

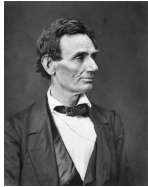
NAME: SOLUTION

AME 20231, Thermodynamics

Examination 1

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“The advantageous use of Steam-power is, unquestionably, a modern discovery. And yet, as much as two thousand years ago the power of steam was not only observed, but an ingenious toy was actually made and put in motion by it, at Alexandria in Egypt.”

Abraham Lincoln, 6 April 1858
Bloomington, Illinois

Happy 204th Birthday!

1. (10) Ideal gases have compressibility of a) Zero, b) One, c) Some positive real number.

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Solution

By the definition of an ideal gas, $Pv = RT$, and compressibility, $Z = Pv/(RT)$, an ideal gas must have compressibility of $Z =$

a) Zero
 b) One
 c) Some positive real number

2. (10) Oxygen is an ideal gas. a) True, b) False, c) Maybe.

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Solution

Oxygen can be a solid, liquid, or gas. When its gaseous state is far from the vapor dome, but at temperatures too low to induce dissociation and/or ionization, it is modeled nicely by an ideal gas law. So the best answer is

a) True
 b) False
 c) Maybe

3. (10) Water at $T = 95^\circ C$, $v = 0.01 \text{ m}^3/\text{kg}$ is a) Liquid, b) Vapor, c) Two-phase mixture.

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Solution

Table B.1.1 shows at $95^\circ C$ that $v_f = 0.001040 \text{ m}^3/\text{kg}$ and $v_g = 1.98186 \text{ m}^3/\text{kg}$. Since $v_f < v < v_g$, we have a

a) Liquid
 b) Vapor
 c) Two-phase mixture

4. (10) Water exists at $P = 1250 \text{ kPa}$, $T = 600^\circ\text{C}$. Find v . Sketch the state in the $T - v$, $P - v$, and $P - T$ planes, taking care to orient the point relative to the vapor dome.

Solution

From Table B.1.3, we find at $P = 1200 \text{ kPa}$, $T = 600^\circ\text{C}$ that $v = 0.33393 \text{ m}^3/\text{kg}$. At $P = 1400 \text{ kPa}$, $T = 600^\circ\text{C}$, $v = 0.28596 \text{ m}^3/\text{kg}$.

Thus, via linear interpolation, we get at 1250 kPa ,

$$v = \left(0.33393 \frac{\text{m}^3}{\text{kg}}\right) + \left(\frac{\left(0.28596 \frac{\text{m}^3}{\text{kg}}\right) - \left(0.33393 \frac{\text{m}^3}{\text{kg}}\right)}{(1400 \text{ kPa}) - (1200 \text{ kPa})}\right) ((1250 \text{ kPa}) - (1200 \text{ kPa})),$$

$$v = 0.321938 \frac{\text{m}^3}{\text{kg}}.$$

Note that the temperature is well above the critical temperature, but the pressure is well below the critical pressure.

The appropriate sketches are given here

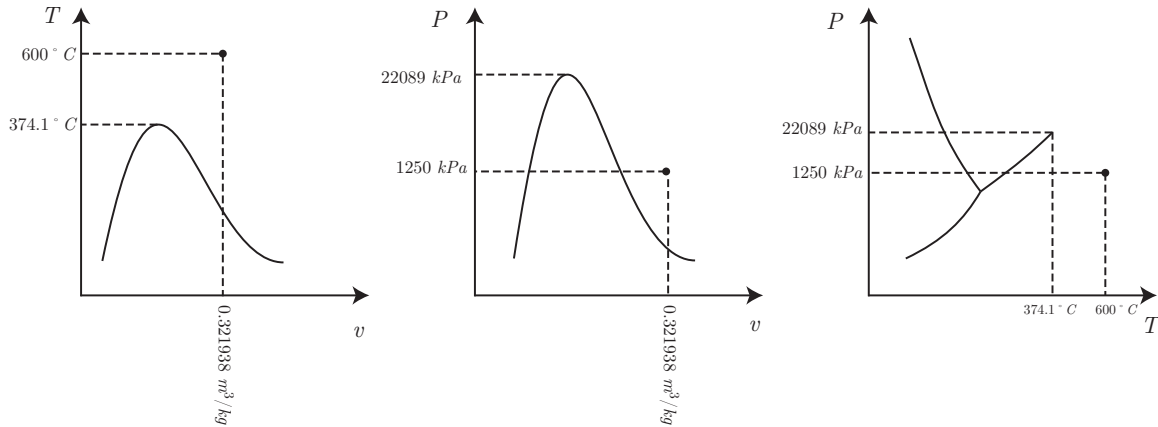


Figure 1: Process path in $T - v$, $P - v$, and $P - T$ planes (not to scale).

5. (25) An ideal gas of mass m and gas constant R at pressure P_1 and temperature T_1 is compressed in a polytropic process to pressure $P_2 = 2P_1$. The polytropic exponent is $n = 1/2$. The gas is isothermally expanded back to $P_3 = P_1$. The gas then undergoes an isobaric process to return to P_1 and T_1 .
- Find T_2 in terms of P_1 , T_1 , m , and R .
 - Find the work of each process in the cycle, ${}_1W_2$, ${}_2W_3$, and ${}_3W_1$ and W_{cycle} in terms of P_1 , T_1 , m , and R .
 - Sketch the cycle in the $P - v$ plane.

Solution

First, we see that

$$v_1 = \frac{RT_1}{P_1}.$$

Because $1 \rightarrow 2$ is polytropic with $n = 1/2$, we have

$$P_2 v_2^{1/2} = P_1 v_1^{1/2}.$$

Thus,

$$v_2 = v_1 \left(\frac{P_1}{P_2} \right)^2 = \frac{RT_1}{P_1} \left(\frac{1}{2} \right)^2 = \frac{1}{4} \frac{RT_1}{P_1}.$$

Also note that

$$v_2 = \frac{v_1}{4}.$$

Now from the ideal gas law, we have

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1}.$$

So

$$T_2 = T_1 \frac{P_2 v_2}{P_1 v_1} = T_1 \left(2 \frac{1}{4} \right) = \boxed{\frac{T_1}{2}}.$$

For the polytropic process, we have

$${}_1W_2 = m \frac{P_2 v_2 - P_1 v_1}{1 - n} = m \frac{(2P_1 \frac{v_1}{4} - P_1 v_1)}{1 - 1/2} = -mP_1 v_1 = \boxed{-mRT_1}.$$

Now we know that

$$v_3 = \frac{RT_3}{P_3}.$$

But we also have $T_3 = T_2 = T_1/2$ and $P_3 = P_1$, so

$$v_3 = \frac{R(T_1/2)}{P_1} = \frac{RT_1}{2P_1}.$$

Thus $v_3/v_2 = 2$. For the isothermal process, we have

$${}_2W_3 = mRT_2 \ln \frac{v_3}{v_2} = \boxed{mRT_1 \frac{\ln 2}{2}}.$$

For the isobaric process from 3 to 1, we have

$${}_3W_1 = mP_1(v_1 - v_3) = mP_1 v_1 \left(1 - \frac{v_3}{v_1} \right) = mRT_1 \left(1 - \frac{1}{2} \right) = \boxed{\frac{mRT_1}{2}}.$$

Therefore, the total work of the cycle is

$$W_{cycle} = -mRT_1 + mRT_1 \frac{\ln 2}{2} + \frac{mRT_1}{2}.$$

$$\boxed{W_{cycle} = \frac{-1 + \ln 2}{2} mRT_1 = -0.153mRT_1.}$$

Note that $W_{cycle} < 0$.

The appropriate sketch is given next.

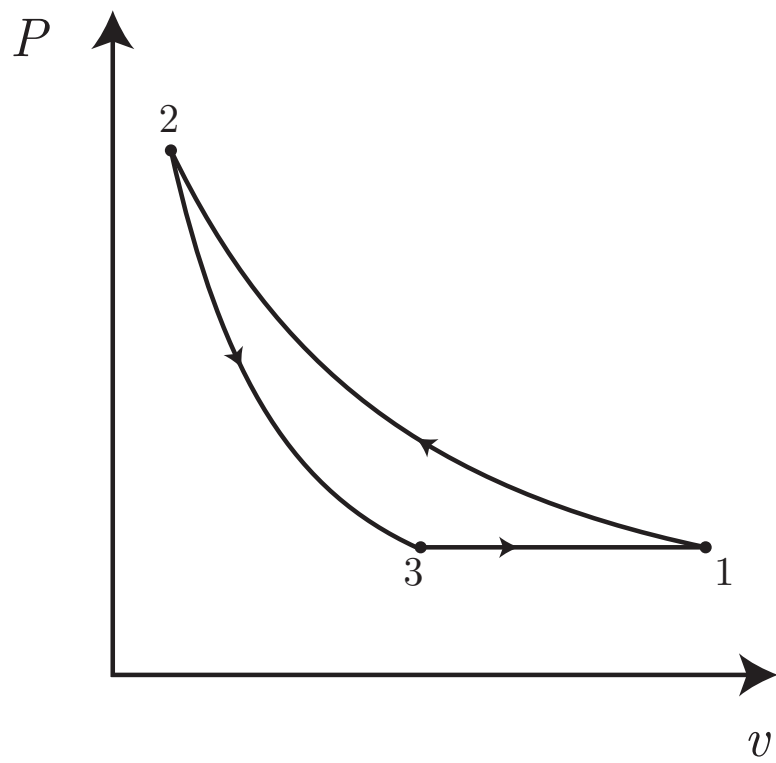
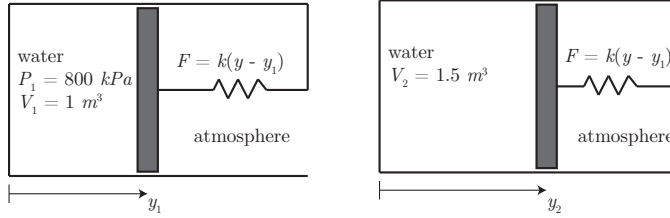


Figure 2: Sketch of cycle in $P - v$ plane.



6. (35) 93.42929 kg of water is enclosed in a cylinder-piston arrangement (see figure). The initial volume is 1 m^3 , and the cross-sectional area of the cylinder is 1 m^2 . A linear spring is attached to the piston. The force from the spring is given as $F = k(y - y_1)$, with $k = 150 \text{ kN/m}$. The spring is initially relaxed, meaning it is neither stretched nor compressed. The water is then heated to expand to a final volume of 1.5 m^3 .
- What is the atmosphere pressure?
 - What is the initial phase and temperature of the water? If it is a two-phase mixture, what is the quality x_1 ?
 - What is the final pressure and temperature of the water?
 - How much work is done on the piston by the water during this expansion process?
 - Accurately sketch the process in the $P - v$ plane with respect to saturation curve.

Solution

Since initially $y = y_1$ and we have an initial force balance, the atmospheric pressure must be

$$P_{atm} = 800 \text{ kPa.}$$

Now

$$v_1 = \frac{V_1}{m} = \frac{1 \text{ m}^3}{93.42929 \text{ kg}} = 0.0107033 \frac{\text{m}^3}{\text{kg}}.$$

From Table B.1.2, we note that at 800 kPa , that $v_f = 0.001115 \text{ m}^3/\text{kg}$ and $v_g = 0.24043 \text{ m}^3/\text{kg}$. So $v_f < v_1 < v_g$, so

$$\text{state 1: two-phase mixture.}$$

From Table B.1.2, we then see that

$$T_1 = 170.43^\circ \text{C,}$$

and that

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{\left(0.0107033 \frac{\text{m}^3}{\text{kg}}\right) - \left(0.001115 \frac{\text{m}^3}{\text{kg}}\right)}{0.23931 \frac{\text{m}^3}{\text{kg}}} = 0.040064.$$

At state 2, we have

$$v_2 = \frac{V_2}{m} = \frac{1.5 \text{ m}^3}{93.42929 \text{ kg}} = 0.0160549 \frac{\text{m}^3}{\text{kg}}.$$

From a force balance, we have

$$PA = P_{atm}A + k(y - y_1).$$

$$P = P_{atm} + \frac{k}{A}(y - y_1).$$

$$P = P_{atm} + \frac{k}{A} \left(\frac{V - V_1}{A} \right).$$

$$P = P_{atm} + \frac{km}{A^2}(v - v_1).$$

Thus, at state 2, we have

$$P_2 = (800 \text{ kPa}) + \frac{(150 \text{ kN/m})(93.42929 \text{ kg})}{(1 \text{ m}^2)^2} \left(\left(0.0160549 \frac{\text{m}^3}{\text{kg}}\right) - \left(0.0107033 \frac{\text{m}^3}{\text{kg}}\right) \right) = 875 \text{ kPa.}$$

Now from Table B.1.2, we do not have direct values at $P = 875 \text{ kPa}$ in the saturation tables. Nevertheless, it is clear that at this pressure $v_2 = 0.0160549 \text{ m}^3/\text{kg}$ lies within the vapor dome and we have a two-phase mixture. Linear interpolation easily gives T_2 :

$$T_2 = \frac{172.96^\circ\text{C} + 175.38^\circ\text{C}}{2} = \boxed{174.17^\circ\text{C}}.$$

We find the work by integration

$${}_1W_2 = m \int_1^2 P dv = m \int_{v_1}^{v_2} \left(P_{atm} + \frac{km}{A^2}(v - v_1) \right) dv.$$

$${}_1W_2 = mP_{atm}(v_2 - v_1) + \frac{km^2}{2A^2}(v_2 - v_1)^2.$$

$${}_1W_2 = P_{atm}(V_2 - V_1) + \frac{k}{2A^2}(V_2 - V_1)^2.$$

$${}_1W_2 = (800 \text{ kPa})((1.5 \text{ m}^3) - (1 \text{ m}^3)) + \frac{150 \frac{\text{kN}}{\text{m}}}{2(1 \text{ m}^2)^2}((1.5 \text{ m}^3) - (1 \text{ m}^3))^2.$$

$${}_1W_2 = \boxed{418.75 \text{ kJ}}.$$

The appropriate sketch is given next. Note that $P_1 < P_2 < P_c$ and $v_c < v_1 < v_2$.

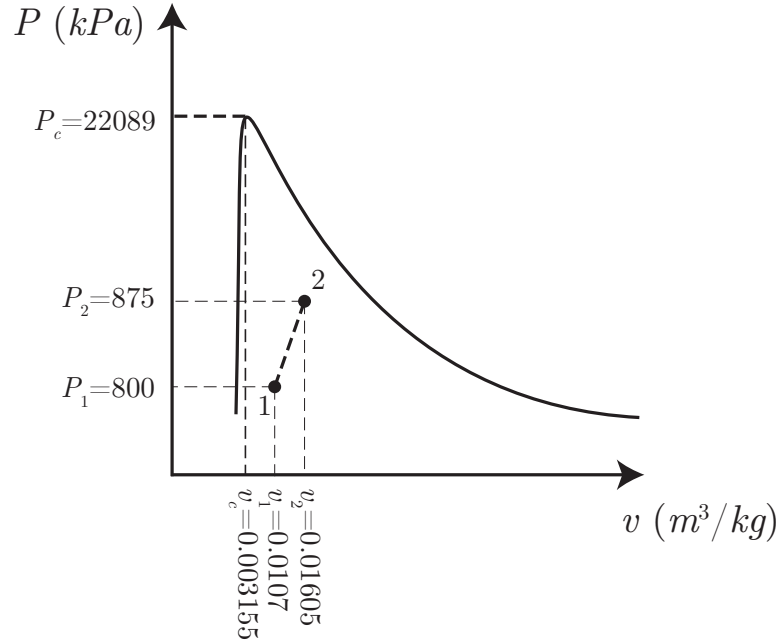


Figure 3: Sketch of process in $P - v$ plane.