AME 20214
Homework 4
Due: Thursday, 20 September 2012, in class

1. (60) Consider the standard model, which can be derived from Newton's second law of motion, of a driven linear mass-spring-damper system:

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F_o \sin(\nu t), \qquad y(0) = y_o, \qquad \left.\frac{dy}{dt}\right|_{t=0} = 0.$$

Here y is the distance with units of m, t is the time with units of s, m is the mass with units of kg, b is the damping coefficient with units of Ns/m, k is the spring constant with units of N/m, F_o is the amplitude of the driving force with units of N, and ν is the frequency of the driving force with units of $Hz = 1/s$. The term y_o is the initial displacement with units of m.

If we define the velocity v to be $v = dy/dt$, we can re-write the governing equation as a system of two first order differential equations:

$$\begin{aligned} \frac{dy}{dt} &= v, \qquad y(0) = y_o, \\ \frac{dv}{dt} &= -\frac{k}{m}y - \frac{b}{m}v + \frac{F_o}{m}\sin(\nu t), \quad v(0) = 0. \end{aligned}$$

This system can be simulated computationally using the Euler method for a system. Simple discretization of the system shows

$$\begin{aligned} \frac{y_{n+1} - y_n}{\Delta t} &= v_n, \\ \frac{v_{n+1} - v_n}{\Delta t} &= -\frac{k}{m}y_n - \frac{b}{m}v_n + \frac{F_o}{m}\sin(\nu t_n), \\ t_{n+1} &= t_n + \Delta t. \end{aligned}$$

For $m = 1\ kg$, $k = 9\ N/m$, $y_o = 1\ m$, $t \in [0, 20\ s]$, numerically estimate $y(t)$ with the Euler method, embodied in a `Fortran` code for the cases:

(a) $b = 0\ Ns/m$, $F_o = 0\ N$.
(b) $b = 1\ Ns/m$, $F_o = 0\ N$.
(c) $b = 1\ Ns/m$, $F_o = 10\ N$, $\nu = 1\ Hz$.
(d) $b = 1\ Ns/m$, $F_o = 10\ N$, $\nu = 3\ Hz$.
(e) $b = 1\ Ns/m$, $F_o = 10\ N$, $\nu = 10\ Hz$.
(f) $b = 1\ Ns/m$, $F_o = 10\ N$, $\nu = 20\ Hz$.

For each case, use $\Delta t = 0.01\ s$, and give a plot of your estimate of $y(t)$ for $t \in [0, 20\ s]$.

A lengthy exact solution is available, but not shown. The exact solution for case c) is plotted in Fig. 1. Discuss what happens to your simulation as Δt is varied from very small to very large, e.g. $\Delta t = \{10^{-6}, 10^{-5}, \ldots, 2\times 10^{-1}, 1\times 10^{-1}, 10^0\}$. Detailed plots are not required, but a reasonable explanation is. Give a brief physical discussion of the effects of unforced damping and damping/forcing/frequency combinations.

2. (40) Use the LaTeX processor to build the paper copy of your submission. You will find it useful to use the file `sample.tex` as a template. No handwritten work should be submitted. All figures should be inserted as `.eps` files. Any equations, and you must include *at least one*, should be fully formatted with proper nomenclature. For example, use Δ, not `Delta`.

Here are a few guidelines to apply for your technical writing:

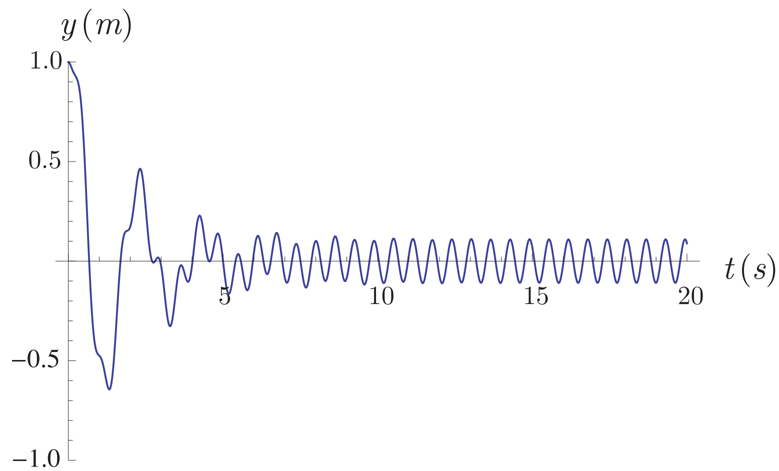

Figure 1: Exact solution for system response for linear forced mass-spring-damper system with $m = 1\ kg$, $k = 9\ N/m$, $y_o = 1\ m$, $b = 1\ Ns/m$, $F_o = 10\ N$, $\nu = 10\ Hz$.

- Your figures should be elegant. All should be inserted into the text as `.eps` files. All must include short, informative captions. All figures must be referred to in the main text.
- Include *at least one* equation.
- All equations and mathematical variables should be formatted in LaTeX mathematical format.
- Use actual Greek letters, e.g. Δ, not "Delta."
- Identify all variables with words of description, e.g. "$y = mx+b$, where y is the dependent variable, x is the independent variable, m is the slope, and b is the intercept."
- All mathematical variables, whether within the text or in a separate equation should be written in math mode:
  - Correct usage: "The variable is named x."
  - Incorrect usage: "The variable is named x."
- English text with equations should be in text mode; use the `mbox` and `qquad` commands for this:

  $$x = 1 \qquad \mbox{when} \qquad y = 0.$$

  The LaTeX script for the above is

  ```
  $$x=1 \qquad \mbox{when} \qquad y=0.$$
  ```
- Run your raw text file through a spelling checker. On the ND linux cluster, this is achieved via the command `ispell` *`filename`*`.tex`.
- Always use complete sentences.
- Use commas or periods at the end of equations, as appropriate.
- Do not use contractions (such as don't).

For this assignment, a) you do not need to repeat all the details of the homework statement; you can focus on the solution, and b) use plain white paper, not engineering paper.