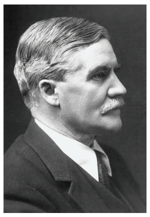
NAME: SOLUTION
AME 20231
Thermodynamics
Examination 2
Prof. J. M. Powers
31 March 2010



Happy 156th birthday, Sir Dugald Clerk,
inventor of the two-stroke engine,
b. 31 March 1854.

1. (25) A calorically imperfect ideal gas, with gas constant R and initially at P_1, T_1, V_1, fills a cylinder which is capped by a frictionless mobile piston. The gas is heated until $V = V_2$. The specific heat is given by

$$c_v(T) = c_{vo} + \alpha T,$$

where c_{vo} and α are constants. Find the final temperature and the heat transferred to the gas in terms of given quantities.

Solution

The mass of the gas, m, is

$$m = \frac{P_1 V_1}{R T_1}.$$

The process is isobaric so

$$P_2 = P_1.$$

From the ideal gas law, we have

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$$

.

$$T_2 = T_1 \underbrace{\frac{P_2}{P_1}}_{=1} \frac{V_2}{V_1}.$$

$$\boxed{T_2 = T_1 \frac{V_2}{V_1}.}$$

The work is

$$_1W_2 = \int_1^2 P dV = P_1(V_2 - V_1).$$

The change in internal energy is

$$u_2 - u_1 = \int_1^2 c_v(T) dT = \int_{T_1}^{T_2} (c_{vo} + \alpha T) dT.$$

$$u_2 - u_1 = c_{vo}(T_2 - T_1) + \alpha \left(\frac{1}{2} T^2 \right) \Big|_{T_1}^{T_2}$$

$$u_2 - u_1 = c_{vo}(T_2 - T_1) + \alpha \left(\frac{1}{2} (T_2^2 - T_1^2) \right)$$

$$u_2 - u_1 = c_{vo}(T_2 - T_1) + \frac{\alpha}{2} (T_2^2 - T_1^2).$$

$$u_2 - u_1 = c_{vo} T_1 \left(\frac{V_2}{V_1} - 1 \right) + \frac{\alpha T_1^2}{2} \left(\left(\frac{V_2}{V_1} \right)^2 - 1 \right).$$

So

$$U_2 - U_1 = \frac{P_1 V_1}{R T_1}\left(c_{vo} T_1 \left(\frac{V_2}{V_1} - 1\right) + \frac{\alpha T_1^2}{2}\left(\left(\frac{V_2}{V_1}\right)^2 - 1\right)\right).$$

$$U_2 - U_1 = \frac{P_1 V_1}{R}\left(c_{vo} \left(\frac{V_2}{V_1} - 1\right) + \frac{\alpha T_1}{2}\left(\left(\frac{V_2}{V_1}\right)^2 - 1\right)\right).$$

From the first law, $U_2 - U_1 = {}_1Q_2 - {}_1W_2$, so

$${}_1Q_2 = U_2 - U_1 + {}_1W_2.$$

$${}_1Q_2 = \frac{P_1 V_1}{R}\left(c_{vo} \left(\frac{V_2}{V_1} - 1\right) + \frac{\alpha T_1}{2}\left(\left(\frac{V_2}{V_1}\right)^2 - 1\right)\right) + P_1 V_1 \left(\frac{V_2}{V_1} - 1\right).$$

$$\boxed{{}_1Q_2 = P_1 V_1 \left(\frac{V_2}{V_1} - 1\right)\left(\frac{c_{vo}}{R} + 1 + \frac{\alpha T_1}{2R}\left(\frac{V_2}{V_1} + 1\right)\right).}$$

Overall performance on this problem was not very good. The biggest problem was simple; many students forgot calculate the total energy U via multiplication by the system mass. Many students also had difficulty in integrating the specific heat to get the internal energy. Also, many students neglected to account for the work, which influenced the heat transfer via the first law. Lastly, a few students failed to realize that the process was isobaric.

2. (25) A sphere of aluminum with radius of 0.01 m is initially at 1500 K. It is suddenly immersed in a very large tub of water at 300 K. The heat transfer coefficient is $\mathsf{h} = 10\ kW/m^2/K$. Assuming the sphere has a spatially uniform temperature and constant material properties, find the time when the sphere's temperature is 400 K.

Solution

The first law holds that

$$\frac{dE}{dt} = \dot{Q} - \dot{W}.$$

There is no work for this problem, so $\dot{W} = 0$. Thus

$$\frac{dE}{dt} = \dot{Q}.$$

Now we neglect kinetic and potential energy changes of the aluminum, so

$$E = mcT + E_o.$$

With m, c and E_o constant, we thus have

$$\frac{dE}{dt} = mc\frac{dT}{dt}.$$

Now we know that

$$\dot{Q} = \mathsf{h}A(T_\infty - T).$$

So the first law reduces to

$$mc\frac{dT}{dt} = \mathsf{h}A(T_\infty - T).$$

$$\frac{dT}{dt} = \frac{\mathsf{h}A}{mc}(T_\infty - T).$$

$$\frac{dT}{dt} = \frac{\mathsf{h}A}{\rho V c}(T_\infty - T).$$

$$\frac{dT}{dt} = \frac{\mathsf{h}(4\pi r^2)}{\rho \frac{4}{3}\pi r^3 c}(T_\infty - T).$$

$$\frac{dT}{dt} = \frac{3\mathsf{h}}{\rho r c}(T_\infty - T).$$

$$\frac{dT}{T_\infty - T} = \frac{3\mathsf{h}}{\rho r c}dt.$$

$$-\ln(T_\infty - T) = \frac{3\mathsf{h}}{\rho r c}t + C.$$

$$T_\infty - T = C' \exp\left(-\frac{3\mathsf{h}}{\rho r c}t\right).$$

At $t = 0$, we have $T = T_o$, so

$$T_\infty - T_o = C'.$$

$$T_\infty - T = (T_\infty - T_o)\exp\left(-\frac{3\mathsf{h}}{\rho r c}t\right).$$

$$\frac{T_\infty - T}{T_\infty - T_o} = \exp\left(-\frac{3\mathsf{h}}{\rho r c}t\right).$$

$$\ln\frac{T_\infty - T}{T_\infty - T_o} = -\frac{3\mathsf{h}}{\rho r c}t.$$

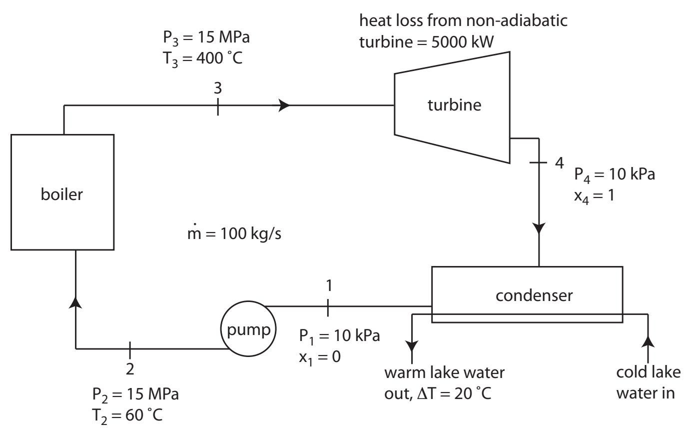
$$t = -\frac{\rho r c}{3\mathsf{h}}\ln\frac{T_\infty - T}{T_\infty - T_o}.$$

From Table A.3, we find for aluminum $\rho = 2700\ kg/m^3$ and $c = 0.90\ kJ/kg/K$. So we look for the time when $T = 400\ K$ and get

$$t = -\frac{\left(2700\ \frac{kg}{m^3}\right)(0.01\ m^2)\left(0.90\ \frac{kJ}{kg\ K}\right)}{3\left(10\ \frac{kW}{m^2\ K}\right)}\ln\frac{(300\ K) - (400\ K)}{(300\ K) - (1500\ K)}.$$

$$\boxed{t = 2.01\ s.}$$

3. (50) Consider the Rankine cycle below. Find



(a) the heat transfer rate to the boiler (kW),

(b) the power output of the turbine (kW),

(c) the overall thermal efficiency,

(d) the thermal efficiency of a Carnot cycle operating between the same temperature limits,

(e) an accurate sketch of the cycle on a $T - s$ diagram,

(f) the mass flow rate of external lake cooling water to exchange heat with the condenser if the lake cooling water temperature rise is designed to be $20\ ^\circ C$.

Solution

At state 1, we have two properties. From the tables, we learn

$$h_1 = 191.81\ \frac{kJ}{kg}, \qquad s_1 = 0.6492\ \frac{kJ}{kg\ K}, \qquad T_1 = 45.81\ ^\circ C.$$

At state 2, the compressed liquid tables give

$$h_2 = 263.65\ \frac{kJ}{kg}, \qquad s_2 = 0.8231\ \frac{kJ}{kg\ K}.$$

After the boiler, we know two properties and find

$$h_3 = 2975.44\ \frac{kJ}{kg}, \qquad s_3 = 5.8810\ \frac{kJ}{kg\ K}.$$

After the turbine, we know two properties and find

$$h_4 = 2584.63\ \frac{kJ}{kg}, \qquad s_4 = 8.1501\ \frac{kJ}{kg\ K}.$$

So the heat transfer rate to the boiler is

$${}_2\dot{Q}_3 = \dot{m}(h_3 - h_2) = \left(100\ \frac{kg}{s}\right)\left(\left(2975.44\ \frac{kJ}{kg}\right) - \left(263.65\ \frac{kJ}{kg}\right)\right) \boxed{= 271179\ kW.}$$

For the turbine we have $dE/dt = {}_3\dot{Q}_4 - {}_3\dot{W}_4 + \dot{m}(h_3 - h_4)$. Now $dE/dt = 0$, so the power output of the turbine is

$${}_3\dot{W}_4 = \dot{m}(h_3 - h_4) - {}_3\dot{Q}_4 = \left(100\ \frac{kg}{s}\right)\left(\left(2975.44\ \frac{kJ}{kg}\right) - \left(2584.63\ \frac{kJ}{kg}\right)\right) - (5000\ kW)$$

$$\boxed{{}_3\dot{W}_4 = 34081\ kW.}$$

The power requirement of the pump, assumed to be adiabatic, is

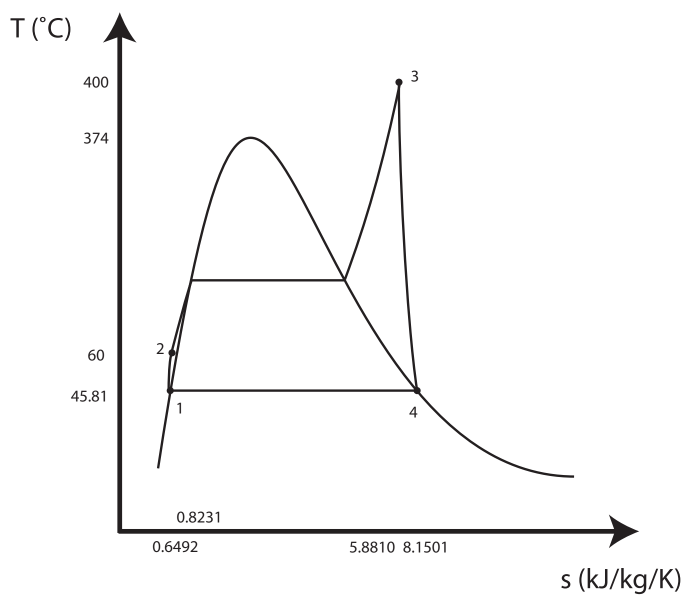
$${}_1\dot{W}_2 = \dot{m}(h_2 - h_1) = \left(100\ \frac{kg}{s}\right)\left(\left(263.65\ \frac{kJ}{kg}\right) - \left(191.81\ \frac{kJ}{kg}\right)\right) = 7184\ kW.$$

The cycle efficiency is

$$\boxed{\eta = \frac{\dot{W}_{net}}{{}_2\dot{Q}_3} = \frac{(34081\ kW) - (7184\ kW)}{271179\ kW} = 0.099185.}$$

The Carnot efficiency for an engine operating between the same temperature limits is

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{45.81 + 273.15}{400 + 273.15} \boxed{= 0.526.}$$



The heat loss in the condenser is

$${}_4\dot{Q}_1 = \dot{m}(h_4 - h_1) = \left(100\ \frac{kg}{s}\right)\left(\left(2584.63\ \frac{kJ}{kg}\right) - \left(191.81\ \frac{kJ}{kg}\right)\right) = 239282\ kW.$$

Now for the lake water we need

$$ {}_4\dot{Q}_1 = \dot{m}_w c(\Delta T). $$

$$ \dot{m}_w = \frac{{}_4\dot{Q}_1}{c\Delta T}. $$

$$ \dot{m}_w = \frac{239282\ kW}{\left(4.186\ \frac{kJ}{kg\ K}\right)(20\ K)} = 2858\ \frac{kg}{s}. $$

Overall performance on this problem was good but not outstanding. Some had fundamental problems identifying the numerical values of the state variables; this was intended to be an easy part of this problem as there was no interpolation involved. A few failed to realize that state 2 was a compressed liquid. Most got the turbine power; a few forgot to multiply by the mass flow rate. Many did not correctly account for the heat transfer in the turbine and had a sign error. Most did not account for the pump work in calculation of thermal efficiency. Surprisingly many people used Celsius and not Kelvin to calculate the ideal efficiency. A few calculated the efficiency for a heat pump, not a power cycle. $T-s$ diagrams were generally bad, with a few very good. Some problems with $T-s$ diagram include: 1) Not showing that the entropy increased in the pump, 2) Not showing that the entropy increased in the turbine, 3) Not showing the vapor dome, 4) Not showing the turbine temperature was greater than the critical temperature, 5) Not showing the isobar was an isotherm under the vapor dome, 6) Not placing states 1 and 4 at the edges of the vapor dome. Most people got the cooling water flow rate correct.