NAME: SOLUTION
AME 20231
Thermodynamics
Examination 1
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1. (10) Argon, Ar, exists at $T = 165.9\ K$, $P = 4.87\ MPa$. Find its specific volume via
   (a) the ideal gas law, and
   (b) the compressibility charts.

Solution

Most people did fine on this. Some had issues with proper units, mixing up MPa with kPa. Others were unsure of how to use the compressibility chart, especially how to look up the critical pressure and temperature, necessary to get the relative pressure and temperature.

At such a low temperature, we must be concerned that the ideal gas model is invalid. For Argon, Table A.5 gives us $R = 0.2081\ kJ/kg/K$. For the ideal gas law, we have

$$v = \frac{RT}{P} = \frac{\left(0.2081\ \frac{kJ}{kg\ K}\right)(165.9\ K)}{(4.87\ MPa)\left(\frac{1000\ kPa}{MPa}\right)} = \boxed{0.00708907\ \frac{m^3}{kg}}.$$

Now let us consider the non-ideal correction via the compressibility charts. From Table A.2, we have critical constants for argon of $T_c = 150.8\ K$, $P_c = 4.87\ MPa$. Thus, the relative temperature and pressure are

$$T_r = \frac{T}{T_c} = \frac{165.9\ K}{150.8\ K} = 1.10,$$

$$P_r = \frac{P}{P_c} = \frac{4.87\ MPa}{4.87\ MPa} = 1.$$

Turning now to Table D.1, we estimate that $Z = 0.68$. Since

$$Z = \frac{Pv}{RT},$$

we get

$$v = \frac{ZRT}{P} = \frac{(0.68)\left(0.2081\ \frac{kJ}{kg\ K}\right)(165.9\ K)}{(4.87\ MPa)\left(\frac{1000\ kPa}{MPa}\right)} = \boxed{0.00482057\ \frac{m^3}{kg}}.$$

2. (10) For H_2O, determine the specific property at the indicated state. Locate the state on a sketch of the $T - v$ diagram.
   (a) $P = 300\ kPa$, $v = 0.5\ m^3/kg$, Find T, in $°C$.
   (b) $P = 1.5\ MPa$, $T = 410\ °C$. Find v.

Solution

Most students did fine on this. The biggest problem was the chart, where the relative positions of the two states (relative to the critical point as well as each other) were often mis-identified.

For part a), we turn to Table B.1.2 and find that at 300 kPa, $v_f = 0.001073\ m^3/kg$ and $v_g = 0.60582\ m^3/kg$. Thus $v_f < v < v_g$, and the water is in a two-phase mixture state, with

$$T = \boxed{133.55^\circ C}.$$

For part b), we turn to Table B.1.3. Unfortunately, there are no values at the precise values of T and P, so we must do a double interpolation. There is more than one way to do this, but we will focus here on one way. First, let us estimate v at $T = 410^\circ C$ and $500^\circ C$ at two nearby pressures. At $P = 1400\ kPa$, we get

$$v = \left(0.21780\ \frac{m^3}{kg}\right) + \frac{\left(0.25215\ \frac{m^3}{kg} - 0.21780\ \frac{m^3}{kg}\right)(410^\circ C - 400^\circ C)}{500^\circ C - 400^\circ C} = 0.221235\ \frac{m^3}{kg}.$$

At $P = 1600\ kPa$, we get

$$v = \left(0.19005\ \frac{m^3}{kg}\right) + \frac{\left(0.22029\ \frac{m^3}{kg} - 0.19005\ \frac{m^3}{kg}\right)(410^\circ C - 400^\circ C)}{500^\circ C - 400^\circ C} = 0.193074\ \frac{m^3}{kg}.$$

Then at our actual pressure of $P = 1500\ kPa$, we can simply average, which amounts to a linear interpolation, to get

$$v = \frac{0.221235\ \frac{m^3}{kg} + 0.190374\ \frac{m^3}{kg}}{2} = \boxed{0.207155\ \frac{m^3}{kg}}.$$

A plot is given in Fig. 1

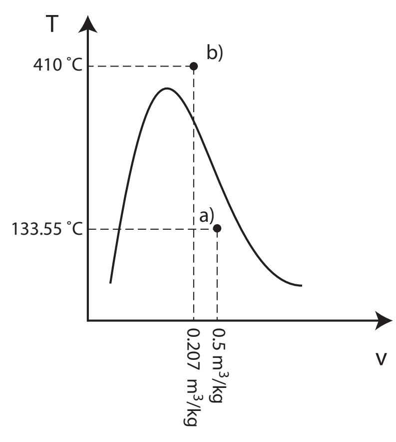

Figure 1: $T - v$ diagram with two appropriate points labeled.

3. (40) A cylinder/piston assembly, see Fig. 2, contains 0.5 kg of air, modeled as an ideal gas, at 100 kPa and volume 0.48 m^3. It is heated so that its volume doubles. Atmospheric pressure is 100 kPa, and the cylinder cross sectional area is 0.06 m^2. The piston has a mass of 184 kg. Gravitational acceleration is 9.8 m/s^2.

(a) What is the final pressure of the air?

(b) Determine the initial and final temperature of the air.

(c) Show the process on a sketch of the $P - v$ diagram.

(d) What is the work done by air during the process?

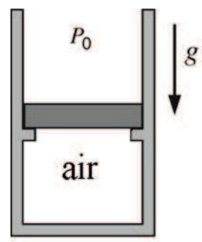

Figure 2: Schematic for piston-cylinder problem.

Solution

We have $P_1 = 100\ kPa$, $V_1 = 0.48\ m^3$, $m = 0.5\ kg$, $V_3 = 2V_1$, $A = 0.06\ m^2$, $m_p = 184\ kg$, $g = 9.81\ m/s^2$.

By inspection the first portion of the process will be isochoric, and the second part isobaric. Let us say 1 is the initial state, 2 is the state where the stops exert no force on the piston, and 3 is the final state. We also have $V_2 = V_1 = 0.48\ m^3$. At state 2, when the stops exert no force on the piston, a force balance gives

$$P_2 A = P_o A + m_p g.$$

$$P_2 = P_o + \frac{m_p g}{A} = (100\ kPa) + \frac{(184\ kg)\left(9.8\ m/s^2\right)}{0.06\ m^2}\left(\frac{kPa}{1000\ N/m^2}\right) = 130.053\ kPa.$$

Now $P_3 = P_2$, so

$$P_3 = \boxed{130.053\ kPa.}$$

The initial temperature of the air is found from the ideal gas law:

$$T_1 = \frac{P_1 V_1}{mR} = \frac{(100\ kPa)\left(0.48\ m^3\right)}{(0.5\ kg)\left(0.287\ \frac{kJ}{kg\ K}\right)} = \boxed{334.495\ K}.$$

Now,

$$V_3 = 2V_1 = 2(0.48\ m^3) = 0.96\ m^3.$$

The final temperature of the air is also found from the ideal gas law:

$$T_3 = \frac{P_3 V_3}{mR} = \frac{(130.053\ kPa)\left(0.96\ m^3\right)}{(0.5\ kg)\left(0.287\ \frac{kJ}{kg\ K}\right)} = \boxed{870.043\ K}.$$

Note also that

$$v_1 = v_2 = \frac{V_1}{m} = \frac{0.48\ m^3}{0.5\ kg} = 0.96\ \frac{m^3}{kg}.$$

$$v_3 = 2v_1 = 2\left(0.96\ \frac{m^3}{kg}\right) = 1.92\ \frac{m^3}{kg}.$$

A plot of the process in the $P - v$ plane is given in Fig. 3

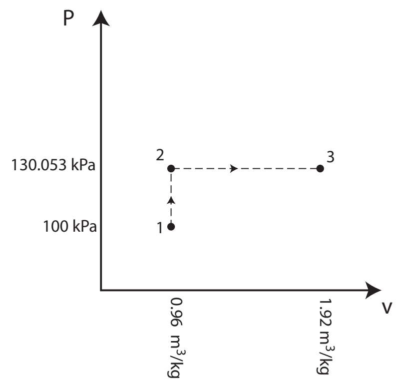

Figure 3: $P-v$ diagram for ideal gas expansion problem.

The total work is given by

$${}_1W_3 = {}_1W_2 + {}_2W_3 = \underbrace{\int_1^2 PdV}_{0} + \int_2^3 PdV = P_2(V_3 - V_2) = (130.053\ kPa)\left(0.96\ m^3 - 0.48\ m^3\right) = \boxed{62.4256\ kJ}.$$

4. (40) A mass, 0.1 kg, of ammonia, NH_3, initially at $T_1 = 0\ °C$, $x_1 = 0.1$ isothermally expands to state 2 where $P_2 = 200\ kPa$.

   (a) Find the initial total volume.

   (b) Find the final total volume.

   (c) Accurately sketch the total process in the $P-v$, $T-v$, and $P-T$ planes. Label each state in your sketch giving numerical values for P, T, v. Include the vapor dome in its correct position.

   (d) Find the total work done in the process.

Solution

Students did well on parts a and b. Most did well on part c. The biggest mistake on part c was to label state 2 to be below the triple point on the $P-T$ diagram. Most students did poorly on part d, and used a variety of crude or incorrect formulæ, instead of a careful piecewise numerical integration formed from a isotherm in $P-v$ space determined from the tables.

From Table B.2, we see $v_c = 0.004255\ m^3/kg$, so $v_1 > v_c$. From Table B.2, we find $P_1 = 429.6\ kPa$, $v_f = 0.001566\ m^3/kg$, $v_g = 0.28920\ m^3/kg$, $v_{fg} = v_g - v_f = 0.287634\ m^3/kg$. We also know $T_2 = T_1 = 0\ °C$. Table B.2.2 then tells us $v_2 = 0.6465\ m^3/kg$.

So v_1 is given by

$$v_1 = v_f + x_1 v_{fg} = \left(0.001566\ \frac{m^3}{kg}\right) + (0.1)\left(0.287634\ \frac{m^3}{kg}\right) = 0.0303294\ \frac{m^3}{kg}.$$

Thus

$$V_1 = mv_1 = (0.1\ kg)\left(0.0303294\ \frac{m^3}{kg}\right) = \boxed{0.00303294\ m^3}.$$

$$V_2 = mv_2 = (0.1\ kg)\left(0.6465\ \frac{m^3}{kg}\right) = \boxed{0.06465\ m^3}.$$

The appropriate sketches are given here

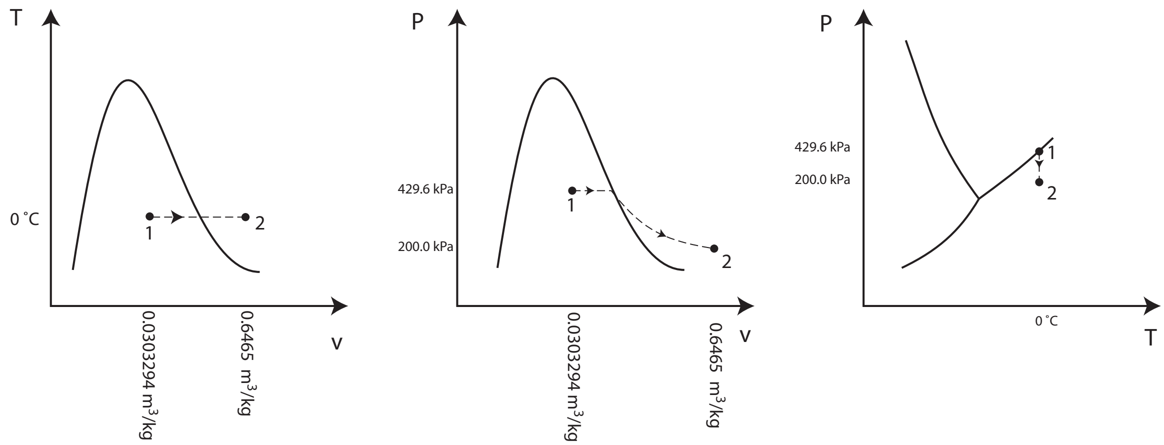

Figure 4: Process path in $T-v$, $P-v$, and $P-T$ planes (not to scale).

To get the work, we must consider the details of the isothermal path in the $P-v$ plane. We can say

$$_1w_2 = \int_1^2 P\ dv = \sum_i P_{ave,i}\Delta v_i.$$

We find values in the tables for all values and summarize the calculations in Table 1. So

Table 1: Tabular calculation of work.

i	P	v	P_i^{ave}	Δv_i	$P_i^{ave}\Delta v_i$
	kPa	m^3/kg	kPa	m^3/kg	kJ/kg
1	429.6	0.0303294	429.6	0.258871	111.211
g	429.6	0.28920	414.8	0.023070	9.56944
a	400.0	0.31227	350.0	0.111550	39.0425
b	300.0	0.42382	250.0	0.222680	55.67
2	200.0	0.64650	-	-	-
					215.493

$_1w_2 = 215.493\ kJ/kg$. Thus the total work is

$$_1W_2 = m_1w_2 = (0.1\ kg)\left(215.493\ \frac{kJ}{kg}\right) = \boxed{21.5493\ kJ}.$$