Examples of bulk-boundary correspondences of field theories from BV pushforwards

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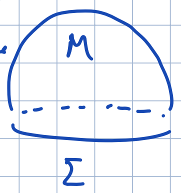
$\boxed{\begin{array}{c|c} \text{dg mfd} & L_\infty\text{-alg.} \\ & \downarrow \text{HT} \\ \text{moduli space of dg mfd} & \text{induced } L_\infty \text{ alg. on a def. retract} \end{array}}$ $\overset{\text{free level}}{\rightsquigarrow}$ BV theory $\xrightarrow{\text{BV pushforward}}$ effective action $\rightsquigarrow$ eff. action in a family over boundary condition ("BV-BFV formalism") $\overset{\text{example}}{\rightsquigarrow}$ CS on $\Sigma\times I \to$ G/G WZW on Σ, 7d abCS on $X\times I$ (X: CY_3) $\to$ BCOV on X

BV-BFV formalism

classically: bulk M, bdry Σ

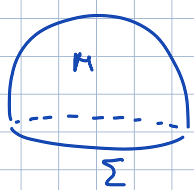


$\to (\mathcal{F}, \underset{+1}{Q}, \underset{-1}{\omega}, \underset{0}{S})$, $Q^2=0$, $\boxed{\delta S = \iota_Q\omega + \pi^*\alpha_\partial}$

$\pi\downarrow \;\; \uparrow\pi_*$

$\rightsquigarrow (\Phi_\partial, \underset{+1}{Q_\partial}, \underset{0}{\omega_\partial = \delta\alpha_\partial}, \underset{+1}{S_\partial})$ ← hamiltonian for Q_∂

quantum BV-BFV:



$\to \begin{cases} \cdot\ (\mathcal{V}_{res}, \underset{-1}{\omega_{res}}) \text{ - space of residual fields} \\ \cdot\ Z \in \mathcal{H}_\partial \otimes \mathrm{Dens}^{1/2}\mathcal{V}_{res} \text{ s.t. } \boxed{\left(\tfrac{i}{\hbar}\Omega_\partial - i\hbar\Delta_{res}\right)Z = 0} \end{cases}$ mQME

$\to (\mathcal{H}^\bullet_\partial, \underset{+1}{\Omega_\partial})$ - chain cx.

idea of quantization: bdry:

$\mathcal{F} \supset \mathcal{F}^b$ - fields subject to b.c. b.

$\downarrow\pi$

$\Phi_\partial \supset p^{-1}(b)$ Lag.

$p\downarrow$ ← fibration with Lag. leaves; require $\boxed{\alpha_\partial\big|_{\text{fiber of } p} = 0}$ (*)

$B_\partial \ni b$

Then: $\mathcal{H}_\partial = \mathrm{Dens}^{1/2} B_\partial$, $\Omega_\partial = \widehat{S_\partial}$

bulk: write $\mathcal{F} \simeq B_\partial \times \underbrace{\mathcal{Y}}_{\mathcal{V}_{res}\times\mathcal{Y}'}$ ← $\mathcal{F}^{b=0}$

$$Z(b, \varphi_{res}) := \text{“}\int_{\mathcal{L}\subset\mathcal{Y}'} e^{\frac{i}{\hbar}S(b+\varphi_{res}+\varphi_{fl})}\, D\varphi_{fl}\text{”} \in \mathcal{H}_\partial\otimes\mathrm{Dens}^{1/2}\mathcal{V}_{res}$$

($\mathcal{L}$: gauge-fixing Lagr.)

Rem: one can change the cl. BV-BFV package by an "f - transformation", $f_\partial \in C^\infty(\Phi_\partial)$:

$S \to S + \pi^* f_\partial$

$\alpha_\partial \to \alpha_\partial + \delta f_\partial$ - can use it to adapt the cl. theory to satisfy (*) for a preferred polarization p.

WARM-UP: 1d ab. CS

$$S(\psi+A)=\int_I \tfrac{1}{2}(\psi,d\psi) \qquad \psi+A\in\Omega^\bullet(I)\otimes\Pi V$$

V: v. sp. w/ inner product $(\,,\,)$

$Q:\ \psi\to 0,\quad A\to d\psi$, $\omega=\int_I(\delta\psi,\delta A)$ | $\Phi_{pt}=\Pi V$, $\alpha_{pt}=\frac{1}{2}(\psi,\delta\psi)$, $Q_\partial=S_\partial=0$

fix a cx. str. on V: $V_{\mathbb{C}}=\underbrace{V^+}_{\text{"hol"}}\oplus\underbrace{V^-}_{\text{"ahol"}}$

$\underset{\psi^+_{in}\ \text{fixed}}{\overset{0}{\bullet}}$ ——— $\underset{\psi^+_{out}\ \text{fixed}}{\overset{1}{\bullet}}$

$\mathcal{B}=\Pi V^+\oplus\Pi V^+$, $\mathcal{Y}=\Omega^\bullet(I,\partial I;\Pi V^+)\oplus\Omega^\bullet(I;\Pi V^-)$

$\mathcal{Y}=$

$(dt\cdot\Pi V^+\oplus 1\cdot\Pi V^-)\ \leftarrow \mathcal{V}_{res}$

$\oplus$

$\Omega^0(I,\partial I;\Pi V^+)\oplus\Omega^0_{\int=0}(I;\Pi V^-)\ \leftarrow \mathcal{V}'_{K\text{-ex}}$

$\oplus$

$\Omega^1_{\int=0}(I,\Pi V^+)\oplus\Omega^1(I,\Pi V^-)\ \leftarrow \mathcal{V}'_{d\text{-ex}}$

Hodge decomposition of $\mathcal{Y}$ (K, d; $K = d^{-1}$)

chain homotopy $K:\mathcal{Y}^\bullet\to\mathcal{Y}^{\bullet-1}$
- int. op. with kernel $\eta(t,t')=\pi^+\otimes(\Theta(t-t')-t)+\pi^-\otimes(t'-\Theta(t'-t))$

$$Z(\psi^+_{in},\psi^+_{out};\underbrace{\psi^{res},A^{res}}_{\in\mathcal{V}_{res}})=\int_{\mathcal{Y}'_{K\text{-ex}}\subset\mathcal{Y}}D\psi^+_{fl}\,D\psi^-_{fl}\ e^{\frac{i}{\hbar}S^f(\widetilde{\psi^+_{in}}+\widetilde{\psi^+_{out}}+\psi^-_{res}+\psi^-_{fl}+dt\,A^+_{res})}=$$

$f=\frac{1}{2}(\psi^+,\psi^-)|_{t=1}-\frac{1}{2}(\psi^+,\psi^-)|_{t=0}$ – f-transf. adapting to chosen polariz.

$$=\dots=\int D\psi^+_{fl}\,D\psi^-_{fl}\ e^{\frac{i}{\hbar}\left(\int_I(\psi^-_{fl},d\psi^+_{fl})+(\psi^+_{out},\psi^-_{res}+\psi^-_{fl}(1))-(\psi^+_{in},\psi^-_{res}+\psi^-_{fl}(0))\right)}=\boxed{e^{\frac{i}{\hbar}(\psi^+_{out}-\psi^+_{in},\psi^-_{res})}}$$

Gaussian integral

$\mathcal{H}_{pt}=\mathrm{Fun}(\psi^+)=\Lambda^\bullet(V^+)^*$, $\Omega_{pt}=0$

Rem: another choice of bdry polarization: $\underset{\psi^-_{in}}{\overset{0}{\bullet}}$ ——— $\underset{\psi^+_{out}}{\overset{1}{\bullet}}$

choice of retract of $\mathcal{Y}$ (res. fields)

$\mathcal{V}^{I}=dt\cdot V\oplus(1-t)\cdot\Pi V^+\oplus t\cdot\Pi V^-$

$$Z\Big(\underset{\psi^-_{in}}{\bullet}\overset{A_{res}}{\underset{\psi^+_{res},\ \psi^-_{res}}{\text{———}}}\underset{\psi^+_{out}}{\bullet}\Big)=e^{\frac{i}{\hbar}\left(\frac{1}{2}(\psi^-_{res},\psi^+_{res})+(\psi^+_{out},\psi^-_{res})-(\psi^-_{in},\psi^+_{res})+(\psi^-_{in},\psi^+_{out})\right)}$$

Feynman diag.: $\psi^-_{in}\,\psi^+_{fl}(0)$ ——— $\psi^-_{fl}(1)\,\psi^+_{out}$, η

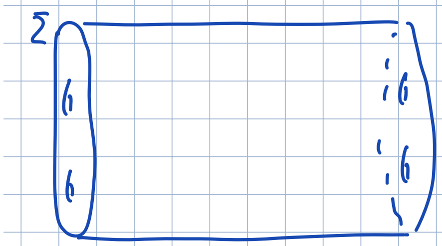
min choice

$\mathcal{V}^{II}=0$

$$Z\Big(\underset{\psi^-_{in}}{\bullet}\text{———}\underset{\psi^+_{out}}{\bullet}\Big)=e^{\frac{i}{\hbar}(\psi^+_{out},\psi^-_{in})}$$

3d abelian Chern-Simons

$M = \Sigma \times I$



$$S = \int_M \tfrac{1}{2}\mathcal{A}\wedge d\mathcal{A} = \int_{\Sigma\times I} \underbrace{\tfrac{1}{2}\mathcal{A}\wedge d_I\mathcal{A}}_{\text{"kin term"}} + \underbrace{\tfrac{1}{2}\mathcal{A}\wedge d_\Sigma \mathcal{A}}_{\text{"perturbation"}}$$

$$\mathcal{A} = c + A + A^* + c^*$$

$$\mathcal{F} = \Omega^\bullet(M)[1] = \Omega^\bullet(I, \Omega^{1,0}_\Sigma \oplus \Omega^{0,1}_\Sigma \oplus \Omega^0_\Sigma[1] \oplus \Omega^2_\Sigma[-1])$$

$p_\partial \Big\{$

$$\mathcal{B} = \underbrace{\left(\Omega^{0,1}\oplus\Omega^0[1]\right)}_{\text{in}} \oplus \underbrace{\left(\Omega^{1,0}\oplus\Omega^0[1]\right)}_{\text{out}}$$

fiber of p_∂ : $\mathcal{Y} = \Omega^\bullet(I,\{0\};\Omega^{0,1}) \oplus \Omega^\bullet(I,\{1\};\Omega^{1,0}) \oplus \Omega^\bullet(I,\partial I;\Omega^0[1]) \oplus \Omega^\bullet(I;\Omega^2[-1])$

$\Big\}$ def. retract

$$\mathcal{V}_{res} = H^\bullet(I,\partial I;\Omega^0[1]) \oplus H^\bullet(I;\Omega^2[-1]) \ni (dt\cdot \underset{gh=0}{\sigma}, \underset{gh=-1}{A^*_{res}})$$

$\mathcal{L}$: set to zero 1-form components in the fiber of $\mathcal{Y} \to \mathcal{V}_{res}$

$A^{1,0}_{fl}$ " $A^{0,1}_{fl}$ "

on $\mathcal{L}$:

$gh=0$: $\mathcal{A}^{(1)} = \tilde{A}^{1,0}_{out} + \tilde{A}^{0,1}_{in} + \underbrace{a^{1,0} + a^{0,1}}_{\text{fluctuations}} + dt\cdot\sigma$

$gh=1$: $\mathcal{A}^{(0)} = \tilde{C}_{in} + \tilde{C}_{out} + c_{fl}$

$gh=-1$: $\mathcal{A}^{(2)} = A^*_{res} + A^*_{fl}$

$gh=-2$: $\mathcal{A}^{(3)} = 0$

$$S^f\big|_{\mathcal{L}} = \int_{\Sigma\times I}\Big(a^{1,0} d_I a^{0,1} + A^*_{fl} d_I c_{fl} + \overbrace{dt(a^{1,0}+a^{0,1}) d_\Sigma \sigma}^{\#}\Big) + \int_\Sigma A^{1,0}_{out} a^{0,1}\big|_{t=1} - \left(A^*_{res} + A^*_{fl}\big|_{t=1}\right) C_{out}$$
$$- \int_\Sigma A^{0,1}_{in} a^{1,0}\big|_{t=1} - \left(A^*_{res} + A^*_{fl}\big|_{t=0}\right) C_{in}$$

propagators :

$$\langle a^{0,1}(t,z)\; a^{1,0}(t',z')\rangle = \Theta(t-t')\,\delta^{(2)}(z-z')\,\tfrac{i}{2}\,d\bar z\, dz' \qquad \text{———}$$

$$\langle c_{fl}(t,z)\; A^*_{fl}(t',z')\rangle = (\Theta(t-t') - t)\,\delta^{(2)}(z-z')\,\tfrac{i}{2}\,dz'\, d\bar z' \qquad \text{- - - -}$$

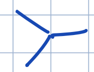
$$S^{eff} = \int_\Sigma A^{1,0}_{out} A^{0,1}_{in} + A^{1,0}_{out}\bar\partial\sigma + A^{0,1}_{in}\partial\sigma - \overbrace{\tfrac{1}{2}\partial\sigma\bar\partial\sigma}^{\text{free 2d boson CFT !}} - A^*_{res}(C_{out} - C_{in})$$

from 

Non-abelian 3d Chern-Simons

$\mathcal{A} \in \Omega^\bullet(M)\otimes\mathfrak{g}[1]$

$$S = \int_{\Sigma\times I} \frac{1}{2}(\mathcal{A}, d\mathcal{A}) + \frac{1}{6}(\mathcal{A},[\mathcal{A},\mathcal{A}])$$

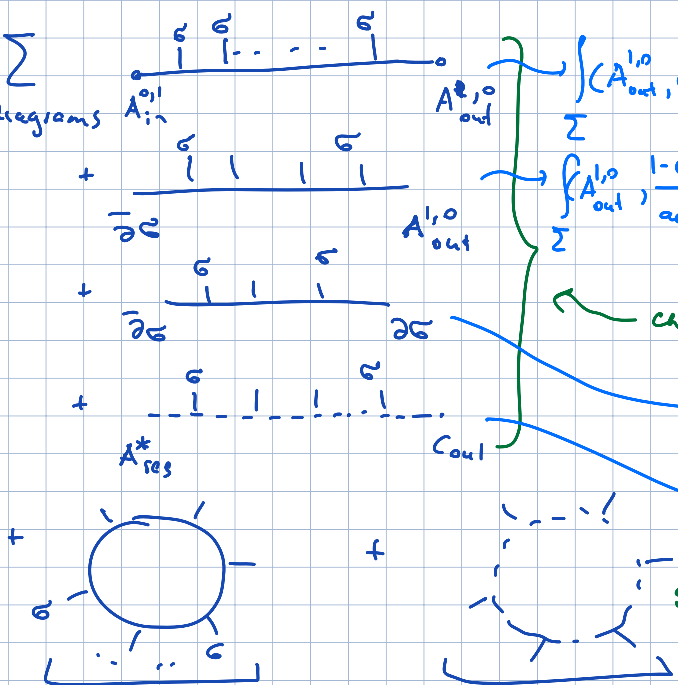
vertex  in Feynman diagrams

$\int$ on $\mathcal{L}$:

$$\int_{\Sigma\times I} dt\,(a^{1,0}, \mathrm{ad}_\sigma a^{0,1}) + \int_{\Sigma\times I} dt\,((c_{fl}, \mathrm{ad}_\sigma(A^*_{res}+A^*_{fl}))$$

polarization, gauge-fixing as before

$S^{eff} = \sum_{\text{F. diagrams}}$



$$\int_\Sigma \left(A^{1,0}_{out}, e^{-\mathrm{ad}_\sigma}\circ A^{0,1}_{in}\right)$$

$$\int_\Sigma \left(A^{1,0}_{out}, \frac{1-e^{-\mathrm{ad}_\sigma}}{\mathrm{ad}_\sigma}\circ\bar\partial\sigma\right)$$

calculated via Bernoulli polynomials

$$-\int_\Sigma \left(\partial\sigma, \frac{e^{-\mathrm{ad}_\sigma}+\mathrm{ad}_\sigma-1}{(\mathrm{ad}_\sigma)^2}\circ\bar\partial\sigma\right)$$

$$-\int_\Sigma \left(A^*_{res}, \frac{\mathrm{ad}_\sigma}{1-e^{-\mathrm{ad}_\sigma}}\circ c_{out}\right)$$

$= 0$

$$-i\hbar \text{"}\sum_{z\in\Sigma}\text{"}\ \underbrace{\mathrm{tr}_\mathfrak{g}\log\frac{\sinh\frac{\mathrm{ad}_{\sigma(z)}}{2}}{\frac{\mathrm{ad}_{\sigma(z)}}{2}}}_{j(\mathrm{ad}_{\sigma(z)})}$$

$$\exp^*\mu_G^{Haar} = e^{j}\cdot\mu_\mathfrak{g}$$

Group-valued parametrization of res. fields

$$g = e^{-\sigma}: \Sigma \longrightarrow G, \qquad g^* = -g^{-1}\left(\frac{\mathrm{ad}_{\log g}}{1-g}\circ A^*_{res}\right) \in \Omega^2(\Sigma, (g^{-1})^*TG)$$

$$\omega_{res} = \int_\Sigma (\delta\sigma, \delta A^*_{res}) = \int_\Sigma (\delta g, \delta g^*)$$

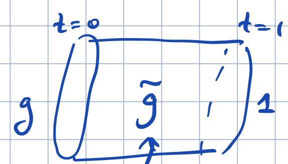
transformation of S^{eff} as a log-half-density

$$Z = e^{\frac{i}{\hbar}S^{eff}_{\sigma,A^*_{res}}} D^{1/2}\sigma\, D^{1/2}A^*_{res} = e^{\frac{i}{\hbar}S^{eff}_{g,g^*}} D^{1/2}g\, D^{1/2}g^*$$

$$S^{eff}_{g,g^*} = \int_\Sigma \langle A^{1,0}_{out}, g A^{0,1}_{in} g^{-1}\rangle - \langle A^{1,0}_{out}, \bar\partial g\cdot g^{-1}\rangle - \langle A^{0,1}_{in}, g^{-1}\partial g\rangle + WZW(g)$$
$$- \langle c_{out}, g g^*\rangle - \langle c_{in}, g^* g\rangle$$

$$= \underbrace{S_{G/G\ WZW}}_{\text{II}} + \text{ghost terms}$$

where
$$WZW(g) = -\frac{1}{2}\int_\Sigma \langle \partial g\cdot g^{-1}, \bar\partial g\cdot g^{-1}\rangle - \frac{1}{12}\int_{\Sigma\times I}\langle d\tilde g\cdot\tilde g^{-1}, [d\tilde g\cdot\tilde g^{-1}, d\tilde g\cdot\tilde g^{-1}]\rangle$$



interpolation between g and 1

Properties · no quantum corrections in S^{eff}_{g,g^*} !

· satisfies mQME $\boxed{\left(\frac{i}{\hbar}\Omega_\partial - i\hbar\Delta_{res}\right) Z = 0}$

where

$$\Omega_\partial = \int_\Sigma \Big\langle c_{out}, \bar\partial A^{1,0}_{out} - i\hbar\left(\partial + [A^{1,0}_{out}, -]\right)\frac{\delta}{\delta A^{1,0}_{out}}\Big\rangle - i\hbar\Big\langle \tfrac{1}{2}[c_{out}, c_{out}], \frac{\delta}{\delta c_{out}}\Big\rangle$$

$$+ \int_\Sigma \Big\langle c_{in}, -\partial A^{0,1}_{in} - i\hbar\left(\bar\partial + [A^{0,1}_{in}, -]\right)\frac{\delta}{\delta A^{0,1}_{in}}\Big\rangle - i\hbar\Big\langle \tfrac{1}{2}[c_{in}, c_{in}], \frac{\delta}{\delta c_{in}}\Big\rangle$$

$$\Delta_{res} = \int_2 \Big\langle \frac{\delta}{\delta g}, \frac{\delta}{\delta g^*}\Big\rangle$$

· mQME (mod $\hbar, g^*$) corresponds to Polyakov-Wiegmann f-la for $\mathbb{I}$:

$$\mathbb{I}\left({}^{h_{out}}(A^{1,0}_{out}), {}^{h_{in}}(A^{0,1}_{in}); h_{out}\, g\, h_{in}^{-1}\right) = \mathbb{I}\left(A^{1,0}_{out}, A^{0,1}_{in}; g\right) - \mathbb{I}\left(A^{1,0}_{out}, 0, h_{out}^{-1}\right) - \mathbb{I}\left(0, A^{0,1}_{in}, h_{in}\right)$$

· $\mathbb{I}$ is a "generalized gen. fun." for the evol. Lagrangian $\longleftrightarrow$ $\mathbb{I}$ is a "Hamilton-Jacobi action" for CS

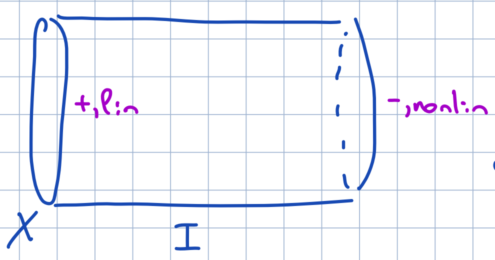
$$L = \{(A_{out}, A_{in}) \mid \exists A \in \mathrm{FlatConn}(\Sigma\times I) \text{ s.t. } A|_{t=1} = A_{out},\ A|_{t=0} = A_{in}\} \subset \overline{\mathrm{Conn}(\Sigma)} \times \mathrm{Conn}(\Sigma)$$

i.e. $$L = \left\{\left(A^{1,0}_{out}, A^{0,1}_{out} = \frac{\partial\mathbb{I}}{\partial A^{1,0}_{out}};\ A^{1,0}_{in} = \frac{\partial\mathbb{I}}{\partial A^{0,1}_{in}}, A^{0,1}_{in}\right) \,\Big|\, \frac{\partial\mathbb{I}}{\partial g} = 0\right\}$$

· $$\int_{\mathcal{L}_{res}\subset\mathcal{V}_{res}} \Big\langle \underset{\substack{A^{1,0}_{out}=0\\ c_{out}=0}}{\psi_{out}} \Big| \underset{\text{"}e^{\frac{i}{\hbar}S^{eff}}\text{"}}{Z} \Big| \underset{\substack{A^{0,1}_{in}=0\\ c_{in}=0}}{\psi_{in}} \Big\rangle = \int Dg\, e^{\frac{i}{\hbar}S_{WZW}(g)}$$ $\leftarrow$ (pure) WZW partition function.

7d abelian CS

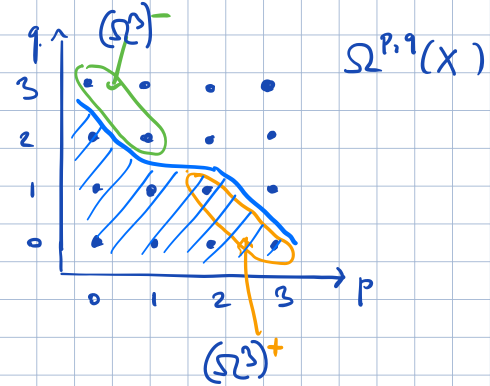
(elaboration on Gerasimov-Shatashvili hep-th/0409238)



$\uparrow$ Calabi-Yau, $\dim_{\mathbb{C}} X = 3$

$M = X \times I$, $S = \int_M \frac{1}{2} \mathcal{A} \wedge d\mathcal{A}$, $\mathcal{A} \in \Omega^\bullet(M)[3]$

on $X \times \{0\}$: "holomorphic" (linear) polarization



on $X \times \{1\}$: fix ghosts $\bigoplus_{p+q \le 2} \Omega^{p,q}$

and "Hitchin polarization" for $\Omega^3(X)$: non-linear!!

$A = A^{+,nl} + \boxed{A^{-,nl}}$ $\leftarrow$ base of polarization

$\uparrow \quad \uparrow$ decomposable 3-forms

$A^{+,nl} = E_1 \wedge E_2 \wedge E_3$

$A^{-,nl} = \bar{E}_1 \wedge \bar{E}_2 \wedge \bar{E}_3$

parametrization: $A^{+,nl} = \rho e^{\mu} \omega_0$ $\quad \mu \in \Omega^{-1,1}(X)$

$A^{-,nl} = \bar{\rho} e^{\bar{\mu}} \bar{\omega}_0$ $\quad \bar{\mu} \in \Omega^{1,-1}(X)$

CY forms

$\Omega^{-1,1} \ni \frac{((\partial b)^\vee)^3 \omega_0}{\omega_0} \cdot \omega_0$

Claim: $\int_{\mathcal{V}_{res}} \langle \psi_{in} | Z\left(\left(\ 7d\, CS\ \right)\right) | \psi_{out} \rangle = \int_{\Omega^{1,1}_X} Db\, e^{\frac{i}{\hbar} \int_X \frac{1}{2} \partial b \bar{\partial} b + \frac{1}{6} \langle \partial b, \partial b, \partial b \rangle}$

lin. hol. $\qquad$ Hitchin

$b \in \Omega^{1,1}_X$

$\| $

$A^{1,1}_{I\, res}$ - res. field of Chern-Simons

"BCOV theory" (aka Kodaira-Spencer gravity)

OR: $\psi(A^{3,0}_{in}, A^{2,1}_{in}) = Z\left(\left(\ l \quad \right) nl\right) | \psi_{out} \rangle$

$\psi(\omega_0, x) \sim \int Db\, e^{\frac{i}{\hbar} \int_X \frac{1}{2} \partial b \bar{\partial} b + \frac{1}{6} \langle \partial b + x, \partial b + x, \partial b + x \rangle}$

$x \in \Omega^{2,1}_{\bar{\partial}\text{-harmonic}}$

Some More details: $Z_{CS} = e^{\frac{i}{\hbar} S^{eff}}$

$$S^{eff} = \int_X \frac{1}{2}\partial A^{1,1}_{I\,res}\,\bar\partial A^{1,1}_{I\,res} + A^{+,\ell}_{in}\, d A^{0,2}_{I\,res} + A^{2,1}_{in}\,\bar\partial A^{1,1}_{I\,res} - G\left(A^{+,\ell}_{in} + dA^{2,0}_{I\,res} + \partial A^{1,1}_{I\,res}, A^{-,n\ell}_{out}\right)$$
$$+ \left(A^{[>0]}_{out} - A^{[>0]}_{in}\right) A^{[<0]}_{res} + A^{[<-1]}_{res}\, d\, A^{[>0]}_{I\,res}$$

Here $G(A^{3,0}, A^{2,1}, \bar\rho, \bar\mu) = \int_X \bar\rho\left(A^{3,0}\bar\omega_0 + A^{2,1}\bar\mu\bar\omega_0\right) + \bar\rho^2\langle\bar\mu^3\rangle\omega_0\bar\omega_0 - \dfrac{\left\langle \left(A^{2,1} - \frac{1}{2}\bar\rho\bar\mu^2\bar\omega_0\right)^{\vee 3}\right\rangle}{(A^{3,0})^\vee - \bar\rho\langle\mu^3\rangle}\,\omega_0\bar\omega_0$

Notations: $\Omega^{p,q} \to \Omega^{p-3,q}$, $A \mapsto A^\vee = A\omega_0^{-1}$ $\qquad \langle\mu^3\rangle = \frac{1}{6}\dfrac{\mu^3\omega_0}{\bar\omega_0}$

$$A^{-\ell}\delta A^{+\ell} = A^{+n\ell}\delta A^{-n\ell} + \delta \underbrace{G}_{\text{generating function for the change of polarization}}$$

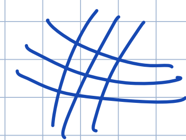
$$\Psi_{out}(A^{-n\ell}) = \delta(\bar\mu\ \ )\,\delta(A^{[>0]})\, e^{\frac{i}{\hbar}\int_X \bar\rho\,\omega_0\bar\omega_0}$$

Toy situation: 1d CS with nonlin. polarization

Map$(T[1]I, \tau^\circ)$ — Cochain cx, ω_Y -symp str.

lin. pol: $\tau^\circ = \tau^+ \oplus \tau^-$

nonlin. pol: $\tau^\circ \simeq \tau^{n\ell,Q} \times \tau^{n\ell,P}$



gen. fun. $\qquad G(\psi^{\ell,+}, \psi^{n\ell,Q})$

$$\psi^{\ell,-} = \frac{\partial G}{\partial \psi^{\ell,+}}$$

Trick: write $\mathcal{F} \simeq \underset{(\psi^+_{in}, \psi^Q_{out})}{\mathcal{B}} \times \mathcal{Y}$ — same as for $(\psi^+_{in}, \psi^-_{out})$ - pol.

$$S^f\left(\psi^+_{in}, \psi^Q_{out}, \psi^+_{fl}, \psi^-_{fl}; dt\cdot A_{res}\right) = \dots + \underbrace{G\left(\psi^+(1), \psi^Q_{out}\right)}_{\text{"bdry observable"}}$$