

AME 20231

Homework 10

Due: Thursday, 2 April 2020, beginning of class

1. 6.59; take instead the initial pressure to be 300 kPa.
2. 6.84, take instead the initial pressure to be 300 kPa.
3. 6.97, take instead the initial temperature to be 100 °C.
4. Consider the ballistics problem as developed in class. We have the governing equation from Newton's second laws of

$$\frac{dx}{dt} = v, \quad x(0) = x_o, \quad (1)$$

$$\frac{dv}{dt} = \frac{P_\infty A}{m} \left( \frac{P_o}{P_\infty} \left( \frac{x_o}{x} \right)^k - 1 \right) - \frac{C}{m} v^3, \quad v(0) = 0. \quad (2)$$

Consider the following parameter values:  $P_\infty = 10^5$  Pa,  $P_o = 2 \times 10^8$  Pa,  $T_o = 300$  K,  $C = 0.01$  N/(m/s)<sup>3</sup>,  $A = 10^{-4}$  m<sup>2</sup>,  $k = 7/5$ ,  $x_o = 0.03$  m,  $m = 0.004$  kg. Consider the gas to be calorically perfect and ideal and let it undergo an isentropic process. Take the length of the tube to be 0.5 m.

- (a) Write a numerical code using the first order explicit Euler method to solve the governing differential equations for this problem.
- (b) Solve these equations and generate plots for  $x(t)$ ,  $v(t)$ ,  $P(t)$ , and  $T(t)$  for the time the projectile is in the tube.
- (c) Report the velocity at the end of the tube and the time the projectile is in the tube.
- (d) *Optional:* Perform any additional analysis you see fit. One might consider the  $x, v$  phase plane and study the equilibrium points and the stability of the equilibrium points via an eigenvalue analysis, one might divide one differential equation by the other and solve explicitly for  $v(x)$  and interpret the results in terms of the kinetic energy of the system, similar to that which you did in an earlier homework, one might integrate numerically for a very long time and study the behavior as  $t \rightarrow \infty$ .
- (e) Include a copy of your source code in your homework.

Remember, Review II is due as well. See the course web site for details.