AME 20214 Homework 12 Due: Thursday, 12 December 2013, in class

1. (10) Use the online help to learn about the Mathematica function FindRoot and use it to identify all real solutions, for  $x \in [-\infty, \infty]$  to the equation

$$x = 2\cos(4x).$$

To aid in determining how many solutions exist, you may wish to plot x and  $2\cos(4x)$  and look for intersections. Inclusion of the actual plot is optional.

2. (40) Consider the differential equation and initial conditions for the mass-spring problem with a time-dependent spring coefficient:

$$\frac{d^2y}{dt^2} + \sqrt[3]{t} y = 0, \qquad y(0) = 1, \quad \frac{dy}{dt}\Big|_{t=0} = 0.$$

- (a) Use the Mathematica function DSolve to get an exact solution. You may wish to use the TeXForm option to help in presenting the exact solution. Give two or three sentences discussing the mathematical meaning of the functions that arise in the exact solution. Use your sleuthing skills to find information on these unusual functions. Plot the solution in Mathematica for  $t \in [0, 50]$ . Does the period of oscillation increase or decrease with t? Give a physics-based explanation.
- (b) Solve the differential equation and initial conditions with the Mathematica function NDSolve to get a numerical solution. Plot the solution in Mathematica for  $t \in [0, 50]$ .

Prepare your report with the  $IAT_EX$  text processor. Include at least one equation, prepare beautiful figures, and include all programs that you wrote. Four page maximum, æsthetics: 50 points, technical content: 50 points.