Chapter 7

Calibration and Response Solutions

Problem 7.1

Problem Objective: To gain an understanding for certain properties of measurement response systems.

Statement: A first-order system has $M(f = 200 \, Hz) = 0.707$. Determine [a] its time constant (in ms), and [b] its phase shift (in $^\circ$).

Solution:

Known:

At $M(\omega) = 0.707$ for a first-order system, $\tau_\omega = 1$.

Also, $\omega = 2\pi f$.

Analysis:

[a] 

\[
\tau_\omega = \tau 2\pi f = 1, \\
\tau = \frac{1}{2\pi f} \\
= \frac{1}{2\pi(200)} \\
= 7.96 \times 10^{-4} \, s \\
\simeq 0.80 \, ms
\]

[b] At $\tau_\omega = 1, \phi = -45^\circ$. 

Problem 7.2

Problem Objective:

Statement: A first-order system with a time constant equal to 10 ms is subjected to a sinusoidal forcing with an input amplitude equal to 8.00 V. When the input forcing frequency equals 100 rad/s, the output amplitude is 5.66 V; when the input forcing frequency equals 1000 rad/s, the output amplitude is 0.80 V. Determine [a] the magnitude ratio for the 100 rad/s forcing case, [b] the roll-off slope (in units of dB/decade) for the $\omega \tau = 1$ to $\omega \tau = 10$ decade, and [c] the phase lag (in °) for the 100 rad/s forcing case.

Solution:

Known:

Input amplitude = 8.00 V = $A_i$

At $\omega = 100$ rad/s, $A_0 = 5.66$ V $\Rightarrow$ $\omega \tau = 1$

At $\omega = 1000$ rad/s, $A_0 = 0.80$ V $\Rightarrow$ $\omega \tau = 10$

$\tau = 10$ ms

Analysis:

[a] $M(\omega) = 1/\sqrt{(\omega \tau)^2 + 1} = 1/\sqrt{2} = 0.707$ for $\omega = 100$ rad/s

[b] $M(\omega = 1000$ rad/s $) = 1/\sqrt{10^2 + 1} = 1/\sqrt{101} = 0.10$

$\frac{dB}{\text{decade}} = 20 \log_{10}(0.707) - 20 \log_{10}(0.10)$

$= -3.0 - (-20)$

$= -17$ dB/decade

[c] $\phi = -\tan^{-1}(\omega \tau) = -\tan^{-1}(1)$

$= -45^\circ$
Problem 7.3

Problem Objective:

Statement: A pressure transducer that behaves as a second order system is supposed to have a damping ratio of 0.7, but some of the damping fluid has leaked out leaving us with an unknown damping ratio. When the transducer is subjected to a harmonic input of 1850 Hz the phase angle between the input and the output is 45°. The manufacturer states that the natural frequency of the transducer is 18500 rad/sec. [a] What is the dynamic error in the transducer output for a harmonic pressure signal of 1200 Hz? [b] If the transducer indicates a pressure amplitude of 50 psi, what is the true pressure amplitude?

Solution:

Analysis:

[a] $\omega/\omega_n = \frac{1850}{18500/2\pi} = (0.1)(2\pi) = 0.628$

$J = \frac{1-(\omega/\omega_n)^2}{2\omega/\omega_n} \geq \frac{1-(0.628)^2}{2(0.628)} = 0.48$

$M = \frac{1}{\sqrt{[1-(\omega/\omega_n)^2]^2 + (2\omega/\omega_n)^2}}$

$= \frac{1}{\sqrt{[1-0.628^2]^2 + [2(0.48)(0.628)]^2}}$

$= \frac{1}{\sqrt{0.367^2 + 0.363^2}} = 1.17$

$\delta = 1 - M = 1 - 1.17 = -0.17$

or 17% error

[b] $50 = 1.17 \cdot p_{\text{actual}} \Rightarrow p_{\text{actual}} = \frac{50}{1.17} = 42.7 \text{ psi}$