Hidden algebraic structure on cohomology of simplicial complexes, and TFT

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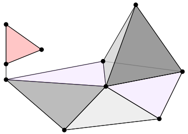
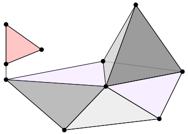


Simplicial complex T



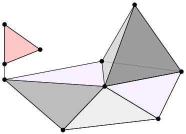
Simplicial complex T

$$\downarrow$$

Simplicial cochains $C^0(T) \to \cdots \to C^{\mathrm{top}}(T)$,

$$C^k(T) = \mathrm{Span}\{k - \text{simplices}\},$$

$$d_k : C^k(T) \to C^{k+1}(T), \qquad \underbrace{e_\sigma}_{\text{basis cochain}} \mapsto \sum_{\sigma' \in T:\ \sigma \in \mathrm{faces}(\sigma')} \pm e_{\sigma'}$$



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$$\downarrow$$

Cohomology $H^\bullet(T)$, $H^k(T) = \ker d_k \;/\; \mathrm{im}\, d_{k-1}$

— a homotopy invariant of T

Cohomology carries a commutative ring structure, coming from (non-commutative) Alexander's product for cochains.

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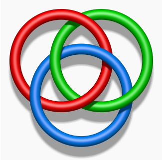

Example of use: linking of Borromean rings is detected by a non-vanishing Massey operation on cohomology of the complement.
$m_3([\alpha],[\beta],[\gamma]) = [u\wedge\gamma + \alpha\wedge v] \in H^2$
where $[\alpha],[\beta],[\gamma] \in H^1$, $du = \alpha\wedge\beta$, $dv = \beta\wedge\gamma$.

Another example: **nilmanifold**

$$M = \mathsf{H}_3(\mathbb{R})/\mathsf{H}_3(\mathbb{Z})$$
$$= \left\{ \begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix} \mid x, y, z \in \mathbb{R} \right\} / \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in \mathbb{Z} \right\}$$

Denote

$$\alpha = dx,\ \beta = dy,\ u = dz - y\,dx \in \Omega^1(M)$$

Important point: $\alpha \wedge \beta = du$. The cohomology is spanned by classes

$$\underbrace{[1]}_{\text{degree }0},\quad \underbrace{[\alpha],[\beta]}_{\text{degree }1},\quad \underbrace{[\alpha\wedge u],[\beta\wedge u]}_{\text{degree }2},\quad \underbrace{[\alpha\wedge\beta\wedge u]}_{\text{degree }3}$$

and

$$m_3([\alpha],[\beta],[\beta]) = [u \wedge \beta] \in H^2(M)$$

is a non-trivial Massey operation.

Fix $\mathfrak{g}$ a unimodular Lie algebra (i.e. with $\mathrm{tr}[x, \bullet] = 0$ for any $x \in \mathfrak{g}$).

Main construction (P.M.)

Simplicial complex T

$\downarrow$ local formula

Unimodular L_∞ algebra structure on $\mathfrak{g} \otimes C^\bullet(T)$

$\downarrow$ homotopy transfer

Unimodular L_∞ algebra structure on $\mathfrak{g} \otimes H^\bullet(T)$

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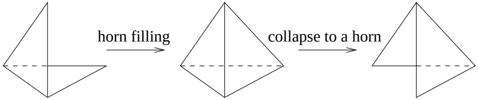
Main theorem (P.M.)

Unimodular L_∞ algebra structure on $\mathfrak{g} \otimes H^\bullet(T)$ (up to isomorphisms) is an invariant of T under simple homotopy equivalence.



Main construction (P.M.)

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$$\Big\downarrow \text{local formula}$$

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$$\Big\downarrow \text{homotopy transfer}$$

Unimodular L_∞ algebra structure on $\mathfrak{g} \otimes H^\bullet(T)$

- Thom's problem: lifting ring structure on $H^\bullet(T)$ to a **commutative** product on cochains. Removing $\mathfrak{g}$, we get a homotopy commutative algebra on $C^\bullet(T)$. This is an improvement of Sullivan's result with cDGA structure on cochains $= \Omega_{\text{poly}}(T)$.
- **Local** formulae for Massey operations.
- Our invariant is strictly stronger than rational homotopy type.

Definition

A unimodular L_∞ algebra is the following collection of data:

(a) a $\mathbb{Z}$-graded vector space $V^\bullet$,

(b) "classical operations" $l_n : \wedge^n V \to V$, $n \geq 1$,

(c) "quantum operations" $q_n : \wedge^n V \to \mathbb{R}$, $n \geq 1$,

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subject to two sequences of quadratic relations:

1. $\sum_{r+s=n} \frac{1}{r!s!} l_{r+1}(\bullet, \cdots, \bullet, l_s(\bullet, \cdots, \bullet)) = 0$, $n \geq 1$ (anti-symmetrization over inputs implied),
2. $\frac{1}{n!} \mathrm{Str}\, l_{n+1}(\bullet, \cdots, \bullet, -) + \sum_{r+s=n} \frac{1}{r!s!} q_{r+1}(\bullet, \cdots, \bullet, l_s(\bullet, \cdots, \bullet)) = 0$

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Note:

- First classical operation satisfies $(l_1)^2 = 0$, so $(V^\bullet, l_1)$ is a complex.
- A unimodular L_∞ algebra is in particular an L_∞ algebra (as introduced by Lada-Stasheff), by ignoring q_n.
- Unimodular Lie algebra is the same as unimodular L_∞ algebra with $l_{\neq 2} = q_\bullet = 0$.

An alternative definition

A unimodular L_∞ algebra is a graded vector space V endowed with

- a vector field Q on $V[1]$ of degree 1,
- a function ρ on $V[1]$ of degree 0,

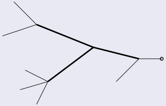
satisfying the following identities:

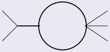
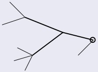
$$[Q, Q] = 0, \qquad \mathrm{div}\, Q = Q(\rho)$$

Homotopy transfer theorem (P.M.)

If $(V, \{l_n\}, \{q_n\})$ is a unimodular L_∞ algebra and $V' \hookrightarrow V$ is a deformation retract of (V, l_1), then

1. V' carries a unimodular L_∞ structure given by

$$l'_n = \sum_{\Gamma_0} \frac{1}{|\mathrm{Aut}(\Gamma_0)|}$$  $$: \wedge^n V' \to V'$$

$$q'_n = \sum_{\Gamma_1} \frac{1}{|\mathrm{Aut}(\Gamma_1)|}$$  $$+ \sum_{\Gamma_0} \frac{1}{|\mathrm{Aut}(\Gamma_0)|}$$  $$: \wedge^n V' \to \mathbb{R}$$

where Γ_0 runs over rooted trees with n leaves and Γ_1 runs over 1-loop graphs with n leaves.

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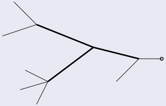
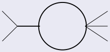
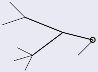
$$l'_n = \sum_{\Gamma_0} \frac{1}{|\mathrm{Aut}(\Gamma_0)|}$$  $$: \wedge^n V' \to V'$$

$$q'_n = \sum_{\Gamma_1} \frac{1}{|\mathrm{Aut}(\Gamma_1)|}$$  $$+ \sum_{\Gamma_0} \frac{1}{|\mathrm{Aut}(\Gamma_0)|}$$  $$: \wedge^n V' \to \mathbb{R}$$

where Γ_0 runs over rooted trees with n leaves and Γ_1 runs over 1-loop graphs with n leaves. **Decorations:**

leaf	$i : V' \hookrightarrow V$	root	$p : V \twoheadrightarrow V'$
edge	$-s : V^\bullet \to V^{\bullet-1}$	$(m+1)$-valent vertex	l_m
cycle	super-trace over V	m-valent $\circ$-vertex	q_m

where s is a chain homotopy, $l_1\, s + s\, l_1 = \mathrm{id} - i\, p$.

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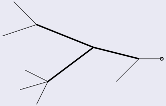
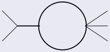
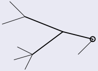
$$l'_n = \sum_{\Gamma_0} \frac{1}{|\mathrm{Aut}(\Gamma_0)|}$$  $$: \wedge^n V' \to V'$$

$$q'_n = \sum_{\Gamma_1} \frac{1}{|\mathrm{Aut}(\Gamma_1)|}$$  $$+ \sum_{\Gamma_0} \frac{1}{|\mathrm{Aut}(\Gamma_0)|}$$  $$: \wedge^n V' \to \mathbb{R}$$

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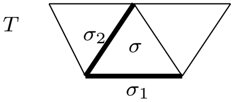
2. Algebra $(V', \{l'_n\}, \{q'_n\})$ changes by isomorphisms under changes of induction data (i, p, s).

Locality of the algebraic structure on simplicial cochains

$$l_n^T(X_{\sigma_1}e_{\sigma_1},\cdots,X_{\sigma_n}e_{\sigma_n}) = \sum_{\sigma\in T\,:\,\sigma_1,\ldots,\sigma_n\in\mathrm{faces}(\sigma)} \bar{l}_n^\sigma(X_{\sigma_1}e_{\sigma_1},\cdots,X_{\sigma_n}e_{\sigma_n})e_\sigma$$

$$q_n^T(X_{\sigma_1}e_{\sigma_1},\cdots,X_{\sigma_n}e_{\sigma_n}) = \sum_{\sigma\in T\,:\,\sigma_1,\ldots,\sigma_n\in\mathrm{faces}(\sigma)} \bar{q}_n^\sigma(X_{\sigma_1}e_{\sigma_1},\cdots,X_{\sigma_n}e_{\sigma_n})$$

Notations: e_σ – basis cochain for a simplex σ, $X_\bullet \in \mathfrak{g}$, $Xe_\sigma := X\otimes e_\sigma$.



Here $\bar{l}_n^\sigma : \wedge^n(\mathfrak{g}\otimes C^\bullet(T)) \to \mathfrak{g}$, $\bar{q}_n^\sigma : \wedge^n(\mathfrak{g}\otimes C^\bullet(T)) \to \mathbb{R}$ are universal local building blocks, depending on dimension of σ only, not on combinatorics of T.

Zero-dimensional simplex $\sigma = [A]$:
$\bar{l}_2(Xe_A, Ye_A) = [X, Y]$, all other operations vanish.

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One-dimensional simplex $\sigma = [AB]$:

$$\bar{l}_{n+1}(X_1 e_{AB}, \cdots, X_n e_{AB}, Y e_B) = \frac{B_n}{n!} \sum_{\theta \in S_n} [X_{\theta_1}, \cdots, [X_{\theta_n}, Y] \cdots]$$

$$\bar{l}_{n+1}(X_1 e_{AB}, \cdots, X_n e_{AB}, Y e_A) = (-1)^{n+1} \frac{B_n}{n!} \sum_{\theta \in S_n} [X_{\theta_1}, \cdots, [X_{\theta_n}, Y] \cdots]$$

$$\bar{q}_n(X_1 e_{AB}, \cdots, X_n e_{AB}) = \frac{B_n}{n \cdot n!} \sum_{\theta \in S_n} \mathrm{tr}_{\mathfrak{g}} \, [X_{\theta_1}, \cdots, [X_{\theta_n}, \bullet] \cdots]$$

where $B_0 = 1,\ B_1 = -1/2,\ B_2 = 1/6,\ B_3 = 0,\ B_4 = -1/30, \ldots$ are Bernoulli numbers.

Higher-dimensional simplices, $\sigma = \Delta^N$**,** $N \geq 2$**:** $\bar{l}_n, \bar{q}_n$ are given by a *regularized* homotopy transfer formula for transfer $\mathfrak{g} \otimes \Omega^\bullet(\Delta^N) \to \mathfrak{g} \otimes C^\bullet(\Delta^N)$

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$$\left.\begin{array}{c} \bar{l}_n^\sigma \\ \bar{q}_n^\sigma \end{array}\right\} (X_{\sigma_1} e_{\sigma_1}, \cdots, X_{\sigma_n} e_{\sigma_n}) = \sum_\Gamma C(\Gamma)^\sigma_{\sigma_1 \cdots \sigma_n} \mathrm{Jacobi}_\mathfrak{g}(\Gamma; X_{\sigma_1}, \cdots, X_{\sigma_n})$$

where Γ runs over **binary** rooted trees with n leaves for $\bar{l}_n^\sigma$ and over **trivalent** 1-loop graphs with n leaves for $\bar{q}_n^\sigma$; $C(\Gamma)^\sigma_{\sigma_1 \cdots \sigma_n} \in \mathbb{R}$ are structure constants.

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$C(\Gamma)^\sigma_{\sigma_1 \cdots \sigma_n} \in \mathbb{R}$ are structure constants.
There are explicit formulae for structure constants for small n.

Summary: logic of the construction

building blocks $\bar{l}_n, \bar{q}_n$ on Δ^N

$\downarrow$ combinatorics of T

algebraic structure on cochains

$\downarrow$ homotopy transfer

algebraic structure on cohomology

Summary: logic of the construction

$$\text{building blocks } \bar{l}_n, \bar{q}_n \text{ on } \Delta^N$$
$$\downarrow \text{combinatorics of } T$$
$$\text{algebraic structure on cochains}$$
$$\downarrow \text{homotopy transfer}$$
$$\text{algebraic structure on cohomology}$$

- Operations l_n on $\mathfrak{g} \otimes H^\bullet(T)$ are **Massey brackets** on cohomology and are a complete invariant of **rational homotopy type** in simply-connected case.
- Operations q_n on $\mathfrak{g} \otimes H^\bullet(T)$ give a version of **Reidemeister torsion** of T.
- Construction above yields new local combinatorial formulae for Massey brackets (in other words: Massey brackets lift to a local algebraic structure on simplicial cochains).

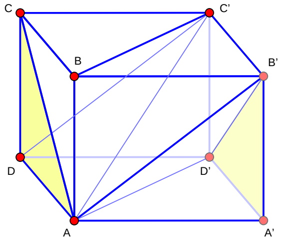
Example: for a circle and a Klein bottle, $H^\bullet(S^1) \simeq H^\bullet(KB)$ as rings, but $\mathfrak{g} \otimes H^\bullet(S^1) \not\simeq \mathfrak{g} \otimes H^\bullet(KB)$ as unimodular L_∞ algebras (distinguished by quantum operations).

$$e^{\sum_n \frac{1}{n!} q_n(X\otimes\varepsilon, \cdots X\otimes\varepsilon)} =$$

$\det_{\mathfrak{g}} \left(\frac{\sinh \frac{\mathrm{ad}_X}{2}}{\frac{\mathrm{ad}_X}{2}} \right)$ for S^1	$\det_{\mathfrak{g}} \left(\frac{\mathrm{ad}_X}{2} \cdot \coth \frac{\mathrm{ad}_X}{2} \right)^{-1}$ for Klein bottle

where $\varepsilon \in H^1$ – generator, $X \in \mathfrak{g}$ – variable.

Triangulation of the nilmanifold:



one **0-simplex:** A=B=C=D=A'=B'=C'=D'

seven **1-simplices:** AD=BC=A'D'=B'C', AA'=BB'=CC'=DD', AB=DC=D'B', AC=A'B'=D'C', AB'=DC', AD'=BC', AC'

twelve **2-simplices:** AA'B'=DD'C', AB'B=DC'C, AA'D'=BB'C', AD'D=BC'C, ACD=AB'D', ABC=D'B'C', AB'D', AC'D', ACC', ABC'

six **3-simplices:** AA'B'D', AB'C'D', ADC'D', ABB'C', ABCC', ACDC'

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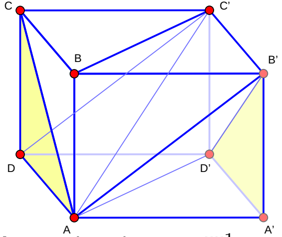

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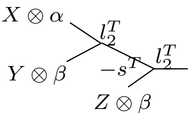
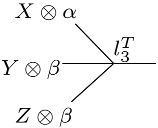
Massey bracket on H^1:

$$l_3(X\otimes[\alpha],Y\otimes[\beta],Z\otimes[\beta])=$$

$= \frac{1}{2}$ 

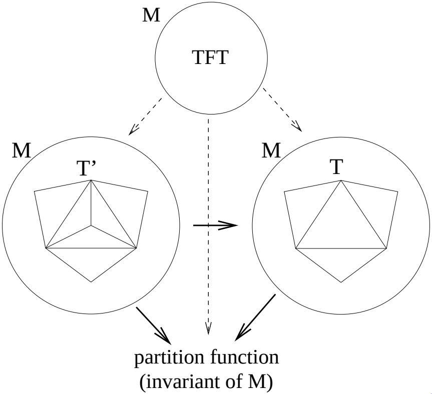
$+\frac{1}{6}$ 

+ permutations of inputs

$$=([[X,Y],Z]+[[X,Z],Y])\otimes[\eta]\in\mathfrak{g}\otimes H^2(T)$$

where $s^T=d^\vee/(dd^\vee+d^\vee d)$;
$\alpha=e_{AC}+e_{AD}+e_{AC'}+e_{AD'}$, $\beta=e_{AA'}+e_{AB'}+e_{AC'}+e_{AD'}$
– representatives of cohomology classes $[\alpha]$, $[\beta]$ in simplicial cochains.

Simplicial program for TFTs: Given a TFT on a manifold M with space of fields F_M and action $S_M \in C^\infty(F_M)[[\hbar]]$, construct an **exact discretization** associating to a triangulation T of M a fin.dim. space F_T and a **local** action $S_T \in C^\infty(F_T)[[\hbar]]$, such that partition function Z_M and correlation functions can be obtained from (F_T, S_T) by fin.dim. integrals. Also, if T' is a subdivision of T, S_T is an effective action for $S_{T'}$.



Example of a TFT for which the exact discretization exists:
BF **theory:**

- fields: $F_M = \underbrace{\mathfrak{g} \otimes \Omega^1(M)}_{A} \oplus \underbrace{\mathfrak{g}^* \otimes \Omega^{\dim M - 2}(M)}_{B}$,
- action: $S_M = \int_M \langle B, dA + A \wedge A \rangle$,
- equations of motion: $dA + A \wedge A = 0$, $d_A B = 0$.

Algebra – TFT dictionary

de Rham algebra $\mathfrak{g} \otimes \Omega^\bullet(M)$ (as a dg Lie algebra)	BF theory
unimodular L_∞ algebra $(V, \{l_n\}, \{q_n\})$	BF_∞ theory, $F = V[1] \oplus V^*[-2]$, $S = \sum_n \frac{1}{n!}\langle B, l_n(A, \cdots, A)\rangle +$ $+\hbar \sum_n \frac{1}{n!} q_n(A, \cdots, A)$
quadratic relations on operations	Batalin-Vilkoviski master equation $\underbrace{\Delta}_{\frac{\partial}{\partial A}\frac{\partial}{\partial B}} e^{S/\hbar} = 0$
homotopy transfer $V \to V'$	effective action $e^{S'/\hbar} = \int_{L \subset F''} e^{S/\hbar}$, $F = F' \oplus F''$
choice of chain homotopy s	gauge-fixing (choice of Lagrangian $L \subset F''$)

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- Construct other simplicial TFTs, in particular simplicial Chern-Simons theory.
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Steps:

- Construct simplicial 1-dimensional Chern-Simons theory as Atiyah's TFT on triangulated 1-cobordisms (**complete**, with Anton Alekseev).
- Construct a finite-dimensional algebraic model of 3-dimensional Chern-Simons theory; study effective action induced on de Rham cohomology and corresponding 3-manifold invariants (**complete**, with Alberto Cattaneo).
- Extend cohomological Batalin-Vilkovisky formalism for treating gauge symmetry of TFTs to allow spacetime manifolds with boundary or corners in a way consistent with gluing (**complete**, with Alberto Cattaneo and Nicolai Reshetikhin).
- Construct the quantization of TFTs on manifolds with boundary in BV formalism by perturbative path integral (**in progress**).
- Extend previous step to manifolds with corners (**in progress**).

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