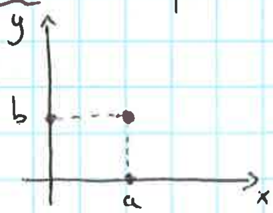
12.1 3D Coordinates

Recall: a point on a plane $\mathbb{R}^2$ is given by a pair of numbers (a, b)

x-coordinate, y-coordinate

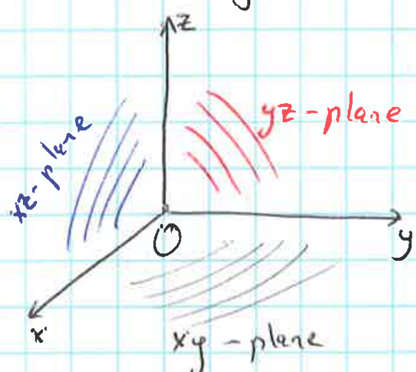


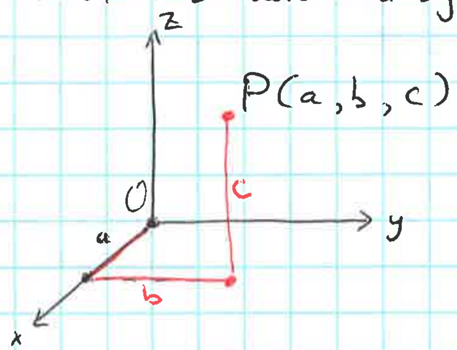
A point P in space $\mathbb{R}^3$ is given by a triple of numbers (a, b, c)

x-coord, y-coord, z-coord.

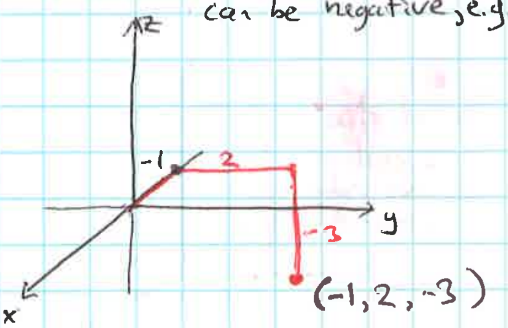
- ~~choose~~ fix O - origin and 3 directed mutually perpendicular lines through O - x-axis, y-axis, z-axis

direction of z - determined by "right-hand rule"

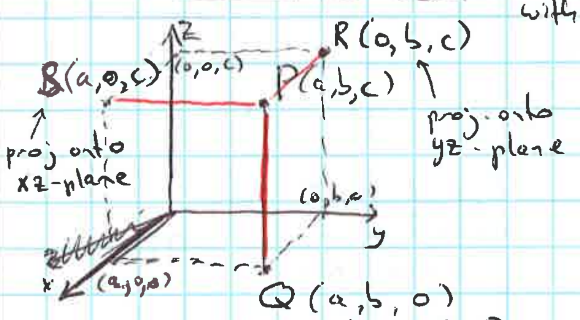




coordinates can be negative, e.g.:



coordinate planes divide $\mathbb{R}^3$ into 8 octants. 1st octant = all points (a, b, c) with $a, b, c > 0$



← point P determines a rectangular box

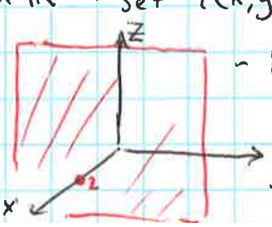
Q (a, b, 0)
- projection of P onto xy-plane

- On $\mathbb{R}^2$, graph of an equation involving x, y represents a curve in $\mathbb{R}^2$.
  In $\mathbb{R}^3$, an eq. involving x, y, z represents a surface in $\mathbb{R}^3$.

Ex: $x = 2$ • in $\mathbb{R}^2$ - set of points $\{(x,y) \mid x = 2\}$

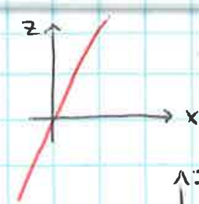
- a line parallel to y-axis

• in $\mathbb{R}^3$ - set $\{(x,y,z) \mid x = 2\}$



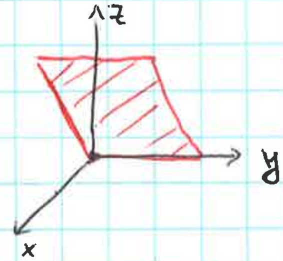
- plane parallel to yz-plane

Ex: $z = 2x$ · on xz plane



- line through origin with slope 2

· in $\mathbb{R}^3$, the eq. describes the set $\{(x, y, 2x) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$

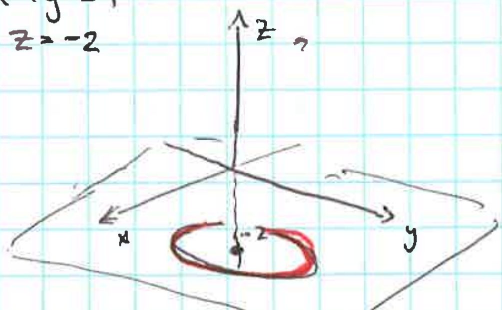


- a plane perpendicular to xz-plane and intersecting it with the line $\begin{cases} z = 2x \\ y = 0 \end{cases}$

Ex:

a) graph $\begin{cases} x^2+y^2=1 \\ z=-2 \end{cases}$

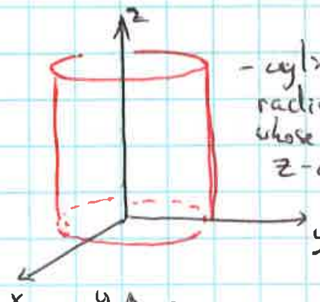
~~b) graph x^2~~



- circle of radius 1 centred on z-axis, in the horizontal plane $z=-2$

(b) $x^2+y^2=1$ in $\mathbb{R}^3$

- all points (x,y,z) s.t. (x,y) on a circle in $\mathbb{R}^2$ and z any



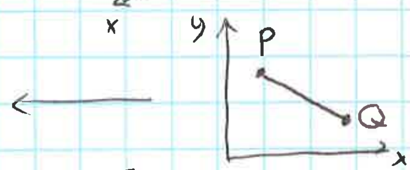
- cylinder of radius 1 whose axis is z-axis

Distance

Recall:

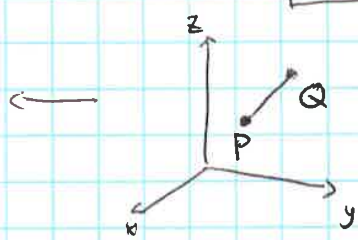
On a plane: distance between $P(x_1,y_1)$ and $Q(x_2,y_2)$

is $|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$



in $\mathbb{R}^3$: for $P(x_1,y_1,z_1)$, $Q(x_2,y_2,z_2)$,

$|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

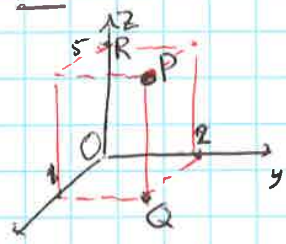


Ex: $P(-2,0,1)$, $Q(0,1,3)$

$|PQ| = \sqrt{(0-(-2))^2+(1-0)^2+(3-1)^2} = \sqrt{4+1+4} = 3$

Ex: $P(1,2,5)$ $Q(1,2,0)$ - projection to xy plane

$R(0,0,5)$ - proj. to z axis



distance from P to xy-plane

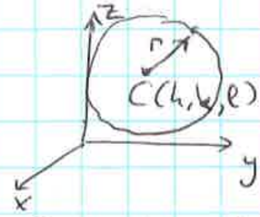
$|PQ| = \sqrt{0^2+0^2+5^2} = 5$

$|PR| = \sqrt{1^2+2^2+0^2} = \sqrt{5}$

$|PO| = \sqrt{1^2+2^2+5^2} = \sqrt{30}$

distance from P to z-axis

A sphere ~~with radius~~ of radius r and with center $C(h,k,\ell)$ is given by equation $\boxed{(x-h)^2+(y-k)^2+(z-\ell)^2=r^2}$



In case $C=O$ · origin: $x^2+y^2+z^2=r^2$

Ex: $x^2+y^2+z^2+4x-6y+2z+6=0$ $(*)$ - show that this is an eq. of a sphere; find its center and radius.

Sol: rewrite: (complete squares) $(x^2+4x+4)+(y^2-6y+9)+(z^2+2z+1)=\underbrace{-6+4+9+1}_{8}$

or $(x+2)^2+(y-3)^2+(z+1)^2=8$

so, $(*)$ defines a sphere of radius $r=\sqrt{8}$, center $C(-2,3,-1)$

* intersection with xy plane: $z=0$, $(x+2)^2+(y-3)^2+1=8$ $\Leftrightarrow (x+2)^2+(y-3)^2=7$

* intersection with xz plane: $y=0$, $(x+2)^2+(z+1)^2=8-3^2=-1$ - empty!

Ex: What region in $\mathbb{R}^3$ is represented by inequalities

$$\begin{cases} 1\le x^2+y^2+z^2\le 9 & (1)\\ z\le 0 & (2)\end{cases}$$

Sol: (1) describes points between (or on) spheres centered at O with radii $r=1$ and $r=3$

(2) implies that point should be below (or on) horizontal xy plane.