AME 20214
Homework 12
Due: Thursday, 6 December 2012, in class

1. Use `Mathematica` alone to reproduce the results of Homework 10, Problem 4 via two methods:

   - a second order Runge-Kutta method, and
   - the `Mathematica` function `NDSolve`.

   Note, you only need to reproduce the results of Homework 10, Problem 4. There is no need to introduce a complicated super-structure of subroutines within a `Mathematica` framework.

2. Use the online help to learn about the `Mathematica` function `FindRoot` and use it to identify *all* solutions, for $x \in [0, 5]$ to the equation

   $$x = 4\sin(4x).$$

   To aid in determining how many solutions exist, you may wish to plot x and $4\sin(4x)$ and look for intersections. Inclusion of the actual plot is optional.

3. Use the `Mathematica` function `DSolve` to get an exact solution to the mass-spring problem with a time-dependent spring coefficient

   $$\frac{d^2y}{dt^2} + \sqrt{t}\, y = 0, \qquad y(0) = 1, \quad \left.\frac{dy}{dt}\right|_{t=0} = 0.$$

   You may wish to use the `TexForm` option to help in presenting the exact solution. Give two or three sentences discussing the mathematical meaning of the functions that arise in the exact solution. Use your sleuthing skills to find information on these unusual functions. Plot the solution for $t \in [0, 50]$. Does the period of oscillation increase or decrease with t? Give a physics-based explanation.

Prepare your report with the LaTeX text processor. Make all plots with `Mathematica`. Include at least one equation, prepare beautiful figures, and include all programs that you wrote. Four page maximum.