AME 20214
Homework 12
Due: Thursday, 11 December 2014, in class

1. (10) Use the online help to learn about the `Mathematica` function `FindRoot` and use it to identify *all* real solutions, for $x \in [-\infty, \infty]$ to the equation

$$-x^2 = 4\sin(4x - 1).$$

   To aid in determining how many solutions exist, you may wish to plot $-x^2$ and $4\sin(4x-1)$ and look for intersections. Inclusion of the actual plot is optional.

2. (40) Consider the differential equation and initial conditions for the forced mass-spring-damper problem with a time-dependent damping coefficient:

$$\frac{d^2y}{dt^2} + t^n \frac{dy}{dt} + y = 10\sin(4t), \qquad y(0) = 1, \quad \left.\frac{dy}{dt}\right|_{t=0} = 0.$$

   (a) Imposing $n = 0$ so as to simulate a time-independent damping coefficient, use the `Mathematica` function `DSolve` to get an exact solution. You may wish to use the `TeXForm` option to help in presenting the exact solution. Give two or three sentences of physical interpretation of your solution. Plot the solution in `Mathematica` for $t \in [0, 50]$.

   (b) Imposing $n = 1$ so as to simulate a time-dependent damping coefficient, use the `Mathematica` function `DSolve` to get an exact solution. You may wish to use the `TeXForm` option to help in presenting the exact solution. Give two or three sentences discussing the mathematical meaning of the functions that arise in the exact solution. Use your sleuthing skills to find information on these unusual functions. Plot the solution in `Mathematica` for $t \in [0, 50]$. Give two or three sentences comparing results with a time-dependent damping coefficient with those of a time-independent damping coefficient.

   (c) For $n = 1$, solve the differential equation and initial conditions with the `Mathematica` function `NDSolve` to get a numerical solution. Plot the solution in `Mathematica` for $t \in [0, 50]$.

Prepare your report with the LaTeX text processor. Include at least one equation, prepare beautiful figures, and include all programs that you wrote. You should be able to save your `Mathematica` plots in `.eps` form directly from `Mathematica`. Four page maximum, æsthetics: 50 points, technical content: 50 points.