AME 20214
Homework 7
Due: Thursday, 25 October 2012, in class

Consider the Taylor series expansion of $\sin x$ about $x = 0$:

$$\sin x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!}. \tag{1}$$

Based on Eq. (1), we see that a five-term Taylor series expansion approximates $\sin x$ by

$$\sin x \sim x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880}. \tag{2}$$

1. (50) Write and execute a `Fortran` program with a similar structure to that given in `ch15g.f90` to generate data to prepare plots of $\sin x$ and its five-term approximation, Eq. (2), within the domain $x \in [x_{min}, x_{max}]$.
   - The program must read input data of x_{min}, x_{max}, and the number of points to plot from an input file named `input.txt`.
   - The program must write the output data to a file named `output.txt`.
   - The program must draw upon a function subroutine to evaluate the approximation of Eq. (2). The `module` formalism, as used in `ch15gf.90` must be employed.
2. (50) Use the LaTeX processor to communicate your results.
   - There is a *two-page maximum, strictly enforced.*
   - Include a concise amount of prose to efficiently describe the problem.
   - Include at least one equation, properly formatted and properly described.
   - Include one elegantly prepared figure, giving on a single plot $\sin x$ and its five-term Taylor series approximation for $x \in [0, 5]$. Take special care that
     – The font size of all terms within the figure is of comparable size to that of the main text–so that the reader can actually read your plot. If for some reason, you are unsure how to achieve this in `MATLAB`, try reading its help page, or simply google "how to increase font size in matlab plots."
     – The reader knows which curve corresponds to which data.
     – For this exercise, use a sufficiently large number of points that both curves appear smooth; do not use identifiers such as small open circles for individual points. The principle is that plots of continuous functions, such as those of this homework, should be simple smooth curves. For other problems which do have a discrete character, e.g. experimental data points, or numerical problems with large Δx, one should use small open circles on the graphs.
   - Include a copy of a) your `Fortran` program, and b) your input file, both embedded within the `verbatim` environment: e.g.

```
\begin{verbatim}
Fortran code here.
\end{verbatim}
```