Interacting theory: φ^4

Consider the scalar theory with "φ^4" interaction

$$S(\varphi) = \int_{M=\mathbb{R}^{1,3}} d\,\mathrm{vol}\left(\frac{1}{2}\sum_{\mu,\nu=0}^{3}\eta^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4\right) = S_0(\varphi) + S_{int}(\varphi)$$

Hamiltonian: $\Sigma = \mathbb{R}^3$

$$\underset{\mathrm{Fun}(\underbrace{C^\infty(\Sigma)\oplus C^\infty(\Sigma)}_{\Phi})}{\overset{\in}{H(\varphi,\pi)}} = \underbrace{\int_\Sigma d^3x\left(\frac{\pi^2}{2} + \sum_{i=1}^{3}\frac{1}{2}(\partial_i\varphi)^2 + \frac{m^2}{2}\varphi^2\right)}_{H_0(\varphi,\pi)} + \underbrace{\frac{\lambda}{4!}\int_\Sigma d^3x\cdot\varphi^4}_{H_{int}(\varphi)}$$

Quantization: Use same $\mathcal{H}$ as in free theory

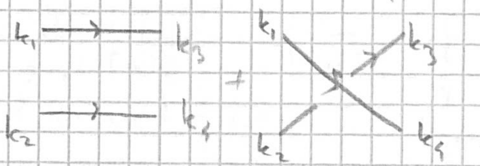
$$\hat{H} = \hat{H}_0 + \underbrace{\frac{\lambda}{4!}\int_\Sigma d^3x\ \hat{\varphi}(x)^4}_{\hat{H}_{int}}$$

We are interested in the evolution operator $U(t_{out} - t_{in}) = e^{-\frac{i}{\hbar}\hat{H}\cdot(t_{out}-t_{in})} : \mathcal{H}\to\mathcal{H}$ in the asymptotics $t_{in}\to-\infty$, $t_{out}\to+\infty$, as a (formal) power series in λ.

E.g. the block $U^{2\to2}:\mathcal{H}_2\to\mathcal{H}_2$ corresponds to two-particle scattering

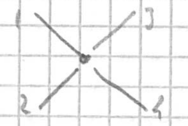
mathematically: we are calculating $\underbrace{\langle k_3,k_4|}_{\text{out-state}}U(t_{out}-t_{in})\underbrace{|k_1,k_2\rangle}_{\text{in-state}}$ - the "S-matrix element"

=



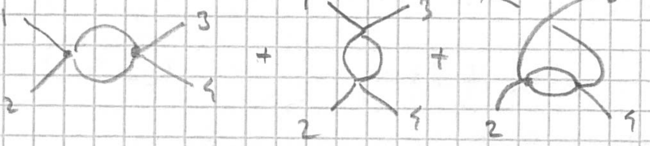
} trivial part corresponding to free evolution

+



$O(\lambda)$

+



$O(\lambda^2)$

+ . . . . .

← Feynman diagrams for the S-matrix element; each graph represents a function of $k_{1\ldots4}$, t_{in}, t_{out}.

More precisely, we want to "substract" the trivial part (corresp to $\hat{H}_0$) from the evolution

→ "Interaction picture": $\tilde{U}(t-t_0) := \underbrace{U_0^{-1}(t-t_0)}_{e^{\frac{i}{\hbar}(t-t_0)\hat{H}_0}}\cdot\underbrace{U_{full}(t-t_0)}_{e^{-\frac{i}{\hbar}(t-t_0)\hat{H}}}$

$$\varphi(t,\vec{x}) = \tilde{U}^{-1}(t,t_0)\underbrace{\varphi_I(t,\vec{x})}_{U_0^{-1}\varphi(t_0,\vec{x})\cdot U_0}\tilde{U}(t,t_0)$$

$\tilde{U}$ satisfies

$$i\hbar\frac{\partial}{\partial t}\tilde{U}(t,t_0) = \hat{H}_I(t)\tilde{U}(t,t_0)$$

where $\hat{H}_I(t) := U_0^{-1}H_{int}U_0 = \int_\Sigma d^3x\,\frac{\lambda}{4!}\hat{\varphi}_I^4$

Solution of ↗ : $\tilde{U}(t,t_0) = \sum_{n\geq0}\left(-\frac{i}{\hbar}\right)^n\int_{t_0<t_1<\ldots<t_n<t}dt_1\ldots dt_n\,\hat{H}_I(t_n)\ldots\hat{H}_I(t_1) = \sum_{n\geq0}\frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n\int_{[t_0,t]^n}dt_1\ldots dt_n\,\overset{\text{time-ordering}}{\swarrow}T\hat{H}_I(t_n)\ldots\hat{H}_I$

$= T\exp -\frac{i}{\hbar}\int_{t_0}^{t}\hat{H}_I(t')dt'$

Feynman diagrams: the (rough) idea

Expectation values $\langle 0 | T \varphi_I(x_1) \cdots \varphi_I(x_n) | 0 \rangle$ can be calculated by Wick's lemma

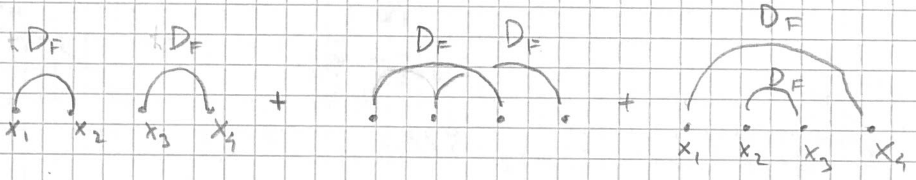
- $\langle 0 | T \varphi_I(x_1) \varphi_I(x_2) | 0 \rangle = \hbar D_F(x_1, x_2)$ - Feynman propagator

$$\overset{"}{=} \lim_{\varepsilon \to +0} \int \frac{d^4 k}{(2\pi)^4} \frac{i\hbar}{k^2 - m^2 + i\varepsilon} e^{-ik\cdot(x_1 - x_2)}$$

- $$\langle 0 | T \varphi_I(x_1) \cdots \varphi_I(x_n) | 0 \rangle = \begin{cases} 0 & \text{if } n \text{ odd} \\ \displaystyle\sum_{\substack{\text{partitions of } \{1 \ldots 2m\} \\ \text{into pairs } \{i_1 j_1\} \ldots \{i_m, j_m\}}} \hbar D_F(x_{i_1}, x_{j_1}) \cdots \hbar D_F(x_{i_m}, x_{j_m}) & \text{if } n = 2m \text{ even} \end{cases}$$

$\nearrow$ unordered

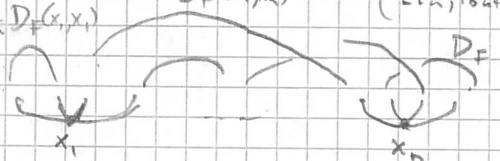
Pictorially:
$n = 4$
$\langle 0 | T \varphi_1 \cdots \varphi_4 | 0 \rangle$



- sum over "matchings" on $\{1 \ldots n\}$

[vacuum]
Graphs:

$$\langle 0 | \frac{1}{n!} \left(-\frac{i}{\hbar}\right)^n \underbrace{\int_{[t_{in}, t_{out}]^n} dt_1 \cdots dt_n \, T \hat{H}_I(t_n) \cdots \hat{H}_I(t_1)}_{T \int_{([t_{in}, t_{out}] \times \Sigma)^{\times n}} d^4x_1 \cdots d^4 x_n \, \varphi_I(x_n)^4 \cdots \varphi_I(x_1)^4 \cdot \left(\frac{\lambda}{4!}\right)^n} | 0 \rangle$$ - appears in $\langle 0 | \tilde{U} | 0 \rangle$

$= \sum_{\text{matchings}}$  $\cdot \frac{1}{n!} \left(-\frac{i}{\hbar}\right)^n \cdot \left(\frac{\lambda}{4!}\right)^n =$

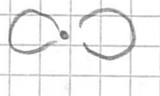
$$= \sum_{\substack{\text{graphs } \Gamma \text{ with} \\ n \text{ 4-valent vertices}}} \frac{1}{|\mathrm{Aut}\,\Gamma|} \Phi(\Gamma)$$

$\uparrow$ Feynman weight of the graph defined by Feynman rules

 $\to D_F(x_1, x_2)$

 $\to -\frac{i\lambda}{\hbar} \int d^4 x$

E.g. contribution to $\langle 0 | \tilde{U} | 0 \rangle$ in $O(\lambda)$:

 $= \frac{1}{8} \cdot \left(-\frac{i\lambda}{\hbar}\right) \cdot \int_{M = [t_{in}, t_{out}] \times \Sigma} d^4 x \; D_F(x, x)^2$

$\nearrow \frac{3}{24}$

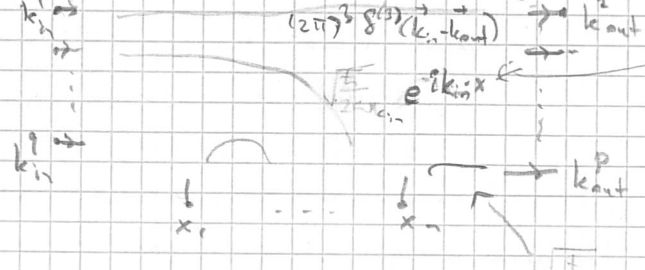
 $= \frac{1}{48} \left(-\frac{i\lambda}{\hbar}\right)^2 \int d^4x \, d^4 y \; D_F(x, y)$

S-matrix elements

modification of Wick's lemma

$$\langle k^1_{out}, \ldots, k^p_{out} | T \varphi_I(x_1) \cdots \varphi_I(x_n) | k^1_{in} \ldots k^q_{in} \rangle =$$

$\sum_{\text{matchings}}$



$\sqrt{\frac{\hbar}{2\omega_{k_{out}}}} \cdot e^{-ik_{out} \cdot x}$

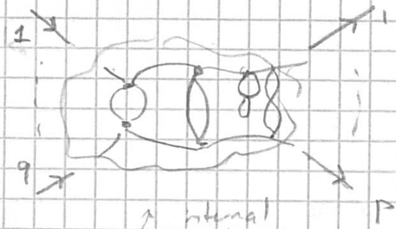
supplementary Feynman rules for "external legs" coming from $\langle 0 | \hat{\varphi}_I(x) \hat{a}^+_k | 0 \rangle$,
$\langle 0 | \hat{a}_{\vec{\ell}} \hat{a}^+_{\vec{k}} | 0 \rangle$

Rem: $|\vec{k}\rangle = \sqrt{2\omega_k} \, a^+_k | 0 \rangle$
- a convention giving Lorentz-invariant CCR

Finally, S-matrix elements:

$\lim_{\substack{t_{in}\to-\infty \\ t_{out}\to+\infty}} \langle k^1_{out} \dots k^p_{out} | \tilde{U}(t_{out}, t_{in}) | k^1_{in} \dots k^q_{in} \rangle =$

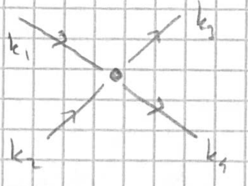
$= \sum$ graphs Γ with
q "in" 1-val. vertices
p "out" 1-val. vertices
other vertices 4-valent



internal

$O(\lambda^{\#\text{vertices}})$

E.g.



$$= -\frac{i\lambda}{\hbar} \int_{[-T,T]\times\mathbb{R}^3} d^4x \cdot \sqrt{\frac{\hbar}{2\omega_1}} \cdots \sqrt{\frac{\hbar}{2\omega_4}} \cdot e^{i(k_3+k_4-k_1-k_2)\cdot x} =$$

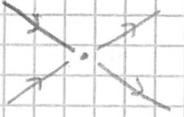
$-T$: t_{in}, T: t_{out}

imposing $k_i^2 = m$

$$= \underbrace{\frac{-i\lambda\hbar}{\sqrt{2\omega_{k_1}} \cdots \sqrt{2\omega_{k_4}}}\,(2\pi)^3}_{\text{amplitude of scattering in a unit of time}} \delta^{(3)}(\vec{k}_3+\vec{k}_4-\vec{k}_1-\vec{k}_2) \cdot \overset{t_{out}-t_{in}}{\boxed{2T}} \qquad (*)$$

conservation of energy-momentum in the process of scattering.

If $\vec{k}_1+\vec{k}_2=\vec{k}_3+\vec{k}_4$, we get $\delta(0)$ in $(*)$, but in lattice (discretized) version we have

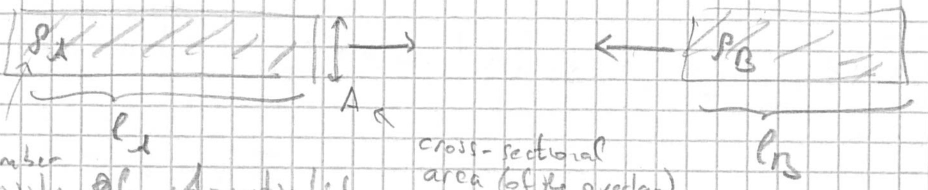


$$= \boxed{-\frac{i\lambda\hbar}{\prod_{\alpha=1}^{4}\sqrt{2\omega_{k_\alpha}}}} \cdot L^3 \cdot 2T$$

amplitude of a scattering event per unit of time per unit of space volume.

(Scattering) cross-section.

2 incident beams of particles (in a collider)



number density of A-particles

cross-sectional area (of the overlap)

$$\sigma = \frac{\text{number of scattering events}}{A \cdot \rho_A l_A \rho_B l_B}$$ - cross-section : "effective" cross-sectional area of beam B "as seen" by a particle of beam A.

no. of events $= \dfrac{\sigma \cdot N_A \cdot N_B}{A}$ - for beams of const density.

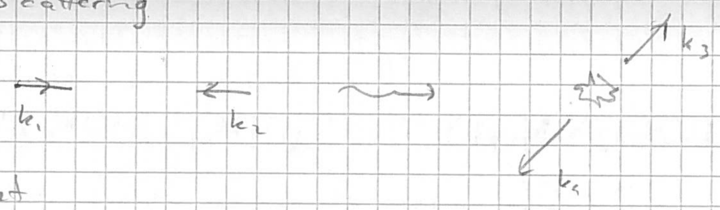
in e^-e^+ scattering, we can measure cross-sections for production of $\mu^+\mu^-$, $\tau^+\tau^-$, $\mu^+\mu^-\gamma$, $\gamma\gamma$, e^+e^-

We can also measure the final momenta of the particles produced: $\vec{p}_1 \dots \vec{p}_n$

Then we are measuring the "differential cross-section" $\dfrac{d\sigma}{d^3\vec{p}_1 \dots d^3\vec{p}_n}$

conservation of energy-momentum imposes constraints

2 → 2 scattering



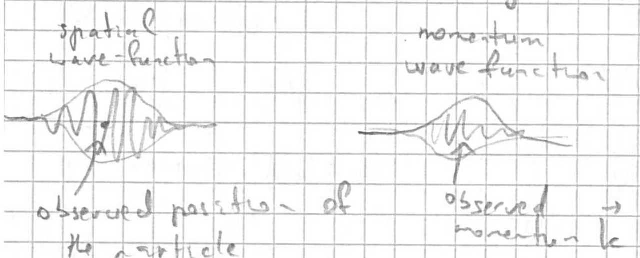
- measure spherical angles θ, φ for momentum e.g. $\vec k_3$

so we get the diff. cross-section $\frac{d\sigma}{d\Omega}(\theta, \varphi)$

In a lab we have "wave-packets" for incoming states

$$|\varphi\rangle = \int \frac{d^3\vec k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \underset{\substack{\uparrow \\ \text{Fourier transform of} \\ \text{spatial wave-function}}}{\varphi(\vec k)} \overset{\text{1-particle state of momentum k in interacting theory}}{|\vec k\rangle}$$

$$\int \frac{d^3k}{(2\pi)^3} |\varphi(\vec k)|^2 = 1$$



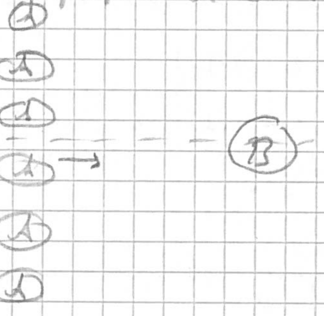
We are interested in probability $\mathcal{P} = \Big| {}_{\text{out}}\langle \underbrace{\varphi_1 \varphi_2 \ldots}_{\text{future}} | \underbrace{\varphi_A \varphi_B}_{\text{past}} \rangle_{\text{in}} \Big|^2$ ← spatially distant, non-interacting wave-packets

we can consider the limit of localized $\varphi(\vec k)$, so that $|\varphi\rangle \longrightarrow |\vec k\rangle$,

$$|\varphi_A, \varphi_B\rangle \longrightarrow |k_A k_B\rangle \quad \text{- definite-momentum state}$$

realistic in-state is a linear superposition of such.

in-wavepackets are uniformly distributed in impact parameter $\vec b$



realistically, we have in-wavepackets, but for out-states the detector measures only momentum, so we are interested in probability of transition to a state with momenta in small region $d^3p_1 \ldots d^3p_n$

$$\mathcal{P}(A, B \to 1 \ldots n) = \left(\prod_f \frac{d^3p_f}{(2\pi)^3} \frac{1}{2E_f}\right) \cdot$$
$$\cdot \left| {}_{\text{out}}\langle p_1 \ldots p_n | \varphi_A \varphi_B \rangle_{\text{in}} \right|^2$$

S-matrix:

$${}_{\text{out}}\langle p_1 p_2 \ldots | k_A k_B \rangle_{\text{in}} = \lim_{T \to \infty} \langle p_1 p_2 \ldots | \underbrace{e^{-i\hat H \cdot 2T}}_{\text{S-matrix}} | k_A k_B \rangle$$

- unitary operator on $\mathcal{H}$

$$S = \underset{\substack{\uparrow \\ \text{no interaction}}}{id} + i\underset{\substack{\uparrow \\ \text{interesting part}}}{T}$$

momentum conservation $\Rightarrow$ $\langle p_1 p_2 \ldots | iT | k_A k_B \rangle = (2\pi)^4 \delta^{(4)}(k_A + k_B - \sum p_f) \cdot i \underbrace{\mathcal{M}(k_A, k_B \to p_f)}_{\text{"invariant matrix element"}}$

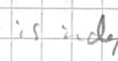
cross-section via $\mathcal{M}$

case of 2 → 2 scattering, all masses equal; in center of mass frame

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{|\mathcal{M}|^2}{64\pi^2 E_{cm}^2}$$

E_{cm} = $\omega_1 + \omega_2 = \omega_3 + \omega_4$ - total center-of mass energy.

Our calculation for φ^4, contribution of



to $\mathcal{M}$ is $\mathcal{M} = -\lambda$

so the corresponding $\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{\lambda^2}{64\pi^2 E_{cm}^2}$ - is independent on angles φ, θ (boring! - changes for electrons!)

$$\sigma_{\text{total}} = \frac{\lambda^2}{32\pi E_{cm}^2}$$

Dirac field

$= \mathbb{C}^4$ - an auxiliary space

introduce matrices $\gamma^\mu \in \mathrm{End}(E)$

s.th. $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\cdot\eta^{\mu\nu}\cdot \mathrm{id}_E$ (Dirac / Clifford algebra)

then E carries a $\overset{\text{projective}}{\text{rep.}}$ of Lorentz group $SO(1,3)$

generators $\underset{so(1,3)}{\underset{\cap}{S^{\mu\nu}}} \longmapsto \frac{i}{4}[\gamma^\mu,\gamma^\nu] \in \mathrm{End}(E)$

Explicitly:

$\gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$ $\gamma^i = \left(\begin{matrix} & \sigma \\ -\sigma^i & \end{matrix} \right.$

$\psi \in E$ a Dirac spinor $\qquad \bar\psi := \psi^\dagger \underbrace{\gamma^0}_{\substack{\text{comes from}\\ \text{non-unitarity of rep on } E \leftarrow \text{non-compactness of } SO(1,3)}} \in E^*$; $\bar\psi\psi$ is a Lorentz-invariant

Dirac equation:

$(i\gamma^\mu\partial_\mu - m)\psi = 0 \qquad \not\partial := \gamma^\mu\partial_\mu : C^\infty(M,E) \circlearrowleft$ - Dirac operator

$\rightsquigarrow$ corresp. field action: $\int_{\mathbb{R}^{1,3}} \bar\psi(i\not\partial - m)\psi = S_{\text{Dirac}}(\psi,\bar\psi)$

EL eq.: and the conjugate: $-i\partial_\mu\bar\psi\gamma^\mu - m\bar\psi = 0$

Global symmetry: $U(1)$ $\quad \psi \mapsto e^{i\alpha e}\psi$, $e^{i\alpha e} \in U(1)$ ← charge of electron

$\bar\psi \mapsto e^{-i\alpha e}\bar\psi$

corresponding Nöther current:

$j^\mu = e\cdot\bar\psi\gamma^\mu\psi$, it is conserved:

$\partial_\mu j^\mu = 0$ mod EL eq.

Quantization of Dirac field:

Heisenberg CCR ⟶ Dirac anti-CCR

$\mathcal{H} = \bigoplus_{n,m=0}^{\infty} \mathcal{H}_{n,m}$ ← n-electron, m-positron states

$\mathcal{H}_{n,m} \ni \sum_{s_1 \ldots s_m, s'_1 \ldots s'_m \in \{\pm\frac{1}{2}\}} \int d^3\vec{k}_1 \ldots d^3\vec{k}_n\, d^3\vec{k}'_1 \ldots d^3\vec{k}'_m\, \psi\left(\begin{matrix} k_1 \ldots k_n \\ s_1 \ldots s_n \end{matrix} \middle| \begin{matrix} k'_1 \ldots k'_m \\ s'_1 \ldots s'_m \end{matrix}\right) \cdot \left| \begin{matrix} k_1 \ldots k_m ; & k'_1 \ldots k'_m \\ s_1 \ldots s_m & s'_1 \ldots s'_m \end{matrix} \right\rangle$

- "helicities" – spin projected to $\vec{k}$

"Pauli exclusion principle" ⟶ wave-function anti-symmetric w.r.t. permutations of (k_i, s_i) and perm. of (k'_i, s'_i) - separately.

field ψ "creates" electrons (fermions), charge $= -1$

— " — $\bar\psi$ — " — positrons (anti-fermions) — " — $+1$

Action functional of classical electrodynamics

$$S_{ED} = \frac{1}{4}\underbrace{\int_{M=\mathbb{R}^{1,3}} F_{\mu\nu}F^{\mu\nu}\, d\mathrm{vol}}_{\int F\wedge * F} + \int_M \bar\psi(i\not\partial - e\gamma^\mu A_\mu)\psi$$

Fields: $A \in \Omega^1(M)$ - a connection 1-form in triv. circle bundle $\quad \begin{matrix} \Pi = M \times S^1 \circlearrowleft U(1) \\ \downarrow \\ M \end{matrix}$ ← of radius $1/e$

$F = dA$ - curvature $\in \Omega^2(M)$

$\psi \in \Gamma(M, \overset{\text{assoc. bundle}}{\Pi \times_{U(1)} E})$ - spinor field

$\not\partial_A = \not\partial + ie\gamma^\mu A_\mu$ - Dirac operator twisted by connection A.

Instead of global $\bar{U}(1)$ - symmetry $\psi \mapsto e^{ie\alpha}\psi$

$\bar{\Psi} \mapsto e^{-ie\alpha}\bar{\psi}$

we now have a local $U(1)$ "gauge" symmetry

$\psi \longmapsto e^{ie\alpha(x)}\psi$

$\bar{\Psi} \longmapsto e^{-ie\alpha(x)}\psi$ for $\alpha \in C^{\infty}(M)$

$A \longmapsto A + d\alpha$

- action of automorphisms of $\mathbb{T}$ (= fiberwise rotations) on fields.

Quantization

$$H = H_{\text{free photon}} + H_{\text{Dirac}} + \underbrace{H_{int}}_{\int d^3x \; e\bar{\Psi}\gamma^{\mu}\psi \cdot A_{\mu}}$$

$\overset{?}{H_{\text{scalar field}}}$

Space of states: $\mathcal{H} = \bigoplus_{n_{\gamma}, n_{e^-}, n_{e^+} \geq 0} \mathcal{H}_{n_{\gamma} n_{e^-} n_{e^+}}$ ← states with fixed no. of photons, electrons, positrons

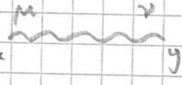
photons: have polarization (↻ or ↺), wave fun. - symmetric ← Bose-Einstein statistics

(pol. vector $\mathcal{E}^{\mu}$ s.t. $\mathcal{E}^{\mu}_{\mu} p_{\mu} = 0$; $\mathcal{E}^{0} = 0$ - "axial gauge")

electrons } have helicity, wave fun anti-sym

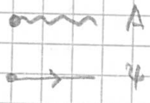
positrons }

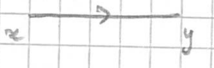
Feynman rules: edges



$\longmapsto \int \frac{d^4k}{(2\pi)^4} \frac{-i\eta_{\mu\nu}}{k^2+i\varepsilon} e^{-ik\cdot(x-y)}$

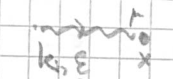
half-edges





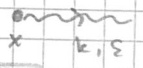
$\longmapsto \int \frac{d^4k}{(2\pi)^4} \frac{i(\not{k}+m)}{k^2-m^2+i\varepsilon} e^{-ik(x-y)}$

external edges:



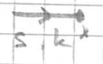
polarization vector

$\longmapsto \mathcal{E}_{\mu} \cdot e^{ikx}$



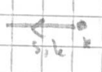
$\longmapsto \mathcal{E}^{*}_{\mu} \; e^{-ikx}$

$s = 1, 2$



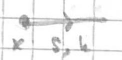
$u^{s}(k) \cdot e^{ikx}$

$\overset{\cap}{E}$

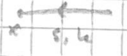


$\bar{v}^{s}(k) \cdot e^{ikx}$

$\overset{\cap}{E^{*}}$

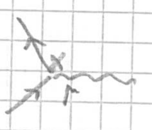


$\bar{u}^{s}(k) e^{-ikx}$



$v^{s}(k) e^{-ikx}$

vertex:



$\longmapsto -ie\gamma^{\mu} \int d^4x$

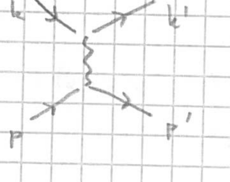
$u(p), v(p)$ comes from plane-wave sol. to Dirac eq.

positive frequency: $\psi(x) = u(p)e^{-ip\cdot x}$, $p^2 = m^2$ $p^0 > 0$

$u^{s}(p)$ $s = 1, 2$ - basis ↙ $v^{s}(p)$ - basis

neg-frequency: $\psi(x) = \boxed{v(p)} e^{ip\cdot x}$, $p^2 = m^2$, $p^0 > 0$

Example: electron-electron scattering (leading order)



$$i\mathcal{M} = (-ie)^2 \bar{u}(p')\gamma^{\mu}u(p) \frac{-i\eta_{\mu\nu}}{(p'-p)^2} \; \bar{u}(k')\gamma^{\mu}u(k)$$

(Yukawa) ← and QM scattering amplitude

in nonrel. limit, one recovers Rutherford scattering formula for cross-section

(= formula from Born approximation for scattering in QM)

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin^4\frac{\theta}{2}}$$

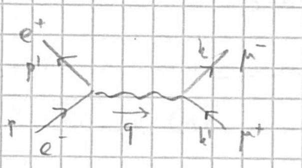
corresp. to repulsive Coulomb potential $\frac{e^2}{4\pi r} = \frac{\alpha}{r}$ between electrons.

for e^-, e^+ - we get attractive potential $-\frac{\alpha}{r}$.

$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$ - fine structure constant

Inelastic scattering example

$e^- e^+ \to \mu^- \mu^+$

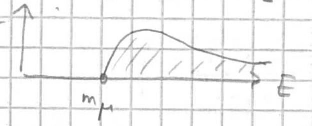


$$\rightsquigarrow i\mathcal{M} = \frac{ie^2}{q^2} \left(\bar{v}(p')\gamma^\mu u(p)\right)\left(\bar{u}(k)\gamma_\mu v(k')\right)$$

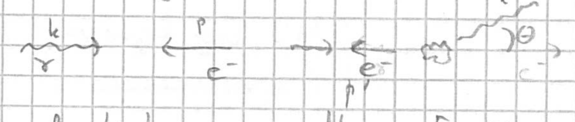
$\overset{\rightsquigarrow}{\text{unpolarized}}$ (averaged over helicities) cross-section

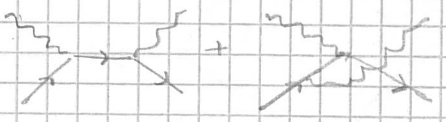
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_{cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}} \cdot \left(\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\Theta\right)$$

$E_{cm} = 2E$



Compton scattering





for high energy scattering, $E_{cm} \gg m_e$,

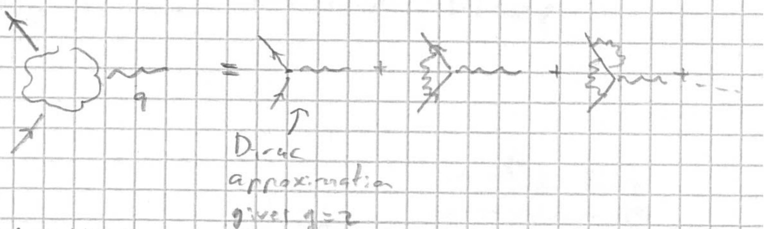
$$\sigma_{tot} \approx \frac{2\pi\alpha^2}{E_{cm}} \log\frac{E_{cm}}{m^2}$$ and the photon is almost surely scattered backwards, $\Theta \approx \pi$.

Anomalous magnetic moment of electron

scattering of electron in slowly-changing magnetic field: $V(x) = -\underset{\substack{\text{magn. moment}\\\text{of electron}}}{\vec{\mu}} \cdot \underset{\substack{\text{external}\\\text{magnetic field}}}{\vec{B}(x)}$ = "magn. moment interaction"

$\vec{\mu} = g \cdot \frac{e}{2m} \cdot \underset{\text{spin}}{\vec{S}}$ g = Landé g-factor

- comes from $q \to 0$ asympt. of diagrams



$$a_e = \frac{g-2}{2} = \underset{\text{Schwinger '48}}{\overset{\approx 0{,}0011614}{\frac{\alpha}{2\pi}}} + \mathcal{O}(\alpha^2)$$

experimentally: $a_e = 0{,}0011527$

g is known experimentally up to 10^{-12}
and calculated from QED up to coeff. of α^4.

electron vertex function: "formfactors"

$$\Gamma^\mu(p',p) = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)$$

$$g = 2\cdot(F_1(0)+F_2(0)) = 2+2F_2(0)$$

Systematics of particles

	photon γ	electron e^-	positron e^+	muon μ^-	anti muon μ^+	tau τ^-	τ^+
nass	0	m_e	m_e	m_μ		m_τ	
pin	1	$1/2$	$1/2$	$1/2$		$1/2$	
harge ne)	0	-1	$+1$	-1	$+1$	-1	$+1$

$e \approx 1{,}6\cdot10^{-19}$ C

$m_e \approx 9{,}1\cdot10^{-31}$ kg
$\approx 0{,}511$ MeV$/c^2$

$m_\mu \approx 106$ MeV$/c^2$

$m_\tau \approx 1778$ MeV$/c^2$

leptons: e μ τ $q=e$, spin=½
ν_e ν_μ ν_τ ← neutrino $q=0$

drons mesons: π, K, ...   spin = 0, 1
baryons ~~hadrons~~: p, n, ...   spin = ½, 3/2
← "made of" quarks

force carriers (gauge bosons): γ, $W^\pm$, Z, g spin = 1