

# Quiz 4

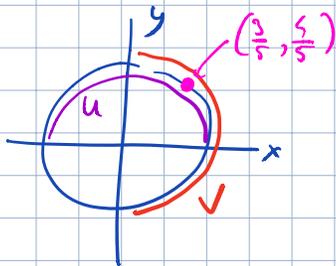
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(I) What is the derivative of a smooth map  $F: M \rightarrow N$ ?  
(Definition)

(II) Given a map  $F: M \rightarrow N$  and a point  $c \in N$ ,  
under which condition  $F^{-1}(c)$  is guaranteed to be a smooth manifold?

(III) Consider  $S^1$  and coord. charts

(1)  $U = \{(x, y) \in S^1 \mid y > 0\}$   $\xrightarrow{\varphi_U}$   $\mathbb{R}$   
 $(x, y)$   $\xrightarrow{\quad}$   $x$



(2)  $V = \{(x, y) \in S^1 \mid x > 0\}$   $\xrightarrow{\varphi_V}$   $\mathbb{R}$   
 $(x, y)$   $\xrightarrow{\quad}$   $y$

transition map:  $\varphi_V \varphi_U^{-1}: x \mapsto y = \sqrt{1-x^2}$

Let  $a = (\frac{3}{5}, \frac{4}{5}) \in S^1$  and  $v = \left(\frac{\partial}{\partial x}\right)_a \in T_a$

Then  $v = C \left(\frac{\partial}{\partial y}\right)_a$  with  $C = ?$