1. Consider the problem of Section 19.3.2 for a forced, damped Duffing equation:

$$\begin{aligned} \frac{dy_1}{dt} &= y_2, \qquad y_1(0) = 1, \\ \frac{dy_2}{dt} &= -\beta y_1 - \delta y_2 - \alpha y_1^3 + f \cos y_3, \qquad y_2(0) = 0, \\ \frac{dy_3}{dt} &= 1, \qquad y_3(0) = 0, \end{aligned}$$

with $\alpha = 1$, $\beta = -1$, $\delta = 0.22$, and f = 0.3. Using your fourth-order Runge-Kutta method, reproduce the results of Figs. 19.3, 19.4, and 19.5. You do not need to show source code.

- 2. Now take $\alpha = 0$, $\beta = 10$, $\delta = 0.22$, and f = 0.3 and reproduce the results of Fig. 19.8. You do not need to show source code. Use your fourth order Runge-Kutta method. Compare $y_1(t = 100)$ as predicted by the first, second, and fourth order Runge-Kutta methods. Give a few sentences of physical interpretations for your results.
- 3. Following the procedure outlined in Chapter 23 of the course notes, write a Fortran 90 function subroutine to evaluate the Taylor series expans ion of $\cosh x$ about x = 0 with seven non-zero terms. Process this subroutine with the f2py Fortran to Python software, and demonstrate its execution within the Python environment much as done in Chapter 23. Include a copy of your subroutine as well as a listing of the Python operations to approximate $\cosh(3)$.

Prepare your solution with the LATEX text processor. As always, use at least one equation, use concise language, prepare beautiful figures. Take a *three page maximum*. Grading: technical merit, 50 points; æesthetics, 50 points.