AME 20214
Homework 3
Due: Thursday, 19 September 2013, in class

1. (60) Consider the standard model, which can be derived from Newton's second law of motion, of a linear mass-spring system:

$$m\frac{d^2y}{dt^2} + ky = 0, \qquad y(0) = 0, \qquad \left.\frac{dy}{dt}\right|_{t=0} = \dot{y}_o.$$

Here y is the distance with units of m, t is the time with units of s, m is the mass with units of kg, and k is the spring constant with units of N/m. The term $\dot{y}_o$ is the initial velocity with units of m/s. The exact solution is easily seen by direct substitution to be

$$y(t) = \dot{y}_o\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right).$$

If we define the velocity v to be $v = dy/dt$, we can re-write the governing equation as a system of two first order differential equations:

$$\begin{aligned} \frac{dy}{dt} &= v, \qquad y(0) = 0, \\ \frac{dv}{dt} &= -\frac{k}{m}y, \quad v(0) = \dot{y}_o. \end{aligned}$$

This system can be simulated computationally using the Euler method for a system. Simple discretization of the system shows

$$\begin{aligned} \frac{y_{n+1} - y_n}{\Delta t} &= v_n, \\ \frac{v_{n+1} - v_n}{\Delta t} &= -\frac{k}{m}y_n. \end{aligned}$$

For $m = 1\ kg$, $k = 9\ N/m$, $\dot{y}_o = 1\ m/s$, $t \in [0, 10\ s]$, numerically estimate $y(t)$ with the Euler method, embodied in a `Fortran` code. For $\Delta t = 0.01\ s$, plot on the same plot the exact solution and your computational estimate for $t \in [0, 10\ s]$. You can choose to copy and paste your `Fortran` output into `matlab` for plotting, or you can read ahead and learn how to read and write from files. Choose your favorite precision, and plot on a log-log scale the error at $t = 10\ s$ as a function of Δt. Comment on the stability of the method as a function of Δt.

2. (20) Create a home page in your Notre Dame-maintained space on the world wide web. You can go to the AME 20214 homepage, documents section. Once there, *right* click on the file `Template` for the file `index.html`. Next choose browser option "View Source," usually found under the "View" toolbar on your browser (under "Develop" in Safari). The next step is important. Save the raw text file, which has the temporary placeholder name of `index.template.html`, as the permanently named `index.html` in your `netfile` space *in* the directory `www`. Note that the `netfile` space is the Notre Dame default for student web pages, not `afs` space. Test to see if it works, then use a text editor to modify it so that your name is identified on the home page in whatever fashion you see fit. Add any æesthetics you wish.

3. (20) Retrieve document `sample.tex` and `sample.figure.eps` from the "documents" link at the course home page,

   `http://www.nd.edu/~powers/ame.20214/`,

   edit it to add your name, process it with a LaTeX processor, convert it to `.pdf` format, and print a copy of the compiled file. Use instead `sample2.tex` and `sample_figure.pdf` if you intend to use `pdflatex`. Post your `.pdf` file on your Notre Dame personal web page and check to see that the general public can view it.

Be sure to follow the homework format specified in the course syllabus. All plots must adhere to course standards. For this homework, there is a *three page maximum*.