AME 20214
Homework 8
Due: Thursday, 7 November 2013, in class

Consider again the Taylor series approximation of $\cos x$ about $x = 0$:

$$\cos x \sim \sum_{n=0}^{m} (-1)^n \frac{x^{2n}}{(2n)!}. \qquad (1)$$

As $m \to \infty$, the approximation becomes exact.

1. (50) Write a set of `Fortran` codes to approximate $\cos x$ that has the following features:
   - It approximates $\cos x$ at a large number of uniformly distributed points within the domain $x \in [x_{min}, x_{max}]$ with the m-term Taylor series of Eq. (1). The main program will read from a file values of x_{min}, x_{max}, m, and the number of points at which the function is to be evaluated.
   - The Taylor series approximation will be evaluated from a user-defined real function subroutine, called upon by the main program.
   - The real function subroutine to evaluate the Taylor series approximation will itself call upon a user-defined function to evaluate the factorial function. While the factorial of an integer is an integer, it will be advantageous to approximate it as a real function because for even moderate values of n, $n!$ is not able to be represented on our machines as an integer because of upper bounds on integer size.
   - The main program and all function subroutines will be written as distinct files, in the same fashion as Section 12.2.4 of the course notes.
   - The compilation and execution of the main program and function subroutines will be achieved via an executable script file in the same fashion as Section 12.2.5 of the course notes.

2. (50) Use the LaTeX processor to communicate your results.
   - There is a *three-page maximum, strictly enforced*.
   - Include a concise amount of prose to efficiently describe the problem.
   - Include at least one equation, properly formatted and properly described.
   - Include one elegantly prepared figure, giving on a single plot $\cos x$ and its five-term Taylor series approximation for $x \in [0, 5]$. Take special care that
     - The font size of all terms within the figure is of comparable size to that of the main text–so that the reader can actually read your plot.
     - The reader knows which curve corresponds to which data.
     - For this figure, use a sufficiently large number of points that both curves appear smooth; do not use identifiers such as small open circles for individual points.
   - Include another elegantly prepared figure, giving a log-log plot of the magnitude of the error in the approximation at $x = 5$ as a function of the number of terms in the approximation, m. Include several points, and identify each point with a small open circle. See large you can take m to see if you can reduce the error to machine roundoff. Take special care to see that the font size is of comparable magnitude as that of your main text, so that the figure is readable.
   - Include a copy of a) your `Fortran` programs, b) your input file, and c) your executable script, all embedded within the `verbatim` environment: e.g.

```
\begin{verbatim}
Fortran codes, input files and script file here.
\end{verbatim}
```