AME 20231
Homework 7
Due: Friday, 9 March 2012, in class

1. 6.15

2. Consider flow in a pipe with constant cross-sectional area A. Flow enters a fixed control volume at the inlet i and exits at the exit e. The velocity in the x direction is v. Derive the control volume version of the linear x-momentum equation for a fluid in a fashion similar to that used in lecture for the mass and energy equations. The only force you need to consider is a pressure force; neglect all wall shear forces and gravity forces. The final form should be of the form

$$\frac{d}{dt}\int_V \rho \mathsf{v} dV = \dot{m}_i \mathsf{v}_i - \dot{m}_e \mathsf{v}_e + P_i A - P_e A.$$

   You may wish to consult any of a variety of undergraduate fluid mechanics textbooks for more guidance.

   Additionally, aficionados may want to examine the interesting paper, link provided below, of Thorpe.[1] The paper uses a control volume approach to clarify that the equation often used to describe Newton's second law for rocket dynamics previous to the 1960s was fundamentally incorrect. Panton[2] notes in his well known textbook "The proper form of the momentum equation for a deforming particle (control region) of variable mass is a relatively recent advance (see, e.g. Thorpe, 1962). It was clarified only after problems of rockets and space vehicles became important." While the direct causality is unclear, one can view the consequences of not understanding all aspects of rocket dynamics here.

3. 6.30

4. 6.60

5. 6.159E

6. Take data from Table A.8 for O_2 and develop your own third order polynomial curve fit for $u(T)$. That is find a_1, a_2, a_3 such that

$$u(T) \sim a_0 + a_1 T + a_2 T^2 + a_3 T^3$$

   well describes the data in the range $200\ K < T < 3000\ K$. Give a plot which gives the predictions of your curve fit $u(T)$ as a continuous curve for $200\ K < T < 3000\ K$. Superpose on this plot discrete points of the actual data. Take an appropriate derivative of the curve fit for $u(T)$ to estimate $c_v(T)$. Give a plot which gives your curve fit prediction of $c_v(T)$ for $200\ K < T < 3000\ K$. Superpose discrete estimates from a simple finite difference model $c_v \sim \frac{\Delta u}{\Delta T}$, where the finite difference estimates come from the data in Table A.8, onto your plot. You will find a discussion on least squares curve fitting in the online course notes to be useful for this exercise.

[1] J. F. Thorpe, 1962, "On the momentum theorem for a continuous system of variable mass," *American Journal of Physics*, 30(9): 637-640.

[2] R. L. Panton, 2005, *Incompressible Flow*, Wiley, Hoboken, NJ, p. 100.