathbf{v}(k)$ is identically zero, let the control law be given by $\tilde{\mathbf{u}}(k)$. Then, the system in Mode 1 evolves as the zero dynamics of the closed loop system or as

$$\begin{aligned} \mathbf{x}(k+1) &= f(\mathbf{x}(k), \tilde{\mathbf{u}}(\mathbf{x}(k)) \triangleq \tilde{f}(\mathbf{x}(k)) \\ \mathbf{y}(k) &= \mathbf{0}. \end{aligned} \tag{5}$$

Denote the switched system defined by (5) and (2) by $\mathcal{S}_2$. We note that since system (1) is passive, the zero dynamics of system (5) are also passive and hence stable [15], [16]. Since for the system $\mathcal{S}_2$, either the input $\tilde{\mathbf{u}}(k)$ or the output $\mathbf{y}(k)$ is zero at every time step, Definition 2.1 implies that the system $\mathcal{S}_2$ is passive if there exists a positive semidefinite storage function $V(\mathbf{x}(.)) \geq 0$ ($V(\mathbf{x}(.)) = 0$ if and only if $\mathbf{x}(.) = \mathbf{0}$) such that for any given $T \in \mathbb{Z}^+$, the following inequality holds:

$$V(\mathbf{x}(T)) - V(\mathbf{x}(1)) \leq \sum_{k=1}^{T-1} \mathbf{u}^{\mathrm{T}}(k)\mathbf{y}(k) = 0. \tag{6}$$

From now on, we will additionally assume that the determinant of Hessian matrix (square matrix of second-order partial derivatives) of the storage function $V(\mathbf{x})$ at $\mathbf{x} = \mathbf{0}$ is non-zero.

Our first result shows that there is a frequency of the steps at which the system $\mathcal{S}_2$ evolves in closed loop that guarantees that the system remains passive.

Lemma 3.1: Consider the switched system $\mathcal{S}_2$. Let there exists a positive semidefinite storage function

$V(\mathbf{x}) \geq 0,\ V(\mathbf{x}) = \mathbf{0}$ if and only if $\mathbf{x} = \mathbf{0}$, and constants $\zeta > 1$ and $0 < \sigma < 1$ such that

$$V(f(\mathbf{x}(k), \mathbf{0})) \leq \zeta V(\mathbf{x}(k)) \tag{7}$$
$$V(\tilde{f}(\mathbf{x}(k))) \leq \sigma V(\mathbf{x}(k)).$$

If for any time T, the ratio $r(T)$ satisfies

$$r(T) > \frac{(T-1)\ln\zeta}{\ln\zeta - T\ln\sigma}, \tag{8}$$

the system $\mathcal{S}_2$ is passive according to Definition 2.1.

Proof: For any time T, (7) implies that $V(\mathbf{x}(T)) \leq \sigma^{T-\tau}\zeta^{\tau-1}V(\mathbf{x}(1))$. Since (8) implies $\sigma^{T-\tau}\zeta^{\tau-1} < 1$, we obtain that $V(\mathbf{x}(T) < V(\mathbf{x}(1))$ for any T, if the conditions (7) in the theorem are met. From Definition 2.1, the system $\mathcal{S}_2$ is passive. ■

Remark 3.1: As discussed earlier, the system (5) is locally stable. A natural candidate for V is the Lyapunov function for the system. Note that the storage function for $\mathcal{S}_2$ may not be the storage function for the original system $\mathcal{S}$.

Remark 3.2: The choice of ζ and σ determines how conservative the condition (8) is. The minimum ζ and σ that satisfy the inequality (7) will result in the least conservative bound.

Remark 3.3: Note that the right hand side of the equation (8) is an increasing function of T. Thus, the condition on the frequency of Mode 2 becomes progressively less stringent. Note also that the condition does not require a constant ratio $r(T)$.

We now prove an intuitive result on the effect of increasing $r(T)$.

Corollary 3.1: Consider the system $\mathcal{S}_2$ with the conditions (7) being satisfied. If the system is passive with a ratio $r(T)$, it is passive with a ratio $r'(T) > r(T)$. Thus, decreasing the frequency of uncontrolled time steps preserves passivity.

Proof: At time T, denote the number of time steps for which the system evolves open loop with the ratio $r(T)$ by $\tau(r, T)$ and with the ratio $r'(T)$ by $\tau(r', T)$. Conditions (7) yield

$$V(\mathbf{x}(T)) \leq \sigma^{T-\tau(r,T)}\zeta^{\tau(r,T)-1}V(\mathbf{x}(1)) \ \text{ and } \ V(\mathbf{x}(T)) \leq \sigma^{T-\tau(r',T)}\zeta^{\tau(r',T)-1}V(\mathbf{x}(1)).$$

Since the system is passive with ratio $r(T)$, $\sigma^{T-\tau(r,T)}\zeta^{\tau(r,T)-1} < 1$. The proof follows by noting that $\tau(r', T) < \tau(r, T)$ and thus, $\sigma^{T-\tau(r',T)}\zeta^{\tau(r',T)-1} < \sigma^{T-\tau(r,T)}\zeta^{\tau(r,T)-1} < 1$. ■

B. Passivity Analysis for the Original System

We now prove that if the zero dynamics are passive, then the original switched system $\mathcal{S}$ is locally passive near the equilibrium point $(\mathbf{x}(k), \mathbf{v}(k)) = (\mathbf{0}, \mathbf{0})$. To this end, we first prove the following result.

Theorem 3.1: Let the system $\mathcal{S}_2$ be passive such that the inequalities (7) hold. Furthermore, let the system $\mathcal{S}_1$ evolve from the same initial condition and with the same mode switching signal as the system $\mathcal{S}_2$. Then, for the system $\mathcal{S}_1$ there exists a positive semidefinite storage function $\tilde{V}(\mathbf{x}(k)) = aV(\mathbf{x}(k)) \geq 0$ ($\tilde{V}(\mathbf{x}(.)) = 0$ if and only if $\mathbf{x}(.) = \mathbf{0}$, and $a > 0$), such that for any $T \in \mathbb{Z}^+$, the following inequality is true in a neighborhood of the equilibrium point $(\mathbf{x}(k), \mathbf{v}(k)) = (\mathbf{0}, \mathbf{0})$.

$$\tilde{V}(\mathbf{x}(T)) - \tilde{V}(\mathbf{x}(1)) \leq \sum_{\substack{k:d(k)=2 \\ k \leq T-1}} \mathbf{v}^{\mathrm{T}}(k)\mathbf{v}(k). \tag{9}$$

Proof: Since $\mathcal{S}_2$ is passive, there exists a positive semidefinite storage function $V(\mathbf{x}(.)) \geq 0$ ($V(\mathbf{x}(.)) = 0$ if and only if $\mathbf{x}(.) = \mathbf{0}$), such that for any $T \in \mathbb{Z}^+$, $V(\mathbf{x}(T)) - V(\mathbf{x}(1)) \leq 0$, when $\mathbf{x}$ evolves according to the switched system $\mathcal{S}_2$. For system $\mathcal{S}_1$, consider the storage function $\tilde{V}(\mathbf{x}(.)) = aV(\mathbf{x}(.))$ for a constant $a > 0$. We first prove that with a suitable choice of the constant a, this storage function guarantees that, for every vector sequence $\{\mathbf{v}(k)\}$, if time k is such that the mode $d(k) = 2$, then

$$\tilde{V}(\bar{f}^{\mathbf{v}(k)}(\mathbf{x}(k))) - \tilde{V}(\mathbf{x}(k)) \leq \mathbf{v}^{\mathrm{T}}(k)\mathbf{v}(k). \tag{10}$$

For the times k where the mode $d(k) = 2$, define the function

$$\phi(\mathbf{x}(k), \mathbf{v}(k)) = \sum_{i=1}^{m} v_i^2(k) + \tilde{V}(\mathbf{x}(k)) - \tilde{V}(\bar{f}^{\mathbf{v}(k)}(\mathbf{x}(k))). \tag{11}$$

We shall prove that $\phi(\mathbf{x}(k), \mathbf{v}(k))$ has a local minimum at $\mathbf{x}(k) = \mathbf{0}$ and $\mathbf{v}(k) = \mathbf{0}$. For notational convenience, we denote this pair by $(\mathbf{0}, \mathbf{0})$ and suppress the dependence on k of the terms in (11). Thus, consider the first order derivatives of $\phi(\mathbf{x}, \mathbf{v})$ at $(\mathbf{0}, \mathbf{0})$. We have for $i = 1, \cdots, n$, $r = 1, \cdots, m$,

$$\begin{aligned}
\left.\frac{\partial \phi(\mathbf{x}, \mathbf{v})}{\partial x_i}\right|_{\mathbf{x}=\mathbf{0},\mathbf{v}=\mathbf{0}} &= \left[\frac{\partial \tilde{V}}{\partial x_i} - \sum_{h=1}^{n} \frac{\partial \tilde{V}}{\partial \bar{f}_h^{\mathbf{v}(k)}} \frac{\partial \bar{f}_h^{\mathbf{v}(k)}(\mathbf{x}, \mathbf{v})}{\partial x_i}\right]_{\mathbf{x}=\mathbf{0},\mathbf{v}=\mathbf{0}} \\
\left.\frac{\partial \phi(\mathbf{x}, \mathbf{v})}{\partial v_r}\right|_{\mathbf{x}=\mathbf{0},\mathbf{v}=\mathbf{0}} &= \left[2v_r - \sum_{h=1}^{n} \frac{\partial \tilde{V}}{\partial \bar{f}_h^{\mathbf{v}(k)}} \frac{\partial \bar{f}_h^{\mathbf{v}(k)}(\mathbf{x}, \mathbf{v})}{\partial v_r}\right]_{\mathbf{x}=\mathbf{0},\mathbf{v}=\mathbf{0}}.
\end{aligned}$$

The storage function $V(\mathbf{x}(k))$, and hence the function $\tilde{V}(\mathbf{x}(k)) = aV(\mathbf{x}(k))$ has a local minimum at $\mathbf{x}(k) = \mathbf{0}$ because V is positive semidefinite with $V(\mathbf{x}) = 0$ if and only if $\mathbf{x} = \mathbf{0}$. Moreover, origin is a local equilibrium of the system; thus, at $\mathbf{x}(k) = \mathbf{v}(k) = \mathbf{0}$, $\bar{f}^{\mathbf{v}(k)}(\mathbf{x}(k), \mathbf{v}(k)) = \mathbf{0}$. Combining these facts, we see that

$$\left.\frac{\partial \phi(\mathbf{x}, \mathbf{v})}{\partial x_i}\right|_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}} = 0, \qquad i = 1, \cdots, n$$

$$\left.\frac{\partial \phi(\mathbf{x}, \mathbf{v})}{\partial v_r}\right|_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}} = 0, \qquad r = 1, \cdots, m.$$

Next, we check the elements of the Hessian matrix of $\phi(\mathbf{x}, \mathbf{v})$ at $(\mathbf{0}, \mathbf{0})$. We have for $i, j = 1, \cdots, n$ and $r, s = 1, \cdots, m$,

$$\left.\frac{\partial^2 \phi(\mathbf{x}, \mathbf{v})}{\partial x_j \partial x_i}\right|_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}} = a \left[\frac{\partial^2 V}{\partial x_j \partial x_i} - \sum_{h,l=1}^{n} \frac{\partial^2 V}{\partial \bar{f}_h^{\mathbf{v}(k)} \partial \bar{f}_l^{\mathbf{v}(k)}} \frac{\partial \bar{f}_h^{\mathbf{v}(k)}}{\partial x_i} \frac{\partial \bar{f}_l^{\mathbf{v}(k)}}{\partial x_j} \right]_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}}$$

$$\left.\frac{\partial^2 \phi(\mathbf{x}, \mathbf{v})}{\partial v_r \partial x_i}\right|_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}} = -a \left[\sum_{h,l=1}^{n} \frac{\partial^2 V}{\partial \bar{f}_h^{\mathbf{v}(k)} \partial \bar{f}_l^{\mathbf{v}(k)}} \frac{\partial \bar{f}_h^{\mathbf{v}(k)}}{\partial x_i} \frac{\partial \bar{f}_l^{\mathbf{v}(k)}}{\partial v_r} \right]_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}}$$

$$\left.\frac{\partial^2 \phi(\mathbf{x}, \mathbf{v})}{\partial v_s \partial v_r}\right|_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}} = 2\delta_{rs} - a \left[\sum_{h,l=1}^{n} \frac{\partial^2 V}{\partial \bar{f}_h^{\mathbf{v}(k)} \partial \bar{f}_l^{\mathbf{v}(k)}} \frac{\partial \bar{f}_h^{\mathbf{v}(k)}}{\partial v_r} \frac{\partial \bar{f}_l^{\mathbf{v}(k)}}{\partial v_s} \right]_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}}.$$

Denote $\tilde{\phi}(\mathbf{x}(k)) = \phi(\mathbf{x}(k), \mathbf{0}) = a\left(V(\mathbf{x}(k)) - V(\bar{f}^{\mathbf{0}}(\mathbf{x}(k))\right)$, so that

$$\left.\frac{\partial^2 \phi(\mathbf{x}, \mathbf{v})}{\partial x_j \partial x_i}\right|_{\mathbf{x}=\mathbf{0}, \mathbf{v}=\mathbf{0}} = \left.\frac{\partial^2 \tilde{\phi}(\mathbf{x})}{\partial x_j \partial x_i}\right|_{\mathbf{x}=\mathbf{0}}. \tag{12}$$

Since $\tilde{\phi}(\mathbf{x})$ has a local minimum at $\mathbf{x} = \mathbf{0}$, and by assumption, the determinant of Hessian matrix of the storage function $V(\mathbf{x})$ at $\mathbf{x} = \mathbf{0}$ is non-zero, we obtain that the eigenvalues of the Hessian matrix of $\tilde{\phi}(\mathbf{x})$ at $\mathbf{x} = \mathbf{0}$ are all positive. Denote these eigenvalues by λ_i, $\forall i = 1, 2, \cdots, n$. Furthermore, the Hessian matrix of $\tilde{\phi}(\mathbf{x})$ at $\mathbf{x} = \mathbf{0}$ is symmetric and can be diagonalized. Thus, with an appropriate choice of coordinates, the Hessian matrix of $\phi(\mathbf{x}, \mathbf{v})$ at $(\mathbf{0}, \mathbf{0})$ can be evaluated to be of the form

$$\begin{bmatrix} a\lambda_1 & \cdots & 0 & ab_{11} & \cdots & ab_{1m} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & a\lambda_n & ab_{n1} & \cdots & ab_{nm} \\ ab_{11} & \cdots & ab_{n1} & 2 + ac_{11} & \cdots & ac_{1m} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ ab_{1m} & \cdots & ab_{nm} & ac_{m1} & \cdots & 2 + ac_{mm} \end{bmatrix}. \tag{13}$$

Now, we apply [17, Lemma 12] which states that for $\lambda_i > 0$ and $\forall a = (0, \hat{a}),\ \hat{a} = \min_j a_j^u$ where

$$a_j^u = \min\left\{1, \frac{2^j \lambda_1 \cdots \lambda_n - \epsilon}{|\alpha_1| + \cdots + |\alpha_j|}\right\},\ j = 1, \cdots, m \tag{14}$$

with $0 < \epsilon \ll 1$ and $\alpha_l,\ l = 1, \cdots, j$ being some constants related to λ_i, b_{il} and $c_{rl},\ i = 1, \cdots, n$, $r = 1, \cdots, j$, $l = 1, \cdots, j$, the determinant of matrix (13) is greater than zero. Sylester's criterion now readily yields that the Hessian matrix of $\phi(\mathbf{x}, \mathbf{v})$ at $(\mathbf{0}, \mathbf{0})$ as evaluated in (13) is positive definite. Therefore, $\phi(\mathbf{x}, \mathbf{v})$ has a local minimum at $(\mathbf{0}, \mathbf{0})$. Thus, at the times when $d(k) = 2$, the relation (10) holds. Summing (10) for all the time steps k in the closed loops, we then obtain the following inequality

$$\sum_{\substack{k:d(k)=2\\ k\le T-1}} \tilde{V}(\bar{f}^{\mathbf{v}(k)}(\mathbf{x}(k)) - \tilde{V}(\mathbf{x}(k)) \le \sum_{\substack{k:d(k)=2\\ k\le T-1}} \mathbf{v}^{\mathrm{T}}(k)\mathbf{v}(k). \tag{15}$$

with the equality holds at $(\mathbf{0}, \mathbf{0})$.

When $d(k) = 1$, systems $\mathcal{S}_1$ and $\mathcal{S}_2$ evolve in an identical manner and therefore (7) yields that

$$\begin{aligned}\tilde{V}(\bar{f}^{\mathbf{0}}(\mathbf{x}(k))) - \tilde{V}(\mathbf{x}(k)) &= \tilde{V}(f(\mathbf{x}(k), \mathbf{0})) - \tilde{V}(\mathbf{x}(k)) = a\left(V(f(\mathbf{x}(k), \mathbf{0})) - V(\mathbf{x}(k))\right)\\ &\le a(\zeta - 1)V(\mathbf{x}(k)).\end{aligned} \tag{16}$$

We now choose a in the interval $(0, \tilde{a})$ where

$$\tilde{a} = \min_T \frac{\sum_{\substack{k:d(k)=2\\ k\le T-1}} \phi(\mathbf{x}(k), \mathbf{v}(k))}{(\zeta - 1)\sum_{\substack{k:d(k)=1\\ k\le T-1}} V(\mathbf{x}(k))}, \forall T \in \mathbb{Z}^+,$$

then the following inequality is satisfied,

$$a(\zeta - 1)\sum_{\substack{k:d(k)=1\\ k\le T-1}} V(\mathbf{x}(k)) + \sum_{\substack{k:d(k)=2\\ k\le T-1}} \left[\tilde{V}(\bar{f}^{\mathbf{v}(k)}(\mathbf{x}(k)) - \tilde{V}(\mathbf{x}(k))\right] \le \sum_{\substack{k:d(k)=2\\ k\le T-1}} \mathbf{v}^{\mathrm{T}}(k)\mathbf{v}(k). \tag{17}$$

Since $\sum_{\substack{k:d(k)=1\\ k\le T-1}} \left[\tilde{V}(\bar{f}^{\mathbf{0}}(\mathbf{x}(k))) - \tilde{V}(\mathbf{x}(k))\right] + \sum_{\substack{k:d(k)=2\\ k\le T-1}} \left[\tilde{V}(\bar{f}^{\mathbf{v}(k)}(\mathbf{x}(k)) - \tilde{V}(\mathbf{x}(k))\right] = \tilde{V}(\mathbf{x}(T)) - \tilde{V}(\mathbf{x}(1))$,

then according to the inequalities (16) and (17), there exists $a \in (0, \min(\hat{a}, \tilde{a}))$, such that the inequality (9) holds with the equality holding if and only if $(\mathbf{x}, \mathbf{v}) = (\mathbf{0}, \mathbf{0})$. ■

Given this result, we can now establish that passivity of $\mathcal{S}_2$ implies local passivity of $\mathcal{S}$.

Theorem 3.2: Let system $\mathcal{S}_2$ be passive such that the inequalities (7) hold. Furthermore, let system $\mathcal{S}$

evolve from the same initial condition and with the same mode switching signal as the system $\mathcal{S}_2$. Then, the system $\mathcal{S}$ is locally passive.

Proof: Under the stated assumptions, we know from Theorem 3.1 that for the system $\mathcal{S}_1$ there exists a positive semidefinite storage function $\tilde{V}(\mathbf{x}(k)) \geq 0$ with $\tilde{V}(\mathbf{x}(k)) = 0$ if and only if $\mathbf{x}(k) = \mathbf{0}$, such that for any $T \in \mathbb{Z}^+$, $\tilde{V}(\mathbf{x}(T)) - \tilde{V}(\mathbf{x}(1)) \leq \sum_{\substack{k:d(k)=2 \\ k \leq T-1}} \mathbf{v}^{\mathrm{T}}(k)\mathbf{v}(k)$. For the system $\mathcal{S}$, consider the storage function $\tilde{\tilde{V}} = \rho\tilde{V}$ where $\rho > 0$ is a constant to be suitably designed. Also, define the term $\eta(T) = \sum_{k=1}^{T-1} \mathbf{u}^{\mathrm{T}}(k)\mathbf{y}(k)$. Since both $\mathbf{u}(k)$ and $\mathbf{y}(k)$ are bounded in the neighborhood of $\mathbf{x}(k) = \mathbf{0}$ and $\mathbf{v}(k) = \mathbf{0}$, we see that $\eta(T)$ is also bounded. Now, there are two cases.

1) If $\eta(T) \geq 0$, we have $\tilde{\tilde{V}}(\mathbf{x}(T)) - \tilde{\tilde{V}}(\mathbf{x}(1)) = \rho(\tilde{V}(\mathbf{x}(T)) - \tilde{V}(\mathbf{x}(1))) \leq \rho \sum_{\substack{k:d(k)=2 \\ k \leq T-1}} \mathbf{v}^{\mathrm{T}}(k)\mathbf{v}(k)$. The inequality (3) holds if $0 < \rho \leq \frac{\eta(T)}{\sum_{\substack{k:d(k)=2 \\ k \leq T-1}} \mathbf{v}^{\mathrm{T}}(k)\mathbf{v}(k)}$.
2) If $\eta(T) < 0$, this corresponds to the case when the system $\mathcal{S}$ is Lyapunov stable as well. Because $\eta(T)$ is bounded, we can guarantee that with a sufficiently large choice of ρ (that depends on $\mathbf{x}(1)$ and $\{\mathbf{v}(k)\}$), the following inequality holds: $\tilde{\tilde{V}}(\mathbf{x}(T)) - \tilde{\tilde{V}}(\mathbf{x}(1)) = \rho(\tilde{V}(\mathbf{x}(T)) - \tilde{V}(\mathbf{x}(1))) \leq \eta(T) = \sum_{k=1}^{T-1} \mathbf{u}^{\mathrm{T}}(k)\mathbf{y}(k) \leq 0, \ \forall \rho > 0$, where we choose ρ

$$\rho \geq \frac{\eta(T)}{\tilde{V}(\mathbf{x}(T)) - \tilde{V}(\mathbf{x}(1))}. \tag{18}$$

Thus, given any T, we can design the constant $\rho > 0$ and the corresponding storage function $\tilde{\tilde{V}} = \rho\tilde{V}, \ \rho > 0$ such that the system $\mathcal{S}$ is locally passive in the neighborhood of $\mathbf{x}(k) = \mathbf{0}$ and $\mathbf{v}(k) = \mathbf{0}$. ■

C. Stability and Interconnections of Passive Systems

We now prove that the Definition 2.1 preserves some of the important properties of the classical passivity.

Theorem 3.3: If system $\mathcal{S}$ is passive and locally ZSD, under a feedback control law of the form $\mathbf{u}(k) = -\psi(\mathbf{y}(k))$ where $\psi(\mathbf{0}) = \mathbf{0}$ and $\mathbf{y}^{\mathrm{T}}(k)\psi(\mathbf{y}(k)) > 0, \ \forall \mathbf{y} \neq \mathbf{0}$, then the equilibrium $(\mathbf{0}, \mathbf{0})$ is locally asymptotically stable.

Proof: According to the passivity definition, for every time step k in the closed loop, we have

$$\tilde{\tilde{V}}(f(\mathbf{x}(k), \mathbf{u}(\mathbf{x}(k)))) - \tilde{\tilde{V}}(\mathbf{x}(k)) \leq \mathbf{u}^{\mathrm{T}}(k)\mathbf{y}(k) = -\mathbf{y}^{\mathrm{T}}(k)\psi(\mathbf{y}(k)) \leq 0$$

with equality holding if and only if $\mathbf{y}(k) = \mathbf{0}$. The total increase in the storage function during open loops in a period $[1, T]$ is always bounded (conditions (7)). Under the control $\mathbf{u}(k)$, we have $\tilde{\tilde{V}}(f(\mathbf{x}(T))) - \tilde{\tilde{V}}(\mathbf{x}(1)) \leq 0$ i.e., the storage function is non-increasing as compared with its initial value. Therefore, the system $\mathcal{S}$ is stable as $T \to +\infty$. Moreover, according to ZSD, $\mathbf{y}(T) = \mathbf{0}$ implies that $\mathbf{x}(T) \to \mathbf{0}$ and the system is locally asymptotically stable. ■

Theorem 3.4: If two switched nonlinear systems $\mathcal{S}^1$ and $\mathcal{S}^2$ are both passive, then their parallel and negative feedback interconnections (as defined in Figure 1) are both passive.

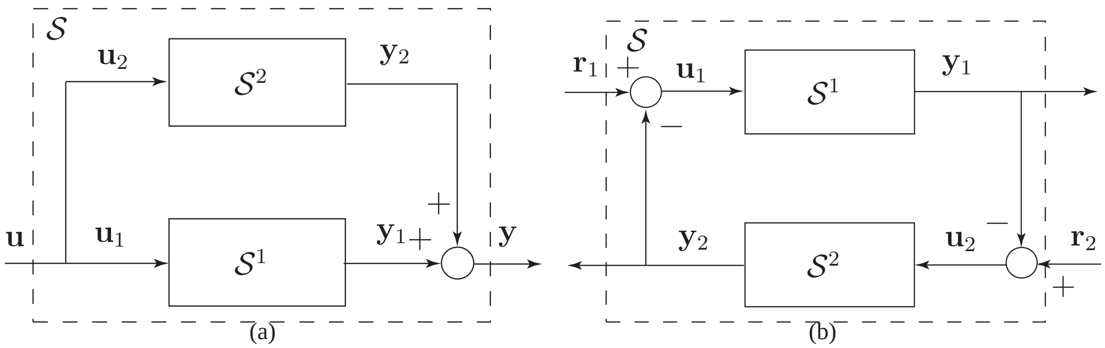


Fig. 1. (a) Parallel, and (b) negative feedback interconnections for two passive switched nonlinear systems $\mathcal{S}^1$ and $\mathcal{S}^2$.

Proof: Let the control inputs for $\mathcal{S}^i$ be $\mathbf{u}_i(k)$, the corresponding output be $\mathbf{y}_i(k)$ and the storage function be $\tilde{\tilde{V}}_i(k)$. For the parallel interconnection, we have for the interconnected system $\mathcal{S}$, the control input $\mathbf{u}(k) = \mathbf{u}_1(k) = \mathbf{u}_2(k)$ and the output $\mathbf{y}(k) = \mathbf{y}_1(k) + \mathbf{y}_2(k)$. For $\mathcal{S}$, consider the storage function $\tilde{\tilde{V}}(k) = \tilde{\tilde{V}}_1(k) + \tilde{\tilde{V}}_2(k)$. For any time $T \in \mathbb{Z}^+$, we have

$$\begin{aligned}\tilde{\tilde{V}}(\mathbf{x}(T)) - \tilde{\tilde{V}}(\mathbf{x}(1)) &= (\tilde{\tilde{V}}_1(\mathbf{x}(T)) - \tilde{\tilde{V}}_1(\mathbf{x}(1))) + (\tilde{\tilde{V}}_2(\mathbf{x}(T)) - \tilde{\tilde{V}}_2(\mathbf{x}(1))) \\ &\le \sum_{k=1}^{T-1} \mathbf{u}_1^{\mathrm{T}}(k)\mathbf{y}_1(k) + \sum_{k=1}^{T-1} \mathbf{u}_2^{\mathrm{T}}(k)\mathbf{y}_2(k) \le \sum_{k=1}^{T-1} \mathbf{u}^{\mathrm{T}}(k)\mathbf{y}(k).\end{aligned}$$

Similarly, for the negative feedback interconnection, we have for the interconnected system $\mathcal{S}$, the control inputs and outputs as $\mathbf{r}_1(k) = \mathbf{u}_1(k) + \mathbf{y}_2(k)$ and $\mathbf{r}_2(k) = \mathbf{u}_2(k) + \mathbf{y}_1(k)$. Consider the storage function $\tilde{\tilde{V}}(k) = \tilde{\tilde{V}}_1(k) + \tilde{\tilde{V}}_2(k)$. For any time $T \in \mathbb{Z}^+$, we have $\tilde{\tilde{V}}(\mathbf{x}(T)) - \tilde{\tilde{V}}(\mathbf{x}(1)) \le \sum_{k=1}^{T-1}(\mathbf{r}_1^{\mathrm{T}}(k)\mathbf{y}_1(k) + \mathbf{r}_2^{\mathrm{T}}(k)\mathbf{y}_2(k))$. ■

Remark 3.4: The main results in this section can be shown to hold as long as the switched system $\mathcal{S}$ satisfies Equation (3) at a given time instant T. In this case, the value of $\tilde{a}$ can be chosen as

$$\tilde{a} = \frac{\sum_{\substack{k:d(k)=2 \\ k \le T-1}} \phi(\mathbf{x}(k), \mathbf{v}(k))}{(\zeta - 1)\sum_{\substack{k:d(k)=1 \\ k \le T-1}} V(\mathbf{x}(k))}.$$

This implies a more general case when the increase in storage function may exceed the accumulated energy supplied to it at closed loops as well. The results can be applied to a periodically controlled system that achieves passivity periodically [21].

IV. Numerical Example

Consider a closed loop system of the form

$$\begin{aligned} x_1(k+1) &= -0.3x_1^2(k)x_2(k) + 1.5x_2(k) + u(k) \\ x_2(k+1) &= x_1(k) - u^2(k) \\ y(k) &= 2x_2(k) + u(k), \end{aligned}$$

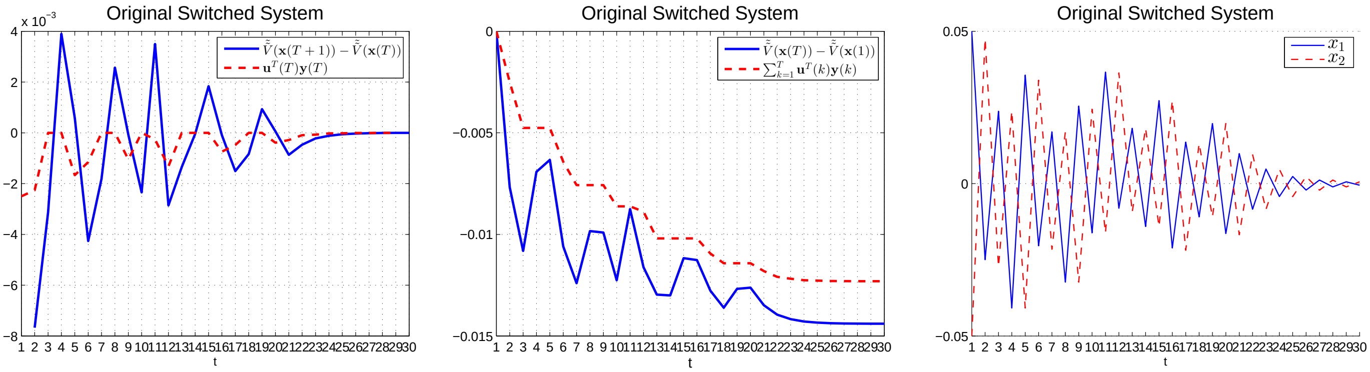
with the controller $u(k) = -y(k) = -x_2(k)$. Note that the system is locally ZSD and has zero relative degree. The evolution of the system in Mode 2 is given when $u(k) = \mathbf{0}$. The transformed dynamics and the zero dynamics remain identical in Mode 2. In Mode 1, the transformed dynamics and the zero dynamics can be obtained as Equations (4) and (5) in Section III-A. Given the zero dynamics, we choose a quadratic storage function $V(\mathbf{x}(k)) = \mathbf{x}(k)^{\mathrm{T}}P\mathbf{x}(k) = x_1^2 + 0.5x_2^2$. We can verify that the determinant of the Hessian matrix of $V(\mathbf{x}(k))$ at $\mathbf{x}(k) = [0\ 0]^{\mathrm{T}}$ is not zero. The parameters in the condition (7) are $\zeta = 2.8$ and $\sigma = 0.6$. According to (8) then,

$$r(T) > \frac{1.0296(T-1)}{1.0296 + 0.5108T} \tag{19}$$

would guarantee system passivity. This condition is satisfied, e.g., by a periodic system in which at every third time step (i.e., at $k = 3, 6, 9, \cdots$) the system is in Mode 2. However, the system need not be periodic to satisfy (19). If the system starts in Mode 1, then any communication protocol that guarantees that out of every 3 consecutive control packets, at most one packet is not delivered would guarantee passivity. Thus the maximum allowable transmission interval (MATI) is 2 [22], [23].

The storage function $\tilde{V}(\mathbf{x}(k)$ for the transformed system is chosen as $0.48V(\mathbf{x}(k))$ with $\hat{a} = 0.48$ and $\tilde{a} = 0.9939$. The storage function for the original switched nonlinear system can be chosen as $12\tilde{V}(\mathbf{x}(k))$ with $\rho = 8$ satisfies inequality (18) under the case when $\eta(T) < 0$.

More insight can be obtained if we consider the system to operate over a finite horizon. Consider the system operation from $k = 1$ to 30. We consider the system to be in Mode 2, i.e., non-passive according to the classical passivity definition, at time steps $k = 4, 5, 8, 9, 11, 14, 15, 16, 19, 20$ as shown in Figure 2(a). Figure 2(b) shows the corresponding passivity inequality for the system. We can see that unlike the classical case, the storage function is now allowed to increase; however, all the passivity inequalities are satisfied at every time till T. Figure 2(c) shows the evolution of the state dynamics of the switched system. Both states are locally asymptotic stable at the origin.



(a) (b) (c)

Fig. 2. (a) Passivity check for the switched system in Example 1 in the time interval $[1, 30]$ according to classical passivity definition, (b) Passivity check for the switched system according to the generalized passivity definition (3), and (c) State dynamics of the switched system.

V. Conclusions and Future Work

We analyzed passivity for a class of discrete-time switched nonlinear systems that switch between two modes - an uncontrolled mode in which the system evolves open loop, and a controlled mode in which a control is applied to the system. This situation is of interest in, e.g., networked control systems where the communication network can erase control packets transmitted to the plant. For the case when the open loop system is non-passive, classical passivity theory considers the switched system to be non-passive as well. We give a new generalized definition of passivity for such a system and show that if the ratio of the time steps for which the system evolves open loop versus the time steps for which the system evolves closed loop is bounded below a critical ratio, then the system is locally passive in this sense. Moreover, we show that this generalized definition is useful since it preserves two important properties of the classical passivity concept - that passivity implies asymptotic stability for zero state detectable systems using feedback and that passivity is preserved in parallel and feedback interconnections.

There are multiple directions in which this work can be extended. The most obvious is relaxing the requirement for systems to have relative degree zero. More general hybrid system, including systems with state dependent and event-triggered switching modes can also be considered. Finally, networked control systems with random delays and data loss can also be considered under this framework.

VI. Acknowledgement

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Appendix

Background on Passivity

Consider a system of the form

$$\begin{cases} \mathbf{x}(k+1) = f(\mathbf{x}(k), \mathbf{u}(k)) \\ \mathbf{y}(k) = h(\mathbf{x}(k), \mathbf{u}(k)) \end{cases}, \tag{20}$$

where $\mathbf{x} \in \mathbf{X} = \mathbb{R}^n$, $\mathbf{u} \in \mathbf{U} = \mathbb{R}^m$ and $\mathbf{y} \in \mathbf{Y} = \mathbb{R}^m$ are the state, input, and output variables, respectively. $\mathbf{X}, \mathbf{U}$ and $\mathbf{Y}$ are the state, input, and output spaces, respectively. $k \in \mathbb{Z}^+$, f and h are smooth. All consideration are restricted to an open set of $\mathbf{X} \times \mathbf{U}$ containing the equilibrium point $(\mathbf{x}^*, \mathbf{u}^*)$ having $\mathbf{x}^* = f(\mathbf{x}^*, \mathbf{u}^*)$. Without loss of generality, it is assumed that $(\mathbf{x}^*, \mathbf{u}^*) = (\mathbf{0}, \mathbf{0})$ and $h(\mathbf{0}, \mathbf{0}) = \mathbf{0}$.

Definition A.1. (Dissipative Systems) [18] A system of the form (20) is said to be *dissipative* with respect to the *supply rate* $w \in \mathbf{U} \times \mathbf{Y} \to \mathbb{R}$ if there exists a positive semidefinite function $V : \mathbf{X} \to \mathbb{R}^+$, $V(0) = 0$, called the *storage function*, such that

$$V(f(\mathbf{x}(k), \mathbf{u}(k))) - V(\mathbf{x}(k)) \leq w(\mathbf{y}(k), \mathbf{u}(k)),\ \forall (\mathbf{x}(k), \mathbf{u}(k)) \in \mathbf{X} \times \mathbf{U},\ \forall k.$$

Note that the above inequality holds if and only if

$$V(f(\mathbf{x}(k), \mathbf{u}(k))) - V(\mathbf{x}(0)) \leq \sum_{\theta=0}^{k} w(\mathbf{y}(\theta), \mathbf{u}(\theta)),\ \forall (\mathbf{x}(k), \mathbf{u}(k)) \in \mathbf{X} \times \mathbf{U},\ \forall k.$$

Definition A.2. (Passive Systems) [18] A system of the form (20) is said to be *passive* if it is dissipative with respect to the supply rate $w(\mathbf{y}(k), \mathbf{u}(k)) = \mathbf{u}^{\mathrm{T}}(k)\mathbf{y}(k)$. That is,

$$V(f(\mathbf{x}(k), \mathbf{u}(k))) - V(\mathbf{x}(k)) \leq \mathbf{u}^{\mathrm{T}}(k)\mathbf{y}(k),\ \forall (\mathbf{x}(k), \mathbf{u}(k)) \in \mathbf{X} \times \mathbf{U},\ \forall k.$$

References

[1] J. Baillieul and P. J. Antsaklis, “Control and Communication Challenges in Networked Real-Time Systems,” *Proceedings of the IEEE*, vol. 95, no. 1, pp. 9–27, 2007.

[2] M. Yu, L. Wang, T. Chu, and F. Hao, “An LMI Approach to Networked Control Systems with Data Packet Dropout and Transmission Delays,” *43rd IEEE Conference on Decision and Control*, vol. 4, pp. 3545 – 3550, December 2004.

[3] J. Xiong and J. Lam, “Stabilization of Linear Systems over Networks with Bounded Packet Loss,” *Automatica*, vol. 43, pp. 80–87, January 2007.

[4] J. C. Willems, “Dissipative Dynamical Systems Part I: General Theory,” *Archive for Rational Mechanics and Analysis*, vol. 45, no. 5, pp. 321–351, 1972.

[5] J. C. Willems, “Dissipative Dynamical Systems Part II: Linear Systems with Quadratic Supply Rates,” *Archive for Rational Mechanics and Analysis*, vol. 45, no. 5, pp. 352–393, 1972.

[6] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, 2002.

[7] J. Bao and P. L. Lee, *Process Control: the Passive Systems Approach*. Springer, 2007.

[8] X. Koutsoukos, N. Kottenstette, J. Hall, P. J. Antsaklis, and J. Sztipanovits, “Passivity-Based Control Design for Cyber-Physical Systems,” *International Workshop on Cyber-Physical Systems Challenges and Applications*, June 2008.

[9] M. J. McCourt and P. J. Antsaklis, “Stability of Networked Passive Switched Systems,” *49th IEEE Conference on Decision and Control*, pp. 1263–1268, December 2010.

[10] H. Yu and P. J. Antsaklis, “Event-Triggered Real-Time Scheduling for Stabilization of Passive/Output Feedback Passive Systems,” *American Control Conference*, pp. 1674–1679, 2011.

[11] R. Naldi and R. G. Sanfelice, “Passivity-based Controllers for a Class of Hybrid Systems with Applications to Mechanical Systems Interacting with their Environment,” *Proc. Joint Conference on Decision and Control and European Control Conference*, pp. 7416–7421, December 2011.

[12] M. Žefran, F. Bullo, and M. Stein, “A Notion of Passivity for Hybrid Systems,” *40th IEEE Conference on Decision and Control*, 2001.

[13] J. Zhao and D. J. Hill, “Dissipativity theory for switched systems,” *IEEE Transactions on Automatic Control*, vol. 53, pp. 941–953, May 2008.

[14] A. N. Michel and L. Hou, “Relaxation of Hypotheses in Lassalle-Krasovskii Type Invariance Results,” *SIAM Journal on Control and Optimization*, vol. 49, pp. 1383–1403, July 2011.

[15] C. I. Byrnes, A. Isidiri, and J. C. Willems, “Passivity, Feedback Equivalence, and the Global Stabilization of Minimum Phase Nonlinear Systems,” *IEEE Transactions on Automatic Control*, vol. 36, pp. 1228–1240, November 1991.

[16] E. M. Eavarro-López, *Dissipativity and Passivity-Related Properties in Nonlinear Discrete-Time Systems*. PhD thesis, Universitat Politècnica de Catalunya, 2002.

[17] E. M. Eavarro-López and E. Foddas-Colet, “Feedback Passivity of Nonlinear Discrete-Time Systems with Direct Input-Output Link,” *Automatica*, vol. 40, no. 8, pp. 1423–1428, 2004.

[18] C. I. Byrnes and W. Lin, “Lossless, Feedback Equivalence, and the Global Stabilization of Discrete-Time Nonlinear Systems,” *IEEE Transactions on Automatic Control*, vol. 39, pp. 83–98, January 1994.

[19] W. Lin and C. I. Byrness, “Passivity and Absolute Stabilization of a Class of Discrete-time Nonlinear Systems,” *Automatica*, vol. 31, no. 2, pp. 263–267, 1995.

[20] R. C. James and G. James, *Mathematics Dictionary*. Springer, 1992.

[21] Y. Wang, V. Gupta, and P. J. Antsaklis, “Passivity Analysis for Discrete-Time Periodically Controlled Systems,” *American Control Conference (ACC)*, June 2012. under review.

[22] D. Nešić and A. R. Teel, “Input-Output Stability Properties of Networked Control Systems,” *IEEE Transactions on Automatic Control*, vol. 49, pp. 1650–1667, October 2004.

[23] X. Wang and M. Lemmon, “Event-Triggering in Distributed Networked Control Systems,” *IEEE Transactions on Automatic Control*, vol. 56, pp. 586–601, March 2011.