1st Edition Appendix H: Laboratory Exercises
to Accompany
MEASUREMENT AND DATA ANALYSIS FOR
ENGINEERING AND SCIENCE
ISBN: 9781439825686

Patrick F. Dunn
pdunn@nd.edu

107 Hessert Laboratory
Department of Aerospace and Mechanical Engineering
University of Notre Dame
Notre Dame, IN 46556

August 2009
Laboratory Exercises

This appendix presents 12 laboratory exercises that were designed to supplement the material in this text. Each section describes a particular laboratory exercise. An accompanying Laboratory Exercises Manual presents the actual student hand-out for each exercise. A companion Laboratory Exercise Solutions Manual provides the answers to all of the questions posed in the laboratory exercise handout and the data acquired for each exercise. In this manner, a virtual laboratory exercise can be performed by providing students with data for analysis and reporting. The laboratory exercises can be performed as written. All have been tested many times by students over the past several years and have been refined. One intent in offering these descriptions is to provide a base for instructors to extrapolate from and generate new exercises.

Typically 6 to 10 exercises are conducted during a one-semester, three-credit-hour undergraduate measurements course. The purpose of these exercises is to introduce the student to the process of conducting experiments and analyzing their results. Some exercises are oriented towards learning about instrumentation and measurement system hardware; others towards examining an actual physical process. The overall objective is to provide students with a variety of measurement and data analysis experiences such that they are fully prepared for subsequent laboratory courses that focus on investigating physical processes, such as those in fluid mechanics, aerodynamics or heat transfer laboratory courses.

Some of the exercises were designed to be performed in series, although each exercise stands alone. In particular, Exercises 2 through 6 progressively introduce the student to the foundational concepts and use of strain gages for both static and dynamic force measurements. Exercises 1, 7 and 10 involve the comparison of measurements with theory within the context of uncertainty. Exercises 8 through 11 introduce the student to various instrumentation and measurement systems. Finally, the Exercise 12 focuses on post-experiment data analysis using files of provided data.

Table 1 lists the instrumentation used for each exercise.


1 Exercise 1

Measurement, Modeling and Uncertainty

1.1 Introduction and Objectives

This laboratory exercise demonstrates the roles that modeling and empirical uncertainties play in determining the outcome of a simple experiment. The experiment involves launching a ball from a pendulum apparatus and measuring the vertical and horizontal distances that the ball travels. The experiment is repeated at different pendulum head release angles, with a specified number of times for each angle. The average results for each angle are compared with the theoretical predictions within the context of the uncertainties involved in the model and in the experiment.

As part of this exercise, a model must be developed that predicts the horizontal distance, $x$, ± its uncertainty (estimated at 95% confidence) where the ball will land based upon the release angle of the pendulum head with respect to the vertical top position, $\theta_{rel}$. The values of some of

<table>
<thead>
<tr>
<th>Instrumentation</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>multimeter</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dial indicator</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wheatstone bridge</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cantilever beam</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>oscilloscope</td>
<td>✓</td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>strain gage</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>manometer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>barometer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dynamometer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DC power supply</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>calibration weights</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stroboscope</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>function generator</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>data acquisition system</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>thermocouples</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>RLC circuit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Helium-Neon laser</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>diode/detector pair</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>optics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Laboratory Exercise Instrumentation
the model’s variables, such as the coefficient of restitution of the ball, in turn, rely upon other empirical information that may need be gathered by performing subsidiary experiments. A subsidiary experiment is any experiment other than the actual one that needs to be performed to obtain input information for the model.

A schematic of the pendulum apparatus is shown in Figure 1. There is a large pendulum that consists of an Al 2024 shaft (46.60 cm long ±0.05 cm; 0.95 cm diameter ±0.01 cm; mass = 89.8 g ±0.5 g) that extends into a rectangular yellow brass strike head (length = 6.36 cm ±0.01 cm; width = 3.18 cm ±0.01 cm; height = 3.18 cm ±0.01 cm; mass = 528.8 g ±0.5 g). The pendulum is swung about a top pivot point, which contains an angle indicator (resolution = 1 °). The ball having mass \( m_2 \) is located on a tee and placed such that contact with the strike head is made at the bottom of the swing. The distance between the center of the strike head at the top of the swing and the bottom of the swing, \( h_1 \), is 90.0 cm ±0.2 cm. The center of mass of the system consisting of the rod and the strike head lies at 42.00 cm ±0.01 cm from the center bearing. Figure 2 shows schematically the pendulum at the top and bottom of its swing with its nomenclature.

1.2 Instrumentation

The following equipment will be used:
Figure 2: Pendulum nomenclature
• Pendulum apparatus to launch the ball
• a standard racquet ball with a mass of 40.60 g ±0.05 g and a diameter of 5.61 cm ±0.05 cm or a standard golf ball with a mass of 45.30 g ±0.05 g and a diameter of 4.27 cm ±0.05 cm
• English/metric tape measure

1.3 Measurements

The objective is to obtain data relating the horizontal distance that a golf ball travels to the pendulum head release angle, which then can be compared to theoretical predictions. The experiment should be repeated at each release angle 5 times to obtain an average horizontal distance. A minimum of 4 release angles must be examined. More data will give a better comparison between experiment and theory.

The pendulum launching apparatus should be placed in a stable position on a table top such that the pendulum can travel freely in a complete circle and that there are no obstructions for approximately 20 ft in front of it for the ball to travel. Start by performing a trial experiment to identify where the ball will land on the lab floor. Have one person launch the ball and the other note where it lands on the floor. Repeat the trial several times to identify the approximate impact point. Tape a piece of carbon paper on the floor to mark on the paper where the ball lands each time. Now perform the experiment 5 times, always noting the impact point. When complete, repeat for another pendulum head release angle. Measure the vertical distance and all horizontal distances using the provided tape measure. Make sure that all the necessary subsidiary experiments and measurements have been performed.

1.4 What to Report

Present the results of this effort in a technical memo. The memo at a minimum must include the following: [1] a statement that summarizes the agreement or disagreement between the measured and predicted distances and plausible, scientifically-based reasons for any disagreement; [2] a table of the predicted and the measured (average) horizontal distances from the launch point (in cm) to the center of the landing
impact point and their associated distance uncertainties. The uncertainty estimates must be supported by detailed calculations (present these in an attached appendix) using standard uncertainty analysis at 95 % confidence (a plot of the same would also be helpful); [3] a brief description of the model developed, stating all parameters and assumptions (any detailed calculations can be put in an appendix); and [4] a table that presents the symbols and lists the values in both SI and Technical English units of all of the model inputs and outputs.
2 Exercise 2

2.1 Introduction and Objectives

The primary purpose of this exercise is to determine the relationship between the relative change in resistance of a fine wire and the relative change in its length. This concept is the fundamental principle by which strain gages operate. A strain gage (see Figure ?? in Chapter ??) basically consists of a metallic pattern bonded to an insulating backing. This can be attached to a surface to provide a method to measure strains induced by loading. The important concept is that when a wire is stretched (strained), its resistance changes.

These objectives will be accomplished by stretching a wire and measuring its resistance at various lengths. In the process of performing this experiment, a digital multimeter will be used to measure the resistance. The experimental results will be examined by plotting the relative change in resistance versus the relative change in length. From this information, a gage factor can be determined for this wire. The uncertainties in the measurements also will be estimated and related to the results. Local and engineering gage factors are explained in Chapter ???. The engineering gage factor will be determined here.

2.2 Instrumentation

The following instruments will be used:

- Hewlett Packard 3468A Multimeter (resolution: 1 $\mu$Ω in the ohm range)
- Starrett dial indicator (resolution: 0.0005 in.)
- A metal meter stick (resolution: 0.5 mm)
- A wire stretcher for wires approximately 1 m in length
2.3 Measurements

First and foremost - a safety note. It is imperative that safety glasses be worn when stretching the wire. It can break and snap back.

A length of wire is to be mounted between the two clamps of the tension device and loaded using the screw mechanism. First, cut a piece of wire approximately 4 ft long from the spool. Figure 3 shows a schematic of the clamping mechanism.

![Figure 3: Clamping of wire in mechanism.](image)

With the top of the clamp removed, loop the wire once about the end post before directing it to the other clamp. Then replace the top of the clamp and tighten the set screw to prevent slipping. Do likewise with the other clamp. Wire slippage while applying tension it will result in an indication of displacement without an expected increase in resistance. When the wire is mounted correctly, it should be straight, not sagging, and the tension end with the brass thumb wheel should have about 0.5 in. of travel available.

The four-wire resistance measurement will be used (see Chapter ??). Connect the multimeter to the wire as follows:

1. Using one pair of banana-alligator test leads, connect the HI and LO pair of terminals of the multimeter under INPUT to the wire near the clamps, one at each end.

2. Using a second pair of test leads, connect the HI and LO terminals under Ω-Sense to the wire just inside of the two leads of Step (1).

3. Place the multimeter into the 4-wire resistance mode by pressing the ”4 WIRE” button (4 Ω should be appear on the LCD display). Also depress with the AUTO/MAN button to put the meter in the manual mode (M RNG should be appear on the display). This will yield a resistance reading resolution of 10 mΩ. The KOHM
range should be displayed. If not, depress the up arrow button to obtain its indication. Finally, depress the blue button, then the INT TRIG button to set the multimeter in the auto zero mode. If the auto zero mode is NOT set, then AZ OFF will appear on the display (no indication means it’s set correctly). The indicated reading should be approximately 150 \( \Omega \) to 200 \( \Omega \). If a negative resistance is indicated, the two inner wires can be switched to make it positive. Allow a couple of minutes for the meter to warm up before starting to take data.

Slowly tension the wire by turning the brass thumb wheel until the resistance starts to increase (watch the least and second least significant digits for some consistent increase). This shall be the zero point. Record the reading on the dial gage and the resistance. (The dial indicator scale goes from 0 to 50, corresponding to 0.000 to 0.050 in. of travel). Measure the initial length of wire between the measuring points, the leads of Step (2), using the metal meter stick.

At increments of approximately 0.01 in. (increments of 10 on the dial indicator), record the elongation (inches) and the resistance (ohms) until the wire has been stretched about 0.20 in. The resistance changes approximately 0.1 \( \Omega \) for every 0.01 in. of stretch. Return to a couple of data points and repeat those measurements to see if they have changed at all. Now, try repeating the experiment all the way out to failure of the wire. This should take on the order of 0.50 in. of travel. Try taking around 20 data points, with larger intervals at the beginning, becoming smaller as the wire approaches failure.

When finished, turn off the multimeter, bring the dial indicator back to the zero starting point, disconnect the test leads from the wire and the multimeter and remove the wire.

2.4 What to Report

Plot the relative resistance change versus the relative length change, for both the first case and the case when the wire was stretched to failure. Estimate the uncertainties of \( \Delta R/R \) and of \( \Delta L/L \) following the procedures detailed in Chapter ???. Calculate the engineering gage
factors for both cases. Are the values the same? Explain this in the context of the measurement uncertainties. Try approximating some local gage factors by calculating slopes over a few data points in the data sets, especially at lower strains. What can be said about the relation between these local gage factors and the extent to which the wire was strained at that point? Plot the local gage factor versus the strain to illustrate this. Compare the local with the engineering gage factors from the two cases, always being aware of the uncertainties involved.

Perform a least-squares linear regression analysis of the relative resistance change versus strain (see Chapter ??). Determine the correlation coefficient and the percent confidence associated with that correlation coefficient. How does the slope of the best-fit line compare with the gage factors that were calculated earlier?

All important experimental results and answers to the posed questions must be presented as a technical memo. Answers to the posed questions should be contained in the explanation of the results and not listed item-for-item.
3 Exercise 3

Strain-Gage-Instrumented Beam: Calibration and Use

3.1 Introduction and Objectives

This laboratory exercise involves the static calibration of a system consisting of four strain gages mounted on a cantilever beam. Once calibrated, this system can be used in either a static or a dynamic mode to determine the weight of an object, mass flow rate of a material and the frequency of a vibrating beam. In this exercise it will be seen how uncertainties enter into the calibration process and how they subsequently enter into the uncertainty when determining such quantities weight, mass flow rate and frequency.

The concept that the change of a wire’s resistance with strain can be utilized in a practical measurement system will be examined. It is possible to take small changes in resistance and, using an electrical circuit, transform the signal into a change in voltage. By making a strain gage one of the resistors in a Wheatstone bridge circuit, a voltage is measured that is proportional to strain. Specifically, in this lab exercise, four strain gages bonded to a cantilever beam will be used. Each of the four gages will serve as one resistor in a leg of a Wheatstone bridge.

A static calibration will be performed to obtain a mathematical relationship between bridge output voltage and force. Once this expression is known, the measurement system can be used for many static and dynamic applications.

3.2 Instrumentation

The following equipment will be used:

- Wheatstone bridge and operational amplifier measurement system
- Cantilever beam load cell
- Tektronix TDS 210 two-channel digital real-time oscilloscope
- Four Micro-Measurement 120 Ω CEA-13-125UW-120 strain gages (bonded to the beam)
• Four unknown materials: circular pipe, rectangular pipe, cylinder and hexagonal cylinder
• Plastic bottle with fine-grained sand
• 1000 ml plastic beaker

3.3 Measurements

![Wheatstone bridge and op amp configuration](image)

Figure 4: Wheatstone bridge and op amp configuration.

**Part 1: Static Calibration**

1. Record the lab set-up number. The set-up number is located on the top of the cantilever beam.

2. Make sure that the power is ON (the warm-up period for stable readings is about 1 hour).

3. Connect the two strain gages on the tension side of the beam to the Wheatstone bridge as follows: connect one between Bridge Excitation and + Bridge Out, and then other between - Bridge Out and Ground. The tension side gages have yellow end connectors.

4. Connect the two strain gages on the compression side of the beam to the Wheatstone bridge as follows: connect one between Bridge Excitation and - Bridge Out, and then other between + Bridge Out and Ground. The compression side gages have blue end connectors.

5. Set the Bridge Excitation voltage to $4.50 \pm 0.01$ V by adjusting the Bridge Excitation dial and observing the output on the panel meter (be sure that the selector switch is set to Bridge Excite).

6. Set the Amplifier Gain dial position to 4.
7. With no weights attached to the beam end, adjust the Balance dial to obtain an Amplifier Out signal of 0.00 ± 0.02 V (be sure that the selector switch is set to Amplifier Out). Wait for about 30 to 60 seconds after the dial is adjusted to be sure that the voltage stays where it was set. Reset the dial if necessary.

8. Add 450 g of weights to the hanger at the end of the beam. Note that the mass of the hanger is 50 g, so the total mass is 500 g. Make sure not to apply any total mass greater than 1 kg to avoid damaging the strain gages.

9. Adjust the Amplifier Gain dial to achieve 4.00 ± 0.02 V. Wait for about 30 to 60 seconds after adjusting the dial to be sure that the voltage stays where it was set. Reset the dial if necessary.

10. Take the hanger and weights off of the end of the beam. Check the Amplifier Output. It should be 0.00 ± 0.02V. If not, repeat steps 6 through 8.

11. Now begin a full calibration. First record the actual zero weight reading. Then, start with just the hanger attached. Record the Amplifier Out reading after waiting about 30 to 60 seconds. Make sure that the hanger is not moving (small movements will cause variations in the readings). Now proceed to add weights progressively up to 450 g, recording the mass and the Amplifier Out reading each time. Next, progressively take off the weights and record the values, ending up with no weight.

12. Then repeat a few of the measurements to determine how repeatable the measurements are.

13. Remove the weights and hanger and record the voltage. It should be at 0 V (the initial bridge-balanced no-weight condition). If it is not within an acceptable range about 0 V ±0.02 V, the bridge may have to be re-balanced and the measurements taken over again.

**Part 2: Unknown Object Measurements**

1. Once all of the calibration data has been taken, select one of the objects of unknown weight. Using the can at the end of the beam to hold the object (make sure the object is close to the center of the
can), determine and record the Amplifier Output voltage. Repeat for each object.

2. Obtain a plastic beaker from the TA. Fill the beaker with 600 ml of water. Place the object in the water and measure the displacement of the water. Repeat for EVERY ONE of the objects.

3. Measure and record the dimensions of each of the unknown objects with calipers. This, in conjunction with the previous water displacement measurements, will give two separate measurements of the volume for each of the four objects.

**Part 3: Measurement of Mass Flow Rate and Oscilloscope Set-up**

1. Obtain a squeeze bottle full of sand and a plastic bag.

2. Turn on the oscilloscope and connect CH. 1 to the Amplifier Output from the bridge using a cable. Now press the Autoset button. Next, Press the CH. 1 button under the vertical section twice. A menu should appear in the display window. Under the menu heading “Coupling”, choose “DC”. This is done by pressing menu keys next to the display to the immediate right of the display.

3. Using the Volts/Div knob for CH. 1, change the voltage per division to 200 mV/div. The volts per division is displayed in the lower left hand corner of the display.

4. Now change the seconds per division to 5 s/div using the SEC/DIV knob under the Horizontal section of the oscilloscope. Again, the seconds per division is displayed at the bottom of the screen.

5. Move the vertical position of CH.1 down to one division above the bottom of the screen using the vertical position knob for CH. 1.

6. Place the plastic bag in the can.

7. Hold but do not invert the squeeze bottle filled with sand over the can at the end of the beam. Wait until the trace on the oscilloscope reaches the end of the first time division. Remove the end cover of the spout from the squeeze bottle, invert the bottle and let the sand flow freely (do NOT squeeze the bottle) into the bag lining the can.
8. When the signal has reached the end of the oscilloscope’s display area or the sand begins to run out, press the Run/Stop button located at the top right of the oscilloscope. The data should be frozen on the screen. Press the Cursor button. Set the “type” to time using the top menu button. Two vertical lines should appear on the screen. Using the vertical position knobs for CH. 1 and CH. 2 to move the lines, measure the horizontal displacement (Time difference) of the data. Measure only the linear region. The time difference is displayed on the screen under the heading “Delta Record”.

9. On the cursor menu, select “Voltage” under the menu Heading “Type”. Using the vertical knobs for CH. 1 and CH. 2 to move the horizontal lines, measure and record the horizontal displacement (Voltage difference) of the data.

10. Carefully remove the bag from the beam, ensuring that sand does not spill out of the bag. Empty all sand back into the squeeze bottle.

11. Using a dry plastic beaker, measure 100 ml of sand.

12. Place the plastic bag in the can.

13. Place the 100 ml of sand in the plastic bag lining the can and record the Amplifier Out Voltage.

14. Carefully remove the bag from the beam, ensuring that sand does not spill out of the plastic bag. Empty all sand back into the squeeze bottle.

**Part 4: Dynamical Measurement**

1. Press the Run/Stop button if necessary to view the signal again. Move the vertical position of the signal to the center of the screen using the vertical position knob for CH. 1.

2. Now change the SEC/DIV under the Horizontal section of the oscilloscope to 50 ms/div. Gently tap the end of the beam. Wait until the entire signal is visible within the oscilloscope display, then press the Run/Stop button on the oscilloscope to freeze the signal.
3. Press the Cursor button and select “Time” under the “Type” menu. Using the vertical knobs, measure and record the time for several periods of the signal. The frequency of vibration (in Hz) is the inverse of ONE period (in seconds) of the signal.

3.4 What to Report

The following must be included in a technical memo (use SI units and 95 % confidence throughout):

1. Plot up the Amplifier Out (V) versus Weight (N), where Amplifier Out is on the ordinate and Weight is on the abscissa. Use different symbols for the “up” calibration sequence (when weights were added), and for “down” sequence (when weights were removed). Note that the masses (in g) must be converted into force units (N). Note in the technical memo any differences between the “up” and “down” calibration sequences.

2. Plot a linear least-squares regression analysis of the data (see Chapter ??).

3. Based upon the information provided by the least-squares regression analysis, determine the uncertainty in the voltage that is related to the standard error of the fit (the “y estimate”).

4. Find the uncertainty in determining weight from a voltage measurement (the “x-from-y estimate”). To determine this uncertainty use a voltage near the mid-point of the calibration voltages and project it back through the calibration curve to the x-axis to determine the expected x value. Also project back through the appropriate confidence limits to the x-axis to find the minimum and maximum possible x values. Find the differences between these values and the expected x value. Use the greater of the two as the uncertainty.

5. Determine the volume of each unknown material using both the water displacement and caliper measurements.

6. Use both water-displacement and caliper-determined volumes and the weights determined using the calibration, determine the density of each material. For three of the objects there will be two density values (one from each measurement method).
7. In a table report the volume determined by each method and the density for each object. Compare the density values with those found from the literature or on the web. Be sure to cite any sources.

8. Determine the uncertainty in the volume and density values. This should be done for both types of volume measurement. Which method has the least uncertainty?

9. Assume that there is no access to calipers and a low cost experiment must be designed to determine the volume and density of the unknown objects. Design a new water-displacement based experiment that costs less than $100.00 and has a volume uncertainty less than 0.061 in.³. Be specific and cite sources.

10. Determine the mass flow rate of the sand using the calibration information and the recorded time difference.

11. Determine the uncertainty in the mass flow rate.

12. Present the weight, volume, density and mass flow rate uncertainties all in one table.

13. Determine the density of the dry sand and calculate the uncertainty of this dry sand density.

14. Assume an hourglass is to be made based on the collected data. How much sand would be needed to measure an hour?

15. Determine the natural frequency of the beam using the data collected from lab (see Supplemental Information). Calculate a theoretical natural frequency for the beam. Compare the experimental and theoretical values and give reasons for any possible differences.

All of the important experimental results and answers to the posed questions must be presented as a technical memo. Answers should be contained in the explanation of the results and not listed item-for-item.

3.5 Supplemental Information

All solid objects vibrate to some extent when they are hit. Determining the frequency of vibration is very important in design. Vibration can cause wear, reliability problems, and induce unwanted noise. In some
instances, vibration is desired and even necessary. A vibrating conveyor belt is a good example of this. The frequency at which an object vibrates depends on both its shape and material properties. An object’s unforced (natural) frequency experimentally can be determined by giving the object a short impulse and measuring its frequency of vibration, as is done in this lab. For simple configurations, it is also possible to determine the natural frequency using theoretical approximations. For the cantilever beam configuration used in this lab, the frequency of vibration can be determined by

\[ \omega_r = \frac{A_r}{l^2} \left( \frac{EI}{\rho A} \right)^{1/2}, \]  

where \( \omega_r \) is the natural frequency, \( E \) is the modulus of elasticity, \( I \) is the moment of inertia, \( A \) is the cross-sectional area, \( A_r \) is the mode coefficient, \( l \) is length, and \( \rho \) is the density. The cantilever beams used in this lab are made of 6061-T65 aluminum with a modulus of elasticity of approximately \( 10^7 \) psi and a density of 0.00305 slugs/in.\(^3\). The length, \( l \), is the distance for the fixed point to the free end of the beam. For the lab set-up, \( l \) is

\[ l = (L - D), \]  

where \( L \) and \( D \) are the distances shown in Figure 5.

![Figure 5: Cantilever Beam Diagram](image)

Recall, that the moment of inertia for a rectangular cross-section is

\[ I = \frac{bh^3}{12}, \]  

where \( b \) is the width of the beam and \( h \) is its height. Figure 6 shows a diagram of relevant dimensions. Every beam in this lab has slightly different dimensions; this is true with any engineered object. Table 2
contains the measurements for each experimental set-up. When the theoretical values are calculated for this lab, use the appropriate dimensions for the particular station. The accuracy of each measurement is also given.

![Figure 6: Cantilever beam with four strain gages](image)

Table 2: Beam Dimensions (all units are in inches)

<table>
<thead>
<tr>
<th>Beam</th>
<th>L (±0.002)</th>
<th>I (±0.020)</th>
<th>d (±0.015)</th>
<th>b (±0.001)</th>
<th>h (±0.0005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.913</td>
<td>9.930</td>
<td>0.491</td>
<td>1.504</td>
<td>0.1275</td>
</tr>
<tr>
<td>2</td>
<td>11.933</td>
<td>9.910</td>
<td>0.492</td>
<td>1.504</td>
<td>0.1275</td>
</tr>
<tr>
<td>3</td>
<td>11.919</td>
<td>9.920</td>
<td>0.495</td>
<td>1.503</td>
<td>0.1275</td>
</tr>
<tr>
<td>4</td>
<td>11.924</td>
<td>9.920</td>
<td>0.492</td>
<td>1.504</td>
<td>0.1275</td>
</tr>
<tr>
<td>5</td>
<td>11.943</td>
<td>9.950</td>
<td>0.488</td>
<td>1.503</td>
<td>0.1275</td>
</tr>
<tr>
<td>6</td>
<td>11.944</td>
<td>9.940</td>
<td>0.491</td>
<td>1.503</td>
<td>0.1275</td>
</tr>
</tbody>
</table>

Table 3: Mode Coefficients for the first three vibration modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$A_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>$A_1$</td>
</tr>
<tr>
<td>Mode 2</td>
<td>$A_2$</td>
</tr>
<tr>
<td>Mode 3</td>
<td>$A_3$</td>
</tr>
</tbody>
</table>

Notice that the dimensions for $a$ and $d$ are not listed in the table. These dimensions are not critical to the calculations and were not measured.

The mode coefficient, $A_r$, depends upon the mode of vibration, $r$. The natural frequency is the first mode (i.e., $r=1$). As the mode of vibration increases so does the frequency of vibration and the mode coefficient. Table 3 gives the mode coefficients for the first three vibration modes.
4 Exercise 4

Propeller Dynamometer:
Static Thrust, Torque and rpm Measurement

4.1 Introduction and Objectives

In this lab exercise, two strain gages, a Wheatstone bridge and amplifiers will be used to determine the thrust and torque generated by a radio-controlled aircraft propeller under static operating conditions. More specifically, the power into the motor, the thrust and torque output of the system, and the rpm of the propeller will be determined.

The data taken from a thrust stand such as this can be used to gather performance data on different propellers. This in turn can help engineers make more informed design choices when selecting a propeller for a given airframe and propulsion system. If this experiment were taken a step further, the measurement system could be easily modified and be placed in a wind tunnel in order to gather dynamic propeller data. Also, such a measurement system could be used to examine the performance of a fan in a heating, ventilating and air-conditioning system. More information regarding propellers and how they work is included in the supplemental information section of this handout.

4.2 Instrumentation

The following equipment will be used:

- Zinger 11-7 propeller (d = 11 in.)
- Two Wheatstone bridge and operational amplifier instrument systems
- propeller dynamometer
- Motor power supply
- Voltmeter/ammeter readout box
- Calibration weights and hanger
- Stroboscope
4.3 Measurements

Before starting on any of the measurements, a word about safety; this is the first exercise that has the potential of being extremely dangerous. The propeller turning at several thousand rpm will not hesitate to remove fingers if one is careless enough to place them in the plane of the blade. Therefore, the rule of operation for the dynamometer is: NEVER REMOVE THE SAFETY CAGE WITHOUT FIRST DISCONNECTING THE POWER SUPPLY TO THE MOTOR. Do this by disconnecting the red and black banana plug power leads from the motor power supply.

1. Connect and adjust the bridges and amplifiers.

2. Calibration:
   - First check to make sure that the motor power supply is disconnected. Once this is done, remove the safety cage from the propeller. Attach the hanging wire to the screw at the center and immediately in front of the propeller, and hang it over the pulley at the front of the test stand.
   - Zero the bridges for both the thrust and torque readouts by adjusting the “BRIDGE BALANCE ADJUST” knobs on the appropriate panels.
   - Perform both thrust and torque calibrations, one at a time, by adding weights to the respective hanger and recording the corresponding voltage output. This data will be used later to convert the voltage measurements taken during the actual running of the propeller to thrust and torque readings. For the thrust calibration, add weights in 50 g increments up to around 1.0 kg. For torque calibration, increase by about 5 g to 10 g up to around 100 g. The little blue basket should be used as the initial torque calibration weight. Its mass is 7 g. The length of the torque calibration moment arm is 8.50 ± 0.05 in.
   - After completing the calibration, remove the wire hanger from the front of the propeller shaft and replace the safety cage.

3. Thrust, Torque, and rpm measurements:
• Reconnect the motor power supply. Turn on the supply. If the red overload light is flashing, depress the button on the voltmeter/ammeter box. Turn the voltage increase dial up just enough to get the prop spinning. Now, depress the voltmeter/ammeter button again. You should hear the propeller increase its RPM.

• Going in 1 volt increments from 1 V to 12 V, record the following data: voltage into the motor, current into the motor, thrust voltage, torque voltage, and propeller RPM. Repeat the measurements at several motor voltages after the first set is completed. The voltages and currents in are read from the voltmeter/ammeter box panel meters. Thrust and torque voltages are the output voltages from the appropriate bridge/amplifier system. The rpm is measured using the strobe. It is best to turn the room lights out during these measurements so that the standing image of the propeller can be seen better.

The stroboscope has three scales: LOW (100-700 RPM), MEDIUM (600-4200 RPM) and HIGH (3600-25000 RPM). During the course of the measurements, all three scales will be used. One can shift from one to another by depressing the appropriate button on the back of the stroboscope. The propeller is marked near its tip with distinct black lines. Basically, the strobe light will be adjusted until a stationary image of the propeller is seen, with the marks on the ends of the propeller appearing identical to what they would if the propeller were not moving. This is a little tricky because standing images with the correct marks (two horizontal lines on one side; two vertical lines on the other) occur at even integer fractions of the correct rpm for a two-blade propeller as well as at the correct RPM. However, a standing image obtained at a strobe frequency of twice the correct propeller rpm will not show the correct marks. So, once a correct image is obtained, keep doubling the strobe frequency until the correct rpm is identified. For the present system, at a motor voltage of 1V, standing images should be seen at approximately 170, 340 and 680 RPM, with correct images at 170 and 340. So, the correct propeller RPM is approximately 340 RPM.
The correct rpm at 2V should be approximately 1000 RPM.

- Disconnect the motor power supply once all measurements are finished.

4.4 What to Report

Submit the information in the form of a technical memo. Be sure to include at the very least the following (with some discussion of each): [1] plots of the thrust and torque calibrations (T and Q versus the voltage outputs of the measurement system), [2] plots of T, Q and $P_{prop,in}$ versus $N_{prop}$, [3] a plot of $\eta_m$ versus $N_{prop}$, and [4] plots of $C_T$, $C_Q$ and $C_P$ versus $N_{prop}$. Decide which plots are the important ones to put in the body of the memo. Put the other plots in an appendix. Remember to construct all plots according to the format presented in the text. Be very careful with the units when calculating the values of all these parameters. Include a sample calculation of each parameter in an appendix to demonstrate proper unit conversion.

4.5 Supplemental Information

An understanding of how a propeller produces thrust rests in a knowledge of how an airfoil generates lift, L, and drag, D. An airfoil in motion generates lift and drag. If we consider the propeller as a rotating airfoil, we can understand how it generates a forward thrust, T, and a torque, Q, where Q results from a force, $F_Q$, acting perpendicular to the forward direction.

Examine this in more detail. Refer to Figure 7, which shows a propeller consisting of two blade elements of pitch angle, $\beta$, each located at distance R from the axis of rotation. The velocity $V_o$ is that of the air through which the propeller advances. Because the blade element also is rotating with an angular velocity $\omega$, it will have a rotational velocity of $\omega R$. The velocities $V_o$ and $\omega r$ are vectors that combine to yield the relative velocity $V_e$. This is the velocity of the air relative to the rotating blade element. Its approach angle is $\phi$. This implies that the actual angle of attack $\alpha$ equals $\beta - \phi$. Therefore

$$ \phi = \tan^{-1} \left( \frac{V_o}{\omega R} \right) $$

(4)
and

\[ V_e = \sqrt{V_o^2 + (\omega R)^2} \]  \hspace{1cm} (5)

Now examine the view along the propeller blade axis. From trigonometry

\[ T = 2[L \cos \phi - D \sin \phi] \]  \hspace{1cm} (6)

and

\[ Q = 2RFQ = 2R[L \sin \phi + D \cos \phi]. \]  \hspace{1cm} (7)

Further, the power required to turn the propeller is

\[ P_{req} = Q\omega. \]  \hspace{1cm} (8)

When \( V_o = 0, \phi = 0 \), giving \( \alpha = \beta \). This leads to

\[ T_{static} = 2L_{static}, \]  \hspace{1cm} (9)

\[ Q_{static} = 2RD_{static} \]  \hspace{1cm} (10)

and

\[ P_{req,static} = Q_o\omega = 2\omega RD_{static}. \]  \hspace{1cm} (11)
Thus, for both static ($V_o = 0$) and dynamic ($V_o \neq 0$) conditions, a rotating propeller generates thrust and torque from its lift and drag. Further, the power required to turn the propeller is related to its torque and rotational velocity and, hence, to its lift and drag.

Now examine the power required to turn the propeller in the experimental set-up. Some expressions that relate the measured variables to those that characterize the performance of some of its components need to be developed. Refer to Figure 8.

The power into the motor is simply the product of its input current, $i$, and voltage, $V$, both of which we measure. That is,

$$P_{motor, in} = i \cdot V. \quad (12)$$

Some of this power will be lost inside the motor and eventually dissipated as heat. This is quantified by the motor efficiency, $\eta_m$, where

$$\eta_m = \frac{P_{motor, out}}{P_{motor, in}}. \quad (13)$$

Now, the power out of the motor equals the power into the gear box, $P_{gearbox, in}$. In a similar manner, a gear box efficiency, $\eta_g$, can be defined where

$$\eta_g = \frac{P_{gearbox, out}}{P_{gearbox, in}}. \quad (14)$$

Assume that $\eta_{gearbox} = 0.95$. $\eta_m$ will be determined through experiment.

The purpose of the gear box is to reduce the revolutions per minute of the motor to one that allows the propeller to operate within its most efficient range. The gear box reduces the motor speed (in rad/s), $N_{motor}$
to the propeller speed (in rad/s), $N_{prop}$, by a factor known as the gear ratio, GR. This is given by

$$N_{motor} = GR \cdot N_{prop}, \quad (15)$$

where for the experiments $GR = 2.21$.

From the above equations it can be determined that

$$P_{motor, out} = \frac{Q_{prop, in} \cdot N_{prop}}{\eta_g}. \quad (16)$$

This yields

$$\eta_m = \frac{Q_{prop, in} \cdot N_{prop}}{\eta_g \cdot i \cdot V}. \quad (17)$$

Every term on the right hand side is known or determined from measurements. Hence, the motor efficiency can be determined using the data and Equation 17.

Finally, the data can be used to determine the values of three coefficients that are commonly employed to characterize propeller performance. These are the thrust coefficient, $C_T$, the torque coefficient, $C_Q$, and the power coefficient, $C_P$. These are defined by the following equations:

$$C_T = \frac{T}{\rho \cdot n^2 \cdot d^4}, \quad (18)$$

$$C_Q = \frac{Q_{in}}{\rho \cdot n^2 \cdot d^5} \quad (19)$$

and

$$C_P = \frac{P_{in}}{\rho \cdot n^3 \cdot d^4}; \quad (20)$$

where $T$, $Q_{in}$ and $P_{in}$ are for the propeller, $d$ is its diameter, $\rho$ is the density of air, and $n$ is the propeller’s revolutions per second.
5 Exercise 5

Solid Rocket Motor:
Transient Thrust Measurement

5.1 Introduction and Objectives

In this exercise, a load cell consisting of four strain gages mounted on a cantilever beam will be utilized to make dynamic thrust measurements of a solid rocket motor. These measurements in turn will be used in another exercise to make altitude predictions of a model rocket during ascent.

The first step in this process will be to calibrate the measurement system. Then, a digital oscilloscope will be used in conjunction with a Wheatstone bridge setup to record the thrust-time history for the model rocket engine.

5.2 Instrumentation

The following instrumentation will be used:

- Wheatstone bridge and operational amplifier measurement system
- Cantilever load cell
- Calibration weights and hanger
- Fluke PM3380-A Combiscope (Analog and Digital Oscilloscope)
- Estes A-83 Solid Propellant Rocket Motor
- Estes launch controller and igniters

5.3 Measurements

Unlike any of the previous lab exercises, these thrust measurements will be made in teams. Before the start of the lab, all groups will be given their rocket motor, and will take turns in acquiring their thrust data. For safety, it is important that no one be near the rocket motors when they are installed in the thrust stand and configured to be lit.

1. Before firing, weight the motor and record its initial mass.
2. The first step will be a calibration of the load cell. Calibrate by placing the calibration masses in the cup at the end of the beam and recording the output voltage of the system. Calibrate up to a mass of 1 kg in order to cover the full range of the rocket motor thrust output.

3. Once the system is calibrated, make sure that the rocket motor sleeve is in the can at the end of the beam. Now configure the scope to make the appropriate measurements. The appropriate settings are 0.1 V DC, with a time base of 100 ms. As with the dynamic beam response, set the trigger settings to trigger off of the beam response by adjusting the trigger level to around one division above ground. Give a delay of about one division as well. Depress the “SINGLE” button to arm the scope. Tap on the beam to check that the system is connected properly and triggering as planned. Then arm the scope again.

4. Now we are ready to set up the firing. Insert the motor into the sleeve at the end of the beam. Insert an igniter into the engine and cap it off with an engine plug. Make sure the safety key is removed from the launch controller, and then attach the launch leads to the igniter wires.

5. Once everything is connected, arm the scope and clear out of the firing area. Insert the safety key into the launch controller. The light bulb on the front of the controller should be lit. Fire the motor by depressing the button on the controller.

6. Check the scope to see if a reasonable thrust trace was obtained. Save the signal into one of the scope’s memory locations. Downloaded this information onto the laboratory computer to save the data for subsequent analysis.

7. Finally, take the empty motor casing and weigh it.

5.4 What to Report

Report the results in a short technical memo.
6 Exercise 6

Rocket Launch: 
Altitude Prediction and Measurement

6.1 Introduction and Objectives

Using the thrust curve for the solid rocket motors in another exercise, predictions can be made of the maximum altitude that a model rocket can reach. This can be accomplished by deriving the appropriate equations of motion, developing appropriate models for all of the force terms, and solving this differential equation for the altitude. An actual model rocket also can be launched and its maximum altitude measured.

6.2 Instrumentation

The following equipment will be used:

- Estes model rocket
- Estes launch pad
- Estes altimeter
- video camera

6.3 Measurements

Several launches of similar rockets will be conducted. For each launch, the altitude will be determined using the altimeter, which uses a triangulation method to determine the height.

6.4 Equations of Motion

Simplify the analysis by assuming that the rocket travels only along a straight line in the vertical direction. Drawing an appropriate free body diagram for the rocket and applying Newton’s Second Law yields

$$\sum F = T - mg - D = m \frac{d^2y}{dt^2}. \quad (21)$$
In the above equation, $T$ is thrust, $m$ the mass, $g$ gravity, $D$ the drag, and $y$ is the vertical displacement. If the drag is written in terms of a drag coefficient, $C_D$,

$$T - mg - \frac{1}{2} \rho S C_D \left( \frac{dy}{dt} \right)^2 = m \frac{d^2y}{dt^2}. \quad (22)$$

The quantity $S$ is the appropriate reference area, which in this case is the body tube cross-sectional area. The drag coefficient can be obtained from the handouts on the drag of model rockets. Note that during the burn phase of the rocket motor, the thrust varies with time, as determined in the previous lab exercise. After the burn phase, the thrust becomes zero. The rocket, however, still continues to travel vertically upward until it reaches its maximum altitude where its velocity equals zero. Hence, there are two phases in the rocket’s ascent, the burn phase and the coast phase. Both phases can be described by the same equation of motion by specifying the value of the thrust to become zero at the end of the burn phase.

Equation 22 is a non-linear, second-order differential equation. This equation is complicated by the fact that both the thrust and mass are quantities that are changing with time. This equation cannot be solved analytically unless some simplifying assumptions are made. Alternatively, through the use of numerical techniques and using MATLAB, this equation can be integrated directly to give us displacement as a function of time.

6.5 Solution Approaches

6.6 Analytical Solution

An analytical solution to the above equation of motion can be obtained if the simplifying assumptions that the mass and thrust are constant in time during the burn phase of the rocket motor are made. These constant values can be calculated from the data that was acquired in the previous solid rocket motor firing lab exercise. The average thrust value should be obtained using the MATLAB `trapz` function and the thrust data file. The following equations result from integrating the equation of motion by parts.

The altitude, $h_b$, at the end of the burn phase will be:
\( h_b = \frac{\beta_o}{g} \log_e \left[ \cosh \left( gt_b \sqrt{a_o/\beta_o} \right) \right], \)  \( (23) \)

where \( a_o \) denotes the drag free acceleration in g’s as given by

\[ a_o = \frac{T}{W} - 1, \]  \( (24) \)

with \( T \) being the thrust and \( W \) the weight. Also \( \beta_o \), known as the density ballistic coefficient, is defined as

\[ \beta_o = \frac{W}{0.5 \rho C_D S}. \]  \( (25) \)

Further, the velocity, \( V_b \), at the end of the burn phase is

\[ V_b = \sqrt{a_o \cdot \beta_o \tanh \left( gt_b \sqrt{a_o/\beta_o} \right)}. \]  \( (26) \)

The altitude gained up to the maximum altitude during the coast phase, \( h_c \), will be

\[ h_c = \frac{\beta_o}{2g} \ln \left( 1 + \frac{V_b^2}{\beta_o} \right). \]  \( (27) \)

Thus, the maximum altitude, \( h_{\text{max}} \), is

\[ h_{\text{max}} = h_b + h_c. \]  \( (28) \)

### 6.7 Numerical Solution

The second-order differential equation can be written as a system of two first-order equations. If \( y_2 \) to is defined as the vertical displacement and \( y_1 \) as the vertical velocity, the above second order equation reduces to

\[ \dot{y}_1 = \frac{1}{m} \left( T - mg - \frac{1}{2} \rho C_D y_1^2 \right) \]  \( (29) \)

and

\[ \dot{y}_2 = y_1. \]  \( (30) \)

These equations can be integrated using a numerical integration algorithm.

The problem of the changing mass and thrust values with time still has to be addressed. The mass can be approximated by assuming
that the motor burns at a constant rate. Thus, the mass will decrease linearly from the initial mass of the rocket to the final mass after the motor is spent. If $m_o$ is the initial rocket mass and $m_f$ the final mass, then

$$m(t) = m_o - \frac{(m_o - m_f)}{t_b} t,$$

where $t_b$ is the burn time of the rocket motor. This expression is only valid up until the motor stops firing. After the thrust stops, the mass is constant and equal to $m_f$. This expression easily is incorporated into any solution algorithm. The thrust presents a unique problem in that the thrust data is at discrete values of time. There are several ways to approach this problem. The simplest method would be to obtain some average thrust value and assume that the thrust assumes this constant average value for the duration of the burn time. More involved methods might be to curve fit a polynomial curve to the thrust data to arrive at a continuous representation of the thrust over the time period of interest. Finally, an interpolation algorithm could be written to find an approximation for the thrust value at any time during the calculation.

There are MATLAB commands that will help demonstrate how to make these calculations. The command \texttt{ode23} is a numerical integration algorithm that can be used to solve systems of differential equations. Additional information, as well as additional examples, can be found in the MATLAB manual.

To proceed, first create an M-file called \texttt{launch.m}. In that M-file, type the command \texttt{[t,y]=ode23(’alt’,t0,tf,y0)}. Before that command, specify the values for $t_0$ (the initial time, here set equal to zero), $t_f$ (the final computation time, on the order of several seconds based upon the type of rocket motor used), and $y_0$ (the initial altitude, here set equal to zero). The command calls another M-file (name it \texttt{alt.m}) that will numerically integrate the equations that are set up in \texttt{alt.m} and then pass the results back to \texttt{launch.m} for subsequent plotting. Altitude can be plotted versus time in \texttt{launch.m} by the command \texttt{plot(t,y(:,2))}. The maximum altitude is given by the command \texttt{max(y(:,2))}.

Remember to create \texttt{alt.m}. The essential lines in \texttt{alt.m} are the first line: function \texttt{ydot=alt(y,t)} and the last line: \texttt{ydot=([1/m]*(T-}
\[ m g - (0.5 \rho C_D S) y(1)^2; y(1)]. \] In between the first and last lines, T, m, g, \( \rho \), \( C_D \) and S must be defined. Here, use average values for T and m or compute them.

Finally, to obtain the solution plot, simply type “launch”.

6.8 What to Report

The results of the rocket motor firing and rocket launch laboratory exercises are to be submitted together in the form of a full technical report by each team. The report should highlight the altitude predictions and how the data that relates to these calculations. A comparison between the predicted and actual measured altitudes must be made. Rational, scientifically based explanations, supported by additional calculations, must be presented to explain any differences between the predicted and measured altitudes.
7 Exercise 7

Cylinder in Cross-Flow:
Pressure and Velocity Measurement

7.1 Introduction and Objectives

The main objective of this lab is to become familiar with the techniques and equipment for making pressure measurements on a circular cylinder. The cylinder is placed in a cross-flow in a subsonic in-draft wind tunnel. Velocity measurements also are made. In addition, concepts of uncertainty are addressed both in the taking of the measurements and the propagation of these uncertainties to obtain estimates for the lift and drag coefficients and the drag of the cylinder.

7.2 Instrumentation

- Dwyer Model 246, 0 in. H₂O to 6 in. H₂O inclined manometer (resolution: 0.02 in. H₂O)
- Microswitch Model 163PC01D36, -5 in. H₂O to +5 in. H₂O differential pressure transducer
- Tenma Model 72-4025 digital multimeter (resolution: 0.01 V on 20 V full scale; 0.001 V on 2 V full scale)
- Princo barometer (resolutions: 0.01 in. Hg and 1 °C)
- wind tunnel rpm indicator (resolution: 20 RPM)
- cylinder rotating position indicator (resolution: 1 ° angle)

The test section for this exercise contains a pitot-static tube and a cylinder fitted with pressure taps. The pitot-static probe is located in the front of the test section and will be used to determine the free-stream centerline velocity of the wind tunnel. The cylinder is 1.675 ±0.005 in. in diameter, 16.750 ±0.005 in. in length, and has several pressure taps located in a line along its span. For this experiment, we will be using the tap in the middle of the cylinder to minimize any possible wind tunnel wall effects. The cylinder and, more importantly, the
pressure tap can be rotated through 360° using the position indicator on the side of the test section.

In this exercise, both an inclined manometer and differential pressure transducers will be used for measuring pressure. Each pressure transducer is connected to a voltmeter to measure its output. The transducers used have a linear 1.01 V DC to 6.05 V DC range (corresponding to a range of -5.0 in. H₂O to +5.0 in. H₂O). Thus, the transducer output will be 3.52 V when the pressure difference is 0 in. H₂O. Any negative differential pressure will be less than 3.52 V. It is important to remember that both the inclined manometer and the transducers measure the difference in the pressure between the two lines connected to them. Be sure that all of the pressure lines are connected in the appropriate manner.

7.3 Measurements

First check to see that all of the lines are set up properly. One of the pressure transducers will be connected hydraulically in parallel with the inclined manometer to measure the pressure difference from the pitot-static probe. The other transducer will measure the pressure difference between the pressure tap on the cylinder and an adjacent static port. The static port for the cylinder can be located on the side wall of the test section, just above the cylinder.

Adjust the voltmeters to the appropriate scales. With the tunnel off, what should the voltmeters display? If the output is not as expected, be sure to make a note of it so that the bias can be accounted for later when reducing the data. Now, check the level at the top of the inclined manometer and adjust the manometer until it is level. If necessary, zero the manometer by loosening and sliding the scale until the bottom of the meniscus is set at zero.

Record the room temperature and pressure using the Princo barometer. Record the room temperature in °C and the pressure in in. Hg. Also record the % correction factor. This factor corrects for the thermal expansion of the metal scale that is used to determine the pressure. For example, the correction factor is 0.38505 % at 22 °C. So, the actual pressure equals the recorded pressure times (1 - 0.0038 505). The actual pressure and temperature values will be used later to compute the
density of the air in the lab assuming ideal gas behavior. Subsequently, the density value is needed to compute velocities and the Reynolds number.

First, calibrate the wind tunnel rpm indicator with respect to the wind tunnel velocity. Do this by setting the tunnel fan at various RPM and record the pressure difference measured using both the inclined manometer and the pressure transducer connected to the pitot-static tube. Start the wind tunnel fan by following the directions on the control panel stand. Make sure the circuit breaker is turned to on and push the start button. Set the rpm indicator to 100 rpm and wait a minute for the tunnel to come to steady state. Record the pressure difference indicated on the inclined manometer and the voltage from the voltmeter connected to the output of the pressure transducer that is connected to the pitot-static tube. Proceed through all the rpm settings in increments of 100 rpm up to and including 900 RPM. Repeat several RPM measurements to assure reproducibility. While taking data, convert a recorded pressure transducer voltage to in. H$_2$O. Are the inclined manometer and pressure transducer readings in agreement?

Now perform pressure measurements on the cylinder. Check that the cylinder’s pressure tap orientation is at 0° as indicated on the rotating position indicator. Now set the tunnel rpm to that RPM specified. Record the dynamic pressure from the pitot-static tube from both the voltmeter and the inclined manometer. Then, in increments of 10°, rotate the cylinder and record the output from the pressure transducer connected to the cylinder. Also record the voltage from the pressure transducer connected to the pitot-static tube. This reading is a good indication of how constant the wind tunnel velocity is during the measurements. When the entire 360° range has been covered, go back and make a couple of spot checks at various angles to check repeatability.

When finished collecting the data, turn the dial indicator on the wind tunnel fan control panel back to zero and stop the tunnel. Again, follow the instructions on the control panel. Finally, make sure that all equipment is turned off.
7.4 What to Report

Submit the results in the form of a technical memo. Be sure to include (as a minimum) the following information:

- Calculations of the air density, operating tunnel velocity (both in SI units) and the Reynolds number.

- Calculation of the temporal precision error \( (S_x/\sqrt{N}) \) of the pitot-static tube pressure transducer voltage taken at the various \( \theta \) during the cylinder measurements. Compare this value to the mean value by determining the percentage of the precision error with respect to the mean value. Ideally, this should be zero if the wind tunnel velocity remained constant during the measurements period.

- Two plots of the wind tunnel RPM calibration, one with the velocity (in ft/s) calculated from the measured in. H₂O from the inclined manometer and the other with that calculated from the pressure transducer voltage. Plot each manometer measurement or voltage along the ordinate versus the tunnel rpm along the abscissa. Perform the necessary regression analysis and display the proper error bars. In performing the regression analysis, keep in mind the actual relationship between differential pressure and velocity.

- Plot of the pressure coefficient, \( C_p \), on the y-axis as a function of azimuthal angle, \( \theta \), on the x-axis (include on this plot the analytical, inviscid solution for comparison (see the Supplemental Information section).

- Calculations of the lift and drag coefficient of the cylinder, \( C_L \) and \( C_D \), and the drag force on the cylinder, \( D \), in units of N. Does this calculated drag force appear reasonable?

- Uncertainty estimates presented in the form of tables supported by example calculations. Two tables are required, one for the measured uncertainties and the other for the result uncertainties. Supporting calculations of all of uncertainty estimates should be contained in an appendix.

Calculations of the lift and drag coefficients will require some numerical integrations of the \( C_p \) data. The appropriate equations are included in
the next section. A spreadsheet can be used to perform a simple trape- 
zoidal rule integration. What should be the $C_L$ value of the cylinder? Use the $C_L$ calculation to check the calculations.

Some possible areas of discussion for this lab might include: What does the analytical solution predict for a drag coefficient? Does the experimental value confirm this? What might some possible reasons be for this? Try to think of the assumptions made in the analytical solution. Are these assumptions valid?

7.5 Supplemental Information

7.6 Velocity Calculation

If incompressible, inviscid, irrotational flow is assumed, then the complete form of the momentum equation reduces to Bernoulli’s equation,

$$P - P_\infty = \frac{1}{2} \rho u_\infty^2,$$  \hspace{1cm} (32)

where $P - P_\infty$ is the pressure difference measured by the pitot-static tube, $\rho$ is the density, and $u_\infty$ is the free stream velocity. Solving for $u_\infty$ gives

$$u_\infty = \sqrt{\frac{2\Delta P}{\rho}}.$$  \hspace{1cm} (33)

This relation can be used to calculate the wind tunnel velocity for a given rpm setting. This relation is also used to estimate the uncertainty in the calculated velocity based on the experimental uncertainties in both the density and pressure difference. Combining uncertainties, the appropriate formula for this velocity calculation would be

$$\partial u_\infty = \sqrt{\left(\frac{\partial u_\infty}{\partial \Delta P} u_\Delta P\right)^2 + \left(\frac{\partial u_\infty}{\partial \rho} u_\rho\right)^2}.$$  \hspace{1cm} (34)

7.7 Reynolds Number

The Reynolds number is defined as

$$Re = \frac{\rho u_\infty D}{\mu}$$  \hspace{1cm} (35)
This is based on the cylinder diameter, \( D \), the free stream velocity, \( u_\infty \), the density, \( \rho \), and the absolute (dynamic) viscosity, \( \mu \). For air, the absolute viscosity is given by the Equation ?? in Chapter ??.

\[
\mu = \frac{b \cdot T^{3/2}}{S + T}
\]  

(36)

where \( \mu \) is in units of N\( \cdot \)s/m\(^2\), \( T \) in K, \( S = 110.4 \) K, and \( b = 1.458E-06 \) kg/(m\( \cdot \)s\( \cdot \)K\(^{1/2}\)).

### 7.8 Pressure Coefficient

The pressure coefficient (\( C_p \)) is defined as

\[
C_p = \frac{P_\theta - P_\infty}{\frac{1}{2} \rho u_\infty^2}.
\]

(37)

The pressure difference in this equation is the \( \Delta P \) measured for each individual rotation angle, \( \theta \). Thus, for every angle, a \( C_p \) value can be calculated. At \( \theta = 0 \), a stagnation point exists, for which \( C_p = 1 \). If \( C_p \) is calculated based on the dynamic pressure measured by the pitot-static tube upstream of the cylinder, at \( \theta = 0 \) the \( C_p \) value may be slightly less than one. This mainly is due to a small pressure decrease through the tunnel between the pitot-static tube and the cylinder. In calculating the \( C_p \) values from the data, it is often easiest to simply assume \( C_p = 1 \) at \( \theta = 0 \), and calculate the rest of the \( C_p \) values based on the \( \theta = 0 \) pressure measurement. That is, the corrected \( C_p \) value is given by

\[
C_{p,\text{corrected}} = \frac{C_{p,\theta}}{C_{p,\theta=0}} = \frac{V_{\text{trans},\theta}}{V_{\text{trans},\theta=0}},
\]

(38)

where \( V_{\text{trans}} \) denotes the voltage of the pressure transducer after being corrected for its offset voltage at zero velocity. Equation 38 is valid because the differential pressure is related linearly to the transducer voltage after the offset correction.

The derivation of the analytic solution for \( C_p \) for this situation can be found in any standard aerodynamics text. The final result is

\[
C_p = 1 - 4 \sin^2 \theta.
\]

(39)
7.9 Lift and Drag Coefficients

The formulas for the lift and drag coefficients of a circular cylinder in cross-flow are derived in many aerodynamic texts. The results are presented here:

\[
C_D = -\frac{1}{2} \int_0^{2\pi} C_p(\theta) \cos(\theta) d\theta \tag{40}
\]

\[
C_L = -\frac{1}{2} \int_0^{2\pi} C_p(\theta) \sin(\theta) d\theta. \tag{41}
\]

These can be calculated for the data using a numerical integration algorithm. The \texttt{trapz} function in MATLAB. The command \texttt{z=trapz(x,y)} computes the integral of \(y\) with respect to \(x\) using trapezoidal integration, where \(x\) and \(y\) are vectors of the same length. Once \(C_D\) is known, the actual drag force on the cylinder can be found using \(C_D\), the dynamic pressure and the frontal area of the cylinder.
8 Exercise 8

Digital Oscilloscope and Function Generator

8.1 Objectives
The objective of this laboratory exercise is to introduce the capabilities of a function generator and digital oscilloscope, and their use in basic measurements.

8.2 Instrumentation
The following equipment will be used:

- Hewlett Packard HP 33120A Function Generator
- Fluke PM3380-A CombiScope [Analog and Digital Oscilloscope]

8.3 Measurements
In this laboratory exercise the capabilities of a function generator (FG) and a digital oscilloscope (DO) will be demonstrated. The DO will be used to observe and analyze various signals produced by the FG. The triggering capabilities of the DO also will be studied.

The FG is an electronic instrument that generates waveforms of preset shape, amplitude and frequency. Often it is used in a laboratory setting to provide a known input to data acquisition devices such as the DO or a computer. This helps the investigator debug and calibrate measurement systems. The DO is perhaps the most used piece of electronic equipment in a laboratory. It is the experimenter’s electronic eye. It has the basic capability to acquire, store, display and analyze signals, and download them to other devices. The typical DO has at two amplifiers (with variable gains), a sample and hold circuit and an A/D converter. The digitized signal is stored in its random access memory (RAM), and the output is sent to the video display. Most digital scopes use a single CCD (charge coupled device) array per channel to sample the signal and hold that value until the A/D has time to convert the signal. Once data has been acquired by the DO, it can be overwritten to display a new signal or it can be saved and analyzed using on-board
software programs. Also, it can be downloaded to another device such as a plotter or computer for further analysis.

Figure 9: Schematic of the front panel of the PM3380A CombiScope.

The front panel of the DO is divided into seven functional areas, as shown in Figure 9. First, examine these areas, referring to their functions listed below.

Area 1: Basic screen and power controls with self-explanatory labels.

Area 2: Screen text control buttons and menu buttons.

Area 3: Basic controls for input Channels 1 and 2. There are controls for amplitude scaling (volts per division), positioning the signal, establishing the signal coupling (AC, DC or GND), turning the channel ON or OFF, scaling the signal, AUTO RANGE, and determining whether the signal will be increasing or decreasing for triggering. Each area also has two additional keys that have special application. The first is the VERT MENU key (not used at this time) and an AVERAGE key which averages the signals on both channels simultaneously. The last key is the INV key which is applicable to Channel 2 only (it inverts the signal).

Area 4: The time and trigger control section for the main time base (to be presented later). Again, there is an AUTO RANGE control plus controls for time scaling (per division), trace position, magnification and several trigger controls that will be discussed later.

Area 5: The cursor control section. The TRACK control knob has a dual purpose. If it is used for measurements of a voltage versus
time trace, it sets the reference x-cursor, while the delta control knob (the one with the \( \Delta \) above it) positions the measurement x-cursor. A reading of the value of the reference x-cursor or the difference between the x-cursors (in volts, time, or both) is provided in text on the screen if chosen. The TRACK control also is used for selection in the menu items and the requirement for their use is indicated by a small ‘T’ inside of a circle.

Area 6: The delayed time base control area has a special application that won’t be covered in this exercise.

Area 7: External trigger section which allows one to use a signal of choice as the control for initiating acquisition of data other than the signals of either Channels 1 or 2 or the line signal.

Area 8: This is the extended function area. The simplest function is the AUTOSET button. This automatically finds the signal and adjusts the settings to produce a properly-proportioned signal on the screen. The other buttons provide the user with a host of powerful built-in functions of math, measurement and presentation.

Lastly, there are the hard wired inputs along the bottom that are clearly labeled.

**Part 1: Viewing a periodic signal on the DO.**

Connect the OUTPUT of the FG to Channel 1 of the DO. Set the FG to deliver a square wave having 150 Hz and VPP (peak-to-peak) amplitude of 4 V, as observed on the DSO (which is 2 VPP on the FG). (NOTE: The VPP amplitude set on the FG appears as twice that amplitude on the DSO. This is because of an impedance mismatch which won’t be dealt with at this point - see Chapter ?? for an explanation). Make sure that the DO is in digital mode by pressing the yellow ANALOG button which toggles between the two modes and indicates briefly the mode on the screen. The trace should show one or at most two complete cycles of the signal while maximizing the vertical display. Do NOT change the FG setting from what was set initially. Use the DO’s vertical gain control, specified in V/div, and horizontal time control, specified in s/div. Center the trace vertically using the
position control for that channel. Place the start of the trace on the left edge of the screen grid by adjusting the X POS knob. Record the following using visual observation not using the cursors.

1. Vertical scaling per division (V peak-to-peak):

2. Time scaling per division (s):

3. Frequency (Hz):

   Did the display show an actual square wave, top and bottom parallel with the horizontal grid with little connection between the two (very faint compared to the horizontal lines)? If not, correct it by changing the signal coupling on the DO. Immediately to the right of the vertical scaling value displayed on the screen is a = sign for DC, a ~ sign for AC and a ⊥ sign for ground. Observe the signal first with DC coupling and then with AC coupling. Sketch each of the two traces.

   Now repeat the measurements using the cursors. To do so, simply press the CURSORS key. Select the second of the =, ||, #, or ‘auto’ choices. Then select and follow the READOUT menu, selecting reading of V1 (voltage measured using first cursor) and 1/ΔT (frequency). Use the cursors to determine the following:

4. Minimum voltage (V):

5. Maximum voltage (V):

6. Maximum - Minimum voltage (V):

7. Frequency (Hz):

   Now examine more closely the function of the types of couplings, AC, DC and GND. First set the coupling on the DO to GND. This grounds the input to the channel, resulting in a horizontal line trace at 0 V. Next set the coupling to DC. Note the shape and position of the trace. Now add a DC offset of +0.5 V on the FG to the generated signal. Adjust the scaling or position of the signal on the DO as necessary to keep it in view. What has happened to the trace?

   Now, change the coupling to AC. What change in the display occurred? Sketch the DC and AC coupled traces.
Often the signal being observed has a midpoint, zero voltage level, that is not as obvious. Then, it is necessary to establish a zero reference line. On the DO, this reference is arbitrary. To establish a zero reference, simply ground the input signal. Establish a zero reference on the center horizontal graticule from the bottom by rotating the vertical position knob and then unground the signal.

Now go back and observe the original signal with DC coupling. Is it centered on the reference line? Now set the FG offset back to 0 volts. Is it centered on the reference line? Try this also with AC coupling with and without offset. How are these different than with DC coupling?

An easier approach is to use the AUTOSET button which, based up some preset criteria, will select the appropriate time and amplitude scaling and coupling. Another function is the magnification function, MAGNIFY. Before using this, change the time base to get approximately 20 cycles on the screen without changing the FG settings. Now press the right directional arrow MAGNIFY button. Notice that a horizontal bar temporarily indicates what portion of the trace in memory is currently being displayed. Now keep pressing the right arrow MAGNIFY button until only one to two cycles is seen on the screen. The amount of time amplification is indicated temporarily on the screen by a * followed by a number indicating the amount. Note that the signal doesn’t look like a perfect square wave any more. This is because the signal stored in the RAM consists of a fixed number of digital points. Magnifying it produces a signal constructed by drawing lines in between the points. Finally, using the left directional arrow MAGNIFY button, bring the time amplification back to 1.

**Part 2: Using the trigger on the DO**

In most situations, the user wants to see what the continuous signal looks like. Therefore, the DO is normally operated in a continuous sweep mode. However, there is something happening in the background that is not readily apparent to the user. In most instances, if the user has selected a time scale that allows viewing of several cycles of a standing wave, it appears that the signal is starting at the same point in its cycle. This is because the DO, in its default mode, is being triggered by the signal itself. The trigger on a DO determines when the trace will
begin. On most DOs, there are two conditions that must be met for a trigger to occur: a specified voltage relative to ground and a direction of change. In addition, there are several choices for the source of the trigger, be it the signal into Channel 1 or 2, an external signal, or the (power) line signal (in this country, a 60 Hz signal). In all cases, the trigger initiates the trace which then continues until it is completed. Then, based upon the trigger options selected, the trace will start over on the next trigger received, stop all together, or execute some other user selected option.

To explore the trigger, start with something already familiar. Establish a square wave as done before with the same characteristics (150 Hz, 4 V peak-to-peak on the DO, no DC offset). Check the settings (press the STATUS button).

The first thing is to observe the effect of direction of change, or slope, on the trace output. In the control box of the channel selected for input, press the TRIG button and observe the change. The slope direction is indicated on the far right, bottom corner of the screen. Keep the STATUS text on screen during the process to observe the settings.

Next, in the time and trigger control section, press the TRIGGER button. Scan through the menu to see its options. Note that when the trigger source (the second item down) is changed, an important thing happens. If the trigger source and the input signal do not have a common frequency, the trace is no longer stable. For each trigger source, observe the stability of the trace. Now connect the SYNC of the FG to the EXT TRIG INPUT of the DO. Go back and put the trigger source on 'ext.trig'. The square wave should be stable now on the screen. This is because the FG sends out a sharply rising pulse at the instant the square wave begins, which is an excellent source for an external trigger.

Now, set the trigger for edge, ch1, level-pp off, noise off, and AC coupling. Then press the TB MODE button (Time Base) and toggle through the selectable trigger options for the top item of the menu. Note that when on 'single', the trace freezes. As the name indicates, it will select a trigger only once in this mode. To obtain a new trace, press the SINGLE button in the time and trigger section. As this is
done, note that a red light to the right of the button briefly comes on. When it is lit, a trigger is armed, indicating that a trigger signal has not been received. After the trigger occurs, the light goes off.

As stated earlier, the DO is normally used to observe periodic signals and the trigger not as much. But, with the trigger option, the DO can be a valuable data acquisition and analysis tool. Consider the following situation. Assume that a signal from a single-event test, such as the firing of a rocket motor, is expected to have an amplitude of 1.5 V to 5 V and a duration of 1.5 ms to 6 ms. To see the complete event from start to finish, view the trace starting 1 ms before the event occurs and ending 1 ms after it ends.

To do this, set up a known signal having an initially rapid amplitude change, a square wave. Change the settings on the FG to obtain a peak-to-peak amplitude of 2.5 VPP on the FG display with a DC offset of 1.0 V. Set the FG frequency to 250 Hz to obtain a period of one square wave equal to 4 ms. Be sure that the TB MODE is in 'auto'. Verify these requirements using the cursors and record the information below.

8. Maximum voltage (V):
9. Minimum voltage (V):
10. Period of one cycle (ms):

Note that if the trace is moving in an apparent random fashion, the action can be stopped by pressing the RUN/STOP button in the time and trigger section. If any changes are made to the generated signal, allow the DO to run to allow the changes to be displayed. Another method is to use the MEASURE function with MEAS 1 set to measure 'pkpk' and MEAS 2 set to 'freq'. Both must be turned on. Once everything is set, press the MEASURE button again to turn the menu display off.

Choose a time scale to obtain at least one period of the signal plus 1 ms before and after. Record this.

11. Time scale chosen (ms/div):
Remember, the DO provides ranges in a 1, 2, 5 format. Now set the pretrigger (the time-record length prior to receiving the trigger). To do this, turn the TRIGGER POSITION knob until the appropriate reading (this will be indicated by a ‘-dv’ on the screen). The position will be shown by a small triangle only for ‘-dv’s. All data gathered to the left of this symbol occurs prior to the trigger; all the data to the right occurs after the trigger. The time to the left of the symbol should be 1 ms. Now the trigger level must be set. First, make sure that the conditions of the trigger used above are set, namely edge, ch1, level-pp off, noise off, and AC coupling. Set ‘auto’ in the TB MODE. Determine the lowest trigger level that will produce an active, stable trace (constantly being refreshed and remaining steady) by turning the knob labeled TRIGGER LEVEL. The value will be indicated by ‘Level=’ on the screen. Record the value below. Now find the highest trigger level and record its value.

12. Lowest trigger level (V):
13. Highest trigger level (V):
14. Difference between trigger levels (V):

Now repeat the process using a DC coupling on the trigger. Record the values.

15. Lowest trigger level (V):
16. Highest trigger level (V):
17. Difference between trigger levels (V):

Now repeat the process again for DC coupling but with no DC offset on the FG. Record the values below. The AC coupling case would be the same as before.

18. Lowest trigger level (V):
19. Highest trigger level (V):
20. Difference between trigger levels (V):
How do the *ranges* of the trigger levels compare for the three cases? How do the absolute trigger levels compare for the three cases?

Next, with a properly triggered signal, press the SINGLE button in the time and trigger section of the front panel. If all was done correctly, a stationary trace of the square wave signal is seen. If not, go back and repeat before proceeding to the next step.

Now return the DO to 'auto' under the TB MODE. Set the FG to display an amplitude setting of 2 VPP, a frequency of 1 Hz and no DC offset. The low frequency will allow one to see the actual trigger occur. Adjust the amplitude setting to 1 volt DC and the time base setting to 100 ms. Center the trace on the screen by establishing a zero reference. View the signal. It should be repeating over and over in time on the screen. Now set 'single' in under TB MODE and 'ch1', 'level-pp off' and 'dc' under TRIGGER. A 'T-' should be on the screen, indicating the amplitude level of the trigger. Move it up and down by rotating the TRIGGER LEVEL knob, then finally position it about one division above the top level of the square wave. Press the SINGLE button in the time and trigger area. Only a horizontal line should be present on the screen, indicating that the signal has not triggered. The red 'ARM'D' light should be on. Now gradually rotate the knob slowly to bring down the trigger level while watching the 'Level=' value on the screen. Observe and record below the value indicated when the signal triggers. If desired, the triggering process can be repeated by pressing the SINGLE button to arm the trigger and then moving the knob in smaller increments to get a better estimate of the trigger level. Now move the indicator below the bottom of the signal and find the minimum value (by moving the indicator up) where the signal will trigger. Record the value below. Thus, determine the range of the trigger level that will properly trigger the DO for this square wave.

21. Lowest trigger level (V):

22. Highest trigger level (V):

23. Difference between trigger levels (V):
8.4 What to Report

Turn in this document with all questions answered. No technical memo is required for this exercise.
9 Exercise 9

Digital Data Acquisition

9.1 Objectives

The objective of this laboratory exercise is to investigate several aspects of digital data acquisition using both a digital oscilloscope and a computer data acquisition system. Specifically, several ways to acquire, store and analyze various waveforms will be explored and some of the limitations of digital data acquisition will be examined.

9.2 Instrumentation

The following equipment will be used:

- Hewlett Packard HP 33120A Function Generator
- Fluke PM3380-A CombiScope [Analog and Digital Oscilloscope]
- Personal computer with a United Electronics Incorporated (UEI) 12-bit, 16 channel A/D board for -5 to +5 V input, and associated software

9.3 Measurements

This laboratory exercise is divided into three sections. The first considers the use of a computer data acquisition system to digitally sample a simple periodic wave of known frequency. The second and third involve the use of the digital oscilloscope to capture, store and subsequently analyze various waveforms.

9.4 Sampling a Periodic Waveform

For this section, the function generator (FG) will be used to generate a simple sine wave. The output of the FG will be sent to both the digital oscilloscope (DO) and the computer data acquisition system (DAS). The FG and DO will be used to ascertain the amplitude and frequency of the wave. The waveform acquired by the DAS will be viewed using the graphical display of a software package. Then, the sample rate of
the DAS will be varied and the waveform frequency recorded. This will permit investigation of the relation between the sampling frequency and the frequency of the input wave (and thus to observe the effects of signal aliasing). To begin, check to see that the BNC cable from the FG to the DAS is DISCONNECTED (input signal amplitudes greater than 7V will destroy the input circuitry of the A/D board). Also check to see that the FG output is CONNECTED to channel 2 of the DO. Using the FG, generate a 500 Hz sine wave with 100 mv peak-to-peak with zero DC offset. Press the light green AUTOSET button on the DO to view the waveform. Acquire a single trace by pressing the SINGLE button in the time base settings area. This freezes a single trace on the display. Using the cursors (press the CURSORS button), record the observed signal frequency and peak-to-peak amplitude. Connect the DAS BNC cable to the T-connector at the FG output. From this point on in this section the FG or DO settings remain the same. Only the sampling frequency of the DAS is varied and its output recorded. Set the sampling rate on the configuration page of the software package and then measure $\Delta t$ from the software package’s graph. The second and third columns will be recorded during the lab. The fourth and fifth columns should be filled in before coming to lab. $f_{\Delta t}$ is computed directly from $\Delta t$, $f_N$ is one-half of $f_{\text{sample}}$ and $f_{\text{calc}}$ from $f_1$ and $f_{\text{sample}}$ (see Chapter ?? on how to use the folding diagram to calculate the $f_{\text{calc}}$ values). $f_{\text{calc}}$ is the aliased frequency expected to occur at $f_{\text{sample}}$.

Now take data using the DAS. Open the data acquisition software by double-clicking on the UEI icon, then on the UEI Status for Windows icon. Under File select Load configuration and then lab8.cfg. Then

<table>
<thead>
<tr>
<th>$f_{\text{sample}}$ (Hz)</th>
<th>$\Delta t$ (ms)</th>
<th>$f_{\Delta t}$ (Hz)</th>
<th>$f_N$ (Hz)</th>
<th>$f_{\text{calc}}$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: DAS Sampling Data
under Analog select “Configure”. This brings up the data acquisition parameter display. Set the sampling rate by typing it in or selecting it if available. The samples per channel should be set and remain at 128. The duration is simply the samples per channel divided by the sampling rate when only one channel is used, as in this case. After the desired sampling rate is set, press enter, then the F7 key to start the acquisition process. When completed a graph will come up on the screen displaying the acquired signal. Remember that this is the digital representation of the signal, so it will not always look exactly like the input signal. Observe the graph. Many periods of the wave are acquired. Now examine only a few of the periods (between 2 and 5) by using the expansion icon shown immediately under the word Graph. Select a region of interest by dragging the mouse and then clicking it. The selected region should now occupy the entire graph. Next, position the two cursors by single clicking the mouse at each of two points on the graph. Try to position the cursors at the same position on the wave (its top) over several periods such that a more accurate value of the $\Delta t$ for ONE period is obtained and recorded. Record the $\Delta t$ for $f_{\text{sample}} = 20,000$ Hz and its peak-to-peak amplitude. How does this value compare to the DO peak-to-peak amplitude recorded above? Now go back to the parameter display and change the sampling rate. Repeat this process until all of the listed sampling rates have been investigated and have filled in the raw data values in the table. When finished, exit the UEI software. Also, disconnect the DAS BNC cable from the T-connector.

5. When all of the columns in Table 4 are completed after lab, state how $f_{\text{calc}}$ and $f_{\Delta t}$ compare for each $f_{\text{sample}}$.

9.5 Examining Frequency Spectra

Now set the FG to deliver a 1 kHz sine wave with a 100 mV peak-to-peak amplitude with zero DC offset. Press the AUTOSET button on the DO and then adjust the settings on the DO to display around 10 cycles on the screen. Acquire a single trace by pressing the SINGLE button in the time base settings area. This provides a frozen signal to perform the fast Fourier transform (FFT).

Enter the MATH menu by pressing the MATH button along the top of the scope. Then under the MATH 1 feature select “fft” and
“ch2”, then press ENTER. Then select “on” (the Fast Fourier Transform [FFT] of the sine wave should appear on the screen) and “no” for DISPLAY source (this will remove the sine wave trace from the display). The FFT of the input sine wave should be displayed on the screen. Press the CURSORS button and then use the cursors to measure the frequency and amplitude (in dB) of the largest peak in the frequency spectrum. Note that the scope displays the amplitudes by referencing all of the values to the largest peak present. In essence, the amplitude at each peak corresponds to the Fourier coefficient at that frequency (see Chapter ??). If the amplitude of the largest peak is defined as $A_1$ and that of a subsequent i-th peak as $A_i$, then the value reported by the scope (in dB) is found from the definition of the decibel:

$$dB = 20 \log \left( \frac{A_i}{A_1} \right).$$

(42)

Thus, it can be seen that the largest peak will be reported by the scope as having a value of 0 dB because, for this case, $A_i = A_1$. Further, if the actual value of $A_1$ is known, the values of each of the $A_i$’s can be computed using the above equation. Also remember that the DO displays digital information, so there may be two adjacent frequencies having maximum amplitudes. In this case, the actual frequency at maximum amplitude lies in between the two frequencies. Record the single frequency value at maximum amplitude (or the average value if there are two local maxima) in Table 5. Is this frequency expected?

Press the AUTOSET button to start over and then repeat the above procedure for a square wave (the second-from-the-left button on the FG) for the same frequency, peak-to-peak amplitude and zero DC offset. Record both the frequencies and the amplitudes of the first five major peaks in the spectra in Table 5. After lab, compute the amplitudes (in dB) of the first five major peaks in the spectra of a 1 kHz square wave. The amplitudes (in dB) can be found in the same manner as done in the class notes for a step function. Compare these calculated amplitudes with those obtained above. If there are any differences, explain what could be the cause(s).
<table>
<thead>
<tr>
<th>Wave Form</th>
<th>Frequency (kHz)</th>
<th>Amplitude (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sine Wave</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Peak 1</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Square Wave</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Peak 1</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Amplitude-Frequency Data for Sine and Square Waveforms

9.6 Sampling an Aperiodic Waveform

Next, the digital oscilloscope will be used to capture a transient waveform. The event to record is the oscillatory response of the cantilever load cell to an impact loading. This will be accomplished by dropping a golf ball into the can at the end of the cantilever beam.

To start off, make sure that the load cell is connected to the bridge circuit correctly. The panel meter wire with end-connector should be connected to the output of the second op amp on the bridge circuit box. Balance the bridge to zero by adjusting the ZERO ADJUST knob on the panel. Test to make sure the load cell is connected properly by lightly depressing the beam and ensuring the panel meter is responding (there should be approximately a 0.1 V to 0.2 V indication).

Check that the BNC output of the bridge circuit is connected to Channel 1 of the DO. Set the scope to the following settings: DC coupling, 0.2 V, 100 ms. This will ensure capture the full signal. Now, go into the TRIGGER menu and set “edge”, “ch1”, “level-pp” to off, and “dc”. If “ch2” appears instead of “ch1”, simply press the TRIG 1 button in the Ch1 area on the panel. Then “ch1” should appear in the TRIGGER menu. The trigger level should now be marked on the scope with a “T”. Set it at about one division above the centerline of the display using the TRIGGER LEVEL knob at the far right of the DO panel. Now set a delay for the trigger such that a part of the signal prior to the trigger event will be displayed. Do this by turning the TRIGGER POSITION knob counterclockwise. A small Δ should appear on the screen. Set it at approximately -1.00 dv. Finally, press the TB MODE button and select “single”. Then press the TB MODE button again to
exit that menu. Depress the SINGLE button on the scope such that the red arming light comes on and the scope is waiting for the event to trigger. The trigger level knob may have to be adjusted slightly higher such that the scope does not trigger off of electronic noise.

Once the scope settings are correct and the trigger level is set properly, arm the scope again (if needed) by pressing SINGLE. Now, take the golf ball and drop it into the can from a height just above the top of the can. Did the scope trigger and was the desired signal captured? If not, adjust the scope settings until a good oscillatory response from the beam is obtained.

The response of the beam should be an oscillation damped in time. When hit, the beam vibrates at its natural frequency, which can be measured using the strain gauge and Wheatstone bridge configuration. As done in the previous section, use the math function to calculate the FFT of the acquired trace. What is the dominant frequency in this signal?

Set the MATH PLUS menu to “off” and then press the MATH button to exit that menu. The stored trace of the signal should be the only item remaining on the screen.

Now download the data to the laboratory computer to save the information in a text file. Then, using the text file, plot the data to reproduce the trace as seen on the scope screen. Also determine and plot the amplitude-frequency spectrum of the signal.

9.7 What to Report

Report the results as a technical memo, being sure to include the plots requested, answers to all questions posed, and the calculations of the square wave amplitudes and frequencies for comparison with the measured values.
10 Exercise 10

Dynamic Response of Measurement Systems

10.1 Introduction and Objectives

The main objective of this laboratory exercise is to investigate the dynamic response characteristics of first-order and second-order measurement systems. First, the dynamic responses of two different-size thermocouples (first-order systems) to step input changes in temperature will be studied. Then, the second-order system dynamic response characteristics of a RLC circuit to a sinusoidal input will be investigated. All data will be acquired, stored and analyzed using a digital oscilloscope.

10.2 Instrumentation

A schematic of the set-up for part 1 is shown in Figure 10. The instrumentation consists of a thermocouple (TC), an ice bath (IB), a thermocouple reference junction and amplifier (TRJA) and a digital oscilloscope (DO).

- Digital Oscilloscope
- Two type-K (chromel-alumel) thermocouples of different size
- Analog Devices AD595AQ type-K thermocouple reference junction, linearizer and amplifier chip in a box
- Ice bath (beaker filled with crushed ice and water)

A schematic of the set-up for part 2 is shown in Figure 11. The instrumentation consists of a function generator (FG), a RLC circuit box (RLC) and a digital oscilloscope (DO).

- Function Generator
- Digital Oscilloscope
- RLC circuit box
10.3 Measurements

10.4 First-Order System Response

In this part of the lab exercise the DO will be used to determine the time constants of two thermocouples. A thermocouple is a passive temperature sensor. It consists of two dissimilar wires connected together at two junctions, namely the hot junction and the cold junction. When one junction is hotter than the other, an emf (electro-motive-force: a voltage difference) is developed between the two junctions. This voltage difference is proportional to the temperature difference between the two junctions and is generally in the millivolt range. The TRJA simulates a cold junction, amplifies the thermocouple output and linearizes it with the temperature at the hot junction at a 10 mv/C.

In fact, the thermocouple behaves as a first-order system. The equation that describes the energy exchange between the thermocouple’s tip and the environment is

\[ mC_v \frac{dT}{dt} = hA_s [T_\infty - T(t)], \]  

where \( m \) is the mass of the tip, \( C_v \) is the specific heat at constant volume of the tip, \( A_s \) is the surface area of the tip, \( h \) the heat transfer
coefficient, \(T(t)\) the temperature of the tip with respect to time, and \(T_\infty\) the temperature of the liquid at “infinity”. The solution to this first-order, linear differential equation is

\[
T(t) = T_\infty + (T_0 - T_\infty) \exp(-t/\tau), \tag{44}
\]

where \(\tau\) is the time constant of the thermocouple, equal to \(mC_v/hA_s\). This equation can be rearranged to yield

\[
\ln \left( \frac{T(t) - T_\infty}{T_0 - T_\infty} \right) = -t/\tau. \tag{45}
\]

A plot of the \(\ln\) term versus time, \(t\), will yield a line with the decreasing slope equal to \(1/\tau\). The time constant is the characteristic measure of the thermocouple’s rate of response. The time constant \(\tau\) can be determined readily by exposing the thermocouple to a step input in temperature. In this section, two thermocouples (A and B), each having different-sized tips, will be exposed to an ”instantaneous” (step) decrease in temperature (from room temperature to 0 °C).

To start, turn on the DO and the TRJA. Connect thermocouple A to the TRJA and the TRJA’s output to channel 1 on the DO. Note that the TRJA gives a linear output of 10 mV/°C referenced from 0 mV at 0 °C. Thus, its output voltage will decrease \(\sim 200\) mv for a \(\sim 20\) °C decrease in temperature. press auto set on the DO. A steady line with some noise that reads the current temperature in the lab should be observed. Adjust the voltage scale on channel 1 to be 50 mV/div and adjust the time scale to be 250 ms for the thin thermocouple and 2.5 s for the thick one. This assures that the signal can be captured with good details. Adjust the ch1 vertical position knob such that the signal baseline is displayed just approximately one division below the top of the DO display.

Now set up the DO to trigger correctly in response to a step input forcing: Press the trigger menu button. Make sure to read edge, slope = falling (the DO will trigger when the slope falls), source = ch1 (the trigger function is looking for signal from channel 1), mode = single (the trigger is waiting for a single event to occur), coupling = DC.

Move the horizontal position button to point on one division from the left side of the screen. This assures that the original level of the signal on one division is seen on the left and the triggered signal on
<table>
<thead>
<tr>
<th>No.</th>
<th>$V_{tcA}$ (mV)</th>
<th>$Time_{tcA}$ (ms)</th>
<th>$V_{tcB}$ (mV)</th>
<th>$Time_{tcB}$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Thermocouple response data

the remaining part of the screen. Move the trigger level knob to point at about 0.4 divisions below the signal level. This assures the DO will not trigger until the signal slope falls to this level. (decreasing the 0.4 to a smaller value has the risk that the DO can be triggered with noise). Make sure that the thermocouple is away from the ice bath to avoid triggering the DO.

Immerse the thermocouple into the ice bath. After ~30 s the screen should display the triggered signal starting from the room temperature to the ice temperature. Press the cursor button, adjust type = voltage (the cursors will be horizontal to measure voltage), source = ch1 and use the voltage cursors and the screen grid lines to take 10 readings of voltage versus time. Record these values in Table 6. The time constant is the time needed by the thermocouple to reach 63.2% of the final voltage, which can be read directly from a plot of voltage versus time.

Another method to obtain the time constant uses a least-squares regression fit of the data. Because the thermocouple can be represented by a first order system, the voltage changes with time is governed by

$$V(t) = V_\infty + (V_0 - V_\infty) \exp \left(-\frac{t}{\tau}\right).$$

(46)

By taking the logarithm of both sides of Equation 46, the variables can be transformed such that a least-squares linear regression analysis can be performed. From this information the time constant can be determined.

Note any obvious physical differences between thermocouples A and B.
Finally, turn OFF the TRJA’s power and disconnect the TRJA BNC from Ch1 of the DO when finished with this part of the exercise.

10.5 Second-Order System Response

In this part a FG and a DO will be used to determine the response characteristics (the magnitude ratio and the phase lag as functions of the input frequency) of an electrical RLC circuit. This circuit consists of a resistor (R), an inductor (L), and a capacitor (C) and has the response characteristics of a second-order system. The circuit will be characterized by providing an input sinusoidal wave of known amplitude and frequency from the FG to the RLC circuit and measuring the circuit’s output amplitude and time delay using the DO, as depicted schematically in Figure 11.

The electrical diagram of the RLC circuit is shown in Figure 12. The input to the circuit is between the resistor and ground and the output is measured across the capacitor connected to ground. The resistor is the parallel combination of a 1 \( \Omega \)-10k\( \Omega \) variable resistor and a fixed 1k\( \Omega \) resistor, yielding an effective variable resistance between approximately 1 and 910 \( \Omega \) using a knob. The capacitance is fixed at 0.68 \( \mu F \) and the inductance at 5 mH. The passive resistance of the inductor is 9.0 \( \Omega \), so the lowest effective resistance that the circuit can have is approximately 10 \( \Omega \) (9.0\( \Omega \) + 1\( \Omega \)).

The voltage differences, \( V \), across each component in an AC circuit are \( V = RI \) for the resistor, \( V = L \frac{dI}{dt} \) for the inductor and \( V = Q/C \) for the capacitor, where \( I = dQ/dt \). In this circuit, all three components are in series. Thus, application of Kirchoff’s Voltage Law for the circuit

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{RLC_Circuit_Diagram.png}
\caption{RLC Circuit Diagram}
\end{figure}
This second-order, linear differential equation can be solved for \( Q \) to yield the steady state output voltage amplitude

\[
E_o = \frac{Q}{C} = \frac{E_i}{C\sqrt{\left[\frac{1}{C} - L\omega^2\right]^2 + [R\omega]^2}}.
\]

From Equation 48 and the solution equation for \( Q \), the magnitude ratio is

\[
M(\omega) = \frac{E_o}{E_i} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\left(\frac{R}{R_c}\right)\left(\frac{\omega}{\omega_n}\right)\right]^2}}.
\]

and the phase lag is

\[
\phi(\omega) = \tan^{-1}\left(\frac{2\left(\frac{R}{R_c}\right)\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right).
\]

This equation yields positive values of \( \phi(\omega) \). By convention, because \( \phi(\omega) \) is a phase lag, it is plotted as having negative values. Further, for \( \omega > \omega_n \), the phase shift must be referenced correctly. Thus, the conventional plot of \( \phi(\omega) \) (in \(^\circ\)) versus \( \omega \) would actually be \(-\phi(\omega)\) for \( \omega \leq \omega_n \) and \(-180^\circ - \phi(\omega)\) for \( \omega > \omega_n \). Also note that in Equations 49 and 50 the resonant frequency is given by \( \omega_n = \sqrt{1/LC} \) and the critical resistance by \( R_c = 2\sqrt{L/C} \).

To start, make sure that the output cable from the FG is attached to the input of the RLC box and in parallel to channel 1 on the DO. The output of the RLC box should be connected to channel 2 of the DO. In that way, both the input and output signals of the RLC box can be viewed on the DO. Make sure that the toggle switch is set to “2nd order.” Now turn the R knob on the RLC box fully counter-clockwise (to MAX). This sets the resistance in the circuit to its highest value, corresponding to a high damping ratio. Then set the FG and the DO to their initial prescribed settings. These are, for the FG, sine wave with 100 Hz frequency, 4 volt peak-to-peak (V\(_{pp}\)) amplitude and no DC offset; for the DO, Chs 1 and 2, both AC with divisional settings of 2 V and 2 ms (change these settings if needed or press auto set to let the DO select the best settings).
Table 7: RLC-high resistance response data

<table>
<thead>
<tr>
<th>Freq. (Hz)</th>
<th>$E_i$ (V)</th>
<th>$E_o$ (V)</th>
<th>$\Delta t$ (s)</th>
<th>$\omega$ (rad/s)</th>
<th>$M(\omega)$</th>
<th>$\phi(\omega)$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2200</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2650</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2800</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3600</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Data will be analyzed in the final form of $M(\omega)$ and $\phi(\omega)$, each versus the normalized frequency ratio, $\omega/\omega_n$. These values will be determined from the raw data. This includes the input and output amplitudes, $E_i$ and $E_o$, and the phase lag time, $\Delta t$, which is the time between the peak of $E_i$ and the corresponding peak of $E_o$. The phase lag in degrees equals $-(360 \, ^\circ)(\Delta t/T_i)$, where $T_i$ is the inverse of the input frequency in Hz and the minus sign indicates a lag in time.

Once a satisfactory set of signals has been captured on the DO display, use the cursors to record the data. Enter all the raw data in the first four columns in Table 7. The last two columns can be filled in after the lab. When done with an input frequency, set the next one on the FG and repeat the process.

Finally, when done with all the frequencies, rotate the R knob on the RLC box clockwise such that the mark on the knob points to the top of the “I” in “MIN”. This sets the resistance in the circuit to another value, corresponding to a different damping ratio. The damping ratio, $\zeta$, equals $R/R_c$ for this circuit. Then repeat the whole procedure again for all frequencies, recording the raw data in Table 8. When done using the voltmeter, measure the total resistance of the RLC circuit (between the center pins of the IN and the OUT connectors). Subtract $9 \, \Omega$ (the
resistance of the inductor) from this value and record it. This is the value of R for this case, which will be needed later in the calculations.

10.6 What to Report

Turn in the results in a technical memo along with answers to the questions posed. Also attach any pertinent plots and M-file listings.

1. Using the data in Table 6, plot the data for each thermocouple and determine the time constant directly. Then transform the variables and performed a linear least-squares regression analysis. From that determine the each time constant. Compare the time constants obtained from the two methods. Which method is more accurate and why?

2. Compare the time constant of thermocouple A to that of thermocouple B. How and why are the time constants different? Provide a plausible physical explanation for their difference.

3. Complete the columns for $\omega$, $M(\omega)$ and $\phi(\omega)$ in both Tables 7 and 8.

4. Using these results and a program or M-file, construct two plots, one of $M(\omega)$ and other of $\phi(\omega)$ versus the normalized frequency ratio, where each of these two plots contains both of the high R

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Freq. (Hz) & $E_i$ (V) & $E_o$ (V) & $\Delta t$ (s) & $\omega$ (rad/s) & $M(\omega)$ & $\phi(\omega)$ (°) \\
\hline
100 & & & & & & \\
500 & & & & & & \\
1000 & & & & & & \\
1200 & & & & & & \\
1600 & & & & & & \\
2000 & & & & & & \\
2200 & & & & & & \\
2500 & & & & & & \\
2800 & & & & & & \\
3100 & & & & & & \\
3600 & & & & & & \\
4000 & & & & & & \\
5000 & & & & & & \\
7000 & & & & & & \\
10000 & & & & & & \\
\hline
\end{tabular}
\caption{RLC-low-resistance response data.}
\end{table}
and low R cases. These plots must contain the data along with the theoretical curves given by Equations 49 and 50, substituting the appropriate values for R, L and C for each case. Plot the data for each of the two cases using a different set of symbols for each case.

5. Does the data support the conclusion the RLC circuit behaves as a second-order system in both cases? Finally, compare the values of the damping ratio found for each case with each value of $R/R_c$. Do this by comparing the data with the corresponding values determined using various values of $R/R_c$ in Equations 49 and 50. How well do the experimental and theoretical values of $\zeta$ compare?
11 Exercise 11

Optics of Lenses, Lasers and Detectors

11.1 Introduction and Objectives

The main objective of this exercise is to become familiar with concepts in optical design, and to apply basic design techniques to several model problems. The exercises will involve incoherent- and coherent-light sources, lenses, and optical detectors.

11.2 Instrumentation

The following equipment will be used:

- small flashlight, to be used as a white-light source
- ruler and protractor
- lens, double-convex, $\phi = 65$ mm, unknown $f$
- Metrologic ML-211, diode-based laser, of unknown $\lambda$; with mount
- holographic, diffraction grating, 750 lines/mm, with mount
- diode/detector pair, with amplifier circuit and battery
- digital, mini-tachometer
- digital oscilloscope
- spinning wheel, of unknown rotation rate, $N$
- dispersing prism, with unknown index, $n$

The lab exercises will use an optical rail and a full optical bench as convenient platforms to mount various components. Several of the exercises will depend on the careful alignment of the optical elements.

This investigation will be broken into several sections. In the first exercise, a lens will be used to image a source onto a screen. The source will be the bulb of a flashlight. With sufficient magnification, fine details of the filament may be seen. This exercise will demonstrate the mounting and positioning of lenses, based on the thin-lens equation.
In the second exercise, the use of a laser will be explored by passing the laser radiation through a diffraction grating. Two of the most important aspects of laser light are its coherence and its monochromaticity. Both of these features will be utilized in the measurement of laser wavelength, based on the grating equation.

In the third exercise, an LED-photodiode pair will be provided. This circuit, along with an oscilloscope, will allow for the measurement of the rotation rate of a spinning wheel. The measurement can be compared to that of the mini-tachometer, also provided.

In the fourth exercise, the properties of a prism will be investigated. The laser will be used to make a measurement that will allow for the calculation of the index of refraction of the glass.

11.3 Laser Safety

The laser to be used is a relatively safe, low-power, laser pointer. As such, special eye protection is not needed. However, direct, prolonged exposure of the eye to this laser can still cause damage, so common sense and caution should be exercised. The following steps can help provide for a safe lab experience.

- The laser should never be aimed directly into a person’s eyes.
- The laser beam should be blocked off so that it cannot extend beyond the limits of the individual laboratory section. Dull, non-reflective barriers such as dark-colored paper or stacks of books can be used for this.
- Reflective objects in the area should be covered with cloth or blocked, to prevent secondary reflections in the lab.

11.4 Measurements

The goal of the first exercise is to become proficient in the use of simple lenses. To this end, the flashlight will be imaged onto a viewing screen. In order to properly image the filament, the distances, or conjugates, between the source and lens, and between the lens and screen, must satisfy the thin-lens equation, which is provided in the Supplemental-Information section.
Using the optical rail provided, experiment with the positioning of the lens versus the source and viewing screen. For each choice of conjugates, a magnification of the image can be calculated. Try a variety of conjugate-distance combinations, and adjust the optical system to be sure the image is in focus at the screen for each combination. Record the results in the table provided, and calculate the magnification of each attempt. You should find that the calculation for the focal length of the lens is the same in each case; the focal length is a constant property of the lens, not of the system.

In the second exercise, measure the wavelength, $\lambda$, of a laser by utilizing a diffraction grating. By using the grating equation, which is provided in the Supplemental-Information section, and given the spatial frequency of the grating to be 750 lines/mm, the wavelength can be calculated. Carefully measure the position of the first fringe (i.e. for $m = 1$ or $m = -1$), and average several readings. A table has been provided.

In the third exercise, calculate the rotation rate of the spinning wheel. The rotation rate, $N$, of a wheel can be measured by configuring a light source on one side of the wheel, and a detector on the other side, and preparing the wheel so that it periodically obstructs the beam of light. Such a sensor is representative of many simple sensors that are used in industrial areas to anchor process-control loops. In order to calculate $N$, capture the chopped signal on an oscilloscope. Estimate the chopping frequency, and use the geometry of the wheel to infer the rotations per minute. Compare the result to that of the minitachometer. Capture the oscilloscope plot, and import it to the final report.

In the fourth exercise, investigate the properties of a dispersing prism. As detailed in the Supplemental-Information section, a prism can be used to separate, or disperse the various frequencies of the signal. Using the equation given, and knowing the wavelength of the laser light from the second exercise, estimate what must be the refractive index of the prism.
11.5 What to Report

Submit the results in a technical memo. Be sure to include (as a minimum) the following information:

- calculation of the focal length, \( f \), of the double-convex lens
- calculation of laser wavelength, \( \lambda \)
- calculation of wheel-rotation rate, \( N \)
- digitized plot of the oscilloscope trace of the rotation-rate sensor
- calculation of refractive index of the prism, \( n \)
- Uncertainty estimates presented in the form of tables supported by example calculations

Be sure to include any interesting observations from any of the four sections of the lab.

11.6 Supplemental Information

11.7 Imaging an Incoherent Source

The thin-lens equation is given by Smith (p. 20, Smith, W.J. Modern Optical Engineering, McGraw-Hill, New York, 1966) as

\[
-1 + \frac{1}{l'} = \frac{1}{f}
\]  

(51)

according to Figure 13, where \( l \) is the distance from the lens to the object (typically a negative number, as in the above diagram), \( l' \) is the distance from the lens to the image, and \( f \) is the focal length of the lens. Note that in this case, for simplicity, the object is the source of radiation, the lamp. Thus, if the system is arranged with an unknown lens such that the image is in sharp focus, the conjugates, \( l \) and \( l' \) can be measured, and the focal length of the lens can be calculated.

In addition, the magnification of the system can be calculated from the conjugates as

\[
m = \frac{l'}{l} = \frac{h'}{h},
\]  

(52)
where $h$ is the object height and $h'$ is the image height. The image height will typically be negative, by convention, where a negative magnification indicates an inverted image. Note that for larger magnifications, the irradiance of the image will decrease.

As an example, if the distance from the lens to the object is -20 mm, and the distance from the lens to the image is 100 mm, then the focal length will be 16.7 mm, and the magnification will be $-5$. A negative magnification indicates that the image is inverted, compared to the object.

### 11.8 Characterizing a Coherent Source

Laser light in general is both monochromatic and spatially-coherent, and these are the reasons the laser is such an important tool in many fields. The spatial coherence of the laser means that the beam is perfectly collimated, as if it had originated infinitely far away. It is this property that allows for strong interference fringes when the beam is crossed with itself, and this is the basis of interferometry. The monochromaticity of the laser is the purity of its wavelength. Because the laser can deliver significant power at such a narrow bandwidth, it is useful in fields ranging from spectroscopy to fiber-optic communications.

The wavelength of the laser diode used in this lab can be calculated by measuring the positions of the fringes in the diffraction pattern beyond a grating. The grating equation is given by Metrologic (p.22, “Laser-Pointer Education Kit,” Metrologic Instruments, Bellmawr, NJ, 1996) as
according to the Figure 14, where \( m \) is the order of the fringe, \( \lambda \) is the wavelength of radiation, \( d \) is the grating spacing, and \( \theta \) is the angle of deviation off axis of the diffraction fringe.

\[
m\lambda = dsin\theta, \quad (53)
\]

Figure 14: Laser-Beam Diffraction

Now, for the simplified case of \( m = 1 \), and given that for small angles, \( sin\theta \) can be approximated by \( \theta \), the wavelength of the laser can be calculated using the relation

\[
\lambda = d\frac{x}{z}, \quad (54)
\]

where \( x \) is the deflection of the fringe off of the optical axis, and \( z \) is the distance from the grating to the screen.

11.9 Optical Measurement of Wheel-Rotation Rate

In this exercise, a source-detector pair will be used for the measurement of the rotation rate of a spinning wheel. Some simple source and detector circuits are provided by Mims (Mims, F. M. Engineer's Mini-Notebook, Optoelectronic Circuits, Printed by Forrest Mims, 1986). Often light-emitting diodes (LED's) are used as the source of a simple sensor. An LED is typically dc-powered, and, when biased, it radiates light at a relatively narrow bandwidth. This type of source can be controlled very precisely, compared to an incandescent lamp. A simple detector will often be a silicon photodiode, followed by an operational amplifier. Such a detector is based on the photoelectric effect, where a small current is generated by the photodiode upon exposure to light, and the current is amplified and converted to voltage by the op-amp.
In this exercise, an LED source will be coupled with a phototransistor for signal detection. The phototransistor output need not be amplified, but rather just fed directly into an oscilloscope for analysis. The circuits are shown in Figure 15.

Connect the spinning wheel (a muffin fan) to the wall outlet. Couple the detector circuit to channel 1 of the oscilloscope, and switch the detector circuit “on.” When not in use, the detector should be switched off, to conserve the battery. Align the detector circuit, mini-tachometer, and wheel such that both sensors can view the chopping at the same time. The mini-tachometer will need an illuminating source (like a flashlight) on the other side of the fan. Acquire the signal on the oscilloscope; the following notes may be of help:

- trigger the oscilloscope off of the detector signal itself; use the buttons, “trig”, and “chan 1”
- use AC coupling to optimize the signal on the scope
- set the amplitude on the scope to 0.1 V/div
- if at this point a nice square wave is not seen, check the circuit’s battery with a voltmeter; a replacement may be needed
- use the cursors to measure the chopping frequency; use the buttons, “cursors”, “on”, and then the “track” and “arrow” buttons

By measuring the frequency of the chopped signal, estimate the rotation rate of the wheel. Save the scope plot, and also record the measurement.
of the mini-tachometer. How close are the two measurements? If time allows, try a couple different measurement positions along the wheel, and see if the result is repeatable.

11.10 Measurement of Refractive Index of Prism

Because the index of refraction of glass is dependent upon the wavelength of the radiation, prisms have been used as the basis of simple monochrometers. The dispersion of light by a prism is given by Smith (p.72, Smith, W.J. *Modern Optical Engineering*, McGraw-Hill, New York, 1966) as

\[ d = i_1 - a + \arcsin[(n(\lambda)^2 - \sin^2(i_1))^{1/2}\sin(a) - \cos(a)\sin(i_1)], \quad (55) \]

according to Figure 16.

![Figure 16: Refraction by a Prism](image)

In Figure 16, \( d \) is the final angle of exit of the beam, \( i_1 \) is the angle of incidence, \( a \) is the characteristic angle of the prism, and \( n(\lambda) \) is the wavelength-dependent index of refraction of the glass. Direct the laser through the prism, and carefully measure the various geometric quantities. Then, knowing the wavelength of radiation, calculate the index of refraction of the prism. Try a couple different orientations of the prism, varying the angle of incidence slightly, to see if the result is repeatable.
<table>
<thead>
<tr>
<th>reading</th>
<th>(l \text{[cm]})</th>
<th>(l' \text{[cm]})</th>
<th>obj height</th>
<th>im height</th>
<th>mag</th>
<th>(f \text{[cm]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9: Imaging of Flashlight

<table>
<thead>
<tr>
<th>reading</th>
<th>(x \text{[cm]})</th>
<th>(z \text{[cm]})</th>
<th>(d \text{[mm/line]})</th>
<th>(\lambda \text{[nm]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>1/750</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>1/750</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>1/750</td>
</tr>
</tbody>
</table>

Table 10: Measurement of Laser Wavelength

<table>
<thead>
<tr>
<th>reading</th>
<th>chopping freq</th>
<th>[rot/s]</th>
<th>(N \text{[RPM]})</th>
<th>mini tach (\text{[RPM]})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11: Measurement of Wheel Rotation Rate

<table>
<thead>
<tr>
<th>reading</th>
<th>(i_1 \text{[}^{\circ})</th>
<th>(a, \text{[}^{\circ})</th>
<th>(d, \text{[}^{\circ})</th>
<th>index, (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 12: Measurement of Prism Index of Refraction

<table>
<thead>
<tr>
<th>Measureand</th>
<th>Units</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>conjugates (l) and (l')</td>
<td>cm</td>
<td></td>
</tr>
<tr>
<td>object and image height, (h, h')</td>
<td>cm</td>
<td></td>
</tr>
<tr>
<td>diffraction-angle parameters, (x, z)</td>
<td>cm</td>
<td></td>
</tr>
<tr>
<td>wheel chopping frequency</td>
<td>Hz</td>
<td></td>
</tr>
<tr>
<td>index-of-refraction angles, (i_1, a, d)</td>
<td>rad</td>
<td></td>
</tr>
</tbody>
</table>

Table 13: Measureand uncertainties

<table>
<thead>
<tr>
<th>Result</th>
<th>Units</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>focal length of lens, (f)</td>
<td>cm</td>
<td></td>
</tr>
<tr>
<td>laser wavelength, (\lambda)</td>
<td>nm</td>
<td></td>
</tr>
<tr>
<td>rotation-rate of wheel, (N)</td>
<td>rpm</td>
<td></td>
</tr>
<tr>
<td>prism index of refraction, (n)</td>
<td>dimensionless</td>
<td></td>
</tr>
</tbody>
</table>

Table 14: Results

<table>
<thead>
<tr>
<th>Result</th>
<th>Units</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>focal length of lens, (u_f)</td>
<td>cm</td>
<td></td>
</tr>
<tr>
<td>laser wavelength, (u_\lambda)</td>
<td>nm</td>
<td></td>
</tr>
<tr>
<td>rotation-rate of wheel, (u_N)</td>
<td>rpm</td>
<td></td>
</tr>
<tr>
<td>prism index of refraction, (u_n)</td>
<td>dimensionless</td>
<td></td>
</tr>
</tbody>
</table>

Table 15: Result uncertainties
12 Exercise 12

Statistical Analysis of Data Using MATLAB

12.1 Introduction and Objectives

The overall objectives of this exercise are to solidify an understanding of several statistical and probabilistic methods and to provide the opportunity to learn how to use MATLAB to retrieve, analyze and display actual experimental data on a computer system.

There are three different data files that will be used in this exercise. The first file shed.dat consists of two columns of data acquired during an experiment in a subsonic wind tunnel. The first column is the time (in s), t, at which a value of the velocity (the second column, in m/s), U(t), was acquired using a hot-wire sensor located immediately behind a cylinder positioned in the wind tunnel. The second file gball.dat is a single column of the golf ball weights (in N). The third file voltage.dat is a single column of voltage readings from an amplifier/Wheatstone bridge system taken at fixed time increments.

In this exercise, MATLAB must be used to analyze this data. Several M-files must be written. Be sure to provide statements at the beginning of each M-file describing its particular function. The following explains specifically what must be done. Turn in the results in a technical memo.

Some helpful information on using MATLAB for probability and statistical calculations is presented throughout text. Other information can be obtained using MATLAB’s “help” command. Some of the commands that needed are not contained in the student version of MATLAB.

12.2 Temporal Realization of Time Series Data

Using shed.dat, determine the following: [1] the period (in s) of the signal’s third cycle, [2] the average period (in s) of one cycle, [3] the signal’s average cyclic frequency (in Hz), [4] the mean velocity (in m/s), [5] the standard deviation of the velocity (in m/s), [6] the minimum velocity (in m/s) and [7] the maximum velocity (in m/s). Write an
M-file to read in the data from shed.dat and then plot the velocity on
the ordinate versus time on the abscissa. Include in that M-file the
commands to automatically identify on the plot the mean velocity, the
mean velocity +1 standard deviation and the mean velocity -1 standard
deviation. HINTS: These MATLAB commands coded into an M-file
may be useful: [a] eval([’load info.dat’]) loads the data from info.dat
into the MATLAB workspace, [b] col1 = eval([’info(:,1)’]) assigns the
first column of info.dat the name col1, [c] text(xpos,ypos,’zz’) places
“zz”, such as a line or an arrow, at the coordinates xpos,ypos on a
plot. For this part, hand in the results, figure, the written M-file and
proof of the calculations of the seven above quantities (this could be,
for example, the printout of the MATLAB session).

12.3 Distribution Comparisons

Using gball.dat, write an M-file to read in the data and then plot the
histogram and frequency distribution side by side on one page (HINT:
use MATLAB’s “subplot” command). Follow the rules for determining
the number of bins as described in Chapter ??.
Assume that \( u_w \), the
uncertainty in the measurement of the weight, \( w \), equals 0.01 N. Check
to make sure that the bin width is greater than this value. Next, by
either writing another M-file or adding on to the previous one, deter-
mine and then plot the histogram of the data and the expected values
as determined for a Normal distribution (use the sample mean and
standard deviation for the mean and standard deviation of the Normal
pdf). HINTS: The following MATLAB commands coded into an M-file
may be useful: [a] hist(x,k) plots the histogram of x with k bins, [b]
[a,b]=hist(x,k) produces the column matrices a and b, where a contains
the counts in each bin and b contains the center coordinates for each
bin, [c] bar(b,a/N) will plot the frequency distribution, where N is the
total number of x values, [d] q=c:dq:e will produce values of q ranging
from c to e in increments of dq, and [e] plot(q,sin(q)) will plot q on
the abscissa versus sin(q) on the ordinate as a continuous and smooth
curve provided that the increment dq is much smaller than the range
of q. For this part, hand in the figures and the written M-files.
12.4 Finite versus Infinite Samples

Using voltage.dat, write an M-file to plot the running mean of the data from 1 up to all 1000 points (running mean on the ordinate; the number of points on the abscissa). The running mean of N points is simply the mean of those N points. As N gets larger, the running mean should approach a constant value equal to the true mean of the underlying population from which the N points were drawn. On the plot, indicate the running mean of 1000 points and its value using the previously described “text” command. Next, determine the number of measurements, N*, required for the running mean to stay continually within 1% of the running mean of 1000 points. Then, compute the sample mean, the sample standard deviation and the standard deviation of the means for all 1000 points. Write a statement for the estimate of the true mean value of the parent population from which this data was drawn at the 95% confidence level based upon these values. Put the values of N*, the sample mean, the sample standard deviation, the standard deviation of the means and the true mean estimate statement in a table. For this part, hand in the figure, the written M-file and the table.

12.5 \( \chi^2 \) Analysis

Continue with the analysis to construct the plot of the histogram of the gball.dat weights versus the values expected for a normal distribution. Perform a \( \chi^2 \) analysis to determine the % confidence that the golf ball weights are Normally distributed. HINT: The following MATLAB command coded into an M-file may be useful: [a] alpha=100-100*chi2cdf(chisq,nu). Report the % confidence value and hand in the M-file written to calculate this.

12.6 What to Report

Turn in a technical memo with the answers to the posed questions, plots and listings of all written M-files.