Problem 5-1: [Problem Objective:] Use of an amplifier to reduce the uncertainty in A/D conversion.

Statement: Chapter 6, HOMEWORK Problem No. 3

Solution:

Known:

The resolution \((Q)\) of an A/D converter is equal to \(\frac{E_{FSR}}{2^m}\).

Analysis:

[a] \(Q = \frac{E_{FSR}}{2^M} = 10/2^{16} = 0.153\) mV/bit

[b] From the problem statement, we know that the resolution of 1 bit of the converter will equal 1% of the balance’s full scale output with the amplifier is in place. Thus,

\[
\text{Gain, } G = \frac{\text{resolution of 1 bit}}{1\% \text{ of output}} = \frac{(153)}{(0.01)(3500 \mu\text{V})} = 4.37.
\]

Problem 5-2: [Problem Objective:] Use of a simple filter.

Statement: Chapter 6, HOMEWORK Problem No. 5

Solution:

Known:

For low pass filters, \(M(f) = 1/\sqrt{1 + (\omega RC)^2}\).

Analysis:

[a] \(M(f) = 1/\sqrt{1 + (2\pi f\tau)^2} \text{ and } \tau = 1/2\pi f_c.\)

So, \(M(f) = 1/\sqrt{1 + (f/f_c)^2} = 1/\sqrt{101} = 0.0995\)

[b] \(\tau = 1/2\pi f_c = 1/(2\pi)(100) = 1.59\) ms.

[c] \(\tau = RC, \text{ so } C = \tau/R = 15.9\) µF.
Problem 5-3: [Problem Objective:] Use of an amplifier with an A/D converter.

Statement: Chapter 6, HOMEWORK Problem No. 7

Solution:

Known:

The quantification error of an M-bit device is: \( e_Q = \pm 0.5(E_{FSR}/2^M) \).

Analysis:

[a] \( e_Q = \pm 0.5(E_{FSR}/2^M) = \pm 0.5(10 \text{ V}/4096) = \pm 1.22 \text{ mV} \).

[b] One can expect the relative quantification error to vary from:

\[ e_Q/E = 1.22 \text{ mV}/2.585 \text{ mV} = 0.472 \text{ or } 47\% \text{ at } 50 ^\circ C, \text{ to} \]

[c] \( e_Q/E = 1.22 \text{ mV}/3.649 \text{ mV} = 0.33 \text{ or } 33\% \text{ at } 70 ^\circ C. \)

Both values are significantly large.

[d] One means to reduce the quantification error is through amplification of the analog signal prior to quantization.

To achieve 5% or less error requires an input signal of the magnitude, \( E = e_Q/0.05 = 1.22 \text{ mV}/0.05 = 24.40 \text{ mV} \).

At 50 °C (the smallest voltage quantized), this requires a linear amplifier gain of \( G = E_o/E_i = 24.40 \text{ mV}/2.582 \text{ mV} = 9.44 \approx 10 \), or roughly, an amplifier having a linear gain of 10.

Problem 5-4: [Problem Objective:] Use of a simple filter.

Statement: Chapter 6, HOMEWORK Problem No. 8

Solution:

Known:

\[ \tau = \frac{1}{2\pi f_c} \]

\[ M(\omega) = \frac{1}{\sqrt{1+(\omega\tau)}} \]
\[ \beta = \frac{-\tan^{-1}(\omega \tau)}{\omega} \]

**Analysis:**

[a] The cutoff frequency, \( f_c \), is the highest frequency of interest, \( f_{max} \).

So, \( f_{max} = 15 \text{ Hz} \Rightarrow f_s = 30 \text{ Hz} \) and \( f_c = 15 \text{ Hz} \).

[b] The time constant for a single-stage low-pass filter is given by:

\[ \tau = \frac{1}{2\pi f_c} = \frac{1}{(2\pi)(15)} = 0.0106 \text{ s} = 10.6 \text{ ms} \]

[c] \( 10 \text{ Hz} = (2\pi)(10) = 62.8 \text{ rad/s} = \omega \).

\[ M(\omega) = \frac{1}{\sqrt{1 + (\omega \tau)^2}} = \frac{1}{\sqrt{1 + (62.8 \times 0.0106)^2}} = \frac{1}{1.2} = 0.83. \]

So, \( E_{o,\text{filter}} = (0.83)(8) = 6.64 \text{ V} \).

[d] \( \beta = \frac{\phi(\omega)}{\omega} \)

\[ = \frac{-\tan^{-1}(\omega \tau)}{\omega} = \frac{-\tan^{-1}(0.066)}{62.8} = -\frac{33.66^\circ}{62.8 \text{ rads}} \]

\[ = \frac{-33.66}{(57.3 \ ^\circ/\text{rad})(62.8 \text{ rad/s})} = -9.35 \times 10^{-3} \text{ s} = -9.35 \text{ ms} \].