BASIC GEOMETRY AND TOPOLOGY HOMEWORK 2, DUE 8/28/2020

I Show that a closed subspace K of a compact topological space X is compact.

II Prove that
  (a) If a topological space X is Hausdorff then any subspace $Y \subset X$ is also Hausdorff.
  (b) If a topological space X is second countable then any subspace $Y \subset X$ is also second countable.

III Prove that if X is an m-manifold and Y is an n-manifold, then $X \times Y$ is an $(m+n)$-manifold. (In particular, you need to show that $X \times Y$ is Hausdorff and second countable.)

IV (a) Which among the topological groups $GL_n(\mathbb{R}), SL_n(\mathbb{R}), O(n), SO(n)$ are compact topological spaces for $n \geq 2$? (Hint: use Heine-Borel theorem.)
  (b) Show that the topological groups $GL_n(\mathbb{R})$ and $O(n)$ are not connected for any $n \geq 1$.

V Consider the "wedge sum" of two circles, $X = S^1 \vee S^1 := S^1 \cup S^1 / x_1 \sim x_2$ where x_1 is a point in the first circle and x_2 a point in the second circle.
  (a) Show that X can be endowed with the structure of a CW complex (with a single 0-cell and two 1-cells).
  (b) Prove that X is *not* a topological manifold.