AME 20214
Homework 2
Due: Thursday, 6 September 2012, in class

Consider the mathematics problem on the 20214 homepage,

$$\frac{dy}{dt} = -y, \qquad y(0) = 1.$$

We note that by inspection, the exact solution is $y(t) = e^{-t}$.

Now, using standard approximations of the derivative, $dy/dt \sim \Delta y/\Delta t$, we can write a discrete approximation of the differential equation as

$$\frac{y_{n+1} - y_n}{\Delta t} = -y_n, \qquad y_0 = 1.$$

Solving for y_{n+1}, we get

$$y_{n+1} = y_n - \Delta t y_n, \qquad y_0 = 1.$$

If this formula is repeated iteratively within a computer code, we get a discrete approximation for $y(t)$.

1. (10) Download the `Fortran` source code, identical to that on the 20214 primary homepage,

   `http://www.nd.edu/~powers/ame.20214/euler.f90`

   compile and execute it, so as to reproduce the results on the homepage. Plot $y(t)$ for $t \in [0, 1]$ for the exact solution and the numerical approximation with $\Delta t = 0.1$.

2. (70) Run the code for several different values of Δt and generate a log-log plot of the magnitude of the error in your prediction of y at $t = 1$. That is, plot

   $$|y_{exact}(t = 1) - y_{num}(t = 1)| \qquad \text{versus} \qquad \Delta t.$$

   For small values of Δt, you will probably wish to suppress the printing of intermediate values and only print the final values at $t = 1$. See how small a value of Δt you can effectively use before roundoff error corrupts the solution. Note, the exact value is

   $$y_{exact}(t = 1) = e^{-1} = 0.36787944117144232159552377016146086744558...$$

3. (15) Repeat the previous exercise with double the precision and generate the equivalent plot. To adjust the precision from its default value is a tricky task, and several generalizations must be made to `euler.f90`. The many generalizations have already been made and are found in the modified file

   `http://www.nd.edu/~powers/ame.20214/euler2.f90`

   As written, the code `euler2.f90` should give identical results as `eulerf.f90`. The precision in `euler2.f90` can be doubled by simply changing the value of `p` from `4` to `8`. For sufficiently small Δt, the error should be significantly lower than for the previous exercise. See how small a value of Δt you can effectively use before roundoff error corrupts the solution.

4. (5) Repeat the previous exercise with quadruple the precision of the original calculation and generate the equivalent plot. The precision here can be set by simply changing the value of `p` from `8` to `16`. For sufficiently small Δt, the error should be significantly lower than for the previous exercise. See how small a value of Δt you can effectively use before roundoff error corrupts the solution.

   It should be noted that adjusting the precision in `Fortran` can be a messy chore, fraught with machine-dependencies. The text of C&S has an extended dissuasion of the `kind` specification, see p. 90, whose difficulty reflects well the complexities of machine precision.