Lecture 7
Filters

CLASS GAME PLAN: Finish op amps by deriving the input/output relation for one more configuration; work an in-class problem on op amps; begin filters.

• The filter is an electronic signal-conditioning component that removes signal content at unwanted frequencies.

• Filter ‘fundamentals’ include understanding both the amplitude and the frequency content of a signal.

• Consider the waveform
  \[ y(t) = D \cos(n\omega t - \phi) = D \cos(2\pi nf t - \phi). \]

• The variables and their units are:
  \[ D: \]
  \[ n: \]
  \[ t: \]
  \[ \omega: \]
  \[ f: \]
  \[ T: \]
  \[ \phi: \]

• NOTE: Phase is the x-axis (time) shift of a signal in units of °, rad, cycles, or s with respect to either another signal or the original signal with \( \phi = 0 \).
EXAMPLE PROBLEM: For the above signals, approximately determine the values with their units of the
amplitude of signal A:
° phase lag of signal C with respect to signal A:
period of signal B:
cyclic frequency of signal A:
circular frequency of signal C:
Later (in Ch 9), you will cover in more detail how any ‘mathematically well-behaved’ signal (amplitude versus time) can be represented by a Fourier series (FS).

The FS is

\[ y(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos [2\pi n ft] + B_n \sin [2\pi n ft]), \]  

(1)

where \( A_0, A_n, \) and \( B_n \) are the Fourier coefficients of the Fourier amplitudes.

The closeness to which the FS ‘signal’ approximates the original signal depends upon the number of terms included in the series.

Each term in the FS contains a specific amplitude and a specific frequency.

This allows us to view a signal in another way: amplitude versus frequency.

Thus, a filter can be understood as a device that removes specific terms of frequency/amplitude from a signal.
The ratio of the amplitude of the signal leaving the filter to that entering the filter is termed the magnitude ratio, $M(\omega)$, where $\omega$ is the circular frequency. Here, $M(\omega) = E_o / E_i$.

The time constant, $\tau$, of a simple $RC$ filter equals $RC$ (units:...
• A low-pass filter passes low frequencies and removes high frequencies. Its governing equations are:

• A high-pass filter passes high frequencies and removes low frequencies. Its governing equations are:

  • When $\omega \tau = 1$, $M(\omega) = 0.707$.
  • The cut-off frequency occurs when $M(\omega) = 0.707$.

  Thus,

**SUGGESTED PROBLEM(S):** RP 3.4, RP 3.5, HP 3.5
EXAMPLE PROBLEM: Determine values of $R$ and $C$ for a simple low-pass filter to have a signal attenuation of 50 % of the input signal at $F = 200$ Hz. Also determine the filter’s phase lag in radians at $f = 100$ Hz.