This essay appeared in the *Canadian Journal of Philosophy* Vol. 29, No. 2 (June 1999), 205-224

Relative Identity and Cardinality[1]

Patricia A. Blanchette

University of Notre Dame

blanchette.1@nd.edu

Abstract

Because cardinality-statements bear close logical connections to identity-statements, it has been argued that Frege's treatment of cardinality, in which cardinality is "relative to" a concept, implies Geach's treatment of identity, in which identity is "relative to" a predicate. If this is right, then those holding a broadly Fregean account of cardinality must accept Geach's very controversial position on identity. The purpose of this paper is to argue that Frege's position does not imply Geach's, but is instead inconsistent with it. Despite superficial similarities between the two doctrines, the problems with Geach's account give no reason to reject Frege's.

Peter Geach famously holds that there is no such thing as absolute identity. There are rather, as Geach sees it, a variety of relative identity relations, each essentially connected with a particular monadic predicate. Though we can strictly and meaningfully say that an individual a is the same *man* as the individual b, or that a is the same *statue* as b, we cannot, on this view, strictly and meaningfully say that the individual a simply *is* b.

It is difficult to find anything like a persuasive argument for this doctrine in Geach's work. But one claim made by Geach is that his account of identity is the account most naturally aligned with Frege's widely-admired treatment of cardinality. And though this claim of an affinity between Frege's and Geach's doctrines has been challenged (see Perry 1978), the challenge has been resisted (Alston and Bennett 1984). William Alston and Jonathan Bennett, indeed, go further than Geach to argue that Frege's doctrine *implies* Geach's. If Alston and Bennett are right, then those who favor a broadly Fregean treatment of cardinality have no choice but to adopt Geach's doctrine of relative identity. And because contemporary set-theoretic accounts of cardinality are essentially Fregean in all respects relevant to this issue (see §II below), it follows from Alston and Bennett's claim that virtually every standard modern treatment of cardinality requires a "relativized" treatment of identity.

The purpose of this paper is to argue against the implication claimed by Alston and Bennett. Despite some superficial similarities between Frege's treatment of cardinality and Geach's treatment of identity, the two doctrines are fundamentally opposed. Frege's account of cardinality, and its set-theoretic descendents, are all inconsistent with the claim that identity is "relative" in Geach's sense.

The Similarity

There are two important respects in which Frege's and Geach's doctrines are, on the surface at least, quite similar:

First: While Geach holds that a monadic predicate is involved essentially in every meaningful identity-statement, Frege holds that a concept - the kind of thing to which a monadic predicate typically refers - is involved essentially in every meaningful cardinality-statement. That is, while on Geach's view we cannot say simply that x and y are *the same*, but must say instead that they are the *same F* for some appropriate F, Frege says that we cannot say of a collection that it simply numbers *two*, but must say instead that it numbers *two F's*, for some appropriate F. Frege and Geach are further agreed that cardinality-statements and identity-statements bear close logical connections to one another: if F(x) and F(y), then, as both see it, x and y are *two* F's if and only if they are not the *same* F. Thus there seem grounds for sympathy with Geach's view that anyone who takes the "F" to occur essentially in the cardinality-statement ought also to take it to occur essentially in the identity-statement.

The second (apparent) agreement between Frege and Geach concerns cardinality-statements directly. Both Frege and Geach deny that cardinality applies directly to collections; pointing at a grove of trees, for example, we cannot strictly and meaningfully say, on either view, that it has a particular, determinate number. Further, both agree that a given collection can number exhaustively n F's, while also numbering exhaustively m G's, for $n \neq m$. This "relativization" of cardinality on Geach's part is of a piece with his relativization of identity: as detailed below, these theses are two sides of the same coin. Thus it would seem, again, that Frege's doctrine is dangerously close to Geach's.

The burden of the rest of this paper is to demonstrate that these similarities between Frege's and Geach's doctrines are no more than superficial. First, a brief review of each doctrine.

<u>II. Frege on Cardinality</u>

Frege holds that "the content of a statement of number is an assertion about a concept." As he explains,

> ...if I place a pile of playing cards in [someone's] hands with the words: find the number of these, this does not tell him whether I

> wish to know the number of cards, or of complete packs of cards, or even say of honour cards at skat. To have given him the pile in his hands is not yet to have given him completely the object he is to investigate; I must add some further word - cards, or packs, or honours.[2]
>
> While looking at one and the same external phenomenon, I can say with equal truth both "It is a copse" and "It is five trees," or both "Here are four companies" and "Here are 500 men." Now what changes here from one judgement to the other is neither any individual object, nor the whole, the agglomeration of them, but rather my terminology. But that is itself only a sign that one concept has been substituted for another. This suggests ... that the content of a statement of number is an assertion about a concept. This is perhaps clearest with the number 0. If I say "Venus has 0 moons," there simply does not exist any moon or agglomeration of moons for anything to be asserted of; but what happens is that a property is assigned to the *concept* "moon of Venus", namely that of including nothing under it. If I say "the King's carriage is drawn by four horses," then I assign the number four to the concept "horse that draws the King's carriage."[3]

The doctrines to which Frege takes his own view to be opposed, and against which he is primarily concerned to argue, are that cardinality applies to objects, and that cardinality applies to collections, or "agglomerations," of objects. That cardinality doesn't apply to objects individually is clear when we note that in saying "four horses draw the King's carriage," we do not mean to attribute any property to each of the horses. And that cardinality doesn't apply simply to collections of objects is shown by the fact that a given collection can be counted in a number of different ways: to be given the collection by itself is not, yet, to be told which units are to be counted.

The role of the concept is made most clear when Frege points out that not just any concept will do. The concept *red*, unlike, say, *red pen on my desk*, has no determinate number, since it does not "isolate what falls under it in a definite manner."[4] As we would put it, *red* is not a sortal concept. It is only once a sortal concept has been specified, as Frege sees it, that the question of cardinality becomes determinate. The reason a given pile P can be counted in different ways is that there are different ways of dividing it up into units. In Fregean terms, there are different concepts (e.g., *pack in pile P* and *card in pile P*), typically with different cardinalities, each such that the items falling under the concept together make up the pile.

Though Frege's view is opposed to the attribution of cardinality to individuals or to collections, it is not opposed to the attribution of cardinality to sets, in the modern sense. To claim e.g. that a concept F has two objects falling under it is just to claim, as Frege sees it, that F's extension has two members. The cardinality of a set is just the cardinality of every concept whose extension is that set.[5] It is worth noting that the usual post-Fregean treatment of cardinality, which bypasses talk of concepts and assigns cardinality directly to sets, is in agreement with Frege that there can be more than one assignment of cardinality to a given pile. For there will generally be distinct sets (with distinct cardinalities) whose members together constitute that pile. Corresponding to Frege's concept *card in that pile* is the set {x : x is a card in that pile}. Corresponding to Frege's concept *pack in that pile* is the set {x : x is a pack in that pile}. The members of either set, taken together, constitute the same pile, though their cardinalities are different.

When we characterize Frege's view as the view that *collections* have no determinate cardinalities, it is important to emphasize that by "collection" we mean something which can be divided into parts in any number of different ways, but which has no determinate specification of members. A pile of playing-cards is such a collection; a set of playing-cards is not. On both Frege's and the modern set-theoretic view, concepts and sets have determinate absolute cardinalities, while collections, in the sense just noted, do not.

III. Geach on Identity and Cardinality

Geach holds that all identity is relative. As he famously puts it,

> When one says "x is identical with y," this, I hold, is an incomplete expression; it is short for "x is the same A as y," where "A" represents some count noun understood from the context of utterance - or else, it is just a vague expression of a half-formed thought.[6]

and

> ...there is just no such notion as unqualified identity... [7]

To say that a is the same F as b is not, on Geach's view, to say that a is an F and b is an F and a = b. It is rather to say that a bears the (often primitive) relative-identity relation *same-F-as*, which we shall henceforth abbreviate "$=_F$", to b. There are, as Geach sees it, individuals a and b, and predicates F and G, such that a and b are each both F and G, and $a =_F b$ while $a \neq_G b$. Geach illustrates this doctrine via the example of *surmen*. Define the relative-identity relation "same surman as" as follows:

"a is the same surman as b" $=_{df}$ "a is a man and b is a man, and a has the same surname as b."

Then though Arlo Guthrie is not the same man as Woody Guthrie, Arlo Guthrie is indeed the same surman as Woody Guthrie.

We can extract the predicate "is a surman" from the relation-term "same surman as" by defining "x is a surman" as shorthand for "x is the same surman as something."[8] Then though Woody and Arlo are both men and surmen, Woody is the same surman as Arlo without being the same man as Arlo. It is worth noting that this procedure of defining "is an F" in terms of the relation "same F as" does not commit one to the existence of any new individuals or types of individual. There are not surmen *in addition* to men, with their own identity-criteria; there are simply men, most of whom have exactly one surname and hence are also surmen. A variety of different relative-identity relations hold between these individuals, two examples of which are same-man-as, and same-surman-as.

Given individuals a and b, if there is a relation *same F as* such that a is not the same F as b, then there will typically be another relation *same G as* such that a *is* the same G as b. Thus we have not succeeded in asking a determinate question when we ask whether a is simply "the same" as b. Woody and Arlo are not in any sense, as Geach sees it, "absolutely" distinct: they simply bear some relative-identity relations to one another, and fail to bear others.

The absolute-identity theorist sees things quite differently. As AI (for short) sees it, there is a single, absolute relation of identity which holds exactly between each individual and itself. When we say that Cicero is the same man as Tully, on this view, we mean that Cicero is a man, Tully is a man, and Cicero = Tully. Similarly for all ordinary English phrases of the form "same F as"; to say that x is the same F as y is to say: F(x) and F(y) and x=y. Hence a crucial difference over the logic of identity-statements between AI and Geach: for AI, the statements

(1) a is the same F as b

and

(2) G(a)

imply

(3) a is the same G as b,

while for Geach, as we have seen, this implication does not hold.

AI must admit that Geach's relation-term "same surman as" is perfectly well-defined. For there is indeed an equivalence-relation holding between all pairs of men with the same surname, and it violates none of the usual standards of definition to pick out this relation and give it a name. Nevertheless, AI will claim, there is something very misleading about the choice of name in this case. For in standard English usage, relation-terms of the form "same F as" are used to refer to relations which imply identity, while Geach's "same surman as" does nothing of the sort, standing as it does for a relation which relates a given man to, typically, a massive number of other men.

Geach's doctrine of relative identity implies a doctrine of relative cardinality. If there is no absolute sense in which an individual a is distinct from an individual b, then there is no absolute sense in which the set {a, b} numbers two. If, further, there are relative-identity relations "same F" and "same G" such that a is the same F as b without being the same G as b (while G(a) and G(b)), then the set {a, b} has two relative cardinalities: it numbers one F and two G's. Consider for example Geach's surmen: the set {Arlo Guthrie, Woody Guthrie} contains, on Geach's account, two men but only one surman, despite the fact that each member of the set is both a man and a surman. As Geach puts it,

> It is easy to see that if "A" and "B" represent different count nouns, the count of A's in a domain may be different from the count of B's even if everything in the domain both is an A and is a B... [9]

If there is no absolute relation of identity, then sets have no absolute cardinalities, and vice-versa.

This point holds for concepts as well. Consider the concept *man in this room*, and suppose that in this room are precisely Arlo and Woody Guthrie. Frege would claim that the cardinality of the concept is straightforwardly *two*: there are x and y falling under the concept such that: $x \neq y$, and for every z falling under the concept, either $z = x$ or $z = y$. Geach cannot say this. Given Geach's strictures about identity and about counting, he can say that two *men* fall under the concept, but of course that only one *surman* falls under the concept, since for any x and y falling under the concept, x is the same surman as y. Geach will agree with Frege that concepts agree in cardinality with their extensions. But while Frege holds that each sortal concept (and its extension) has a single absolute cardinality, Geach must hold that each such concept (and its extension) has a variety of relative cardinalities.

In short, then, Geach's account of identity gives rise to an account of cardinality on which each relative-identity relation generates a series of relative-cardinality predicates. The set {Arlo, Woody} numbers 2_{man} and 1_{surman}, since $\text{Arlo} \neq_{man} \text{Woody}$, but $\text{Arlo} =_{surman} \text{Woody}$. In general, where S is a set,

- S has cardinality 1_F iff: $\exists x(x \in S \& \forall y(y \in S \rightarrow y=_F x))$;
- S has cardinality $(n+1)_F$ iff either:
  - $\exists x[(Fx \& (x \in S \& S\text{-}\{x\}$ has cardinality $n_F)) \& \forall y(y \in S\text{-}\{x\} \rightarrow x \neq_F y)]$; or
  - $S = S_1 \cup S_2$, and S_1 has cardinality $(n+1)_F$, and $\forall x(x \in S_2 \rightarrow \exists y(y \in S_1 \& y=_F x))$

Similarly for concepts: a concept C has the Geach relative-cardinality n_F iff its extension does. As Geach puts it, though every man in Leeds is a surman in Leeds and vice-versa (assuming each man in Leeds to have exactly one surname), "if we count the men in Leeds and the surmen in Leeds, we shall get different counts; the count of surmen will be smaller"[10] since "'surmen' and 'men' ... give two ways of counting" the adult male inhabitants of Leeds.[11]

Geach's account of cardinality, then, is not Frege's. Concepts and sets have determinate cardinalities on Frege's account, and do not on Geach's. Though both accounts entail that collections lack absolute cardinalities, this agreement masks a deeper underlying disagreement. For Frege, a collection lacks a determinate cardinality because there is more than one way of dividing it up into units to be counted. Once it is clear which units are to be counted, the cardinality is determinate and absolute. For Geach on the other hand, even after the units have been specified, there remains the question of which identity-relation we are to "count by."

On Geach's account, there are cases in which, as he puts it, a given "domain" numbers n F's and m G's, for n≠m, even though everything in that domain is both an F and a G. Such different counts are, on Frege's view, impossible. If by "domain" we mean "set," then every domain has one absolute cardinality, for Frege. If by "domain" we mean "collection," then a domain can indeed number n F's and m G's on Frege's account, but only when F and G are different sortal concepts, with different extensions. In this case, it cannot be that everything in the domain is both an F and a G. To return to Frege's own example: if the count of cards in the pile differs from the count of decks of cards in the pile, it cannot be that each card is a deck and vice-versa.

IV. The Conditional: Geach

The fact that Geach and Frege differ over cardinality does not yet answer our original question, which is whether Frege's account of cardinality implies or supports Geach's account of identity. Geach's view of the matter is as follows:

> Frege emphasized that 'x is one' is an incomplete way of saying 'x is one A, a single A,' or else has no clear sense; since the connection of the concepts *one* and identity comes out just as much in the German '*ein und dasselbe*' as in the English 'one and the same,' it has always surprised me that Frege did not similarly maintain the parallel doctrine of relativized identity... [12]

> I maintain that it makes no sense to judge whether x and y are 'the same,' or whether x remains 'the same,' unless we add or understand some general term - "the same F." ... Frege sees clearly that "one" cannot significantly stand as a predicate of objects unless it is (at least understood as) attached to a general term; I am surprised he did not see that the like holds for the closely allied expression "the same."[13]

Frege does not in fact say that "x is one" is an incomplete way of saying "x is one A." He does say two things which are relevant. First: as part of the argument against the claim that "one" expresses a property of individual objects, Frege notes some salient differences between statements of the form "x is one" and those which, like "x is wise," do attribute properties to individuals:

> If it were correct to take "one man" in the same way as "wise man," we should expect to be able to use "one" also as a grammatical predicate, and to be able to say "Solon was one" just as much as "Solon was wise." It is true that "Solon was one" can actually occur, but not in a way to make it intelligible on its own in isolation. It may, for example, mean "Solon was a wise man," if "wise man" can be supplied from the context. ... This is even clearer if we take the plural. Whereas we can combine "Solon was wise" and "Thales was wise" into "Solon and Thales were wise," we cannot say "Solon and Thales were one." But it is hard to see why this should be impossible, if "one" were a property both of Solon and of Thales in the same way that "wise" is.[14]

Though we can use "Solon was one" as elliptical for "Solon was a wise man" if, for example, we have just been asked to name some wise men, we cannot take "Solon was one" to predicate the purported property *oneness* of Solon. The oddness of such a property is evidenced in part by noting that it would be entirely different from every other property P of objects in failing the inference from "a has P" and "b has P" to "a and b have P." The point of the last example is not that there is nothing we might assert by saying "Solon and Thales are one," but rather that such a statement does not attribute the purported property *oneness* to each of Solon and Thales.

The second thing Frege says which is relevant to statements of the form "... is/are one" is that every cardinality-statement attributes cardinality to a concept. Thus if a particular statement of the form "... is/are one" successfully makes a cardinality-claim (and is not, say, elliptical for "... is a wise man"), then that statement attributes the second-level property *having a single instance* to a concept. The statement "Cicero and Tully are one," for instance, attributes that second-level property to the concept *identical with Cicero or identical with Tully*.[15] Thus it says something logically equivalent with "Cicero = Tully," and not equivalent with any of Geach's relative-identity claims about Cicero and Tully.

Similarly for explicit use of phrases of the form "one A." Geach's own account of these phrases, on which it is possible for a set to number one A without numbering one B, while everything in the set is both an A and a B, is indeed "parallel to" the doctrine of relative identity. The two doctrines come to exactly the same thing, assuming that, for Geach, "a and b are one A" means "$a=_A b$." But Frege's view of phrases of the form "one A" implies no such relativization of identity. Though for Frege a collection can number one A and n B's for $n \neq 1$ (say, one team and n players), this is always because different entities fall under the different concepts in question (teams under one, players under the other). No collection can number one A and n B's for $n \neq 1$ while every member of the collection is both an A and a B. And since there is nothing "relative" or indeterminate, on Frege's view, about whether a set has exactly one member, there is nothing "relative" or indeterminate about whether its members are all identical with one another.

It is difficult to see exactly why Geach takes Frege's view to be "parallel" to his own. Perhaps, as is arguably implied in the passages quoted above, Geach takes Frege to hold that a collection can number one A without numbering one B, while each member of the collection is both an A and a B. If so, Geach attributes to Frege a view he never held, and indeed which is inconsistent with what he did hold. Perhaps, on the other hand, Geach means only to point to Frege's view that a collection can number one A without numbering one B, when the concepts A and B give different ways of dividing the collection into units. But as we have seen, this doctrine of Frege's has no affinity with the doctrine of relative identity.

V. The Conditional: Alston and Bennett

John Perry has argued, against Geach, that Frege's account of cardinality gives no support to Geach's doctrine of relative identity.[16] The above discussion owes a good deal to Perry's paper. In the course of his argument, Perry points out, among other things, a particularly un-Geach-like feature of Frege's doctrine. This is worth examining in some detail, because it is the primary target of Alston and Bennett's criticism of Perry, and the focus of their defense of the Frege-Geach conditional. Having argued that number is not a property of collections, Perry notes, Frege *could* have gone on to adopt the "doctrine of relative numbers," the doctrine that

> "having the number two" is not a *single* property. There is no such thing as having the number two simpliciter. There are just a bunch of relative number properties: having the pack-number two; having the card-number two; having the honours-at-skat number two, and so forth. ... [T]he pile ... has the pack-number two, and the card-number one hundred four. ...
>
> The doctrine of relative numbers would be a reasonable stable-mate for [Geach's] doctrine of relative identity. To say that

> x is identical with y is to say that x and y are one. So the need to ask "which kind of identity," pressed by the doctrine of relative identity, is merely a special case of the need to ask "what kind of number," pressed by the doctrine of relative numbers.[17]

But, continues Perry,

> It seems clear that Frege did not adopt anything like the doctrine of relative numbers. Rather than multiplying the kinds of numbers attributed to the pile, he rejects the idea that the pile has a number at all.[18]

Indeed, it is abundantly clear that Frege did not adopt the doctrine of relative numbers (henceforth, RN). On Frege's view, cardinality-predicates are absolute and not relative, and cardinality-predications apply always to concepts and never to piles.

Alston and Bennett claim that what Perry has pointed out here is a merely superficial difference between Frege and Geach. Though Frege did not in fact adopt RN, that doctrine is, as Alston and Bennett see it, a harmless variant of Frege's actual view.[19] While Frege's preferred route was to view cardinality-assertions as attributions of absolute cardinality to concepts rather than to piles, he could just as easily have viewed these assertions as attributions of cardinality-with-respect-to-concepts, rather than of absolute cardinality, to piles. That is, instead of viewing the concept as the *subject* of cardinality-predications, Frege could equivalently have incorporated the concept into the cardinality-predicate itself, obtaining RN, an "exact parallel" of Geach's doctrine of relative identity. Alston and Bennett continue:

> Just as we have constructed a variant on Frege's doctrine of cardinality [namely, RN] which does make it run parallel to the relative identity thesis, we could instead modify the latter so as to make it parallel to what Frege actually held about cardinality. Having become convinced that "a=b" won't do as it stands and that a general concept must be lurking somewhere in the vicinity, Geach might, in closer emulation of Frege, have gone on to construe identity as a relation between concepts. Instead of requiring the form "a is the same F as b" he might have opted for "The concept *a which is F* is uniquely coextensive with the concept *b which is F.*"
>
> ... It seems clear that for both topics we can move freely between the "changing the subject" version and the "relativizing the predicate" version, that the two versions are motivated by the same considerations, and that they accommodate the same range of data. Thus Geach can still ask: if we adopt one of these 'generality' theses for number, how can we refuse to adopt some generality thesis for identity?[20]

In short, by harmlessly shifting the monadic predicate out of subject-position and incorporating it into the predicate, we get a variant of Frege's theory which is equivalent to Geach's. By shifting it in Geach's case from the identity-relation back to the subject, we get a Geach-variant which is equivalent with Frege's doctrine.

RN is indeed a harmless variant of Frege's view. We can define the first in terms of the second, as follows: Say that a pile P has the relative cardinality "n-F's" iff P is comprised of objects falling under the concept F, and the concept *F-in-pile-P* numbers (in Frege's original sense) n. Assertions of cardinality will indeed, on this variation of Frege's view, be attributions of "relative" cardinality to piles.

But Alston and Bennett, and Perry, are wrong to characterize the doctrine of relative numbers as in any sense a parallel doctrine to Geach's Relative Identity thesis.

First of all, RN is inconsistent with Geach's relative-cardinality thesis. It is essential to Geach's thesis that a given collection of individuals can number n F's and m G's, for n≠m, *when each individual is both an F and a G.* This can never happen, according to RN. If a pile P has the relative cardinality n-F's and m-G's, in RN's sense, for m≠n, then the concepts *F-in-pile-P* and *G-in-pile-P* number m and n respectively (in the original Frege sense), with the result that these concepts are not coextensive: though their extensions form the same pile, they are different ways of "carving up" that pile into units to be counted. In brief: RN is the doctrine that a given pile has different "relative cardinalities" in the sense that it can be divided into parts in different ways; Geach's doctrine is that a given pile has different "relative cardinalities" because even given a particular way of dividing it into parts, these parts are only "relatively" identical or non-identical with one another.

Similarly for the Geach-variant and the original Frege doctrine. Though Geach can move the monadic predicate out of the relation-position and into subject-position, thereby leaving room for an absolute identity-like relation between newly-complex relata, this will not give us anything equivalent to Frege's doctrine. The Geach-variant must countenance the holding of this identity-like relation between the pairs <Arlo, *surman*> and <Woody, *surman*>, while it fails to hold between <Arlo, *man*> and <Woody, *man*>. The concept *man which is Woody or Arlo* will then number two, while the concept *surman which is Woody or Arlo* will number one. This is certainly

Fregean in its absolute attribution of cardinality to concepts. But it is logically inconsistent with Frege's doctrine, on which if Woody and Arlo are both surmen and men, then *man which is Woody or Arlo* numbers two only if Woody ≠ Arlo, in which case *surman which is Woody or Arlo* must also number two. Geach's original theory, and the Geach variant, give us different "counts" of the same individuals. Frege's original theory, and its variant RN, imply that such different counts are logically impossible.

The difference between Frege's and Geach's treatment of the monadic predicate is considerably more important than their bare grammatical differences imply. For Geach, the role of the predicate F in statements of the form "a is the same F as b" is to tell us which relative-identity relation is in question. Similarly, F's role in cardinality-claims is to determine which relation we are to "count by": because the question of x and y's identity has no absolute answer, neither does the question of whether x and y are one (or two). Once we supply a relativizing predicate, both issues become determinate: x and y are one *man* if, and only if, x bears the relation *same man as* to y; x and y are one *surman* if, and only if, x bears the relation *same surman as* to y. As we have seen, it is possible, on Geach's view, for the first relation to hold without the second, which is why the relativizing predicate is essential.

For Frege on the other hand, the concept (and hence, typically, the predicate) is essential in cardinality-statements because without it, there is no specification of the items to be counted. To say, e.g., "that's *one*" is to say something ambiguous; we might mean to claim that there is one *pack* in question, or one *card* in question, or any number of other things. The connection with absolute identity is straightforward: If there is one *pack* on the table, then, for any packs x and y on the table, x = y; if there is one *card* on the table, then for any cards z and w on the table, z = w. Once it is clear which entities are in question, there is nothing left to be specified: there is no variety of identity-like relations between which to choose.

Geach's view that the statements $a=_F b$ and G(a) do not together imply $a=_G b$ entails his view that the set {a, b} can number one F without numbering one G, though F(a), F(b), G(a) and G(b). Frege's view that the set {a, b} has a determinate cardinality implies that either a=b or a≠b, absolutely. Not only does Frege's treatment of cardinality fail to imply Geach's doctrine of relative identity; it implies the negation of that doctrine.

VI. The Common Problem

In arguing that Frege's doctrine implies Geach's, Alston and Bennett intend not to lend support to Geach, but instead to provide an argument against Frege. As Alston and Bennett see it, Frege's treatment of cardinality shares a serious problem with Geach's treatment of identity; the two theories, as they see it, ought to be rejected together. Though as we have seen there are no grounds for the claim that Frege's doctrine implies Geach's, the further claim - that these theories share a serious problem - needs still to be examined.

The problem, as Alston and Bennett see it, arises for Geach as follows:

> We not infrequently succeed in picking out particular items ... by the use of proper names, definite descriptions, and indexical expressions of various sorts. Given that we have succeeded in picking out something by the use of "a" and in picking out something by the use of "b" it is surely a complete determinate proposition that a=b, that is, it is surely either true or false that the item we have picked out with "b" is the item we have picked out with ["a"]; nor do we have to range a and b, covertly or

> overtly, under a common concept in order to form an identity proposition with a determinate truth-value.[21]

This plausible view of reference and of identity-statements does indeed conflict directly with Geach's doctrine. The question is whether it, or any other similarly plausible view, conflicts with Frege's. Alston and Bennett continue,

> But then why shouldn't we say the same thing about cardinality? Aren't two or more successful singular references sufficient to set up a determinate cardinality question? Suppose we pick out some particular item by "a", one by "b", one by "c", one by "d", and one by "e." Can't we then go on to ask how many that is? And won't that have a determinate answer, assuming that our attempted reference was successful in each case? There is nothing in all this about understanding this question to be really "How many F's?" for any F more specific than "item," or "entity." Indeed there may be no such F available. What if a is the greatest prime number less than 38, b is Jim's copy of *Rasselas*, c is yesterday's thunderstorm, d is President Reagan, and e is Syracuse University? ... Because successful singular reference is an adequate basis for identity propositions, it is also an adequate basis for cardinality propositions. Thus does Frege sink with Geach, rather than Geach floating with Frege.[22]

If Frege claimed that every meaningful cardinality-sentence must involve a general predicate, then indeed examples of the above kind would pose serious problems. Examples such as "Cicero and Tully are one" make it clear that meaningful, true cardinality-sentences need not all contain such predicates explicitly. Examples involving such disparate kinds of entities as universities and book-tokens, further, make it clear that any attempt to resuscitate such predicates as parts of fully fleshed-out versions of the offending sentences will look highly implausible.

Frege does not, however, claim that every meaningful cardinality-sentence must involve a general predicate. What he *does* say is that every such sentence attributes cardinality to a concept. Alston and Bennett take examples of the form

(□) $a_1, \ldots, a_n$ are m

to conflict with this claim, since there is just no concept "there" to which a cardinality is being attributed.

The straightforward Fregean response to this objection is to say that there is of course a concept "there;" in asserting a sentence of the form (□), one is attributing the cardinality m to the concept *identical with* a_1 *or with ... or with* a_n[23]. For, after all, one is certainly not attributing this cardinality to the pile composed of $a_1 \ldots a_n$, or to each individual. The claim, then, is that an assertion of the form (□) would normally express what might more perspicuously be expressed by

(□') There are m things falling under the concept: *identical with* a_1 *or with ... or with* a_n.

If this is right, then the problem which arises for Geach does not arise for Frege. That successful singular reference is sufficient for raising a determinate identity-question conflicts with Geach's claim that determinate identity-questions require a relativizing predicate. That successful singular reference is sufficient for raising a determinate cardinality-question does not conflict with Frege's view that determinate cardinality-questions are all about concepts, since the singular reference in question gives us the essential disjunctive concept.

Alston and Bennett, however, claim that the appeal to disjunctive concepts is a trivialization of, or out of the spirit of, Frege's overall treatment of cardinality. As they put it,

> It seems clear ... that if that supposedly general [disjunctive] concept suffices to meet the demands of Frege's theory and of Geach's, each theory is deprived of its intended thrust. [24]

There are good reasons to think that a disjunctive concept of the kind just discussed would be inadequate for the purposes of Geach's theory. Recall that Geach holds that a statement of the form "a=b" is incomplete or ambiguous, and that identity-statements must properly be of the form "a is the same F as b." Because he holds that there is no relation of absolute identity, Geach must rule out some potential substitutions of "F." For example, he cannot hold that the two-place predicate "x is the same *thing* as y" is complete, since such a predicate would, grammatical structure notwithstanding, express absolute identity. Similar considerations affect the substitution of instances of "identical with x or identical with y" for F: "a is the same *thing identical with a or identical with b* as b" says simply that a is identical with b. Further, such instances appeal to the very notion of identity which Geach rejects. On pain of inconsistency, Geach cannot allow the Fregean's disjunctive predicates as appropriate relativizations of identity.

But there is no reason to take the disjunctive concepts to be inappropriate, or trivializing, for Frege. The problem with assigning cardinalities to piles or to collections, as Frege sees it, is that piles and collections offer nothing determinate to be counted; they do not come with members already individuated. A concept, on the other hand, always has a cardinality as long as it individuates the entities to be counted. Disjunctive concepts, like that given above, satisfy this requirement straightforwardly. Suppose each card in the pile has a name ("c_1", "c_2," ... "c_{104}"), and each deck of cards in the pile has a name ("d_1", "d_2"). Then to count the cards in the pile is to determine the number of the concept

identical with c_1 or with c_2 or with ... or with c_{104};

while to count the decks in the pile is to determine the number of the concept

identical with d_1 or with d_2.

Appeal to disjunctive concepts would count as an abandonment of the spirit of Frege's doctrine if such concepts played no role in the relevant cardinality-statements. But, as the example just given suggests, they play an essential role. To say that a statement of (□)'s form expresses an assertion about a concept is, among other things, to deny that (□) expresses an assertion about a pile. And here Frege would seem to be right. Suppose each of the entities a_1 ... a_5 is a company of soldiers. Their collection is then, as in Frege's example, five companies, seventy-five men, one regiment, and so forth. They are not five *simpliciter*, which is to say, the collection specified by the list does not have uniquely the number *five*.

Having been given some entities, even if via a list of names, one has not yet been given, in Frege's words, "the object which is to be investigated." Ordinarily, however, the list gives more than just the collection of entities. When one is asked "how many are a_1, ..., a_n," the expectation is typically that one is being asked not about the number of, say, three-membered teams comprised of things just named, but rather about the number of things just named. That is, the use of the list implies that the concept in question is: *identical with* a_1 *or with... or with* a_n. This implication can of course be overridden; we might ask "how many *crews* are Jones, Smith, Davis, Beck, and White?" But without a quite explicit specification of an alternative concept, a list of singular terms, in most contexts, does seem to make the disjunctive concept the subject of the cardinality-claim. There is, in any case, nothing about examples of the form of (□) which should dissuade us from this view.

VII. Conclusion

The sense in which cardinalities are "relative" for Frege puts him in the same boat not with Geach, but with modern accounts on which cardinality applies absolutely to sets. Though Frege and Geach both deny that collections have absolute cardinalities, it should be clear by now that this agreement is largely superficial. For Frege, collections fail to have absolute cardinalities because they can be divided into countable units in different ways. Those things which *do* have cardinalities - namely, concepts and their extensions - have, as Frege sees it, absolute cardinalities. For Geach on the other hand, collections fail to have absolute cardinalities because the members of the collection are not absolutely identical with or distinct from one another. Thus sets and concepts too, on Geach's view, lack absolute cardinalities.

Though both Geach and Frege take the canonical form of a cardinality-statement to be "there are n F's," with the monadic predicate playing an essential role, here again the doctrines are importantly different. For Geach, a cardinality-claim is indeterminate without that predicate, even if the individuals to be counted have been unambiguously specified. For the predicate tells us which relative-identity relation between those individuals we are to "count by." For Frege on the other hand, the important role of the monadic predicate is to specify which individuals are being counted. Once this is clear, there is only one relation to "count by," and it is the relation of absolute identity.

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[1]An earlier version of this paper was read at the 1997 MMM conference at the University of Notre Dame; thanks to members of that audience, and particularly to John O'Leary-Hawthorne, for helpful comments. Thanks also to Leora Weitzman, Palle Yourgrau, and an anonymous referee for helpful comments.

[2]Frege 1884 §22.

[3]Frege 1884 §46.

[4]Frege 1884 §54.

[5]Though in 1884 Frege takes the notion of "set" to be problematically vague, and hence resists the identification of his extensions with sets (see e.g. 1884 §45), he is by the 1890's ready to treat extensions as equivalent with sets. See e.g. the letter to Russell of 28 July 1902, English version in Frege 1980 139-142.

Regarding Husserl's claim that number belongs primarily to an extension and only secondarily to a concept, Frege replies:

> This actually concedes all that I am maintaining: a statement of number states something about a concept. I will not quibble over whether the statement is directly about the concept and indirectly about its extension, or indirectly about the concept and directly about its extension, for one goes with the other. (Frege 1984 322).

See also Frege's letter to Russell *op cit.* To say that a set S has n members, for Frege, is to say that n things fall under a concept (e.g., *member of S*) of which S is the extension.

Palle Yourgrau has argued (Yourgrau 1985) that since Frege's "relativity argument" applies to sets as well as to other objects, Frege cannot unproblematically take number to apply to sets. Yourgrau is right that sets can be "numbered" in various ways; for each set, we can ask how many *members* it has, how many *subsets* it has, perhaps how many *fingers* its members have, and so on. Sets do not have, intrinsically, a unique number. But this is not an issue for Frege, since Frege's assignment of number to sets simply requires that each set have a unique number of *members*.

Yourgrau has argued further (Yourgrau 1997) that the relativity argument applies to *concepts* as well, with a similarly problematic result for Frege's attribution of number to concepts. This is not the place for a detailed response to this claim, but the important point here is that on Frege's account, an attribution of number to a concept is always a claim about how many things *fall under* that concept. Frege's position is not that each concept (or set) somehow comes along with a unique number intrinsically associated with it. It is rather that all assignments of number can (and should) be treated as claims about the number of entities falling under a given concept.

[6]Geach 1967 238.

[7]Geach 1967 241.

[8]This is Geach's procedure. See e.g. Geach 1962 §109, Geach 1973 291.

[9]Geach 1973 292

[10]Geach 1973 294-5.

[11]Geach 1967 246.

[12]Geach 1967 238.

[13] Geach 1962 39.

[14]Frege 1884 §29.

[15]See §VI for a defense of this claim.

[16]Perry 1978.

[17]Perry 1978 7.

[18]*ibid.*

[19]The doctrine of relative numbers is what Alston and Bennett call the "Relative Cardinality Thesis;" see Alston & Bennett 555.

[20]Alston & Bennett 1984 557.

[21]Alston & Bennett 1984 558.

[22]Alston & Bennett 1984 560.

[23]That there are such disjunctive concepts follows immediately from Frege's view that every (1-place, first-level) predicative phrase refers to a concept. Such a phrase is, as Frege sees it, obtained from a sentence by removing one or more occurrences of a singular term;

thus e.g. "____ = 2 or ____ = 7" refers to a concept under which exactly 2 and 7 fall. See e.g. Frege 1879 §9, Frege 1884 §65n, Frege 1892, Frege 1893 §26. The use of phrases of the form "____ = a" (for "a" a referring singular term) to refer to concepts is essential to Frege's construction of the natural numbers; see e.g. Frege 1884 §77, Frege 1893 §42.

[24]Alston & Bennett 1984 561.