Topics in Topology II, Spring 2019
MATH 80440
"Introduction to Conformal Field Theory"
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Two-dimensional conformal field theory (CFT) was developed in 1980's starting from the seminal work of Belavin-Polyakov-Zamolodchikov (1984). It studies quantum field theory as a set of correlation functions on punctured Riemann surfaces, with punctures decorated by elements of a vector space V (the "space of fields" or "space of states"), subject to constraints imposed by conformal symmetry (in particular, the moduli space of complex structures on the surface plays a central role). Symmetry induces a rich structure on V and on correlators. In particular, V has to carry a representation of a special infinite-dimensional Lie algebra – the Virasoro algebra. Conformal field theory is a setup for quantum field theory in which one can produce highly nontrivial exactly computable answers for correlators and partition functions (e.g. 2-point correlators given by power laws with special rational exponents, 4-point correlators written in terms of hypergeometric functions and genus 1 partition function written in terms of Jacobi theta functions).

The plan of the class is to develop conformal field theory from scratch – its general structure (operator product expansions, stress-energy tensor, central charge, primary fields and descendants etc.), discuss various mathematical pictures/axiomatizations of CFT (Segal's axioms, vertex algebras) and introduce a zoo of examples:

- Free scalar field.
- Free scalar field with values in the circle of radius r. (This model turns out to be sensitive to whether r^2 is a rational number.)
- Free fermion. (The model corresponding the the famous Ising lattice statistical model.)
- Minimal models of rational conformal field theory.
- Wess-Zumino-Witten model. (This model is deeply related to 3-dimensional Chern-Simons theory – a prototypical topological field theory.)
- (Possibly) Witten's A and B models – topological field theories arising as from CFT with supersymmetry. This is the starting point of mirror symmetry (and the associated topics, e.g., Fukaya category).

No background in quantum field theory is assumed in the course; the exposition is intended to be self-contained.

Some references:

- Paul Ginsparg,"Applied conformal field theory," hep-th/9108028.
- Toshitake Kohno, "Conformal field theory and topology," Vol. 210 AMS 2002.
- P. Di Francesco, P. Mathieu, D. Sénéchal, "Conformal Field Theory," Springer 1997.