Lecture 18
Assessing Measurement System Performance:
System Response

Sinusoidal-Input Forcing of Second-Order Systems

• For sinusoidal-input forcing, the solutions typically are re-cast into the expressions for $M(\omega)$ and $\phi(\omega)$ versus $\omega/\omega_n$ (eqns. 4.62 - 4.64).

• For $M(\omega) = f(\omega/\omega_n)$:

$$M(\omega) = \frac{1}{\left\{\left[1 - (\omega/\omega_n)^2\right]^2 + [2\zeta\omega/\omega_n]^2\right\}^{1/2}}. \quad (1)$$

• For $\phi(\omega) = g(\omega/\omega_n)$:

$$\phi(\omega) = -\tan^{-1}\frac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2} \text{ for } \frac{\omega}{\omega_n} \leq 1; \quad (2)$$

or

$$\phi(\omega) = -\pi - \tan^{-1}\frac{2\zeta\omega/\omega_n}{1 - (\omega/\omega_n)^2} \text{ for } \frac{\omega}{\omega_n} > 1. \quad (3)$$

• All of these solutions can be plotted on two graphs.
Figure 1: Magnitude ratio versus nondimensional frequency.

Figure 2: Phase lag versus nondimensional frequency.
EXAMPLE PROBLEM: A RLC circuit with $R = 2 \Omega$, $C = 0.5 \text{ F}$, and $L = 0.5 \text{ H}$ is subjected to a sinusoidal-input forcing of $3 \sin(2t)$. Determine its output waveform mathematically and, then, check this on the $M(\omega)$ and $\phi(\omega)$ plots.

- First, determine the values of $\zeta$ and $\omega/\omega_n$.

- Next, determine the values of $M(\omega)$ and $\phi(\omega)$.

- Noting that this system is linear ($\omega$ does not change through the system), the output waveform will be of the form which becomes

- These calculational results can be checked using the plots.
The Decibel

- In many situations, the magnitude attenuation of a signal is given in units of dB/decade or dB/octave.
- The **decibel** is defined in terms of a power \(P\) ratio or a base measurand \(Q\) ratio or the magnitude ratio \(M\) as

- A **decade** is defined as a 10-fold increase in frequency and an **octave** as a 2-fold increase (doubling) in frequency.
- **Roll-off** refers to the slope (in dB/decade or dB/octave) of the \(M(\omega)\) versus \(\omega/\omega_n\) plot.
- The **one-half power point** (the point at which the output \(power\) is one-half of the input \(power\)) equals

- This corresponds to the magnitude ratio of
**EXAMPLE PROBLEM:** Determine the dB/decade roll-off of the system that has an output of $3 \sin(200t)$ for an input of $4 \sin(200t)$ and an output of $\sin(2000t)$ for an input of $4 \sin(2000t)$.

- First, determine the $\omega$’s and the $M$’s:

- Note that the difference between $\omega_1$ and $\omega_2$ is one decade. So, we need to find the attenuation in dB for that decade.

- Thus, the dB/decade (which conventionally is determined from higher frequency to lower frequency) is