NAME:
AME 20231
Thermodynamics
Examination 2: Solution
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1. (20) A heat pump is used to heat a house in winter. The house's temperature is maintained at $23°C$. When the ambient temperature is $-10°C$, the rate of heat lost from the house to the surroundings is 25 kW. Calculate the minimum electrical power required to run the heat pump under these conditions.

Solution

We have the first law which gives

$$\dot{W} = \dot{Q}_H - \dot{Q}_L,$$
$$\dot{W} = \dot{Q}_H \left(1 - \frac{\dot{Q}_L}{\dot{Q}_H}\right).$$

The best possible heat pump is a Carnot heat pump, for which

$$\frac{\dot{Q}_L}{\dot{Q}_H} = \frac{T_L}{T_H}.$$

So,

$$\dot{W} = \dot{Q}_H \left(1 - \frac{T_L}{\dot{T}_H}\right),$$
$$\dot{W} = (25kW) \left(1 - \frac{-10 + 273.15}{23 + 273.15}\right),$$
$$\boxed{\dot{W} = 2.79\ kW.}$$

The coefficient of performance is

$$COP = \frac{\dot{Q}_H}{\dot{W}},$$
$$COP = 8.974.$$

2. (40) A refrigeration cycle using R-134a as the working fluid consists of a compressor, a condenser, an expansion valve, and an evaporator. See Fig. 1. R134a at $P_2 = 1.000\ MPa$ and $T_2 = 50°C$ enters the condenser. It leaves the condenser as a saturated liquid at the same pressure. The pressure in the evaporator is 133.7 kPa. The processes in the condenser and the evaporator are isobaric. The fluid enters the adiabatic compressor as a saturated vapor.

   (a) Determine for the condenser $\dot{Q}_{out}/\dot{m}$.

   (b) Determine T_4 and x_4.

   (c) Determine for the evaporator $\dot{Q}_{in}/\dot{m}$.

   (d) Determine the coefficient of performance of the cycle.

Solution

With $P_2 = 1000\ kPa$ and $T_2 = 50°C$, we find from the superheated tables that$h_2 = 431.24\ kJ/kg$. We then note that $P_3 = P_2 = 1000\ kPa$, while $x_3 = 0$. We can then interpolate the saturation tables to find that $h_3 = 255.563\ kJ/kg$, $T_3 = 39°C$. After the expansion valve we have $h_4 = h_3 = 255.563\ kJ/kg$. We

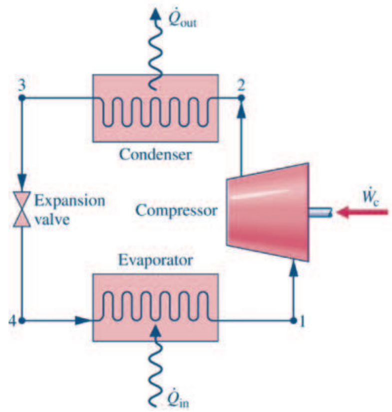

Figure 1: Schematic for refrigeration problem.

are given that $P_4 = 133.7\ kPa$. At this pressure, we note that $h_f < h_4 < h_g$, so we have a saturated mixture. The temperature at this state is

$$\boxed{T_4 = -20°C.}$$

We then get the quality at state 4 via

$$x_4 = \frac{h_4 - h_f}{h_{fg}} = \frac{\left(255.563\ \frac{kJ}{kg}\right) - \left(173.74\ \frac{kJ}{kg}\right)}{212.34\ \frac{kJ}{kg}} = \boxed{0.385.}$$

Then since $x_1 = 1$ and $P_1 = P_4 = 133.7\ kPa$, we find $T_1 = -20°C$, $h_1 = 386.08\ kJ/kg$. Thus,

$$\frac{\dot{W}}{\dot{m}} = h_2 - h_1 = \left(431.24\ \frac{kJ}{kg}\right) - \left(386.08\ \frac{kJ}{kg}\right) = \boxed{45.16\ \frac{kJ}{kg}}.$$

For the condenser, we get

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = \left(431.24\ \frac{kJ}{kg}\right) - \left(255.563\ \frac{kJ}{kg}\right) = \boxed{175.667\ \frac{kJ}{kg}.}$$

For the evaporator, we get

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 = \left(386.08\ \frac{kJ}{kg}\right) - \left(255.563\ \frac{kJ}{kg}\right) = \boxed{130.517\ \frac{kJ}{kg}.}$$

The coefficient of performance is

$$COP = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}/\dot{m}} = \frac{130.517\ \frac{kJ}{kg}}{45.16\ \frac{kJ}{kg}} = \boxed{2.89.}$$

3. (40) A calorically imperfect ideal gas of mass m with gas constant R and specific heat at constant volume $c_v(T) = c_{vo} + aT$ exists in a piston-cylinder configuration at initial pressure and volume P_1 and V_1. The piston, with cross-sectional area A, is restrained by a linear spring, whose spring constant is k_s. At the initial state, the spring exerts no force on the piston. The gas is heated until its final volume is V_2. Find the final temperature T_2, the final pressure P_2, the work done ${}_1W_2$ and the heat transfer ${}_1Q_2$.

Solution

The relationship between pressure and volume for the linear spring is easily shown to be given by

$$P = P_1 + \frac{k_s}{A^2}(V - V_1).$$

Note when $V = V_1$ that $P = P_1$, as required. From the ideal gas law, $PV = mRT$, we get

$$T_1 = \frac{P_1 V_1}{mR}.$$

Now, we have been given V_2, so from the force balance equation we find

$$P_2 = P_1 + \frac{k_s}{A^2}(V_2 - V_1).$$

From the ideal gas law, we have

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}.$$

Thus,

$$T_2 = T_1 \frac{P_2 V_2}{P_1 V_1}.$$

Since we know P_2, we can thus say

$$T_2 = T_1 \frac{\left(P_1 + \frac{k_s}{A^2}(V_2 - V_1)\right) V_2}{P_1 V_1}.$$

And since we know T_1, we can further say

$$T_2 = \frac{\left(P_1 + \frac{k_s}{A^2}(V_2 - V_1)\right) V_2}{mR}.$$

Now we know ${}_1W_2 = \int_{V_1}^{V_2} P dV$, so

$${}_1W_2 = \int_{V_1}^{V_2} \left(P_1 + \frac{k_s}{A^2}(V - V_1)\right) dV.$$

$${}_1W_2 = \int_{V_1}^{V_2} \left(P_1 + \frac{k_s}{A^2}(V - V_1)\right) dV.$$

$${}_1W_2 = P_1(V_2 - V_1) + \frac{k_s}{2A^2}(V_2 - V_1)^2.$$

Now we know for a calorically perfect ideal gas that

$$U_2 - U_1 = \int_{T_1}^{T_2} c_v(T) dT.$$

So for our gas, we have

$$U_2 - U_1 = m \int_{T_1}^{T_2} (c_{vo} + aT)\, dT.$$

Integrating, we find

$$U_2 - U_1 = m\left(c_{vo}(T_2 - T_1) + \frac{a}{2}\left(T_2^2 - T_1^2\right)\right).$$

$$U_2 - U_1 = m(T_2 - T_1)\left(c_{vo} + \frac{a}{2}(T_2 + T_1)\right).$$

Substituting for known values of T_1 and T_2, we can say

$$U_2 - U_1 = \left(\left(\frac{\left(P_1 + \frac{k_s}{A^2}(V_2 - V_1)\right) V_2}{R}\right) - \frac{P_1 V_1}{R}\right)\left(c_{vo} + \frac{a}{2}\left(\left(\frac{\left(P_1 + \frac{k_s}{A^2}(V_2 - V_1)\right) V_2}{mR}\right) + \frac{P_1 V_1}{mR}\right)\right).$$

The first law gives $U_2 - U_1 = {}_1Q_2 - {}_1W_2$, so

$${}_1Q_2 = U_2 - U_1 + {}_1W_2.$$

Thus

$$\boxed{{}_1Q_2 = \left(\left(\frac{\left(P_1 + \frac{k_s}{A^2}(V_2 - V_1)\right) V_2}{R}\right) - \frac{P_1 V_1}{R}\right)\left(c_{vo} + \frac{a}{2}\left(\left(\frac{\left(P_1 + \frac{k_s}{A^2}(V_2 - V_1)\right) V_2}{mR}\right) + \frac{P_1 V_1}{mR}\right)\right) + P_1(V_2 - V_1) + \frac{k_s}{2A^2}(V_2 - V_1)^2.}$$