AME 20231 Homework Solutions[1]
Spring 2012

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Contents

[1] Solutions adapted from Borgnakke, Sonntag (2008) "Solutions Manual," *Fundamentals of Thermodynamics, 7th Edition*, and previous *AME 20231 Homework Solutions* documents.

Homework 1

1. <u>2.41</u> The hydraulic lift in an auto-repair shop has a cylinder diameter of 0.2 m. To what pressure should the hydraulic fluid be pumped to lift 40 kg of piston/arms and 700 kg of a car?
<u>Given:</u> $d = 0.3$ m, $m_{arms} = 40$ kg, $m_{car} = 700$ kg
<u>Assumptions:</u> $P_{atm} = 101$ kPa
<u>Find:</u> P
Gravity force acting on the mass, assuming the y-direction is on the axis of the piston:

$$\sum F_y = ma \rightarrow F = (m_{arms} + m_{car})(9.81 \text{ m/s}^2) = 7256.9 \text{ N}$$

Now balance this force with the pressure force:

$$F = 7256.9 \text{ N} = (P - P_{atm})(A) \rightarrow P = P_{atm} + F/A$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (0.3 \text{ m})^2}{4} = 0.0707 \text{ m}^2$$

$$P = 101 \text{ kPa} + \frac{7256.9 \text{ N}}{0.0707 \text{ m}^2} = \boxed{204 \text{ kPa} = P.}$$

2. <u>2.46</u> A piston/cylinder with cross sectional area of 0.01 m^2 has a piston mass of 200 kg resting on the stops, as shown in Fig. P2.46. With an outside atmospheric pressure of 100 kPa, what should the water pressure be to lift the piston?
<u>Given:</u> $m = 200$ kg, $A_c = 0.01$ m^2, $P_{atm} = 100$ kPa
<u>Assumptions:</u>
<u>Find:</u> P
The force acting down on the piston comes from gravitation and the outside atmospheric pressure acting over the top surface. Force balance:

$$\sum F = 0 \rightarrow PA_c = mg + P_{atm}A_c$$

Now solve for P:

$$\begin{aligned} P &= P_{atm} + \frac{mg}{A_c} = 100 \cdot 10^3 \text{ Pa} + \frac{(200 \text{ kg})(9.81 \text{ m/s}^2)}{0.01 \text{ m}^2} \\ &= 100 \text{ kPa} + 196.2 \text{ kPa} = \boxed{296.2 \text{ kPa} = P.} \end{aligned}$$

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AME 20231
20 January 2012

This is a sample file in the text formatter LaTeX. I require you to use it for the following reasons:

- It produces the best output of text, figures, and equations of any program I've seen.
- It is machine-independent. It runs on Linux, Macintosh (see TeXShop), and Windows (see MiKTeX) machines. You can e-mail ASCII versions of most relevant files.
- It is the tool of choice for many research scientists and engineers. Many journals accept LaTeX submissions, and many books are written in LaTeX.

Some basic instructions are given below. Put your text in here. You can be a little sloppy about spacing. It adjusts the text to look good. You can make the text smaller. You can make the text tiny.

Skip a line for a new paragraph. You can use italics (*e.g. Thermodynamics is everywhere*) or **bold**. Greek letters are a snap: Ψ, ψ, Φ, ϕ. Equations within text are easy— A well known Maxwell thermodynamic relation is $\frac{\partial T}{\partial p}\Big|_s = \frac{\partial v}{\partial s}\Big|_p$. You can also set aside equations like so:

$$du = Tds - pdv, \qquad \text{first law} \tag{1}$$

$$ds \geq \frac{dq}{T}. \qquad \text{second law} \tag{2}$$

Eq. (2) is the second law. References [2] are available. If you have an postscript file, say sample.figure.eps, in the same local directory, you can insert the file as a figure. Figure 1, below, plots an isotherm for air modeled as an ideal gas.

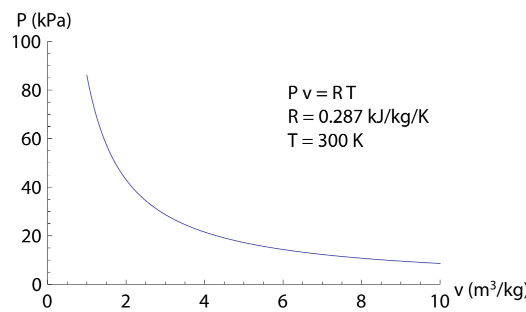


Figure 1: Sample figure plotting $T = 300\ K$ isotherm for air when modeled as an ideal gas.

Running LaTeX

You can create a LaTeX file with any text editor (vi, emacs, gedit, etc.). To get a document, you need to run the LaTeX application on the text file. The text file must have the suffix "`.tex`" On a Linux cluster machine, this is done via the command

```
pdflatex file.tex
```

This generates three files: `file.pdf`, `file.aux`, and `file.log`. The most important is `file.pdf`. This file can be viewed by any application that accepts `.pdf` files, such as Adobe Acrobat reader.

The `.tex` file must have a closing statement as below.

[2] Lamport, L., 1986, *LaTeX: User's Guide & Reference Manual*, Addison-Wesley: Reading, Massachusetts.

Quiz 1

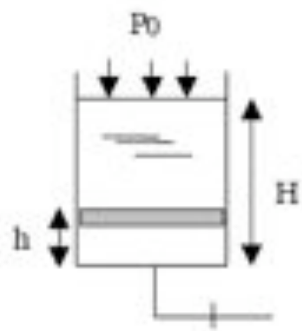
1. Steam turbines, refrigerators, steam power plants, fuel cells, etc.
2. False, *coal, natural gas, nuclear, etc.*
3. True or false (An air separation plant separates air into its various components, which in addition to oxygen and nitrogen include argon and other gases.)

Homework 2

2.56

Liquid water with density ρ is filled on top of a thin piston in a cylinder with cross-sectional area A and total height H, as shown in Fig. P2.56. Air is let in under the piston so it pushes up, spilling the water over the edge. Derive the formula for the air pressure as a function of piston elevation from the bottom, h.

Solution:

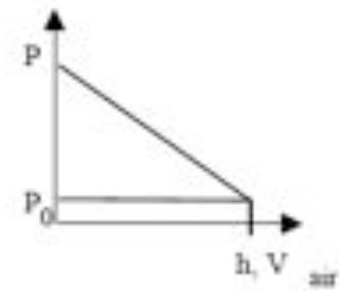


Force balance
Piston: $F\uparrow = F\downarrow$

$$PA = P_0A + m_{H_2O}g$$

$$P = P_0 + m_{H_2O}g/A$$

$$\mathbf{P = P_0 + (H - h)\rho g}$$



2. 2.71 A U-tube manometer filled with water, density 1000 kg/m^3, shows a height difference of 25 cm. What is the gauge pressure? If the right branch is tilted to make an angle of 30 with the horizontal, as shown in Fig. P2.71, what should the length of the column in the tilted tube be relative to the U-tube?
Given: $\rho = 1000$ kg/m^3, $h = 0.25$ m, $\theta = 25\ ^\circ$
Assumptions: $g = 9.81$ m/s^2
Find: P, l

$$\begin{aligned} P &= F/A = mg/A = V\rho g/A = h\rho g \\ &= (0.25\text{ m})(1000\text{ kg/m}^3)(9.81\text{ m/s}^2) \\ &= 2452.5\text{ Pa} \\ &= \boxed{2.45\text{ kPa} = P} \end{aligned}$$

$$h = (l)(\sin 25^\circ)$$

$$l = h/\sin 25^\circ = \boxed{59\text{ cm} = l}$$

2.79

The density of mercury changes approximately linearly with temperature as

$$\rho_{Hg} = 13595 - 2.5\,T \ \text{kg/m}^3 \qquad T \ \text{in Celsius}$$

so the same pressure difference will result in a manometer reading that is influenced by temperature. If a pressure difference of 100 kPa is measured in the summer at 35°C and in the winter at −15°C, what is the difference in column height between the two measurements?

Solution:

The manometer reading h relates to the pressure difference as

$$\Delta P = \rho\, L\, g \quad \Rightarrow \quad L = \frac{\Delta P}{\rho g}$$

The manometer fluid density from the given formula gives

$$\rho_{su} = 13595 - 2.5 \times 35 = 13507.5 \ \text{kg/m}^3$$

$$\rho_{W} = 13595 - 2.5 \times (-15) = 13632.5 \ \text{kg/m}^3$$

The two different heights that we will measure become

$$L_{su} = \frac{100 \times 10^3}{13507.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\ \text{m/s}^2} = 0.7549 \ \text{m}$$

$$L_{W} = \frac{100 \times 10^3}{13632.5 \times 9.807} \frac{\text{kPa (Pa/kPa)}}{(\text{kg/m}^3)\ \text{m/s}^2} = 0.7480 \ \text{m}$$

$$\Delta L = L_{su} - L_{W} = \mathbf{0.0069\ m = 6.9\ mm}$$

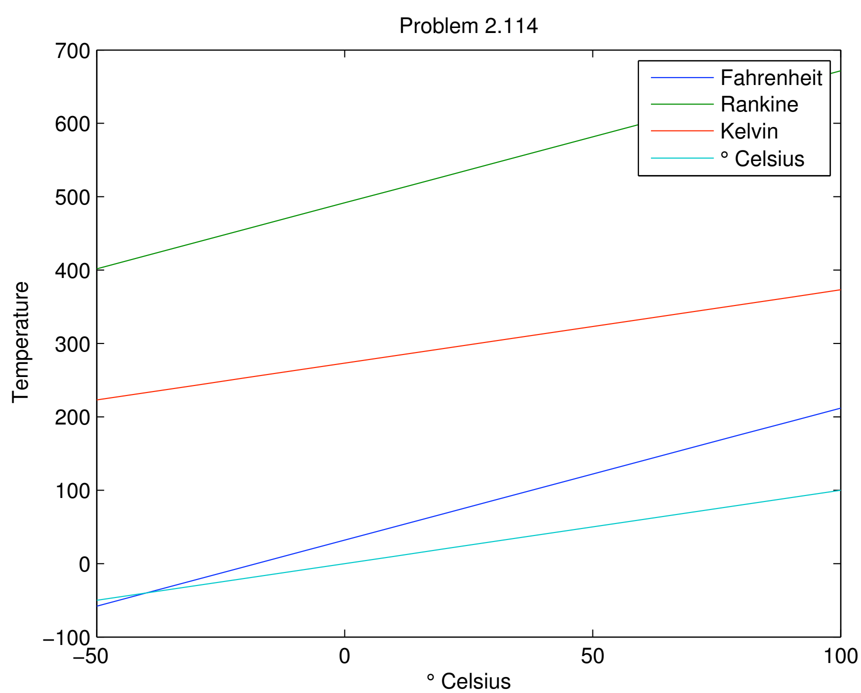
2.102E

A powerplant that separates carbon-dioxide from the exhaust gases compresses it to a density of 8 lbm/ft^3 and stores it in an un-minable coal seam with a porous volume of 3 500 000 ft^3. Find the mass they can store.

Solution:

$$m = \rho V = 8 \text{ lbm/ft}^3 \times 3\,500\,000 \text{ ft}^3 = 2.8 \times 10^7 \text{ lbm}$$

Just to put this in perspective a power plant that generates 2000 MW by burning coal would make about 20 million tons of carbon-dioxide a year. That is 2000 times the above mass so it is nearly impossible to store all the carbon-dioxide being produced.



Quiz 2

1. Water has a density of 997 $\mathrm{kg/m^3}$. Rationally estimate the pressure difference between the difference between the surface and the bottom of a typical Olympic swimming pool located on Earth. Make any necessary assumptions.

Answers will vary. Assuming no atmospheric pressure difference between the surface and bottom of the pool and a depth of 3 m, the pressure difference ΔP is

$$\begin{aligned}\Delta P &= \rho g h = (997\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})(3\ \mathrm{m}) \\ &= \boxed{29\ \mathrm{kPa} = \Delta P.}\end{aligned}$$

It was acceptable to leave the height h in the solution as a variable. Atmospheric pressure acts on the surface *and* the bottom of the pool so it should not factor into the pressure difference of the pool.

Homework 3

1. <u>3.72</u> A spherical helium balloon of 11 m in diameter is at ambient T and P, 15 °C and 100 kPa. How much helium does it contain? It can lift a total mass that equals the mass of displaced atmospheric air. How much mass of the balloon fabric and cage can then be lifted?

<u>Given:</u> $d = 11$ m, $T = 15$ ° C, $P = 100$, kPa
<u>Assumptions:</u>
<u>Find:</u> m_{He}, m_{lift}
We need to find the masses and the balloon volume:

$$V = \frac{\pi}{6} d^3 = \frac{\pi}{6}(11 \text{ m})^3 = 696.9 \text{ m}^3$$

$$m_{He} = \rho V = \frac{V}{v} = \frac{(100 \text{ kPa})(696.9 \text{ m}^3)}{(2.0771 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(288 \text{ K})} = \boxed{116.5 \text{ kg} = m_{He}}$$

$$m_{air} = \frac{(100 \text{ kPa})(696.9 \text{ m}^3)}{(0.287 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(288 \text{ K})} = 843 \text{ kg}$$

$$m_{lift} = m_{air} - m_{He} = \boxed{726.5 \text{ kg} = m_{lift}}$$

3.108

Give the phase and the missing properties of *P*, *T*, *v* and *x*.

Solution:

a. R-410a $T = 10^\circ C$ $\quad v = 0.01\ m^3/kg$

Table B.4.1 $\quad v < v_g = 0.02383\ m^3/kg$

sat. liquid + vapor. $P = P_{sat} = 1085.7$ kPa,

$x = (v - v_f)/v_{fg} = (0.01 - 0.000886)/0.02295 =$ **0.397**

b. H_2O $\quad T = 350^\circ C$ $\quad v = 0.2\ m^3/kg$

Table B.1.1 at given T: $\quad v > v_g = 0.00881\ m^3/kg$

sup. vapor $\quad P \cong 1.40$ MPa, x = undefined

c. R-410a $T = -5\,^\circ C$ $\quad P = 600$ kPa

sup. vapor ($P < P_g = 678.9$ kPa at $-5^\circ C$)

Table B.4.2:

$v = 0.04351\ m^3/kg$ at $-8.67^\circ C$

$v = 0.04595\ m^3/kg$ at $0^\circ C$

=> $\quad v = 0.04454\ m^3/kg$ at $-5^\circ C$

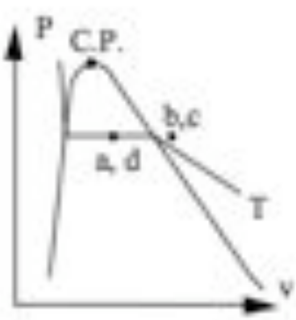
d. R-134a $P = 294$ kPa, $v = 0.05\ m^3/kg$

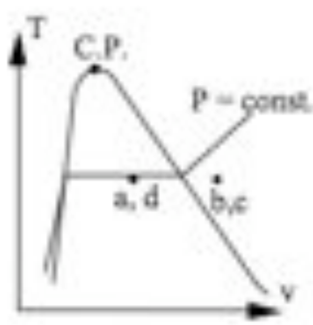
Table B.5.1: $v < v_g = 0.06919\ m^3/kg$

two-phase $\quad T = T_{sat} = 0^\circ C$

$x = (v - v_f)/v_{fg} = (0.05 - 0.000773)/0.06842 = 0.7195$

States shown are placed relative to the two-phase region, not to each other.





3. <u>3.122</u> A cylinder has a thick piston initially held by a pin as shown in Fig. P3.122. The cylinder contains carbon dioxide at 200 kPa and ambient temperature of 290 K. The metal piston has a density of 8000 kg/m^3 and the atmospheric pressure is 101 kPa. The pin is now removed, allowing the piston to move and after a while the gas returns to ambient temperature. Is the piston against the stops?
<u>Given:</u> $\rho_p = 8000$ kg/m^3, $P_1 = 200$ kPa, $P_{atm} = 101$ kPa, $T = 290$ K
<u>Assumptions:</u>
<u>Find:</u> P_2
Do a force balance on piston determines equilibrium float pressure. Piston:

$$m_p = (A_p)(l)(\rho)$$

$$P_{ext} = P_{atm} + \frac{m_p g}{A_p} = 101 \text{ kPa} + \frac{(A_p)(0.1 \text{ m})(8000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)}{(A_p)(1000)} = 108.8 \text{ kPa}$$

The pin is released, and since $P_1 > P_{ext}$, the piston moves up. $T_2 = T_0$, so if the piston stops, then $V_2 = V_1 \times H_2/H_1 = V_1 \times 150/100$. Using an ideal gas model with $T_2 = T_0$ gives

$$P_2 = (P_1)(V_1/V_2) = (200)(100/150) = 133 \text{ kPa} > P_{ext} \rightarrow \boxed{P_2 = 133 \text{ kPa}}$$

Therefore, the piston is at the stops for the ideal gas model.

Now for the tabulated solution:
To find P_2, we must do some extrapolation because Table B.3.2 does not list 200 kPa. We first interpolate to find the specific volume at $T_1 = 290$ K $= 16.85$ °C at 400 kPa and 800 kPa, and then extrapolate using those points to find the specific volume at 200 kPa, v_1. Since $v_1 = \dfrac{V_1}{m_{CO_2}}$ and the area of the piston is $A_p = \pi \dfrac{(100 \text{ mm})^2}{4} = 0.00785$ m^2, the mass of carbon dioxide in the cylinder is

$$m_{CO_2} = \frac{V_1}{v_1} = \frac{(A_p)(100 \text{ mm})}{0.1682 \text{ m}^3/\text{kg}} = \frac{(0.00785 \text{ m}^2)(0.1 \text{ m})}{0.1682 \text{ m}^3/\text{kg}} = \frac{0.000785 \text{ m}^3}{0.1682 \text{ m}^3/\text{kg}} = 0.00467 \text{ kg}$$

Since the cylinder is closed, the mass of the carbon dioxide does not change, but the volume does, so we will assume the piston is at the stops and $V_2 = 0.00118$ m^3, so $v_2 = \dfrac{V_2}{m_{CO_2}} =$ 0.2523 m^3/kg. Now we have two state variables (T_2 and v_2), so let's go back to Table B.3.2 and find a third state variable, P_2. Interpolating between 400 kPa and 800 kPa gives $P_2 = -291$ kPa, which means our original assumption that the piston is against the stops was incorrect. A similar iterative approach could be used to find P_2 in the tables. The piston is not against the stops. This problem was difficult, so it was only worth two points. You received two points for a reasonable attempt with some calculations, one point for just writing something, and no points for not attempting it. +2 indicates you were awarded two points for part (b), and they were not bonus points.

4. <u>3.166E</u> A 36 ft^3 rigid tank has air at 225 psia and ambient 600 R connected by a valve to a piston cylinder. The piston of area 1 ft^2 requires 40 psia below it to float, Fig. P3.99. The valve is opened and the piston moves slowly 7 ft up and the valve is closed. During the process air temperature remains at 600 R. What is the final pressure in the tank?

Given: $V_A = 36$ ft^3, $P_A =$ psia, $T = 600$ R, isothermal process
Assumptions:
Find: P_{A2}

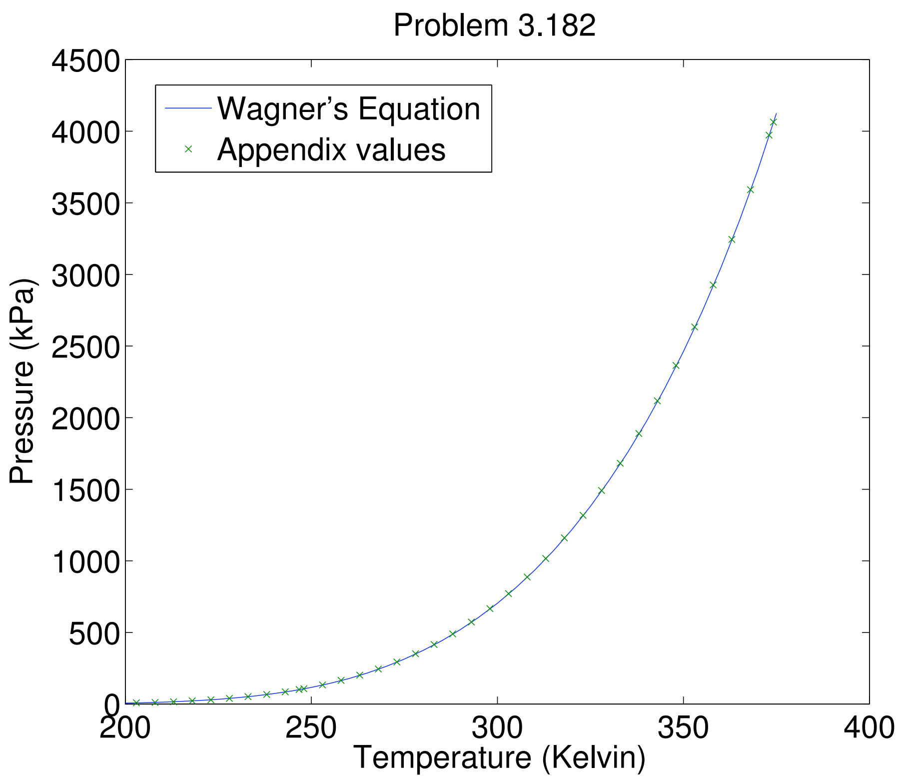
$$m_A = \frac{P_A V_A}{RT} = \frac{(225)(36)(144)}{(53.34)(600)} = 36.4 \text{ lbm}$$

Now find the change in mass during the process:

$$m_{B2} - m_{B1} = \frac{\Delta V_A}{v_B} = \frac{\Delta V_B P_B}{RT} = \frac{(1)(7)(40)(144)}{(53.34)(600)} = 1.26 \text{ lbm}$$

$$M_{A2} = m_A - (m_{B2} - m_{B1}) = 36.4 - 1.26 = 35.1 \text{ lbm}$$

$$P_{A2} = \frac{m_{A2} RT}{V_A} = \frac{(35.1)(53.34)(600)}{(36)(144)} = \boxed{217 \text{ psia} = P_{A2}}$$

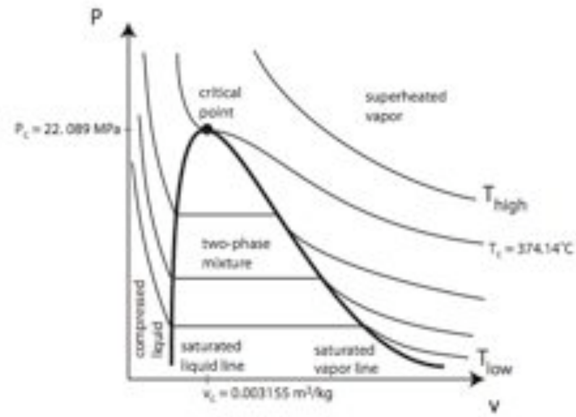


Quiz 3

1. A fixed mass of water exists in a fixed volume at the saturated liquid state with $v = v_f$. Heat is added to the water isochorically. Which of the following are possible final states for the water?

Compressed liquid and supercritical liquid. As seen in the $P - v$ diagram, Figure 3.6 in the notes, the saturated liquid state is the line on the vapor dome to the left of the critical point. As you add heat isochorically, the pressure increases but the volume stays the same. Thus the only possible final states for the water are compressed liquid and supercritical liquid.

You received 2 points for putting your name on the quiz, 4 points for each correct response, and lost 1 point for each incorrect response.



Homework 4

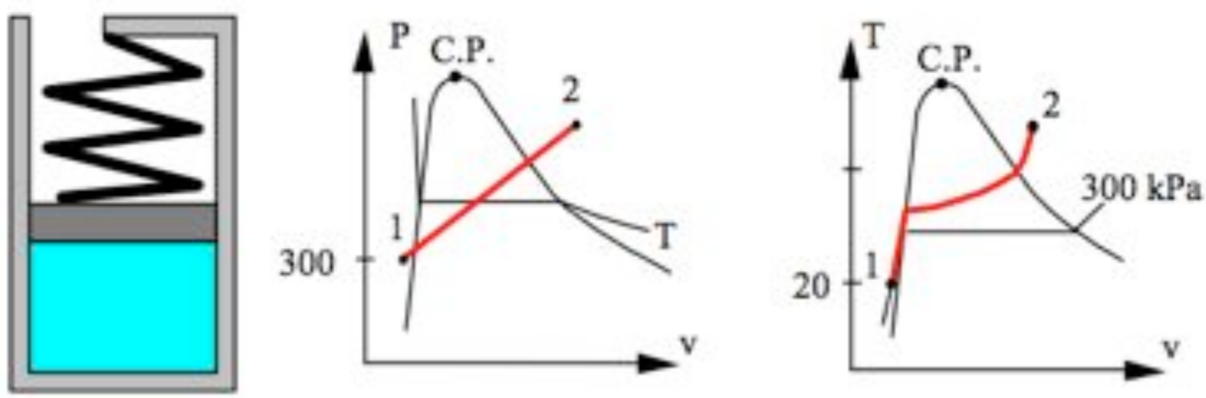
1. 4.38 A piston cylinder contains 1 kg of liquid water at 25 °C and 300 kPa, as shown in Fig. P4.38. There is a linear spring mounted on the piston such that when the water is heated the pressure reaches 3 MPa with a volume of 0.1 m^3.
(a) Find the final temperature
(b) Plot the process in a P-v diagram.
(c) Find the work in the process.
Given:
Assumptions:
Find: T_2, ${}_1W_2$
Solution:
Take CV as the water. This is a constant mass:

$$m_2 = m_1 = m$$

State 1: Compressed liquid, take saturated liquid at same temperature.
Table B.1.1: $v_1 = v_f(25) = 0.001003$ m^3/kg
State 2: $v_2 = V_2/m = 0.1/1 = 0.1$ m^3/kg and P= 3000 kPa from B.1.3 → Superheated vapor, close to $T = 400$°C, Interpolate: $\boxed{T_2 = 404°\text{C}}$
Work is done while piston moves at linearly varying pressure, so we get:

$$\begin{aligned} {}_1W_2 &= \int PdV = P_{avg}(V_2 - V_1) = 1/2(P_1 + P_2)(V_2 - V_1) \\ &= 0.5(300 + 3000)\text{ kPa }(0.1 - 0.001)\text{ m}^3 = \boxed{163.4\text{ kJ} = {}_1W_2} \end{aligned}$$

See the $P - v$ diagram below:



2. 4.64 A piston/cylinder arrangement shown in Fig. P4.64 initially contains air at 150 kPa, 400°C. The setup is allowed to cool to the ambient temperature of 25°C. (a) Is the piston resting on the stops in the final state? What is the final pressure in the cylinder? (b) What is the specific work done by the air during this process?
Given:
Assumptions: For all states air behave as an ideal gas.
Find:

Solution: State 1: $P_1 = 150$ kPa, $T_1 = 400°\text{C} = 673.2$ K
State 2: $T_2 = T_0 = 20°\text{C} = 293.2$ K
(a) If piston at stops at 2, V2 = V1/2 and pressure less than Plift = P1

$$\rightarrow P_2 = P_1 \times \frac{V_1}{V_2} \times T_2 T_1 = (150 \text{ kPa})(2)(\frac{298.2}{673.2}) = \boxed{132.9 \text{ kPa} = P_2} < P_1$$

Since $P_2 < P_1$, $\boxed{\text{the piston is resting on the stops.}}$
(b) Work done while piston is moving at constant $P_{ext} = P_1$.

$$_1W_2 = \int P_{ext} dV = P_1(V_2 - V_1)$$

Since $V_2 = \frac{V_1}{2} = \frac{1}{2}\frac{mRT_1}{P_1}$,

$$_1w_2 =_1 W_2/m = RT_1(1/2 - 1) = (-0.287\frac{\text{kJ}}{\text{kg} \cdot \text{K}})(673.2 \text{ K})(-1/2) = \boxed{-96.6 \text{ kJ/kg} = {}_1w_2}$$

3. <u>4.124</u> A cylinder fitted with a piston contains propane gas at 100 kPa, 300 K with a volume of 0.1 m^3. The gas is now slowly compressed according to the relation PV1.1 = constant to a final temperature of 340 K. Justify the use of the ideal gas model. Find the final pressure and the work done during the process.
<u>Given:</u>
<u>Assumptions:</u>
<u>Find:</u>
Solution:
The process equation and T determines state 2. Use ideal gas law to say

$$P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} = 100 \left(\frac{340}{300}\right)^{\frac{1.1}{1.0}} = \boxed{396 \text{ kPa} = P_2}$$

$$V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}} = 0.1 \left(\frac{100}{396}\right)^{\frac{1}{1.1}} = 0.0286 \text{ m}^3$$

For propane Table A.2: $T_c = 370$ K, $P_c = 4260$ kPa, Figure D.1 gives Z.

$$T_{r1} = 0.81, P_{r1} = 0.023 \rightarrow Z_1 = 0.98$$

$$T_{r2} = 0.92, P_{r2} = 0.093 \rightarrow Z_2 = 0.95$$

Ideal gas model OK for both states, minor corrections could be used. The work is integrated to give Eq. 4.4

$$\begin{aligned} _1W_2 &= \int PdV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{(396 \times 0.0286) - (100 \times 0.1)}{1 - 1.1} \text{ kPa m}^3 \\ &= \boxed{-13.3 \text{ kJ} = {}_1W_2} \end{aligned}$$

4.150E

A piston/cylinder has 2 lbm of R-134a at state 1 with 200 F, 90 lbf/in.^{2}, and is then brought to saturated vapor, state 2, by cooling while the piston is locked with a pin. Now the piston is balanced with an additional constant force and the pin is removed. The cooling continues to a state 3 where the R-134a is saturated liquid. Show the processes in a P-V diagram and find the work in each of the two steps, 1 to 2 and 2 to 3.

Solution :

C.V. R-134a This is a control mass.

Properties from table F.10.1 and 10.2

State 1: (T,P) => $v = 0.7239$ ft^3/lbm

State 2 given by fixed volume and $x_2 = 1.0$

State 2: $v_2 = v_1 = v_g$ => $\mathbf{{}_1W_2 = 0}$

$$T_2 = 50 + 10 \times \frac{0.7239 - 0.7921}{0.6632 - 0.7921} = 55.3 \text{ F}$$

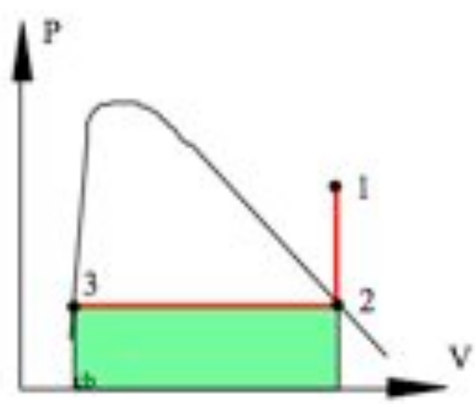
$$P_2 = 60.311 + (72.271 - 60.311) \times 0.5291 = 66.64 \text{ psia}$$

State 3 reached at constant P (F = constant) state 3: $P_3 = P_2$ and

$v_3 = v_f = 0.01271 + (0.01291 - 0.01271) \times 0.5291 = 0.01282$ ft^3/lbm

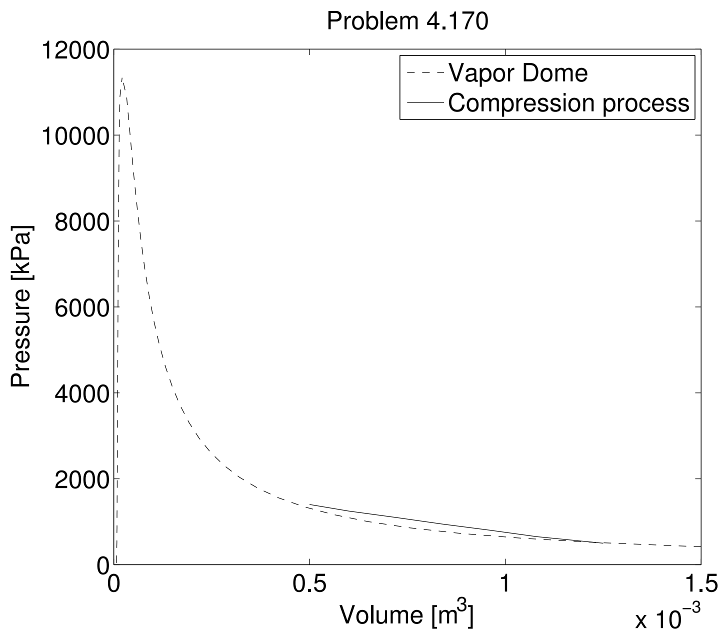
$${}_1W_3 = {}_1W_2 + {}_2W_3 = 0 + {}_2W_3 = \int P\, dV = P(V_3 - V_2) = mP(v_3 - v_2)$$

$$= 2\times 66.64\,(0.01282 - 0.7239)\,\frac{144}{778} = \mathbf{-17.54\ Btu}$$



5. <u>4.170</u> The data from Table B.2.1 was used to create the vapor dome seen in Figure 2. The plot was scaled to make the compression process and vapor dome more visible. Using trapezoidal numerical integration in MATLAB, the work was found to be $\boxed{W = 0.6861 \text{ kJ}}$. The mass of the ammonia was found (using the ideal gas law and initial conditions) to be 0.00507 kg and this was multiplied by the specific volume to find the volume ($V = mv$).

Figure 2: Problem 4.170



Quiz 4

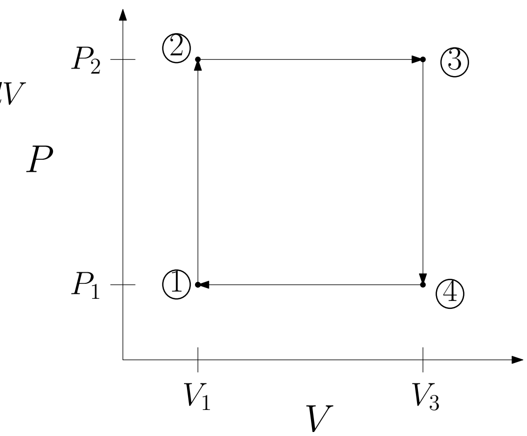
A gas exists with volume V_1 and pressure P_1. It is compressed isochorically to P_2, expands isobarically to V_3, decompresses isochorically back to P_1, and compresses isobarically back to V_1. Sketch on a $P-V$ plane diagram and find the net work.

The process is seen in the figure to the right.
For the net work:

$$\begin{aligned} W_{net} &= \oint P dV \\ &= \int_1^2 P dV + \int_2^3 P dV + \int_3^4 P dV + \int_4^1 P dV \end{aligned}$$

Now because there paths are isochoric, $\int_1^2 P dV = \int_3^4 P dV = 0$. *Thus the net work becomes*

$$\begin{aligned} W_{net} &= \int_2^3 P dV + \int_4^1 P dV \\ &= P_2 \int_2^3 dV + P_1 \int_4^1 dV \\ &= \boxed{P_2(V_3 - V_1) + P_1(V_1 - V_3) = W_{net}} \\ &= (P_2 - P_1)(V_3 - V_1) \end{aligned}$$



Quiz 5

1. We are given that

$$u(T, v) = a_1 T + a_2 T^2 + a_3 v + a_4 v^2.$$

Find c_v.

The specific heat at constant volume, c_v, is defined as follows:

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

so the given specific energy equation becomes

$$\left(\frac{\partial u}{\partial T}\right)_v = \boxed{a_1 + 2a_2 T = c_v}.$$

Homework 5

5.27

Find the phase and the missing properties of P, T, v, u and x

a. Water at 5000 kPa, u = 3000 kJ/kg

b. Ammonia at 50^oC, v = 0.08506 m^3/kg

c. Ammonia at 28^oC, 1200 kPa

d. R-134a at 20^oC, u = 350 kJ/kg

a) Check in Table B.1.2 at 5000 kPa: $u > u_g = 2597$ kJ/kg

Goto B.1.3 it is found very close to **450^oC**, x = undefined, v = 0.0633 m^3/kg

b) Table B.2.1 at 50^oC: $v > v_g = 0.06337\ m^3/kg$, so superheated vapor

Table B.2.2: close to 1600 kPa, u = 1364.9 kJ/kg, x = undefined

c) Table B.2.1 between 25 and 30^oC: We see $P > P_{sat} = 1167$ kPa (30^oC)

We conclude compressed liquid without any interpolation.

$$v = v_f = 0.001658 + \frac{28-25}{5}(0.00168 - 0.001658) = 0.00167\ m^3/kg$$

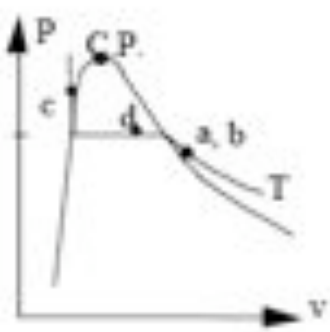
$$u = u_f = 296 + \frac{28-25}{5}(320.46 - 296.59) = 310.91\ kJ/kg$$

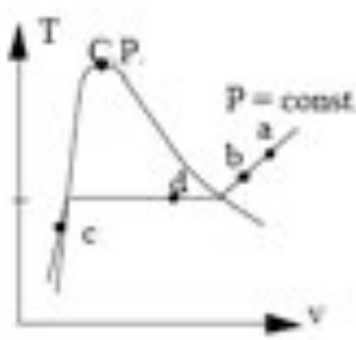
d) Table B.5.1 at 20^oC: $227.03 = u_f < u < u_g = 389.19$ kJ/kg so two-phase

$$x = \frac{u - u_f}{u_{fg}} = \frac{350 - 227.03}{162.16} = 0.7583, \quad P = P_{sat} = 572.8\ kPa$$

$$v = v_f + x\ v_{fg} = 0.000817 + x \times 0.03524 = 0.02754\ m^3/kg$$

all states are relative to the two-phase region, not to each other





5.42

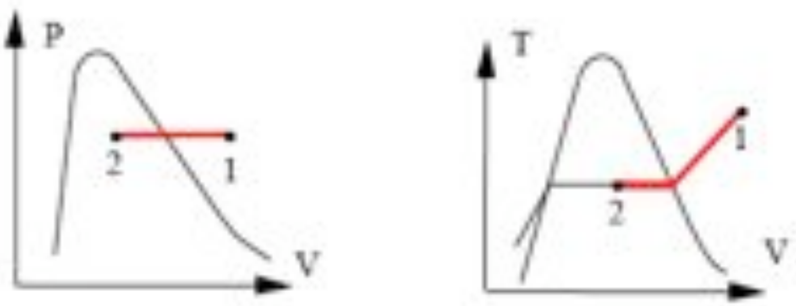
A cylinder fitted with a frictionless piston contains 2 kg of superheated refrigerant R-134a vapor at 350 kPa, 100°C. The cylinder is now cooled so the R-134a remains at constant pressure until it reaches a quality of 75%. Calculate the heat transfer in the process.

Solution:

C.V.: R-134a $\quad m_2 = m_1 = m;$

Energy Eq.5.11 $\quad m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Process: $P = \text{const.} \Rightarrow {}_1W_2 = \int PdV = P\Delta V = P(V_2 - V_1) = Pm(v_2 - v_1)$



State 1: Table B.5.2 $\quad h_1 = (490.48 + 489.52)/2 = 490$ kJ/kg

State 2: Table B.5.1 $\quad h_2 = 206.75 + 0.75 \times 194.57 = 352.7$ kJ/kg (350.9 kPa)

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = m(u_2 - u_1) + Pm(v_2 - v_1) = m(h_2 - h_1)$$

$${}_1Q_2 = 2 \text{ kg} \times (352.7 - 490) \text{ kJ/kg} = \mathbf{-274.6 \text{ kJ}}$$

5.55

A cylinder having a piston restrained by a linear spring (of spring constant 15 kN/m) contains 0.5 kg of saturated vapor water at 120°C, as shown in Fig. P5.55. Heat is transferred to the water, causing the piston to rise. If the piston cross-sectional area is 0.05 m^2, and the pressure varies linearly with volume until a final pressure of 500 kPa is reached. Find the final temperature in the cylinder and the heat transfer for the process.

Solution:

C.V. Water in cylinder.

Continuity: $m_2 = m_1 = m$;

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: (T, x) Table B.1.1 => $v_1 = 0.89186\ m^3/kg$, $u_1 = 2529.2\ kJ/kg$

Process: $P_2 = P_1 + \frac{k_s m}{A_p^2}(v_2 - v_1) = 198.5 + \frac{15 \times 0.5}{(0.05)^2}(v_2 - 0.89186)$

State 2: $P_2 = 500$ kPa and on the process curve (see above equation).

=> $v_2 = 0.89186 + (500 - 198.5) \times (0.05^2/7.5) = 0.9924\ m^3/kg$

(P, v) Table B.1.3 => $T_2 = \mathbf{803°C}$; $u_2 = 3668\ kJ/kg$

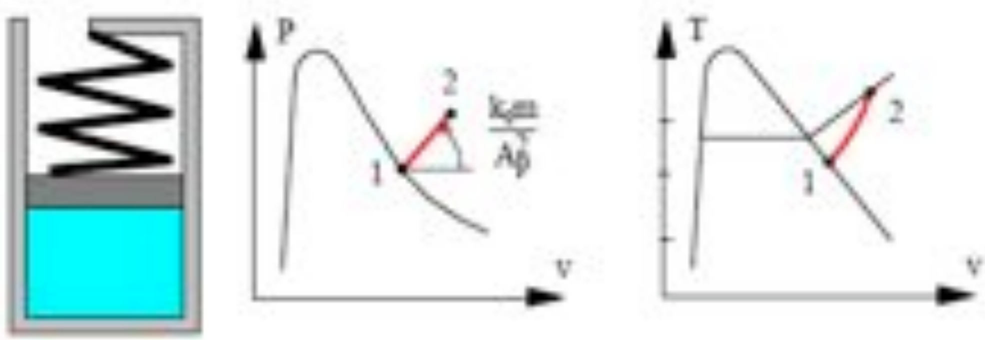
The process equation allows us to evaluate the work

$${}_1W_2 = \int PdV = \left(\frac{P_1 + P_2}{2}\right) m(v_2 - v_1)$$

$$= \left(\frac{198.5 + 500}{2}\right) kPa \times 0.5\ kg \times (0.9924 - 0.89186)\ m^3/kg = 17.56\ kJ$$

Substitute the work into the energy equation and solve for the heat transfer

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 0.5\ kg \times (3668 - 2529.2)\ kJ/kg + 17.56\ kJ = \mathbf{587\ kJ}$$



5.211E

An insulated cylinder is divided into two parts of 10 ft^3 each by an initially locked piston. Side A has air at 2 atm, 600 R and side B has air at 10 atm, 2000 R as shown in Fig. P5.111. The piston is now unlocked so it is free to move, and it conducts heat so the air comes to a uniform temperature $T_A = T_B$. Find the mass in both A and B and also the final T and P.

C.V. A + B . Then ${}_1Q_2 = 0$, ${}_1W_2 = 0$.

Force balance on piston: $P_A A = P_B A$, so final state in A and B is the same.

State 1A: $u_{A1} = 102.457$; $m_A = \frac{PV}{RT} = \frac{29.4 \times 10 \times 144}{53.34 \times 600} =$ **1.323 lbm**

State 1B: $u_{B1} = 367.642$; $m_B = \frac{PV}{RT} = \frac{147 \times 10 \times 144}{53.34 \times 2000} =$ **1.984 lbm**

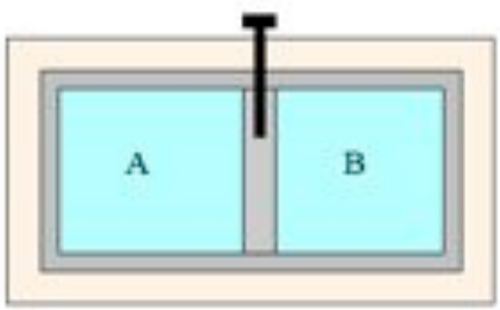
$m_A(u_2 - u_1)_A + m_B(u_2 - u_1)_B = 0$

$(m_A + m_B)u_2 = m_A u_{A1} + m_B u_{B1}$

$= 1.323 \times 102.457 + 1.984 \times 367.642 = 864.95$ Btu

$u_2 = 864.95/3.307 = 261.55 \quad \Rightarrow \quad T_2 =$ **1475 R**

$P = m_{tot}RT_2/V_{tot} = \frac{3.307 \times 53.34 \times 1475}{20 \times 144} =$ **90.34 lbf/in^2**



Quiz 6

1. A calorically imperfect ideal gas with gas constant R and

$$c_v = c_{v0} + a(T - T_0),$$

where c_{v0}, a, and T_0 are constants, isobarically expands from $P = P_0$, $T = T_0$ to $T = T_1$. Find ${}_0q_1$.

$$\begin{aligned} u_1 - u_0 &= {}_0q_1 - {}_0w_1 \\ &= {}_0q_1 - \int_0^1 P dv \\ &= {}_0q_1 - Pv_1 + Pv_0 \end{aligned}$$

$$(u_1 + P_1 v_1) - (u_0 + P_0 v_0) = {}_0q_1$$

$$h_1 - h_0 = {}_0q_1$$

$$\begin{aligned} {}_0q_1 = h_1 - h_0 &= \int_{T_0}^{T_1} c_p(T) dT \\ &= \int_{T_0}^{T_1} (c_v(T) dT + R) dT \\ &= \int_{T_0}^{T_1} (c_{v0} + R + a(T - T_0)) dT \\ &= \left((c_{v0} + R)T\right)_{T_0}^{T_1} + \frac{a}{2}\left((T - T_0)^2\right)_{T_0}^{T_1} \\ &= \boxed{(c_{v0} + R)(T_1 - T_0) + \frac{a}{2}(T_1 - T_0)^2 = {}_0q_1}. \end{aligned}$$

Homework 6

5.81

In a sink 5 liters of water at 70°C is combined with 1 kg aluminum pots, 1 kg of flatware (steel) and 1 kg of glass all put in at 20°C. What is the final uniform temperature neglecting any heat loss and work?

Energy Eq.: $U_2 - U_1 = \sum m_i(u_2 - u_1)_i = {}_1Q_2 - {}_1W_2 = 0$

For the water: $v_f = 0.001023\ m^3/kg$, $V = 5\ L = 0.005\ m^3$; $m = V/v = 4.8876\ kg$

For the liquid and the metal masses we will use the specific heats (Tbl A.3, A.4) so

$$\sum m_i(u_2 - u_1)_i = \sum m_i C_{v\,i}(T_2 - T_1)_i = T_2 \sum m_i C_{v\,i} - \sum m_i C_{v\,i} T_{1\,i}$$

noticing that all masses have the same T_2 but not same initial T.

$$\sum m_i C_{v\,i} = 4.8876 \times 4.18 + 1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8 = 22.59\ kJ/K$$

Energy Eq.: $22.59\ T_2 = 4.8876 \times 4.18 \times 70 + (1 \times 0.9 + 1 \times 0.46 + 1 \times 0.8) \times 20$

$= 1430.11 + 43.2$

$T_2 = \mathbf{65.2°C}$

5.95

Water at 150°C, 400 kPa, is brought to 1200°C in a constant pressure process. Find the change in the specific internal energy, using a) the steam tables, b) the ideal gas water table A.8, and c) =the specific heat from A.5.

Solution:

a)

State 1: Table B.1.3 Superheated vapor $u_1 = 2564.48$ kJ/kg

State 2: Table B.1.3 $u_2 = 4467.23$ kJ/kg

$u_2 - u_1 = 4467.23 - 2564.48 =$ **1902.75 kJ/kg**

b)

Table A.8 at 423.15 K: $u_1 = 591.41$ kJ/kg

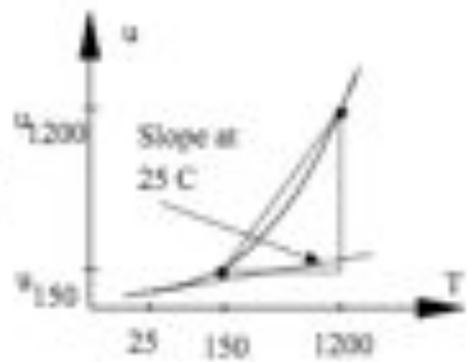
Table A.8 at 1473.15 K: $u_2 = 2474.25$ kJ/kg

$u_2 - u_1 = 2474.25 - 591.41 =$ **1882.8 kJ/kg**

c) Table A.5 : $C_{vo} = 1.41$ kJ/kgK

$u_2 - u_1 = 1.41$ kJ/kgK $(1200 - 150)$ K $=$ **1480.5 kJ/kg**

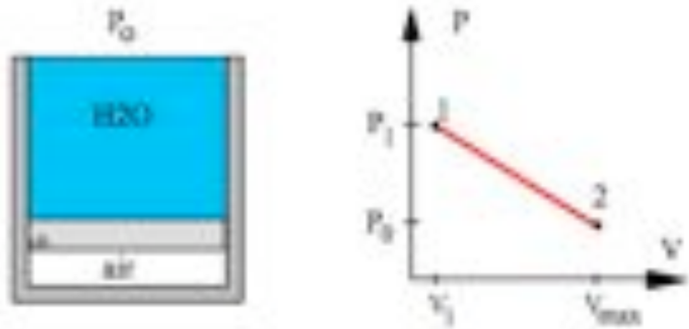
Notice how the average slope from 150 C to 1200 C is higher than the one at 25 C ($= C_{vo}$)



5.109

A 10-m high cylinder, cross-sectional area 0.1 m^2, has a massless piston at the bottom with water at 20°C on top of it, shown in Fig. P5.109. Air at 300 K, volume 0.3 m^3, under the piston is heated so that the piston moves up, spilling the water out over the side. Find the total heat transfer to the air when all the water has been pushed out.

Solution:



The water on top is compressed liquid and has volume and mass

$$V_{H_2O} = V_{tot} - V_{air} = 10 \times 0.1 - 0.3 = 0.7 \text{ m}^3$$

$$m_{H_2O} = V_{H_2O}/v_f = 0.7 / 0.001002 = 698.6 \text{ kg}$$

The initial air pressure is then

$$P_1 = P_0 + m_{H_2O}g/A = 101.325 + \frac{698.6 \times 9.807}{0.1 \times 1000} = \mathbf{169.84\ kPa}$$

and then $\quad m_{air} = PV/RT = \dfrac{169.84 \times 0.3}{0.287 \times 300} \dfrac{\text{kPa m}^3}{(\text{kJ/kg-K}) \times \text{K}} = 0.592 \text{ kg}$

State 2: No liquid over piston: $P_2 = P_0 = 101.325$ kPa, $V_2 = 10 \times 0.1 = 1$ m^3

State 2: $P_2, V_2 \quad \Rightarrow \quad T_2 = \dfrac{T_1 P_2 V_2}{P_1 V_1} = \dfrac{300 \times 101.325 \times 1}{169.84 \times 0.3} = 596.59$ K

The process line shows the work as an area

$${}_1W_2 = \int P dV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(169.84 + 101.325)(1 - 0.3) = 94.91 \text{ kJ}$$

The energy equation solved for the heat transfer becomes

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 \cong mC_v(T_2 - T_1) + {}_1W_2$$

$$= 0.592 \text{ kg} \times 0.717 \text{ kJ/kg-K} \times (596.59 - 300) \text{ K} + 94.91 \text{ kJ} = \mathbf{220.7\ kJ}$$

Remark: we could have used u values from Table A.7:

$u_2 - u_1 = 432.5 - 214.36 = 218.14$ kJ/kg $\quad$ versus 212.5 kJ/kg with C_v.

5.171

A piston/cylinder arrangement B is connected to a 1-m^3 tank A by a line and valve, shown in Fig. P5.171. Initially both contain water, with A at 100 kPa, saturated vapor and B at 400°C, 300 kPa, 1 m^3. The valve is now opened and, the water in both A and B comes to a uniform state.

a. Find the initial mass in A and B.

b. If the process results in T_2 = 200°C, find the heat transfer and work.

Solution:

C.V.: A + B. This is a control mass.

Continuity equation: $m_2 - (m_{A1} + m_{B1}) = 0$;

Energy: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

System: if $V_B \ge 0$ piston floats $\Rightarrow$ $P_B = P_{B1}$ = const.

if $V_B = 0$ then $P_2 < P_{B1}$ and $v = V_A/m_{tot}$ see P-V diagram

$${}_1W_2 = \int P_B dV_B = P_{B1}(V_2 - V_1)_B = P_{B1}(V_2 - V_1)_{tot}$$

State A1: Table B.1.1, x = 1

v_{A1} = 1.694 m^3/kg, u_{A1} = 2506.1 kJ/kg

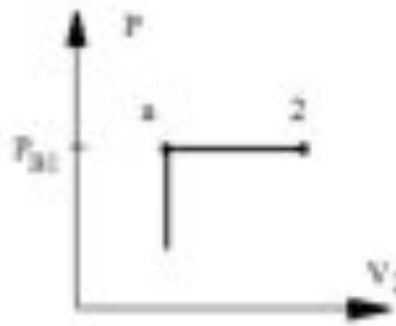
$m_{A1} = V_A/v_{A1}$ = **0.5903 kg**

State B1: Table B.1.2 sup. vapor

v_{B1} = 1.0315 m^3/kg, u_{B1} = 2965.5 kJ/kg

$m_{B1} = V_{B1}/v_{B1}$ = **0.9695 kg**

$m_2 = m_{TOT}$ = 1.56 kg



• At (T_2, P_{B1}) v_2 = 0.7163 > $v_a = V_A/m_{tot}$ = 0.641 so $V_{B2} > 0$

so now state 2: $P_2 = P_{B1}$ = 300 kPa, T_2 = 200 °C

=> u_2 = 2650.7 kJ/kg and $V_2 = m_2 v_2$ = 1.56 × 0.7163 = 1.117 m^3

(we could also have checked T_a at: 300 kPa, 0.641 m^3/kg => T = 155 °C)

${}_1W_2 = P_{B1}(V_2 - V_1)$ = **-264.82 kJ**

${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} + {}_1W_2$ = **-484.7 kJ**

5. You supervise an industrial process which uses forced convection to cool hot 10 g steel ball bearings. In the forced convection environment, the heat transfer coefficient is $h = 0.2\frac{\text{kW}}{\text{m}^2\cdot\text{K}}$. The initial temperature is 1600 K. The ambient temperature is 300 K. Using the method

developed in class, estimate the time constant of cooling, find an expression for $T(t)$, and find the time when $T = 350$ K. Plot $T(t)$. Repeat the analysis for a 1 kg sphere.
Given: $T_0 = 1600$ K, $T_\infty = 300$ K, $m = 10$ g, $h = 0.2 \frac{\text{kW}}{\text{m}^2 \cdot \text{K}}$
Assumptions: Incompressible, Newton's law of cooling
Find: τ, t @ 350 K, plot $T(t)$

$$\frac{dU}{dt} = \dot{Q} - \dot{W}$$

The ball bearing is incompressible, so $\dot{W} = 0$:

$$\frac{dU}{dt} = \dot{Q}$$

$$mc\frac{dT}{dt} = \dot{Q}$$

$$mc\frac{dT}{dt} = -hA(T - T_\infty)$$

$$\rho Vc\frac{dT}{dt} = -hA(T - T_\infty)$$

$$\frac{dT}{dt} = -\frac{hA}{\rho cV}(T - T_\infty)$$

$$\frac{dT}{T - T_\infty} = -\frac{hA}{\rho cV}dt$$

$$\int \frac{dT}{T - T_\infty} = \int -\frac{hA}{\rho cV}dt$$

$$\ln(T - T_\infty) = -\frac{hA}{\rho cV}t + C$$

$$\begin{aligned} T - T_\infty &= C' \exp\left(-\frac{hA}{\rho cV} \cdot 0\right) \\ &= C' \end{aligned}$$

$$T(t) = T_\infty + (T_0 - T_\infty)\exp\left(-\frac{hA}{\rho cV}t\right) \tag{3}$$

Generally, the time constant for a first-order system is the inverse-reciprocal of the exponential term. This gives

$$\tau = \frac{\rho cV}{hA}$$

Now to evaluate. Since the mass is $m = 10$ g and the density of steel was found to be 7850 kg/m^3,[3] the volume of the ball is

$$V = \frac{m}{\rho} = \frac{0.010 \text{ kg}}{7850 \text{ kg/m}^3} = 1.27 \cdot 10^{-6} \text{ m}^3$$

[3] `http://www.engineeringtoolbox.com/metal-alloys-densities-d_50.html`

Knowing the volume, the radius of the ball bearing was found to be

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = 6.72 \cdot 10^{-3} \text{ m}$$

The surface area A of the steel ball is

$$A = 4\pi r^2 = 5.68 \text{ m}^2$$

The specific heat of steel is $c = 0.49 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$[4]. A plot of the 10 g and 1 kg ball bearings is found in Figure 3. The time it takes for the temperature to reach 350 K was found by rearranging Eq. (3):

$$\ln\left(\frac{T - T_\infty}{T_0 - T_\infty}\right)\left(-\frac{\rho c V}{hA}\right) = t$$

A summary of the results for both analyses is found in the table below:

m	τ	t @ 350 K
10 g	43.1 s	140.5 s
1 kg	200.1 s	652.0 s

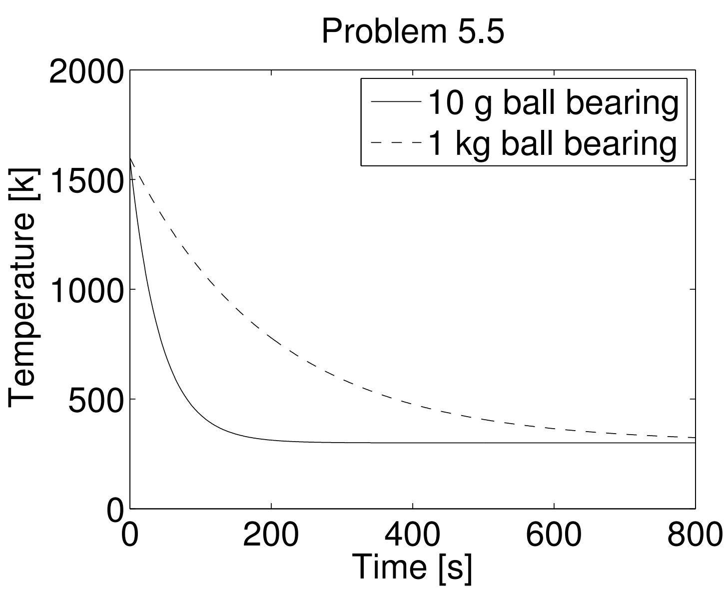


Figure 3: Plot of $T(t)$ for problem 5.5.

6. <u>5.228</u> A car with mass 1275 kg is driven at 60 km/h when the brakes are applied quickly to decrease its speed to 20 km/h. Assume the break pads have a 0.5 kg mass with heat capacity 1.1 kJ/kg K and that the brake disks and drums are 4.0 kg of steel. Further assume that both masses are heated uniformly. Find the temperature increase in the break assembly and produce a plot of the temperature rise as a function of the car mass.
<u>Given:</u> $m_c = 1275$ kg, $v_{ci} = 60$ km/h, $v_{cf} = 20$ km/h, $m_p = 0.5$ kg, $c_p = 1.1$ kJ/kg K, $m_d =$

[4] http://www.engineeringtoolbox.com/specific-heat-metals-d_152.html

4 kg, $c_d = 0.46$ kJ/kg K
Find: ΔT

Starting with the first law,

$$\Delta E = {}_1Q_2 + {}_1W_2$$

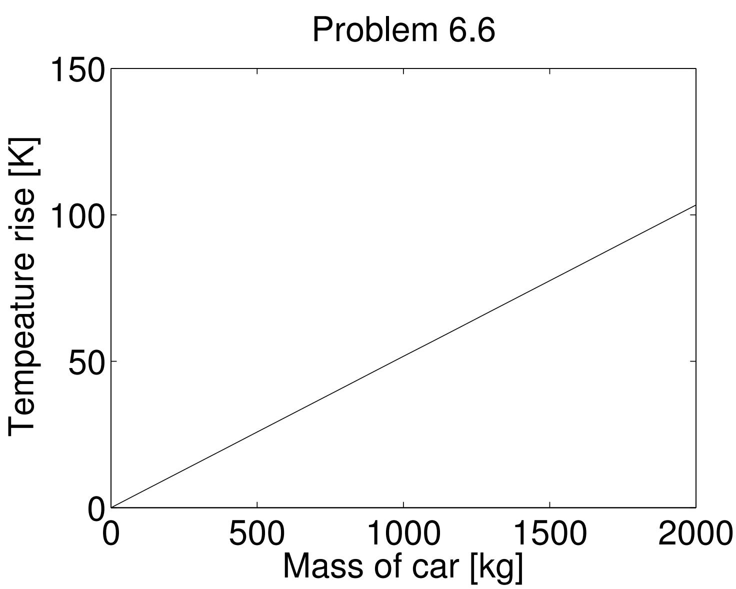
There is no work being done (${}_1W_2 = 0$). The heat transfer of the deceleration is equal to the sum of the heat transfer of the break pads and the break disks/drums. Thus the change in kinetic energy of the car is equal to the heat transfer of those components:

$$\frac{1}{2}m_c v_{ci}^2 - \frac{1}{2}m_c v_{cf}^2 = m_p c_p(\Delta T) + m_d c_d(\Delta T)$$

$$\Delta T = \frac{\frac{1}{2}m_c(v_{ci}^2 - v_{cf}^2)}{m_p c_p + m_d c_d}$$

$$\Delta T(m_c) = 0.05166 m_c \rightarrow \boxed{65.9 \text{ K} = \Delta T}$$

The function $\Delta T(m_c)$ is found in the following figure:



Quiz 7

1. The following ordinary differential equation and initial condition results from a control volume analysis for a fluid entering and exiting a leaky bucket. In contrast to the example problem done in lecture, the equation accounts better for the actual behavior of the fluid exiting:

$$\rho A \frac{dH}{dt} = \dot{m}_i - \rho A_e \sqrt{2gH}, \qquad H(0) = 0.$$

Here we have the following constants: density ρ, cross-sectional area of the tank A, cross-sectional area of the exit hole A_e, inlet mass flow rate $\dot{m}_i$, gravitational constant g. The independent variable is time t, and the dependent variable is height H. Find the fluid height H at the equilibrium state.

Equilibrium state means the tank is in steady state and the amount of water in the tank is not changing, so $\frac{dH}{dt} = 0$. *Taking this into account, the given equation becomes*

$$\dot{m}_i = \rho A_e \sqrt{2gH},$$

and you can solve for H *to find*

$$\boxed{H = \left(\frac{\dot{m}_i}{\rho A_e}\right)^2 \cdot \frac{1}{2g}}$$

Homework 7

6.15

A boiler receives a constant flow of 5000 kg/h liquid water at 5 MPa, 20°C and it heats the flow such that the exit state is 450°C with a pressure of 4.5 MPa. Determine the necessary minimum pipe flow area in both the inlet and exit pipe(s) if there should be no velocities larger than 20 m/s.

Solution:

Mass flow rate from Eq.6.3, both $\mathbf{V} \le 20$ m/s

$$\dot{m}_i = \dot{m}_e = (A\mathbf{V}/v)_i = (A\mathbf{V}/v)_e = 5000 \, \frac{1}{3600} \text{ kg/s}$$

Table B.1.4 $\quad v_i = 0.001 \text{ m}^3/\text{kg},$

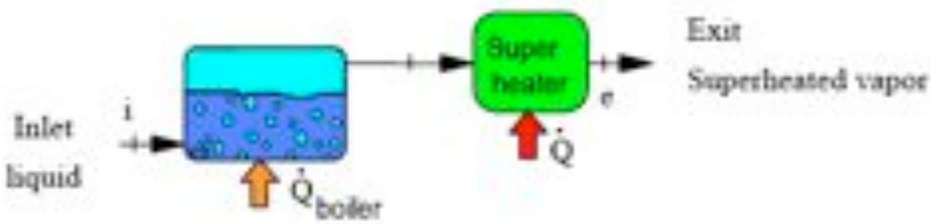
Table B.1.3 $\quad v_e = (0.08003 + 0.00633)/2 = 0.07166 \text{ m}^3/\text{kg},$

$$A_i \ge v_i \dot{m}/\mathbf{V}_i = 0.001 \text{ m}^3/\text{kg} \times \frac{5000}{3600} \text{kg/s} / 20 \text{ m/s}$$

$$= 6.94 \times 10^{-5} \text{ m}^2 = \mathbf{0.69\ cm^2}$$

$$A_e \ge v_e \dot{m}/\mathbf{V}_e = 0.07166 \text{ m}^3/\text{kg} \times \frac{5000}{3600} \text{kg/s} / 20 \text{ m/s}$$

$$= 4.98 \times 10^{-3} \text{ m}^2 = \mathbf{50\ cm^2}$$



2. Consider ow in a pipe with constant cross-sectional area A. Flow enters a xed control volume at the inlet i and exits at the exit e. The velocity in the x-direction is v. Derive the control volume version of the linear x-momentum equation for a uid in a fashion similar to that used in lecture for the mass and energy equations. The only force you need to consider is a pressure force; neglect all wall shear forces and gravity forces. The nal form should be of the form

$$\frac{\partial}{\partial t}\int_V \rho \mathsf{v} dV = \dot{m}_i \mathsf{v}_i - \dot{m}_e \mathsf{v}_e + P_i A - P_e A.$$

You may wish to consult any of a variety of undergraduate uid mechanics textbooks for more guidance.

Consulting any fluid mechanics book gives the following equation for the x-momentum for flow in a pipe:

$$\sum F_x = \frac{\partial}{\partial t}\int_V \rho \mathsf{v} dV + \iint_{cs} \rho \mathsf{v} \bar{\mathsf{v}} \cdot \hat{n} ds$$

Since a pressure force is defined as $P = F/A$, and by convention saying that the pressure force at the inlet is positive and negative at the exit, the sum of the forces $\sum F_x$ can be substituted:

$$P_i A - P_e A = \frac{\partial}{\partial t}\int_V \rho \mathsf{v} dV + \iint_{cs} \rho \mathsf{v} \bar{\mathsf{v}} \cdot \hat{n} ds$$

The normal vector $\hat{n}$ points out perpendicularly for both the inlet and exit control surfaces. Since the velocity v is always in the positive x-direction, the dot product will give the opposite signs for the inlet and exit velocities (one will add and the other will subtract). Taking that dot product, and since the cross-section of the pipe is said to be constant, the double integral $\iint_{cs} ds = A$ and the momentum equation becomes

$$P_i A - P_e A = \frac{\partial}{\partial t}\int_V \rho \mathsf{v} dV + ([\rho \mathsf{v}_e A]\mathsf{v}_e - [\rho \mathsf{v}_i A]\mathsf{v}_i)$$

The mass flow rate can be written as $\dot{m} = \rho \mathsf{v} A$, so the final form of the momentum equation can be rearranged to be

$$\boxed{\frac{\partial}{\partial t}\int_V \rho \mathsf{v} dV = \dot{m}_i \mathsf{v}_i - \dot{m}_e \mathsf{v}_e + P_i A - P_e A.}$$

6.30

The front of a jet engine acts as a diffuser receiving air at 900 km/h, -5°C, 50 kPa, bringing it to 80 m/s relative to the engine before entering the compressor. If the flow area is reduced to 80% of the inlet area find the temperature and pressure in the compressor inlet.

Solution:

C.V. Diffuser, Steady state, 1 inlet, 1 exit flow, no q, no w.

Continuity Eq.6.3: $\dot{m}_i = \dot{m}_e = (A\mathbf{V}/v)$

Energy Eq.6.12: $\dot{m}\,(h_i + \frac{1}{2}\mathbf{V}_i^2) = \dot{m}\,(\frac{1}{2}\mathbf{V}_e^2 + h_e)$

$$h_e - h_i = C_p\,(T_e - T_i) = \frac{1}{2}\mathbf{V}_i^2 - \frac{1}{2}\mathbf{V}_e^2 = \frac{1}{2}\left(\frac{900 \times 1000}{3600}\right)^2 - \frac{1}{2}(80)^2$$

$$= 28050 \text{ J/kg} = 28.05 \text{ kJ/kg}$$

$$\Delta T = 28.05/1.004 = 27.9 \;\Rightarrow\qquad T_e = -5 + 27.9 = \mathbf{22.9°C}$$

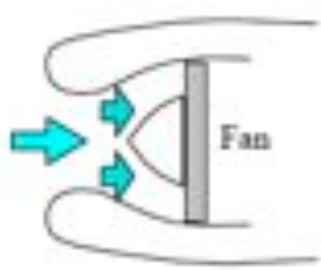
Now use the continuity eq.:

$$A_i\mathbf{V}_i/v_i = A_e\mathbf{V}_e/v_e \;\Rightarrow\qquad v_e = v_i\left(\frac{A_e\mathbf{V}_e}{A_i\mathbf{V}_i}\right)$$

$$v_e = v_i \times \frac{0.8 \times 80}{1 \times 250} = v_i \times 0.256$$

Ideal gas: $Pv = RT \;\Rightarrow\; v_e = RT_e/P_e = RT_i \times 0.256/P_i$

$$P_e = P_i\,(T_e/T_i)/0.256 = 50 \text{ kPa} \times 296/(268 \times 0.256) = \mathbf{215.7\ kPa}$$



6.60

Saturated liquid nitrogen at 600 kPa enters a boiler at a rate of 0.005 kg/s and exits as saturated vapor. It then flows into a super heater also at 600 kPa where it exits at 600 kPa, 280 K. Find the rate of heat transfer in the boiler and the super heater.

Solution:

C.V.: boiler steady single inlet and exit flow, neglect KE, PE energies in flow

Continuity Eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_3$

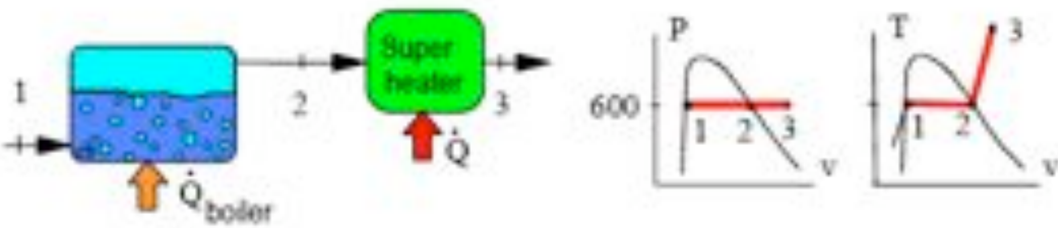


Table B.6.1: $h_1 = -81.469$ kJ/kg, $h_2 = 86.85$ kJ/kg,

Table B.6.2: $h_3 = 289.05$ kJ/kg

Energy Eq.6.13: $q_{boiler} = h_2 - h_1 = 86.85 - (-81.469) = 168.32$ kJ/kg

$\dot{Q}_{boiler} = \dot{m}_1 q_{boiler} = 0.005$ kg/s $\times$ 168.32 kJ/kg = **0.842 kW**

C.V. Superheater (same approximations as for boiler)

Energy Eq.6.13: $q_{sup\ heater} = h_3 - h_2 = 289.05 - 86.85 = 202.2$ kJ/kg

$\dot{Q}_{sup\ heater} = \dot{m}_2 q_{sup\ heater} = 0.005$ kg/s $\times$ 202.2 kJ/kg = **1.01 kW**

6.159E

A small, high-speed turbine operating on compressed air produces a power output of 0.1 hp. The inlet state is 60 lbf/in.^{2}, 120 F, and the exit state is 14.7 lbf/in.^{2}, −20 F. Assuming the velocities to be low and the process to be adiabatic, find the required mass flow rate of air through the turbine.

Solution:

C.V. Turbine, no heat transfer, no ΔKE, no ΔPE

Energy Eq.6.13: $h_{in} = h_{ex} + w_T$

Ideal gas so use constant specific heat from Table A.5

$$w_T = h_{in} - h_{ex} \cong C_p(T_{in} - T_{ex})$$
$$= 0.24(120 - (-20)) = 33.6 \text{ Btu/lbm}$$

$$\dot{W} = \dot{m} w_T \quad \Rightarrow$$

$$\dot{m} = \dot{W}/w_T = \frac{0.1 \text{ hp} \times 550 \text{ lbf-ft/s-hp}}{778 \text{ lbf-ft/Btu} \times 33.6 \text{ Btu/lbm}} = \mathbf{0.0021 \text{ lbm/s} = 7.57 \text{ lbm/h}}$$

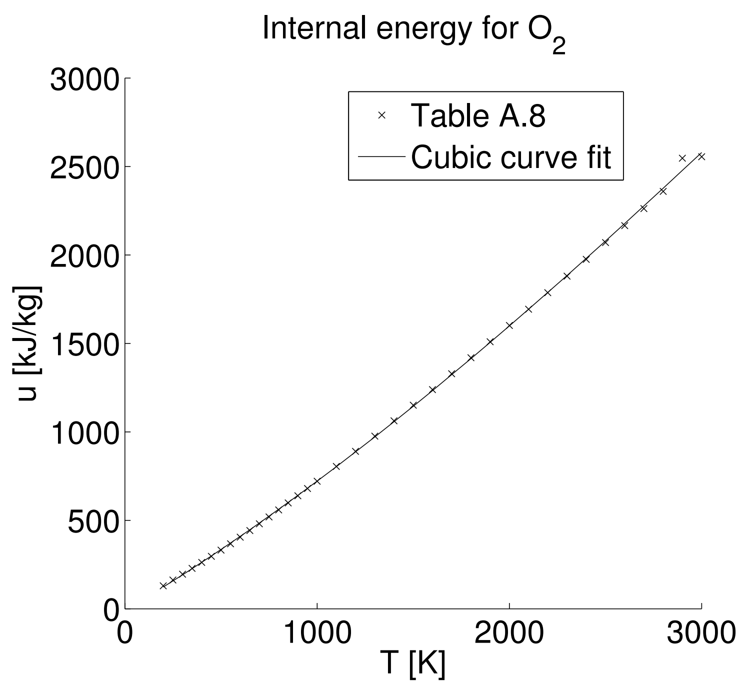
The dentist's drill has a small air flow and is not really adiabatic.

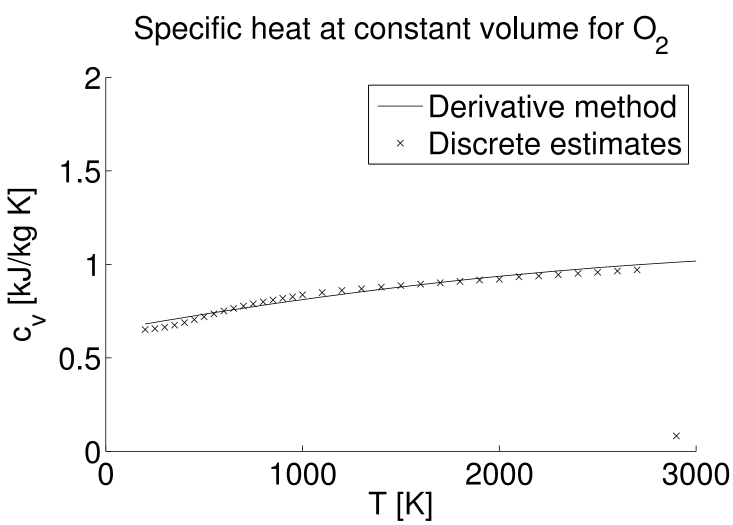
6. Take data from Table A.8 for O2 and develop your own third order polynomial curve t for $u(T)$. That is find a_1, a_2, a_3 such that

$$u(T) = a_0 + a_1 T + a_2 T^2 + a_3 T^3$$

well describes the data in the range 200 K $< T <$ 3000 K. Give a plot which gives the predictions of your curve fit $u(T)$ as a continuous curve for 200 K $< T <$ 3000 K. Superpose on this plot discrete points of the actual data. Take an appropriate derivative of the curve fit for $u(T)$ to estimate $c_v(T)$. Give a plot which gives your curve fit prediction of $c_v(T)$ for 200 K $< T <$ 3000 K. Superpose discrete estimates from a simple finite difference model $c_v = \frac{\Delta u}{\Delta T}$, where the finite difference estimates come from the data in Table A.8, onto your plot. You will find a discussion on least squares curve tting in the online course notes to be useful for this exercise.

See the plots below for the internal energy and the specific heat at constant volume for O_2. The third-order polynomial fit for the data was found to be $u(T) = -9.17 + 0.644T + 0.001T^2 + 0.00T^3$.




Specific heat at constant volume for O2
Derivative method
Discrete estimates
c_v [kJ/kg K]
T [K]
0
0.5
1
1.5
2
0
1000
2000
3000

Quiz 8

1. Mass flows through a duct with variable area. The flow is incompressible and steady. At the inlet, we have $\mathsf{v} = 10$ m/s,$A = 1$ m^2, $\rho = 1000$ kg/m^3. At the exit, we have $A = 0.1$ m^2. Find the flow velocity at the exit.

From mass conservation for steady one-dimensional flow,

$$\dot{m}_i = \dot{m}_e$$

Since $\dot{m} = \rho \mathsf{v} A$, *we can substitute:*

$$\rho \mathsf{v}_i A_i = \rho \mathsf{v}_e A_e$$

The flow is incompressible, so the density cancels and you can solve for the velocity at the exit:

$$\mathsf{v}_e = \frac{\mathsf{v}_i A_i}{A_e} = \frac{(10\text{ m/s})(1\text{ m}^2)}{0.1\text{ m}^2} = \boxed{100\text{ m/s} = \mathsf{v}_e.}$$

Homework 8

6.74

The main waterline into a tall building has a pressure of 600 kPa at 5 m below ground level. A pump brings the pressure up so the water can be delivered at **180** kPa at the top floor 150 m above ground level. Assume a flow rate of 10 kg/s liquid water at 10^0C and neglect any difference in kinetic energy and internal energy u. Find the pump work.

Solution:

C.V. Pipe from inlet at -5 m up to exit at +150 m, **180** kPa.

Energy Eq.6.13: $h_i + \frac{1}{2}V_i^2 + gZ_i = h_e + \frac{1}{2}V_e^2 + gZ_e + w$

With the same u the difference in h's are the Pv terms

$$w = h_i - h_e + \frac{1}{2}(V_i^2 - V_e^2) + g\,(Z_i - Z_e)$$

$$= P_i v_i - P_e v_e + g\,(Z_i - Z_e)$$

$$= 600 \times 0.001 - \mathbf{180} \times 0.001 + 9.806 \times (-5 - 150)/1000$$

$$= \mathbf{.42} - 1.52 = \mathbf{-1.1}\ \text{kJ/kg}$$

$$\dot{W} = \dot{m}w = 10 \times (-1.1\ ) = \mathbf{-11}\ \ \textbf{kW}$$

6.103

The following data are for a simple steam power plant as shown in Fig. P6.103.

State	1	2	3	4	5	6	7
P MPa	6.2	6.1	5.9	5.7	5.5	0.01	0.009
T °C		45	175	500	490		40
h kJ/kg	-	194	744	3426	3404	-	168

State 6 has $x_6 = 0.92$, and velocity of 200 m/s. The rate of steam flow is 25 kg/s, with 300 kW power input to the pump. Piping diameters are 200 mm from steam generator to the turbine and 75 mm from the condenser to the steam generator. Determine the velocity at state 5 and the power output of the turbine.

Solution:

Turbine $A_5 = (\pi/4)(0.2)^2 = 0.031\ 42\ m^2$, $v_5 = 0.06163\ m^3/kg$

$$\mathbf{V}_5 = \dot{m}v_5/A_5 = 25\ kg/s \times 0.061\ 63\ m^3/kg\ /\ 0.031\ 42\ m^2 = \mathbf{49\ m/s}$$

$$h_6 = 191.83 + 0.92 \times 2392.8 = 2393.2\ kJ/kg$$

$$w_T = h_5 - h_6 + \frac{1}{2}(\mathbf{V}_5^2 - \mathbf{V}_6^2)$$

$$= 3404 - 2393.2 + (49^2 - 200^2)/(2 \times 1000) = 992\ kJ/kg$$

$$\dot{W}_T = \dot{m}w_T = 25\ kg/s \times 992\ kJ/kg = \mathbf{24\ 800\ kW}$$

Remark: Notice the kinetic energy change is small relative to enthalpy change.

6.108

A R-410a heat pump cycle shown in Fig. P6.108 has a R-410a flow rate of 0.05 kg/s with 5 kW into the compressor. The following data are given

State	1	2	3	4	5	6
P, kPa	3100	3050	3000	420	400	390
T, °C	130	110	45		-10	-5
h, kJ/kg	377	367	134	-	280	284

Calculate the heat transfer from the compressor, the heat transfer from the R-410a in the condenser and the heat transfer to the R-410a in the evaporator.

Solution:

CV: Compressor

$$\dot{Q}_{COMP} = \dot{m}(h_1 - h_6) + \dot{W}_{COMP} = 0.05\ (377 - 284) - 5.0 = \mathbf{-0.35\ kW}$$

CV: Condenser

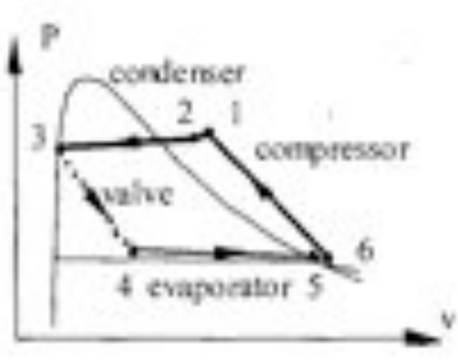
$$\dot{Q}_{COND} = \dot{m}(h_3 - h_2) = 0.05\ \text{kg/s}\ (134 - 367)\ \text{kJ/kg} = \mathbf{-11.65\ kW}$$

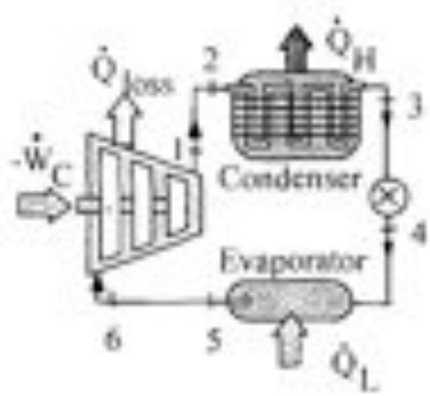
C.V. Valve:

$h_4 = h_3 = 134$ kJ/kg

CV: Evaporator

$$\dot{Q}_{EVAP} = \dot{m}\ (h_5 - h_4) = 0.05\ \text{kg/s}\ (280 - 134)\ \text{kJ/kg} = \mathbf{7.3\ kW}$$





6.109

A modern jet engine has a temperature after combustion of about 1500 K at 3200 kPa as it enters the turbine setion, see state 3 Fig. P.6.109. The compressor inlet is 80 kPa, 260 K state 1 and outlet state 2 is 3300 kPa, 780 K; the turbine outlet state 4 into the nozzle is 400 kPa, 900 K and nozzle exit state 5 at 70 kPa, 640 K. Neglect any heat transfer and neglect kinetic energy except out of the nozzle. Find the compressor and turbine specific work terms and the nozzle exit velocity.

Solution:

The compressor, turbine and nozzle are all steady state single flow devices and they are adiabatic.

We will use air properties from table A.7.1:

$h_1 = 260.32,\ h_2 = 800.28,\ h_3 = 1635.80,\ h_4 = 933.15,\ h_5 = 649.53$ kJ/kg

Energy equation for the compressor gives

$$w_{c\ in} = h_2 - h_1 = 800.28 - 260.32 = \mathbf{539.36\ kJ/kg}$$

Energy equation for the turbine gives

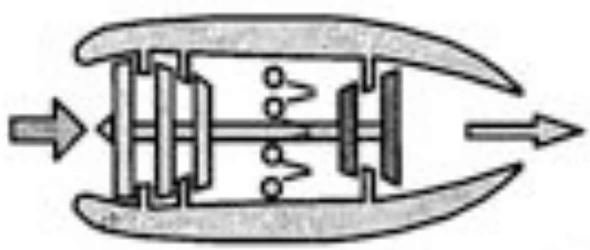
$$w_T = h_3 - h_4 = 1635.80 - 933.15 = \mathbf{702.65\ kJ/kg}$$

Energy equation for the nozzle gives

$$h_4 = h_5 + ½\, \mathbf{V}_5^2$$

$$½\, \mathbf{V}_5^2 = h_4 - h_5 = 933.15 - 649.53 = 283.62 \text{ kJ/kg}$$

$$\mathbf{V}_5 = [2(h_4 - h_5)]^{1/2} = (2\times 283.62 \times 1000)^{1/2} = \mathbf{753\ m/s}$$



6.173E

A condenser, as the heat exchanger shown in Fig. P6.83, brings 1 lbm/s water flow at 1 lbf/in.^{2} from 500 F to saturated liquid at 1 lbf/in.^{2}. The cooling is done by lake water at 70 F that returns to the lake at 90 F. For an insulated condenser, find the flow rate of cooling water.

Solution:

C.V. Heat exchanger

Energy Eq.6.10: $\dot m_{cool} h_{70} + \dot m_{H_2O} h_{500} = \dot m_{cool} h_{90} + \dot m_{H_2O} h_{f\,1}$

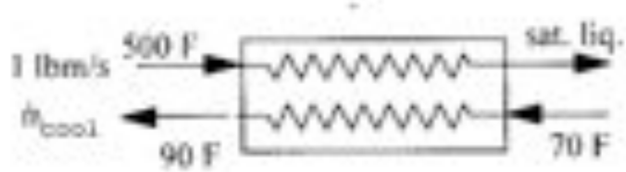
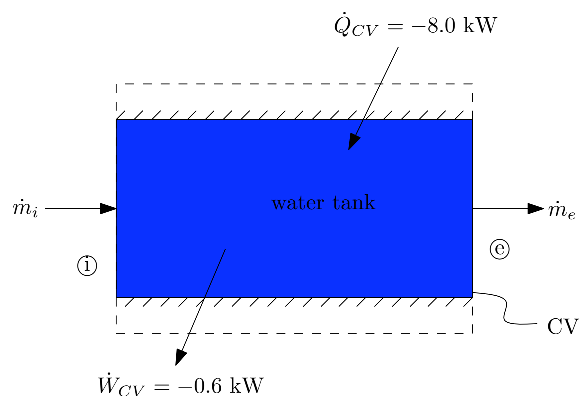


Table F.7.1: $h_{70} = 38.09$ Btu/lbm, $h_{90} = 58.07$ Btu/lbm, $h_{f,1} = 69.74$ Btu/lbm

Table F.7.2: $h_{500,1} = 1288.5$ btu/lbm

$$\dot m_{cool} = \dot m_{H_2O} \frac{h_{500} - h_{f,1}}{h_{90} - h_{70}} = 1 \text{ lbm/s} \times \frac{1288.5 - 69.74}{58.07 - 38.09} = \mathbf{61\ lbm/s}$$

6. A tank containing 50 kg of liquid water initially at 45°C has one inlet and one exit with equal mass flow rates. Liquid water enters at 45°C and a mass flow rate of 270 kg/hr. A cooling coil immersed in the water removes energy at a rate of 8.0 kW. The water is well mixed by a paddle wheel so that the water temperature is uniform throughout. The power input to the water from the paddle wheel is 0.6 kW. The pressures at the inlet and exit are equal and all kinetic and potential energy effects can be ignored. Determine the variation of water temperature with time. Give a computer-generated plot of temperature versus time.



Given: m=50 kg, $T_0 = 45$ °C, $T_i = 45$ °C, $\dot m = 270$ kg/hr, $\dot W_{CV} = -0.6$ kW, $\dot Q_{CV} = -8.0$ kW

Assumptions: $\Delta P = 0$, $\Delta KE = 0$, $\Delta PE = 0$, perfect mixing, $c_p = 4186 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$

Find: $\frac{dT}{dt}$

This solution follows much the same course as Example 6.7 in Professor Powers's notes. The first law is found as Eq. (6.100) in Professor Powers's notes:

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \sum \dot{m}_i h_{tot,i} - \sum \dot{m}_e h_{tot,e} \tag{4}$$

Our control volume is undergoing transient energy transfer, so $\frac{dE_{CV}}{dt} \neq 0$. The energy in the control volume is

$$E_{CV} = U_{CV} = mu_{cv}.$$

We will assume from mass conservation that

$$\dot{m}_i = \dot{m}_e = \dot{m} = 270 \text{ kg/hr}$$

and we will also assume that the total enthalpy for the water is

$$dh_{tot} = c_p dT.$$

Putting all this into Eq. (4),

$$\frac{d}{dt}(mu_{cv}) = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}(h_i - h_e)$$

Since $du_{cv} = c_v dT$ and m and c_v are constant, the above equation becomes

$$mc_v \frac{dT}{dt} = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}(h_i - h_e)$$

where T is the temperature of the tank of water. The enthalpy difference between the inlet and the exit is

$$h_i - h_e = c_p(T_i - T)$$

because the water supplied at the inlet is always a constant temperature of $T_i = 45$ °C. For liquid water,

$$c_p = c_v = c$$

Simplifying, the standard form of the differential equation is

$$\frac{dT}{dt} = \frac{\dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}c(T_i - T)}{mc}$$

To solve this first-order, linear, inhomogeneous differential equation, we can use separation of variables. Consulting a differential equations textbook[5], you will find that you can solve the equation by using separation of variables. The differential equation can be rewritten as

$$\begin{aligned}
\frac{dT}{dt} &= \frac{\dot{Q}_{CV} - \dot{W}_{CV}}{mc} + \frac{\dot{m}}{m}(T_i - T) \\
\frac{dT}{dt} &= a + b(T_i - T)
\end{aligned}$$

[5] Goodwine, Bill, 2010, *Engineering Differential Equations: Theory and Practice*, Springer: New York.

The latter equation is equivalent to the one above it with $a = \dfrac{\dot{Q}_{CV} - \dot{W}_{CV}}{mc}$ and $b = \dfrac{\dot{m}}{m}$. It will make solving the problem much easier. Separating the variables of the latter equation:

$$\frac{dT}{a + b(T_i - T)} = dt \tag{5}$$

Now we must do u'-substitution (I use u' here so that you don't get confused with specific internal energy, u). If $u' = a + b(T_i - T)$,

$$\begin{aligned} du' &= -bdT \\ -\frac{1}{b}du' &= dT. \end{aligned} \tag{6}$$

Substituing Eq. (6) and $u' = a + b(T_i - T)$ into Eq. (5) reveals

$$\begin{aligned} -\frac{1}{b}\frac{du'}{u'} &= dt \\ \ln(u') &= -bt + C \\ u' &= C\exp(-bt) \end{aligned}$$

Substituting for u' gives

$$a + b(T_i - T) = C\exp(-bt)$$

then for b

$$a + \frac{\dot{m}}{m}(T_i - T) = C\exp(-\frac{\dot{m}}{m}t)$$

and finally for a

$$\frac{\dot{Q}_{CV} - \dot{W}_{CV}}{mc} + \frac{\dot{m}}{m}(T_i - T) = C\exp(-\frac{\dot{m}}{m}t).$$

Rearrange the above equation to give T as a function of t:

$$T(t) = T_i + \frac{\dot{Q}_{CV} - \dot{W}_{CV}}{\dot{m}c} - C'\exp(-\frac{\dot{m}}{m}t) \tag{7}$$

where C' is a constant that must be solved for using initial conditions, $T(t = 0 \text{ s}) = T_0 = 45\ ^\circ\text{C} = 318$ K. The units of some material properties have to be made compatible. The mass flow rate $\dot{m}$ is

$$\dot{m} = 270 \text{ kg/hr}\left(\frac{1 \text{ hr}}{3600 \text{ s}}\right) = 3/40 \text{ kg/s}$$

Plugging into Eq. (7),

$$\begin{aligned} T(t = 0) &= T_0 + \frac{\dot{Q}_{CV} - \dot{W}_{CV}}{\dot{m}c} - C'\exp(-\frac{\dot{m}}{m}t) \\ 318 \text{ K} &= 318 \text{ K} + \frac{-8.0 \text{ kW} - (-0.6 \text{ kW})}{(3/40 \text{ kg/s})(4186 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})} - C'\exp\left(-\frac{3/40 \text{ kg/s}}{50 \text{ kg}}(0)\right) \\ 0 &= 0 + \frac{-8.0 \text{ kW} - (-0.6 \text{ kW})}{(3/40 \text{ kg/s})(4186 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})} - C'(1) \end{aligned}$$

Solving for C' we find

$$\begin{aligned} C' &= \frac{-8.0 \text{ kW} - (-0.6 \text{ kW})}{(3/40 \text{ kg/s})(4186 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}})} \\ C' &= \frac{-7.4 \text{ kW}}{313.95 \ \frac{\text{kW}}{\text{K}}} \\ C' &= -23.57 \text{ K} \end{aligned}$$

Thus the expression for T(t) is

$$\boxed{T(t) = T_i + \frac{\dot{Q}_{CV} - \dot{W}_{CV}}{\dot{m}c} + (23.57 \text{ K}) \exp(-\frac{\dot{m}}{m} t).}$$

or, numerically,

$$\boxed{T(t) = 294.4 \text{ K} + (23.57 \text{ K}) \exp(-0.0015 \frac{1}{s} t).}$$

The plot of $T(t)$ is Figure 4.

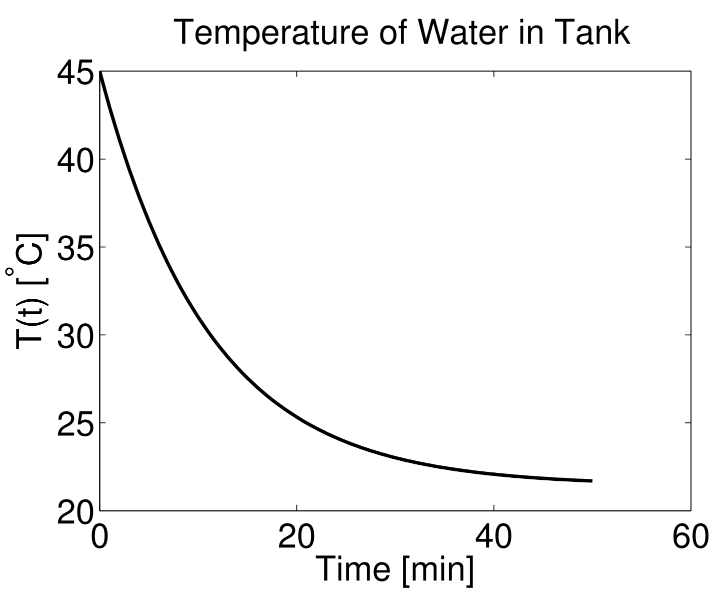


Figure 4: Plot for Problem 8.6.

Quiz 10

1. The indoor temperature of a home is T_H. The winter-time outdoor temperature is T_L. A heat pump maintains this temperature difference. Find the best possible ratio of heat transfer into the home to the work required by the pump.

For a Carnot heat pump,

$$\beta = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} = \frac{T_H}{T_H - T_L} = \boxed{\frac{1}{1 - \dfrac{T_L}{T_H}} = \beta.}$$

Homework 9

7.26

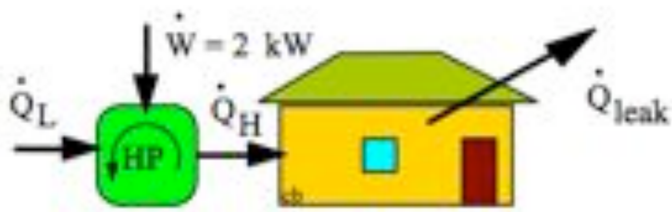
A farmer runs a heat pump with a 2 kW motor. It should keep a chicken hatchery at 30°C, which loses energy at a rate of 10 kW to the colder ambient T_{amb}. What is the minimum coefficient of performance that will be acceptable for the heat pump?

Solution:

Power input: $\dot{W} = 2$ kW

Energy Eq. for hatchery: $\dot{Q}_H = \dot{Q}_{Loss} = 10$ kW

Definition of COP: $\beta = COP = \dfrac{\dot{Q}_H}{\dot{W}} = \dfrac{10}{2} = \mathbf{5}$



7.42

Consider a heat engine and heat pump connected as shown in figure P7.42. Assume $T_{H1} = T_{H2} > T_{amb}$ and determine for each of the three cases if the setup satisfy the first law and/or violates the 2nd law.

	$\dot{Q}_{H1}$	$\dot{Q}_{L1}$	$\dot{W}_1$	$\dot{Q}_{H2}$	$\dot{Q}_{L2}$	$\dot{W}_2$
a	6	4	2	3	2	1
b	6	4	2	5	4	1
c	3	2	1	4	3	1

Solution:

	1st. law	2nd law
a	Yes	Yes (possible)
b	Yes	No, combine Kelvin - Planck
c	Yes	No, combination clausius

7.67

It is proposed to build a 1000-MW electric power plant with steam as the working fluid. The condensers are to be cooled with river water (see Fig. P7.67). The maximum steam temperature is 550°C, and the pressure in the condensers will be 10 kPa. Estimate the temperature rise of the river downstream from the power plant.

Solution:

$$\dot{W}_{NET} = 10^6\ \text{kW},\quad T_H = 550°\text{C} = 823.3\ \text{K}$$

$$P_{COND} = 10\ \text{kPa} \rightarrow T_L = T_G\ (P = 10\ \text{kPa}) = 45.8°\text{C} = 319\ \text{K}$$

$$\eta_{TH\ CARNOT} = \frac{T_H - T_L}{T_H} = \frac{823.2 - 319}{823.2} = 0.6125$$

$$\Rightarrow \dot{Q}_{L\ MIN} = 10^6\left(\frac{1 - 0.6125}{0.6125}\right) = 0.6327 \times 10^6\ \text{kW}$$

But $\dot{m}_{H_2O} = \dfrac{60 \times 8 \times 10/60}{0.001} = 80\ 000$ kg/s having an energy flow of

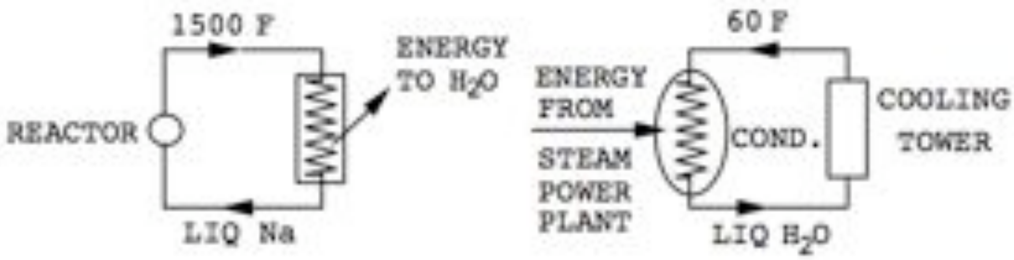
$$\dot{Q}_{L\ MIN} = \dot{m}_{H_2O}\,\Delta h = \dot{m}_{H_2O}\, C_{P\ LIQ\ H_2O}\, \Delta T_{H_2O\ MIN}$$

$$\Rightarrow \Delta T_{H_2O\ MIN} = \frac{\dot{Q}_{L\ MIN}}{\dot{m}_{H_2O} C_{P\ LIQ\ H_2O}} = \frac{0.6327 \times 10^6}{80000 \times 4.184} = \mathbf{1.9°C}$$

7.137E

Liquid sodium leaves a nuclear reactor at 1500 F and is used as the energy source in a steam power plant. The condenser cooling water comes from a cooling tower at 60 F. Determine the maximum thermal efficiency of the power plant. Is it misleading to use the temperatures given to calculate this value?

Solution:



$T_H = 1500\text{ F} = 1960\text{ R}, \quad T_L = 60\text{ F} = 520\text{ R}$

$$\eta_{TH\ MAX} = \frac{T_H - T_L}{T_H} = \frac{1960 - 520}{19860} = 0.735$$

It might be misleading to use 1500 F as the value for T_H, since there is not a supply of energy available at a constant temperature of 1500 F (liquid Na is cooled to a lower temperature in the heat exchanger).

⇒ The Na cannot be used to boil H_2O at 1500 F.

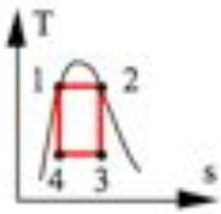
Similarly, the H_2O leaves the cooling tower and enters the condenser at 60 F, and leaves the condenser at some higher temperature.

⇒ The water does not provide for condensing steam at a constant temperature of 60 F.

8.37

In a Carnot engine with ammonia as the working fluid, the high temperature is 60°C and as Q_H is received, the ammonia changes from saturated liquid to saturated vapor. The ammonia pressure at the low temperature is 190 kPa. Find T_L, the cycle thermal efficiency, the heat added per kilogram, and the entropy, *s*, at the beginning of the heat rejection process.

Solution:



Constant T $\Rightarrow$ constant P from 1 to 2, Table B.2.1

$$q_H = \int T ds = T (s_2 - s_1) = T s_{fg}$$

$$= h_2 - h_1 = h_{fg} = \mathbf{997.0\ kJ/kg}$$

States 3 & 4 are two-phase, Table B.2.1

$$\Rightarrow \; T_L = T_3 = T_4 = T_{sat}(P) = \mathbf{-20°C}$$

$$\eta_{cycle} = 1 - \frac{T_L}{T_H} = 1 - \frac{253.2}{333.2} = \mathbf{0.24}$$

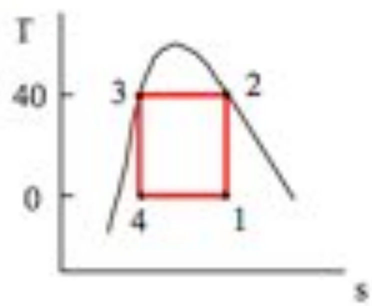
Table B.2.1: $\quad s_3 = s_2 = s_g(60°C) = \mathbf{4.6577\ kJ/kg\ K}$

8.44

Do Problem 8.43 using refrigerant R-134a instead of R-410a.
Consider a Carnot-cycle heat pump with R-410a as the working fluid. Heat is rejected from the R-410a at 40°C, during which process the R-410a changes from saturated vapor to saturated liquid. The heat is transferred to the R-410a at 0°C.

a. Show the cycle on a *T–s* diagram.
b. Find the quality of the R-410a at the beginning and end of the isothermal heat addition process at 0°C.
c. Determine the coefficient of performance for the cycle.

Solution:

a) 

b) From Table B.5.1, state 3 is saturated liquid

$$s_4 = s_3 = 1.1909 \text{ kJ/kg K}$$
$$= 1.0 + x_4(0.7262)$$

$$\Rightarrow \quad x_4 = \mathbf{0.2629}$$

State 2 is saturated vapor so from Table B.5.1

$$s_1 = s_2 = 1.7123 \text{ kJ/kg K} = 1.0 + x_1(0.7262)$$

$$\Rightarrow \quad x_1 = \mathbf{0.981}$$

c) $$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{313.2}{40} = \mathbf{7.83}$$

Quiz 11

1. Write the Gibbs equation, then use it to find s for a calorically perfect incompressible material.

The Gibbs equation is

$$\boxed{du = Tds - Pdv.}$$

By definition, $dv = 0$ *for an incompressible material (an incompressible material cannot change volume). Also, the material is calorically perfect, so* $du = c_v dT$ *and the change in entropy is*

$$\begin{aligned} du &= Tds \\ c_v \frac{dT}{T} &= ds \\ c_v \int_1^2 \frac{dT}{T} &= \int_1^2 ds \end{aligned}$$

$$\boxed{s_2 - s_1 = c_v \ln \frac{T_2}{T_1}}$$

Homework 10

8.94

A handheld pump for a bicycle has a volume of $30\ cm^3$ when fully extended. You now press the plunger (piston) in while holding your thumb over the exit hole so that an air pressure of 300 kPa is obtained. The outside atmosphere is at P_0, T_0. Consider two cases: (1) it is done quickly (~1 s), and (2) it is done very slowly (~1 h).

a. State assumptions about the process for each case.

b. Find the final volume and temperature for both cases.

Solution:

C.V. Air in pump. Assume that both cases result in a reversible process.

State 1: P_0, T_0 State 2: 300 kPa, ?

One piece of information must resolve the ? for a state 2 property.

Case I) Quickly means no time for heat transfer

$Q = 0$, so a reversible adiabatic compression.

$$u_2 - u_1 = -{}_1w_2 \ ; \quad s_2 - s_1 = \int dq/T + {}_1s_{2\ gen} = 0$$

With constant s and constant heat capacity we use Eq.8.23

$$T_2 = T_1 (P_2 / P_1)^{\frac{k-1}{k}} = 298 \left(\frac{300}{101.325}\right)^{\frac{0.4}{1.4}} = \mathbf{405.3\ K}$$

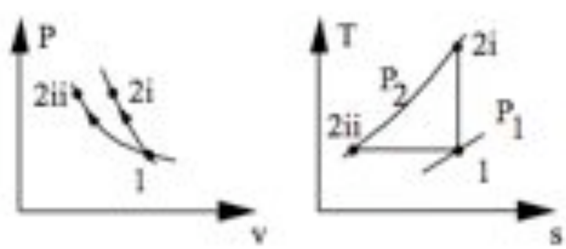
Use ideal gas law $PV = mRT$ at both states so ratio gives

$$\Rightarrow \quad V_2 = P_1V_1T_2/T_1P_2 = \mathbf{13.78\ cm^3}$$

Case II) Slowly, time for heat transfer so T = constant = T_0.

The process is then a reversible isothermal compression.

$$T_2 = T_0 = \mathbf{298\ K} \quad \Rightarrow \quad V_2 = V_1P_1/P_2 = \mathbf{10.1\ cm^3}$$



8.126

A piston/cylinder contains 2 kg water at 150 kPa, 20°C. The piston is loaded so pressure is linear in volume. Heat is added from a 600°C source until the water is at 1 MPa, 500°C. Find the heat transfer and the total change in entropy.

Solution:

CV H_2O out to the source, both $_1Q_2$ and $_1W_2$

Energy Eq.5.11: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $m(s_2 - s_1) = {}_1Q_2 / T_{SOURCE} + {}_1S_{2\,gen}$

Process: $P = A + BV$ => ${}_1W_2 = \int P\,dV = ½\,(P_1 + P_2)\,(V_2 - V_1)$

State 1: B.1.1 Compressed liquid use saturated liquid at same T:

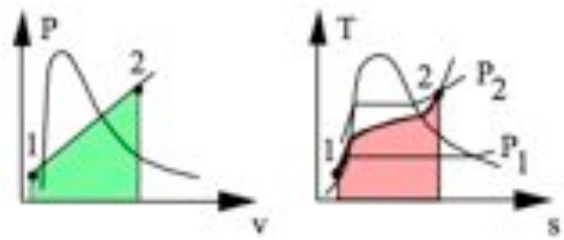
$v_1 = 0.001002$ m^3/kg; $u_1 = 83.94$ kJ/kg; $s_1 = 0.2966$ kJ/kg K

State 2: Table B.1.3 sup. vap.

$v_2 = 0.35411$ m^3/kg

$u_2 = 3124.3$ kJ/kg;

$s_2 = 7.7621$ kJ/kg K



${}_1W_2 = ½\,(1000 + 150)$ kPa × 2 kg (0.35411 - 0.001002) m^3/kg = 406 kJ

${}_1Q_2 = 2(3124.3 - 83.94) + 406 =$ **6486.7 kJ**

$m(s_2 - s_1) = 2$ kg (7.7621 - 0.2968) kJ/kg-K = 14.931 kJ/K

${}_1Q_2 / T_{source} =$ 7.429 kJ/K (for source $Q = -{}_1Q_2$)

${}_1S_{2\,gen} = m(s_2 - s_1) - {}_1Q_2 / T_{SOURCE} = \Delta S_{total}$

$= \Delta S_{H2O} + \Delta S_{source} = 14.931 - 7.429 =$ **7.502 kJ/K**

Remark: This is an external irreversible process (delta T to the source)

8.135

One kilogram of ammonia (NH_3) is contained in a spring-loaded piston/cylinder, Fig. P8.135, as saturated liquid at −20°C. Heat is added from a reservoir at 100°C until a final condition of 800 kPa, 70°C is reached. Find the work, heat transfer, and entropy generation, assuming the process is internally reversible.

Solution:

C.V. = NH_3 out to the reservoir.

Continuity Eq.: $m_2 = m_1 = m$

Energy Eq.5.11: $E_2 - E_1 = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

Entropy Eq.8.37: $S_2 - S_1 = \int dQ/T + {}_1S_{2,gen} = {}_1Q_2/T_{res} + {}_1S_{2\,gen}$

Process: $P = A + BV$ linear in V =>

$${}_1W_2 = \int PdV = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(P_1 + P_2)m(v_2 - v_1)$$

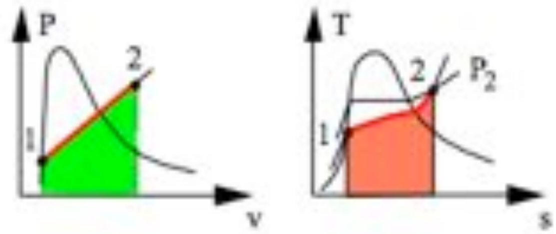
State 1: Table B.2.1

$P_1 = 190.08$ kPa,

$v_1 = 0.001504\ m^3/kg$

$u_1 = 88.76$ kJ/kg,

$s_1 = 0.3657$ kJ/kg K



State 2: Table B.2.2 sup. vapor

$v_2 = 0.199\ m^3/kg$, $u_2 = 1438.3$ kJ/kg, $s_2 = 5.5513$ kJ/kg K

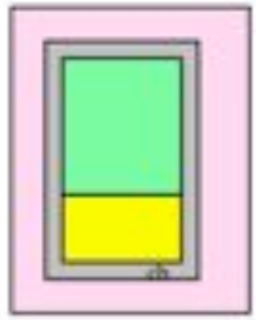
$${}_1W_2 = \tfrac{1}{2}(190.08 + 800)\ \text{kPa} \times 1\ \text{kg}\ (0.1990 - 0.001504)\ m^3/kg = \mathbf{97.768\ kJ}$$

$${}_1Q_2 = m(u_2 - u_1) + {}_1W_2 = 1(1438.3 - 88.76) + 97.768 = \mathbf{1447.3\ kJ}$$

$${}_1S_{2,gen} = m(s_2 - s_1) - {}_1Q_2/T_{res} = 1(5.5513 - 0.3657) - \frac{1447.3}{373.15} = \mathbf{1.307\ kJ/K}$$

8.223E

One lbm of air at 15 psia is mixed with one lbm air at 30 psia, both at 540 R, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process.



C.V. All the air.

Energy Eq.: $U_2 - U_1 = 0 - 0$

Entropy Eq.: $S_2 - S_1 = 0 + {}_1S_{2\,gen}$

Process Eqs.: $V = C$; $W = 0$, $Q = 0$

States A1, B1: $u_{A1} = u_{B1}$

$V_A = m_A RT_1/P_{A1}$; $V_B = m_B RT_1/P_{B1}$

$U_2 - U_1 = m_2u_2 - m_Au_{A1} - m_Bu_{B1} = 0 \Rightarrow u_2 = (u_{A1} + u_{B1})/2 = u_{A1}$

State 2: $\mathbf{T_2 = T_1 = 540\ R}$ (from u_2); $m_2 = m_A + m_B = 2$ kg;

$V_2 = m_2RT_1/P_2 = V_A + V_B = m_ART_1/P_{A1} + m_BRT_1/P_{B1}$

Divide with m_ART_1 and get

$$2/P_2 = 1/P_{A1} + 1/P_{B1} = \frac{1}{15} + \frac{1}{30} = 0.1 \text{ psia}^{-1} \quad \Rightarrow \mathbf{P_2 = 20\ psia}$$

Entropy change from Eq. 8.16 with the same T, so only P changes

$${}_1S_{2\,gen} = S_2 - S_1 = -m_AR \ln \frac{P_2}{P_{A1}} - m_BR \ln \frac{P_2}{P_{B1}}$$

$$= -1 \times 53.34 \left[\ln \frac{20}{15} + \ln \frac{20}{30} \right]$$

$$= -53.34\,(0.2877 - 0.4055) = \mathbf{6.283\ lbf\text{-}ft/R = 0.0081\ Btu/R}$$

5. Consider the ballistics problem as developed in class. We have the governing equation from Newtons second law of

$$\begin{aligned} \frac{dx}{dt} &= \mathsf{v}, \quad x(0) = x_0, \\ \frac{dv}{dt} &= \frac{P_\infty A}{m}\left(\frac{P_0}{P_\infty}\left(\frac{x_0}{x}\right)^k - 1\right) - \frac{C}{m}\mathsf{v}^3, \quad \mathsf{v}(0) = 0. \end{aligned}$$

Consider the following parameter values: $P_1 = 105$ Pa, $P_0 = 2 \times 108$ Pa, $T_0 = 300$ K, $C = 0.01$ N/(m/s)^3, $A = 10^{-4}$ m^2, $k = 7/5$, $x_0 = 0.03$ m, $m = 0.004$ kg. Consider the gas to be calorically perfect and ideal and let it undergo an isentropic process. Take the length of the tube to be 0.5 m.

(a) From Eq. (8.239) of Professor Powers's notes,

$$P = P_0 \left(\frac{x_0}{x} \right)^k ,$$

thus P is a function of x. Then to get a function for the temperature, taking the ideal gas law,

$$\begin{aligned}
PV &= mRT \\
mRT &= PV \\
mRT &= \left(P_0 \left(\frac{x_0}{x} \right)^k \right) V \\
mRT &= \left(P_0 \left(\frac{x_0}{x} \right)^k \right) (Ax) \\
T &= \frac{\left(P_0 \left(\frac{x_0}{x} \right)^k \right) (Ax)}{mR}
\end{aligned}$$

The forward Euler method form, $u(t + \Delta t) = u(t) + \frac{du}{dt}\Delta t$, is used to first integrate $\frac{d\mathsf{v}}{dt}$ and then integrate $\frac{dx}{dt}$, as follows:

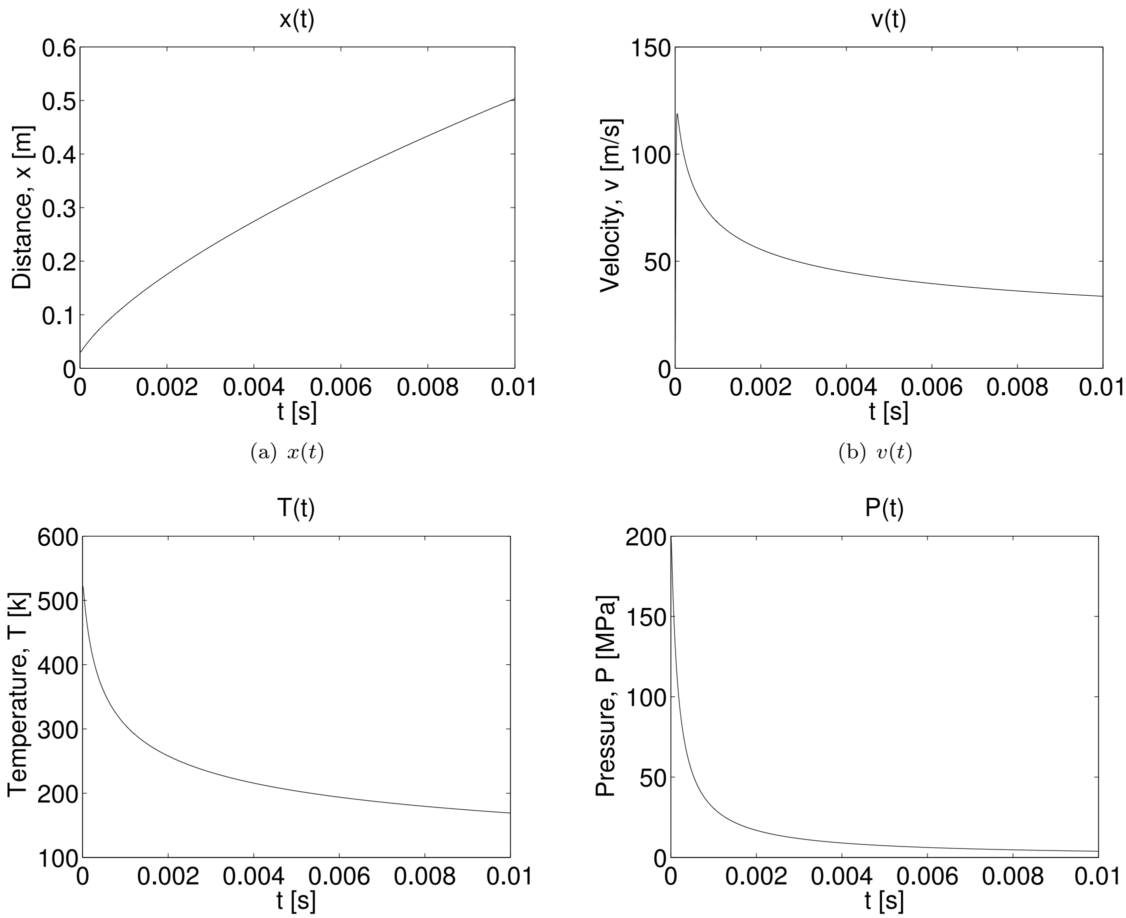
$$\begin{aligned}
\mathsf{v}(t + \Delta t) &= \mathsf{v}(t) + \left(\frac{P_\infty A}{m} \left(\frac{P_0}{P_\infty} \left(\frac{x_0}{x(t)} \right)^k - 1 \right) - \frac{C}{m}\mathsf{v}(t)^3 \right) \times \Delta t, \\
x(t + \Delta t) &= x(t) + \mathsf{v}(t + \Delta t) \times \Delta t.
\end{aligned}$$

From Eq. (8.250) in Professor Powers's notes, in order for the Euler method to provide a stable solution for early time, we need for Δt:

$$\Delta t < \sqrt{\frac{m x_0}{k P_0 A}} = \sqrt{\frac{(0.004 \text{ kg})(0.03 \text{ m})}{(7/5)(2 \times 10^8 \text{ Pa})(10^{-4} \text{ m}^2)}} = 0.0000654 \text{ s}.$$

For my MATLAB program, I used $\Delta t = 0.000001$ s.

(b) Here are the plots:



(a) $x(t)$ (b) $v(t)$ (c) $T(t)$ (d) $P(t)$

Figure 5: Plots for Problem 10.5.

(c) The velocity at the end of the tube is $\boxed{\mathrm{v}_{end} = 33.7 \text{ m/s}}$ and the time for the bullet to reach the end of the tube is $\boxed{t_{end} = 0.0099 \text{ s}}$.

(d) Extra points (up to 5) were awarded for analysis done for part (d).

(e) Here is the source code, presented in two columns to save space:

```
%HW 10, problem 5
clear all; close all; clc;

%given values
Pinf = 10^5;
P0 = 2*10^8;
T0 = 300;
C = 0.01;
A = 10^-4;
k = 7/5;
x0 = 0.03;
m = 0.004;
R = 287;

%initial conditions
x(1) = x0;
v(1) = 0;
dt = 0.000001;
t = 0:dt:.01;

%First-order Euler method below
%cycle through time range, which was
```

```
%found iteratively to be the length
%of time necessary for x to reach 0.5 m
for i = 1:length(t)
    v(i + 1) = v(i) +
        (Pinf*A/m*
         (P0/Pinf*(x0/x(i))^k - 1)
         - C/m*v(i)^3)*dt;
    x(i + 1) = x(i) + v(i)*dt;
end

%Find out velocity and time
% until x = 0.5 m
n=1;
while x(n)<0.5
    n = n+1;
end
%Report velocity and time at x = 0.5 m
v(n)
t(n)

P = P0*(x0./x).^k;
for i = 1:length(t)
    T(i) = P(i)*A*x(i)/(m*R);
end

%Plot results
set(gca,'FontSize',20)
plot(t,x(1,1:length(t)),'k')
axis([0 0.01 0 0.6])
title('x(t)')
ylabel('Distance, x [m]')
xlabel('t [s]')

figure
set(gca,'FontSize',20)
plot(t,v(1,1:length(t)),'k')
axis([0 0.01 0 150])
title('v(t)')
ylabel('Velocity, v [m/s]')
xlabel('t [s]')

figure
set(gca,'FontSize',20)
plot(t,T(1,1:length(t)),'k')
title('T(t)')
ylabel('Temperature, T [k]')
xlabel('t [s]')

figure
P = P/1000000;
set(gca,'FontSize',20)
plot(t,P(1,1:length(t)),'k')
title('P(t)')
ylabel('Pressure, P [MPa]')
xlabel('t [s]')
```

Homework 11

9.22

Atmospheric air at -45°C, 60 kPa enters the front diffuser of a jet engine with a velocity of 1000 km/h and frontal area of 1 m^2. After the adiabatic diffuser the velocity is 20 m/s. Find the diffuser exit temperature and the maximum pressure possible.

Solution:

C.V. Diffuser, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.6.13: $h_i + \mathbf{V}_i^2/2 = h_e + \mathbf{V}_e^2/2$, and $h_e - h_i = C_p(T_e - T_i)$

Entropy Eq.9.9: $s_i + \int dq/T + s_{gen} = s_i + 0 + 0 = s_e$ (Reversible, adiabatic)

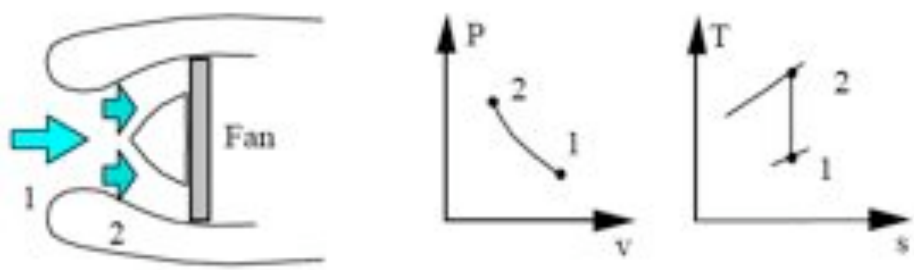
Heat capacity and ratio of specific heats from Table A.5: $C_{Po} = 1.004 \frac{kJ}{kg\,K}$, $k = 1.4$, the energy equation then gives:

$$1.004[\,T_e - (-45)] = 0.5[(1000 \times 1000/3600)^2 - 20^2\,]/1000 = 38.38 \text{ kJ/kg}$$

$$\Rightarrow \; T_e = -6.77 \;°C = \mathbf{266.4\ K}$$

Constant s for an ideal gas is expressed in Eq.8.23 (we need the inverse realation here):

$$P_e = P_i\,(T_e/T_i)^{\frac{k}{k-1}} = 60 \text{ kPa } (266.4\,/228.1)^{3.5} = \mathbf{103.3\ kPa}$$



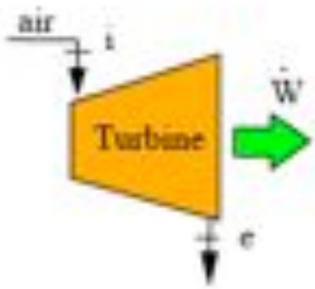
9.30

Air enters a turbine at 800 kPa, 1150 K, and expands in a reversible adiabatic process to 100 kPa. Calculate the exit temperature and the work output per kilogram of air, using

a. The ideal gas tables, Table A.7

b. Constant specific heat, value at 300 K from table A.5

Solution:



C.V. Air turbine.

Adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.6.13: $w_T = h_i - h_e$,

Entropy Eq.9.9: $s_e = s_i$

a) Table A.7: $h_i = 1219.30$ kJ/kg, $s^o_{Ti} = 8.29616$ kJ/kg K

The constant s process is written from Eq.8.19 as

$$\Rightarrow s^o_{Te} = s^o_{Ti} + R\ln\left(\frac{P_e}{P_i}\right) = 8.29616 + 0.287\ln\left(\frac{100}{800}\right) = 7.699 \text{ kJ/kg K}$$

Interpolate in A.7.1 $\Rightarrow T_e = \mathbf{674\ K}$, $h_e = 685.6$ kJ/kg

$w = h_i - h_e = \mathbf{533.7\ kJ/kg}$

b) Table A.5: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$, then from Eq.8.23

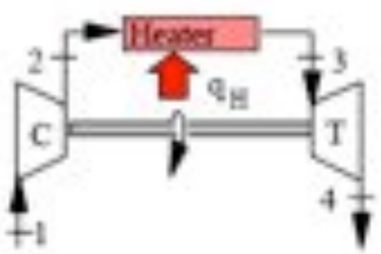
$$T_e = T_i\,(P_e/P_i)^{\frac{k-1}{k}} = 1150\text{ K}\left(\frac{100}{800}\right)^{0.286} = \mathbf{634.5\ K}$$

$$w = C_{Po}(T_i - T_e) = 1.004 \text{ kJ/kg-K } (1150-634.5) \text{ K} = \mathbf{517.6\ kJ/kg}$$

9.43

A heat-powered portable air compressor consists of three components: (a) an adiabatic compressor; (b) a constant pressure heater (heat supplied from an outside source); and (c) an adiabatic turbine. Ambient air enters the compressor at 100 kPa, 300 K, and is compressed to 600 kPa. All of the power from the turbine goes into the compressor, and the turbine exhaust is the supply of compressed air. If this pressure is required to be 200 kPa, what must the temperature be at the exit of the heater?

Solution:



$P_2 = 600$ kPa, $P_4 = 200$ kPa

Adiabatic and reversible compressor:

Process: $q = 0$ and $s_{gen} = 0$

Energy Eq.6.13: $h - w_c = h_2$

Entropy Eq.9.8: $s_2 = s_1$

For constant specific heat the isentropic relation becomes Eq.8.23

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \text{ K } (6)^{0.2857} = 500.8 \text{ K}$$

$$-w_c = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.5 \text{ kJ/kg}$$

Adiabatic and reversible turbine: $q = 0$ and $s_{gen} = 0$

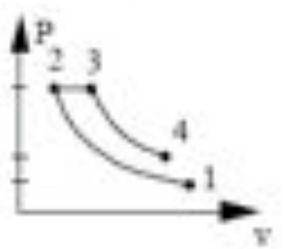
Energy Eq.6.13: $h_3 = w_T + h_4$; Entropy Eq.9.8: $s_4 = s_3$

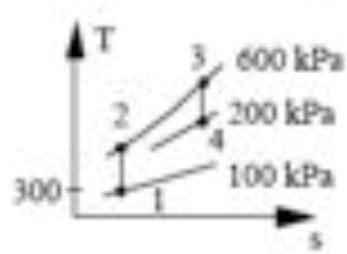
For constant specific heat the isentropic relation becomes Eq.8.23

$$T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = T_3\,(200/600)^{0.2857} = 0.7304\,T_3$$

Energy Eq. for shaft: $-w_c = w_T = C_{P0}(T_3 - T_4)$

$$201.5 \text{ kJ/kg} = 1.004 \text{ kJ/kgK} \times T_3(1 - 0.7304) \;\Rightarrow\; T_3 = \mathbf{744.4\ K}$$





9.62

A small dam has a pipe carrying liquid water at 150 kPa, 20°C with a flow rate of 2000 kg/s in a 0.5 m diameter pipe. The pipe runs to the bottom of the dam 15 m lower into a turbine with pipe diameter 0.35 m. Assume no friction or heat transfer in the pipe and find the pressure of the turbine inlet. If the turbine exhausts to 100 kPa with negligible kinetic energy what is the rate of work?

Solution:

C.V. Pipe. Steady flow no work, no heat transfer.

States: compressed liquid B.1.1 $\quad v_2 = v_1 = v_f = 0.001002 \text{ m}^3/\text{kg}$

Continuity Eq.6.3: $\qquad \dot{m} = \rho AV = AV/v$

$$\mathbf{V}_1 = \dot{m}v_1/A_1 = 2000 \text{ kg/s} \times 0.001002 \text{ m}^3/\text{kg} / (\tfrac{\pi}{4} 0.5^2 \text{ m}^2) = 10.2 \text{ m s}^{-1}$$

$$\mathbf{V}_2 = \dot{m}v_2/A_2 = 2000 \text{ kg/s} \times 0.001002 \text{ m}^3/\text{kg} / (\tfrac{\pi}{4} 0.35^2 \text{ m}^2) = 20.83 \text{ m s}^{-1}$$

From Bernoulli Eq.9.16 for the pipe (incompressible substance):

$$v(P_2 - P_1) + \tfrac{1}{2}(\mathbf{V}_2^2 - \mathbf{V}_1^2) + g(Z_2 - Z_1) = 0 \Rightarrow$$

$$P_2 = P_1 + [\tfrac{1}{2}(\mathbf{V}_1^2 - \mathbf{V}_2^2) + g(Z_1 - Z_2)]/v$$

$$= 150 \text{ kPa} + \frac{\frac{1}{2} \times (10.2^2 - 20.83^2) + 9.80665 \times 15}{1000 \times 0.001002} \; \frac{\text{m}^2\text{s}^{-2}}{\text{J/kJ} \times \text{m}^3/\text{kg}}$$

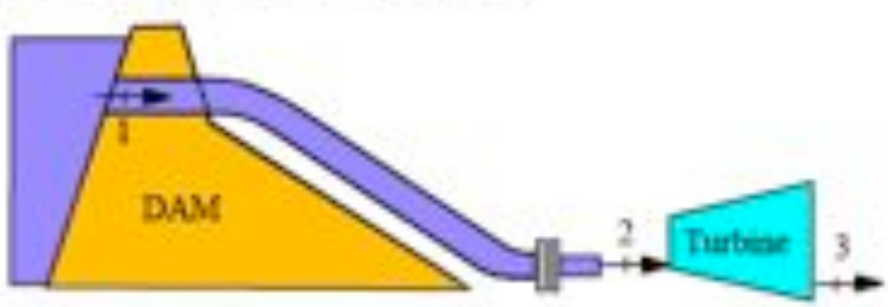
$$= 150 - 17.8 = \mathbf{132.2 \text{ kPa}}$$

Note that the pressure at the bottom should be higher due to the elevation difference but lower due to the acceleration. Now apply the energy equation Eq.9.13 for the total control volume

$$w = -\int v\, dP + \tfrac{1}{2}(\mathbf{V}_1^2 - \mathbf{V}_3^2) + g(Z_1 - Z_3)$$

$$= -0.001002\,(100 - 150) + [\tfrac{1}{2} \times 10.2^2 + 9.80665 \times 15]/1000 = 0.25 \text{ kJ/kg}$$

$$\dot{W} = \dot{m}w = 2000 \text{ kg/s} \times 0.25 \text{ kJ/kg} = \mathbf{500 \text{ kW}}$$



9.133

A refrigerator uses carbon dioxide that is brought from 1 MPa, -20°C to 6 MPa using 2 kW power input to the compressor with a flow rate of 0.02 kg/s. Find the compressor exit temperature and its isentropic efficiency.

C.V. Actual Compressor, assume adiabatic and neglect kinetic energies.

Energy Eq.6.13: $-w_C = h_2 - h_1 = \frac{\dot{W}}{\dot{m}} = \frac{2\text{ kW}}{0.02\text{ kg/s}} = 100\text{ kJ/kg}$

Entropy Eq.9.9: $s_2 = s_1 + s_{gen}$

States: 1: B.3.2 $h_1 = 342.31$ kJ/kg, $s_1 = 1.4655$ kJ/kg-K

2: B.3.2 $h_2 = h_1 - w_C = 442.31$ kJ/kg ⇨ $T_2 = \mathbf{117.7^\circ C}$

Ideal compressor. We find the exit state from (P,s).

State 2s: P_2, $s_{2s} = s_1 = 1.4655$ kJ/kg-K ⇨ $h_{2s} = 437.55$ kJ/kg

$-w_{C\,s} = h_{2s} - h_1 = 437.55 - 342.31 = 95.24$ kJ/kg

$$\eta_C = -w_{C\,s} / -w_C = \frac{95.24}{100} = \mathbf{0.952}$$

9.149

Air flows into an insulated nozzle at 1 MPa, 1200 K with 15 m/s and mass flow rate of 2 kg/s. It expands to 650 kPa and exit temperature is 1100 K. Find the exit velocity, and the nozzle efficiency.

Solution:

C.V. Nozzle. Steady 1 inlet and 1 exit flows, no heat transfer, no work.

Energy Eq.6.13: $h_i + (1/2)\mathbf{V}_i^2 = h_e + (1/2)\mathbf{V}_e^2$

Entropy Eq.9.9: $s_i + s_{gen} = s_e$

Ideal nozzle $s_{gen} = 0$ and assume same exit pressure as actual nozzle. Instead of using the standard entropy from Table A.7 and Eq.8.19 let us use a constant heat capacity at the average T and Eq.8.23. First from A.7.1

$$C_{p\,1150} = \frac{1277.81 - 1161.18}{1200 - 1100} = 1.166 \text{ kJ/kg K};$$

$$C_v = C_{p\,1150} - R = 1.166 - 0.287 = 0.8793, \quad k = C_{p\,1150}/C_v = 1.326$$

Notice how they differ from Table A.5 values.

$$T_{e\,s} = T_i\,(P_e/P_i)^{\frac{k-1}{k}} = 1200\left(\frac{650}{1000}\right)^{0.24585} = 1079.4 \text{ K}$$

$$\frac{1}{2}\mathbf{V}_{e\,s}^2 = \frac{1}{2}\mathbf{V}_i^2 + C(T_i - T_{e\,s}) = \frac{1}{2}\times 15^2 + 1.166(1200 - 1079.4)\times 1000$$

$$= 112.5 + 140619.6 = 140732 \text{ J/kg} \quad \Rightarrow \quad \mathbf{V}_{e\,s} = 530.5 \text{ m/s}$$

Actual nozzle with given exit temperature

$$\frac{1}{2}\mathbf{V}_{e\,ac}^2 = \frac{1}{2}\mathbf{V}_i^2 + h_i - h_{e\,ac} = 112.5 + 1.166(1200 - 1100)\times 1000$$

$$= 116712.5 \text{ J/kg}$$

$$\Rightarrow \quad \mathbf{V}_{e\,ac} = \mathbf{483\ m/s}$$

$$\eta_{noz} = \left(\tfrac{1}{2}\mathbf{V}_{e\,ac}^2 - \tfrac{1}{2}\mathbf{V}_i^2\right) / \left(\tfrac{1}{2}\mathbf{V}_{e\,s}^2 - \tfrac{1}{2}\mathbf{V}_i^2\right) =$$

$$= (h_i - h_{e,AC})/(h_i - h_{e,s}) = \frac{116600}{140619.6} = \mathbf{0.829}$$

Quiz 12

1. Write the Gibbs equation.

The Gibbs equation is

$$\boxed{du = Tds - Pdv}$$

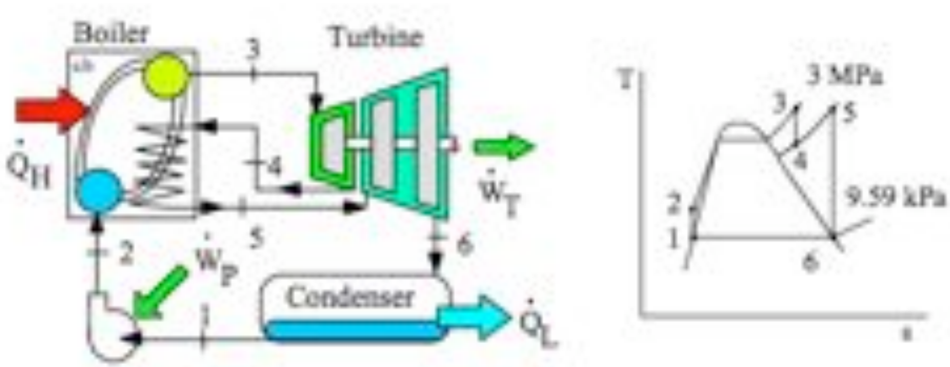
or alternatively

$$\boxed{dh = Tds + vdP.}$$

Homework 12

11.33

A smaller power plant produces steam at 3 MPa, 600°C in the boiler. It keeps the condenser at 45°C by transfer of 10 MW out as heat transfer. The first turbine section expands to 500 kPa and then flow is reheated followed by the expansion in the low pressure turbine. Find the reheat temperature so the turbine output is saturated vapor. For this reheat find the total turbine power output and the boiler heat transfer.



The states properties from Tables B.1.1 and B.1.3

1: 45°C, $x = 0$: $h_1 = 188.42$ kJ/kg, $v_1 = 0.00101$ m³/kg, $P_{sat} = 9.59$ kPa

3: 3.0 MPa, 600°C: $h_3 = 3682.34$ kJ/kg, $s_3 = 7.5084$ kJ/kg K

6: 45°C, $x = 1$: $h_6 = 2583.19$ kJ/kg, $s_6 = 8.1647$ kJ/kg K

C.V. Pump Reversible and adiabatic.

Energy: $w_p = h_2 - h_1$; Entropy: $s_2 = s_1$

since incompressible it is easier to find work (positive in) as

$$w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101\ (3000 - 9.59) = 3.02 \text{ kJ/kg}$$

$$h_2 = h_1 + w_p = 188.42 + 3.02 = 191.44 \text{ kJ/kg}$$

C.V. HP Turbine section

Entropy Eq.: $s_4 = s_3$ => $h_4 = 3093.26$ kJ/kg; $T_4 = 314$°C

C.V. LP Turbine section

Entropy Eq.: $s_6 = s_5 = 8.1647$ kJ/kg K => state 5

State 5: 500 kPa, s_5 => $h_5 = 3547.55$ kJ/kg, $T_5 =$ **529°C**

C.V. Condenser.

Energy Eq.: $q_L = h_6 - h_1 = h_{fg} = 2394.77$ kJ/kg

$$\dot{m} = \dot{Q}_L / q_L = 10\,000 / 2394.77 = 4.176 \text{ kg/s}$$

Both turbine sections

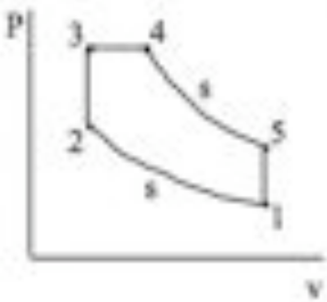
$$\dot{W}_{T,tot} = \dot{m}w_{T,tot} = \dot{m}(h_3 - h_4 + h_5 - h_6)$$
$$= 4.176\ (3682.34 - 3093.26 + 3547.55 - 2583.19) = \mathbf{6487\ kW}$$

Both boiler sections

$$\dot{Q}_H = \dot{m}(h_3 - h_2 + h_5 - h_4)$$
$$= 4.176\ (3682.34 - 191.44 + 3547.55 - 3093.26) = \mathbf{16\,475\ kW}$$

12.82

In the Otto cycle all the heat transfer q_H occurs at constant volume. It is more realistic to assume that part of q_H occurs after the piston has started its downward motion in the expansion stroke. Therefore, consider a cycle identical to the Otto cycle, except that the first two-thirds of the total q_H occurs at constant volume and the last one-third occurs at constant pressure. Assume that the total q_H is 2100 kJ/kg, that the state at the beginning of the compression process is 90 kPa, 20°C, and that the compression ratio is 9. Calculate the maximum pressure and temperature and the thermal efficiency of this cycle. Compare the results with those of a conventional Otto cycle having the same given variables.



$P_1 = 90$ kPa, $T_1 = 20°C$

$r_V = v_1/v_2 = 7$

$q_{23} = (2/3) \times 2100$
$= 1400$ kJ/kg;
$q_{34} = 2100/3 = 700$ kJ/kg

Constant s compression, Eqs.8.24-25

$$P_2 = P_1(v_1/v_2)^k = 90 \text{ kPa } (9)^{1.4} = 1951 \text{ kPa}$$

$$T_2 = T_1(v_1/v_2)^{k-1} = 293.15 \text{ K } (9)^{0.4} = 706 \text{ K}$$

Constant v combustion

$$T_3 = T_2 + q_{23}/C_{V0} = 706 + 1400/0.717 = \mathbf{2660\ K}$$

$$P_3 = P_2T_3/T_2 = 1951 \text{ kPa } (2660/706) = \mathbf{7350.8\ kPa} = P_4$$

Constant P combustion

$$T_4 = T_3 + q_{34}/C_{P0} = 2660 + 700/1.004 = 3357 \text{ K}$$

Remaining expansion: $$\frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{P_4}{P_1} \times \frac{T_1}{T_4} = \frac{7350.8}{90} \times \frac{293.15}{3357} = 7.131$$

$$T_5 = T_4(v_4/v_5)^{k-1} = 3357 \text{ K } (1/7.131)^{0.4} = 1530 \text{ K}$$

$$q_L = C_{V0}(T_5 - T_1) = 0.717 \text{ kJ/kg-K } (1530 - 293.15) \text{ K} = 886.2 \text{ kJ/kg}$$

$$\eta_{TH} = 1 - q_L/q_H = 1 - 886.2/2100 = \mathbf{0.578}$$

Std. Otto Cycle: $\eta_{TH} = 1 - (9)^{-0.4} = \mathbf{0.585}$, small difference

14.37

Start from Gibbs relation $dh = Tds + vdP$ and use one of Maxwell's equation to get $(\partial h/\partial v)_T$ in terms of properties P, v and T. Then use Eq.14.24 to also find an expression for $(\partial h/\partial T)_v$.

Find $\left(\frac{\partial h}{\partial v}\right)_T$ and $\left(\frac{\partial h}{\partial T}\right)_v$

$dh = Tds + vdP$ and use Eq.14.18

$$\Rightarrow \quad \left(\frac{\partial h}{\partial v}\right)_T = T\left(\frac{\partial s}{\partial v}\right)_T + v\left(\frac{\partial P}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_v + v\left(\frac{\partial P}{\partial v}\right)_T$$

Also for the second first derivative use Eq.14.24

$$\left(\frac{\partial h}{\partial T}\right)_v = T\left(\frac{\partial s}{\partial T}\right)_v + v\left(\frac{\partial P}{\partial T}\right)_v = C_v + v\left(\frac{\partial P}{\partial T}\right)_v$$

14.54

Consider the speed of sound as defined in Eq. 14.41. Calculate the speed of sound for liquid water at 20°C, 2 MPa, and for water vapor at 200°C, 400 kPa, using the steam tables.

From Eq. 14.41: $c^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = -v^2\left(\frac{\partial P}{\partial v}\right)_s$

Liquid water at 20°C, 2.5 MPa, assume

$$\left(\frac{\partial P}{\partial v}\right)_s \approx \left(\frac{\Delta P}{\Delta v}\right)_T$$

Using saturated liquid at 20°C and compressed liquid at 20°C, 5 MPa,

$$c^2 = -\left(\frac{0.001\,002+0.001001}{2}\right)^2\left(\frac{2-0.002339}{0.001001-0.001\,002}\right)\frac{MJ}{kg}$$

$$= 2.002\times 10^6\ \frac{J}{kg}$$

$$\Rightarrow \quad c = \mathbf{1416\ m/s}$$

Superheated vapor water at 200°C, 400 kPa

$$v = 0.5342\ m^3/kg, \quad s = 7.1706\ kJ/kg\ K$$

At P = 300 kPa & s = 7.1706 kJ/kg K: T = 170°C, v = 0.6666 m^3/kg

At P = 500 kPa & s = 7.1706 kJ/kg K: T = 226.3°C, v = 0.4509 m^3/kg

$$c^2 = -(.5342)^2\left(\frac{0.500-0.300}{.4509-.6666}\right)\frac{MJ}{kg} = 0.2646\times 10^6\ m^2/s^2$$

$$\Rightarrow \quad c = \mathbf{514\ m/s}$$

14.69

Show that the van der Waals equation can be written as a cubic equation in the compressibility factor involving the reduced pressure and reduced temperature as

$$Z^3 - \left(\frac{P_r}{8T_r} + 1\right) Z^2 + \left(\frac{27\,P_r}{64\,T_r^2}\right) Z - \frac{27\,P_r^{\,2}}{512\,T_r^{\,3}} = 0$$

van der Waals equation, Eq.14.55: $\quad P = \frac{RT}{v-b} - \frac{a}{v^2}$

$$a = \frac{27}{64}\frac{R^2T_C^{\,2}}{P_C} \qquad\qquad b = \frac{RT_C}{8P_C}$$

multiply equation by $\frac{v^2(v-b)}{P}$

Get: $\quad v^3 - (b + \frac{RT}{P})\,v^2 + (\frac{a}{P})\,v - \frac{ab}{P} = 0$

Multiply by $\frac{P^3}{R^3\,T^3}$ and substitute $Z = \frac{Pv}{RT}$

Get: $\quad Z^3 - (\frac{bP}{RT} + 1)\,Z^2 + (\frac{aP}{R^2T^2})\,Z - (\frac{abP^2}{R^3\,T^3}) = 0$

Substitute for a and b, get:

$$Z^3 - \left(\frac{P_r}{8T_r} + 1\right) Z^2 + \left(\frac{27\,P_r}{64\,T_r^2}\right) Z - \frac{27\,P_r^{\,2}}{512\,T_r^{\,3}} = 0$$

Where $\quad P_r = \frac{P}{P_c}, \qquad T_r = \frac{T}{T_c}$

14.81

A flow of oxygen at 230 K, 5 MPa is throttled to 100 kPa in a steady flow process. Find the exit temperature and the specific entropy generation using Redlich-Kwong equation of state and ideal gas heat capacity. Notice this becomes iterative due to the non-linearity coupling *k*, *P*, *v* and *T*.

C.V. Throttle. Steady single flow, no heat transfer and no work.

Energy eq.: $h_1 + 0 = h_2 + 0$ so constant h

Entropy Eq.: $s_1 + s_{gen} = s_2$ so entropy generation

Find the change in h from Eq.14.26 assuming C_p is constant.

Redlich-Kwong equation of state: $P = \frac{RT}{v - b} - \frac{a}{v(v + b)T^{1/2}}$

$$\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{v - b} + \frac{a}{2v(v + b)T^{3/2}}$$

From Eq.14.31

$$(u_2 - u_1)_T = \int_1^2 \frac{3a}{2v(v + b)T^{1/2}} dv = \frac{-3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

We find change in h from change in u, so we do not do the derivative in Eq.14.27. This is due to the form of the EOS.

$$(h_2 - h_1)_T = P_2v_2 - P_1v_1 - \frac{3a}{2bT^{1/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

Entropy follows from Eq.14.35

$$(s_2 - s_1)_T = \int_1^2 \left[\frac{R}{v - b} + \frac{a/2}{v(v + b)T^{3/2}}\right] dv$$

$$= R \ln\left(\frac{v_2 - b}{v_1 - b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\left(\frac{v_2 + b}{v_2}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

$P_c = 5040$ kPa; $T_c = 154.6$ K; $R = 0.2598$ kJ/kg K

$$b = 0.08664 \frac{RT_c}{P_c} = 0.08664 \times \frac{0.2598 \times 154.6}{5040} = 0.000\,690\,5 \text{ m}^3\text{/kg}$$

$$a = 0.427\,48\,\frac{R^2 T_c^{5/2}}{P_c} = 0.427\,48 \times \frac{0.2598^2 \times 154.6^{5/2}}{5040} = 1.7013$$

We need to find T_2 so the energy equation is satisfied

$$h_2 - h_1 = h_2 - h_x + h_x - h_1 = C_p(T_2 - T_1) + (h_2 - h_1)_T = 0$$

and we will evaluate it similar to Fig. 13.4, where the first term is done from state x to 2 and the second term is done from state 1 to state x (at T_1 = 230 K). We do this as we assume state 2 is close to ideal gas, but we do not know T_2.
We first need to find v_1 from the EOS, so guess v and find P

$v_1 = 0.011\ m^3/kg \quad \Rightarrow \quad P = 5796.0 - 872.35 = 4924 \quad$ too low

$v_1 = 0.01082\ m^3/kg \quad \Rightarrow \quad P = 5899.0 - 900.7 = 4998.3 \quad$ OK

Now evaluate the change in h along the 230 K from state 1 to state x, that requires a value for v_x. Guess ideal gas at T_x = 230 K,

$$v_x = RT_x/P_2 = 0.2598 \times 230/100 = 0.59754\ m^3/kg$$

From the EOS: $\quad P_2 = 100.1157 - 0.3138 = 99.802$ kPa (close)

A few more guesses and adjustments gives

$v_x = 0.59635\ m^3/kg; \quad P_2 = 100.3157 - 0.3151 = 100.0006$ kPa $\quad$ OK

$$(h_x - h_1)_T = P_x v_x - P_1 v_1 - \frac{3a}{2bT^{1/2}} \ln\left[\left(\frac{v_x + b}{v_x}\right)\left(\frac{v_1}{v_1 + b}\right)\right]$$

$$= 59.635 - 5000 \times 0.01082 - 243.694 \ln\left[\frac{0.59704}{0.59635} \times \frac{0.01082}{0.01151}\right]$$

$$= 59.635 - 54.1 + 14.78335 = 20.318\ kJ/kg$$

From energy eq.: $\quad T_2 = T_1 - (h_x - h_1)_T/C_p = 230 - 20.318 / 0.922 =$ **208 K**

Now the change in s is done in a similar fashion,

$$s_{gen} = s_2 - s_1 = (s_x - s_1)_T + s_2 - s_x$$

$$= R \ln\left(\frac{v_x - b}{v_1 - b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\left(\frac{v_x + b}{v_x}\right)\left(\frac{v_1}{v_1 + b}\right)\right] + C_p \ln\frac{T_2}{T_x}$$

$$= 0.2598 \ln\left(\frac{0.59566}{0.0101297}\right) - 0.35318 \ln(0.94114) + 0.922 \ln\left(\frac{208}{230}\right)$$

$$= 1.05848 + 0.021425 - 0.092699$$

$$= \mathbf{0.987\ kJ/kg\ K}$$